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Discrepancy based control of systems of population balances

Stefan Palis [1,*], **Andreas Bück** [1], **Achim Kienle**[1,2]

[1] *Otto-von-Guericke-Universität,*
Universitätsplatz 2, D-39106 Magdeburg, Germany
[2] *Max-Planck-Institut für Dynamik komplexer technischer Systeme*
Sandtorstrasse 1, D-39106 Magdeburg, Germany

Abstract: This contribution is concerned with control of systems of population balances, which are frequently used for modeling of particulate processes like granulation or crystallization. Using the model of a pellet coating processes it will be shown that discrepancy based control can be successfully applied for control of systems of population balances. Here, the main idea is to choose a system output being a generalized measure for the distance between the particle size distribution and its desired steady state, which allows a direct Lyapunov design.

1. INTRODUCTION

Systems of population balance equations are frequently used in models of particulate processes as for example fluidized bed spray granulation, drum granulation, spray drying and crystallization. They are used to describe the behavior of a certain particle property (e.g. liquid content, particle size, porosity). Due to the vast range of individual processes (e.g. particle breakage, particle growth, agglomeration, nucleation) the population balance model may be a simple linear first order hyperbolic partial differential equation or a system of nonlinear partial integro-differential equations. Hence, control design for this type of processes is challenging. In order to simplify the control design procedure the discrepancy based control has been proposed in [4, 5, 8]. Although this design has been successfully applied to different particulate processes [4, 5, 8] rigorous proof of stability in a L_2 or L_∞ norm for a concrete process model is still a challenge.

In this contribution the discrepancy based control will be applied to a pellet coating processes, which is often used for production of drugs, fertilizers, foods and detergents. Here, the pellets are coated in a fluidized bed process, where a bed of particles is fluidized, while simultaneously injecting a solid matter solution. Due to high process air temperature, the fluid evaporates and the remaining solid material contributes to growth of already existing particles. As product particles should have a certain size an additional product classification is required. This can achieved by internal classification using an air sifter with countercurrent flow as depicted in Fig. 1. For a film coating processes the Wurster apparatus is the most common configuration. Here, a Wurster tube is located in the center of the process chamber and the solution is injected by a bottom-spray nozzle. A corresponding process model for the pellet coating in a Wurster fluidized bed process has been proposed by Hampel et. al. [2].

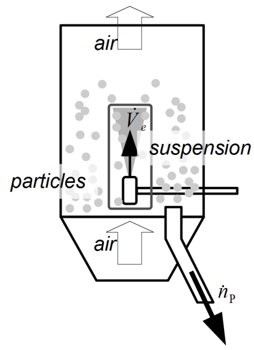

Fig. 1. Scheme of the Wurster fluidized bed process

2. PELLET COATING IN A WURSTER FLUIDIZED BED PROCESS

As has been described, the pellet coating process can be realized in a continuous fluidized bed spray granulator with internal product classification as depicted in Fig. 1. Here, the granulator consists of a granulation chamber, where the particle population is fluidized through an air stream. The solution $\dot{V}_e$ is injected from the granulator bottom in the middle of the Wurster coater, which separates the inner high velocity zone from the outer low velocity zone. Due to this separation the apparatus can be decomposed into two functional zones

- the spraying zone, i.e. the inner high velocity zone, where the solution is supplied to the particles,
- the drying zone, i.e. the outer low velocity zone, where the particles are dried.

This configuration allows under certain operating conditions the suppression of particle breakage and agglomeration, which are highly undesired in a coating process [3].

For the modeling of the particulate phase the aforementioned decomposition can be reflected by introducing two particle size distributions $n_1(L,t)$ and $n_2(L,t)$ for the spraying and the drying zone, where $L \in [0,\infty)$ is the characteristic particle diameter and $t \geq 0$ is the time. The volumetric ratio between spraying and drying zone is represented by introducing a parameter α. The particle growth in the spraying zone associated to the layering process can been described using a surface-proportional growth law [1].

$$G = \frac{2\dot{V}_e}{\pi \int_0^\infty L^2 n_1 dL} = \frac{2\dot{V}_e}{\pi \mu_{2,1}}, \tag{1}$$

Due to fluidization intense particle mixing occurs, which results in particle transport between the two compartments. The associated exchange rates from compartment one to two $\dot{n}_{12}$ and from two to one $\dot{n}_{21}$ can be characterized by their residence time τ_1 and τ_2, which can in turn be related to the relative size of the compartments.

$$\dot{n}_{12} = \frac{1}{\tau_1} n_1 \tag{2}$$

$$\dot{n}_{21} = \frac{1}{\tau_2} n_2 \tag{3}$$

The product particles are continuously removed through an air sifter with countercurrent flow. Due to the particle size specific sinking velocity large particles pass the air sifter while small particles are reblown into the granulation chamber. The associated non-ideal separation function T shown in Fig. 2 depends on the critical separation diameter L_1, which can be directly influenced by the air mass flow rate.

$$T(L) = \frac{\int_0^L e^{-\frac{(L'-L_1)^2}{a^2}} dL'}{\int_0^\infty e^{-\frac{(L-L_1)^2}{a^2}} dL} \tag{4}$$

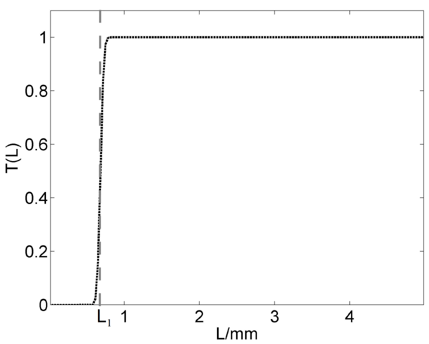

Fig. 2. Non-ideal separation function T due to classifying product removal

It is assumed that product particles are removed from both compartments equally, where the drain is equal to the inverse residence time τ_3

$$\dot{n}_{1,P} = \frac{1}{\tau_3} T(L) n_1 \tag{5}$$

$$\dot{n}_{2,P} = \frac{1}{\tau_3} T(L) n_2 \tag{6}$$

In order to allow a continuous operation nuclei of a predefined size distribution are added. Here, it is assumed that the nuclei size distribution is a normal distribution with mean diameter L_0.

$$n_{nuc}(L) = \frac{e^{-\frac{(L-L_0)^2}{a^2}}}{\int_0^\infty L^3 e^{-\frac{(L-L_0)^2}{a^2}} dL} \tag{7}$$

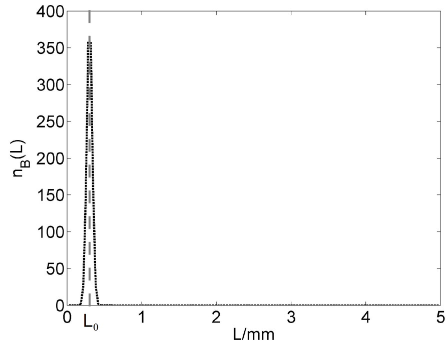

Fig. 3. Nuclei size distribution $n_{nuc}(L)$

The particle fluxes due to external nuclei hence are

$$\dot{n}_{1,nuc} = \alpha K n_{nuc}(L) \tag{8}$$

$$\dot{n}_{1,nuc} = (1-\alpha) K n_{nuc}(L) \tag{9}$$

$$\tag{10}$$

where K is the inlet rate. To describe the process, a population balance model for each particle size distribution has been proposed recently in [2]. Fig. 4 illustrates the coupling of the two population balance models.

$$\frac{\partial n_1}{\partial t} = -G \frac{\partial n_1}{\partial L} - \dot{n}_{12} + \dot{n}_{21} - \dot{n}_{1,P} + \dot{n}_{1,nuc} \tag{11}$$

$$\frac{\partial n_2}{\partial t} = \dot{n}_{12} - \dot{n}_{21} - \dot{n}_{2,P} + \dot{n}_{2,nuc} \tag{12}$$

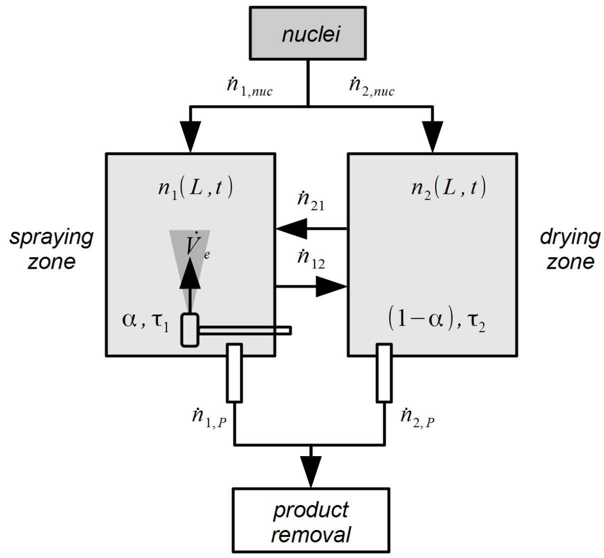

Fig. 4. Coupling of the population balances

For numerical simulation the model equations are semi-discretized with the finite volume method (1st order upwind flux discretization) with 150 grid points. The model parameters used are given in Table 1.

For a continuous process operation the particle size distributions $n_1(L,t)$ and $n_2(L,t)$ should be stabilized. This

τ_1	$0.1s$
τ_2	$9.9s$
τ_3	$1800s$
α	0.01
ε	0.5
$\dot{V}_e$	$201\frac{mm^3}{s}$
a	0.05
L_0	$0.3mm$
K	$60\frac{1}{s}$

Table 1. Plant parameters

can be achieved using for example the critical separation diameter L_1 as a control handle. The main problems here are the non-affinity of the control and the growth related integral term in the population balance equation, which results in a nonlinear partial integro-differential equation. Both problems can be however solved applying discrepancy based control design, which relies on the theory of stability in the sense of Lyapunov with respect to two generalized distance measures, the discrepancies.

3. STABILITY WITH RESPECT TO TWO DISCREPANCIES

Most of the control design methodologies for distributed parameter systems presented in the literature rely on special system properties, as for example boundary actuation, linearity, solvability of the systems equation or at least the desired error system, i.e. the system in closed loop operation. Two popular representatives of them are for example the backstepping approach (e.g. [9]), where the control input is designed such that it maps the original system onto a desired stable error system, or the approach proposed in the works of Bastin et. al. (e.g. [10, 11]), where stability is proven using the solution derived with the method of characteristics.
For the presented system of population balance equations and the population balance models studied in [4, 5, 8] these approaches are obviously not well suited. However, as has been shown in previous contributions [4, 5, 8] this problem is solvable by introducing a generalized stability notion, i.e. stability with respect to two generalized distance measures, the discrepancies. In the following the most important properties and facts on stability with respect to two discrepancies are stated in accordance to [14, 15, 16]. Here, the process $\varphi(.,t)$ is a solution of the distributed parameter system and $\varphi_0 = 0$ an equilibrium of the system.

Definition 1. Discrepancy

A discrepancy is a real valued functional $\rho = \rho[\varphi(.,t),t]$ with the following properties

(1) $\rho(\varphi,t) \geqslant 0$
(2) $\rho(0,t) = 0$
(3) for an arbitrary process $\varphi = \varphi(.,t)$ the discrepancy $\rho(\varphi(.,t),t)$ is continuous with respect to t.
(4) introducing a second discrepancy $\rho_0(\varphi)$ with $\rho_0(\varphi) \geqslant 0$ and $\rho_0(0) = 0$. Than the discrepancy $\rho(\varphi(.,t),t)$ is continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$, if for every $\varepsilon > 0$ and $t_0 > 0$ there exists a $\delta(\varepsilon, t_0) > 0$, such that from $\rho_0 \leqslant \delta(\varepsilon, t_0)$ follows $\rho < \varepsilon$.

According to this definition a discrepancy has not all properties of a metric, e.g. symmetry $d(x,y) = d(y,x)$ or triangular inequality $d(x,y) \leq d(x,z) + d(z,y)$. In addition, it has not to satisfy the important property of definiteness, i.e. a vanishing discrepancy $\rho(\varphi,t) = 0$ does not automatically imply $\varphi = 0$.

Definition 2. Stability with respect to two discrepancies ρ and ρ_0

The equilibrium $\varphi_0 = 0$ is stable in the sense of Lyapunov with respect to the two discrepancies ρ and ρ_0 for all $t \geq t_0$ if for every $\varepsilon > 0$ and $t_0 \geq 0$ there exists a $\delta = \delta(\varepsilon, t_0) > 0$, such that for every process $\varphi(.,t)$ with $\rho_0 < \delta(\varepsilon, t_0)$ follows $\rho < \varepsilon$ for all $t \geq t_0$. If in addition $\lim_{t\to\infty} \rho = 0$, than the equilibrium φ_0 is called asymptotically stable in the sense of Lyapunov with respect to the two discrepancies ρ and ρ_0.

In order to establish a relationship between stability with respect to two discrepancies and the existence of a Lyapunov functional V the notions of positivity and positive definiteness of a functional with respect to a discrepancy have been introduced.

Definition 3. Positivity with respect to a discrepancy ρ

The functional $V = V[\varphi,t]$ is called positive with respect to the discrepancy ρ, if $V \geq 0$ and $V[0,t] = 0$ for all φ with $\rho(\varphi,t) < \infty$.

Definition 4. Positive definiteness with respect to a discrepancy ρ

The functional $V = V[\varphi,t]$ is positive definite with respect to a discrepancy ρ, if $V \geqslant 0$ and $V[0,t] = 0$ for all φ with $\rho(\varphi,t) < \infty$ und for every $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$, such that $V \geq \delta(\varepsilon)$ for all φ with $\rho[\varphi,t] \geq \varepsilon$.

The following two theorems state the conditions for a function V guaranteeing (asymptotical) stability with respect to two discrepancies.

Theorem 5. [15] The process φ with the equilibrium $\varphi_0 = 0$ is stable with respect to the two discrepancies ρ and ρ_0 if and only if there exists a functional $V = V[\varphi,t]$ positive definite with respect to the discrepancy ρ, continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$ and not increasing along the process φ, i.e. $\dot{V} \leq 0$.

Theorem 6. [15] The process φ with the equilibrium $\varphi_0 = 0$ is asymptotically stable with respect to the two discrepancies ρ and ρ_0 if and only if there exists a functional $V = V[\varphi,t]$ positive definite with respect to the discrepancy ρ, continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$ and not increasing along the process φ, i.e. $\dot{V} \leq 0$, with $\lim\limits_{t\to\infty} V = 0$.

It has to be mentioned that stability with respect to two discrepancies is necessary but in general not sufficient for stability with respect to a L_p norm or L_∞ norm.

4. DISCREPANCY BASED CONTROL DESIGN CONTROL DESIGN

In the following a stabilizing control is derived for the pellet coating process in a fluidized bed (11) and (12). The control input is the critical particle diameter L_1, which can be adjusted directly via the air mass flow. In order to derive a stabilizing controller the above presented stability concept is applied. Here, we choose the discrepancy ρ as follows

$$\rho = \frac{1}{2}\left(\int_0^\infty L^3\left(n_d - n\right)dL\right)^2. \qquad (13)$$

where $n = n_1+n_2$ and $n_d = n_{1,d}+n_{2,d}$ is the desired steady state particle size distribution. Obviously, the above requirements on a discrepancy are met. In order to guarantee continuity at time $t = t_0$ at $\rho_0 = 0$ the second discrepancy ρ_0 is simply chosen as follows

$$\rho_0 = \rho(t = 0). \qquad (14)$$

The associated error is

$$e = \int_0^\infty L^3\left(n_d - n\right)dL. \qquad (15)$$

According to Theorem 6 existence of an appropriate functional V is sufficient to guarantee asymptotic stability with respect to the two discrepancies ρ and ρ_0. For this purpose the following candidate Lyapunov functional is introduced

$$V = \frac{1}{2}\left(\int_0^\infty L^3\left(n_d - n\right)dL\right)^2. \qquad (16)$$

In order to achieve stability in the sense described above the control variable has to be chosen such that the time derivative of V along the system trajectories (11) and (12) is negative definite for all times and vanishs only for $V = 0$. Calculating the time derivative $\dot{V}$ yields

$$\dot{V} = -e\int_0^\infty L^3\left(-G\frac{\partial n_1}{\partial L} - \dot{n}_P + \dot{n}_{nuc}\right)dL \qquad (17)$$

$$= -e\int_0^\infty L^3\left(-G\frac{\partial n}{\partial L} + \dot{n}_{nuc}\right)dL - e\frac{1}{\tau_3}\int_0^\infty L^3 KT(L) n dL \qquad (18)$$

with $\dot{n}_P = \dot{n}_{1,P}+\dot{n}_{2,P}$ and $\dot{n}_{nuc} = \dot{n}_{1,nuc}+\dot{n}_{2,nuc}$. In order to achieve affinity in the control a virtual control u_{virt} is introduced.

$$u_{virt} = \int_0^\infty L^3 KT(L) n dL \qquad (19)$$

Using this virtual control negative definiteness of the time derivative of the candidate Lyapunov functional V (18) can be achieved choosing the following control law.

$$u_{virt} = \tau_3\left[ce + \int_0^\infty L^3\left(-G\frac{\partial n_1}{\partial L} + \dot{n}_{nuc}\right)dL\right] \qquad (20)$$

For an application the virtual control u_{virt} has to be transformed into the associated critical particle diameter L_1, which leads to the following zero-finding problem.

$$0 = f(L_1) = u_{virt} - \int_0^\infty L^3 K\frac{\int_0^L e^{-\frac{(L'-L_1)^2}{a^2}}dL'}{\int_0^\infty e^{-\frac{(L-L_1)^2}{a^2}}dL} n dL \qquad (21)$$

In addition to stability with respected to the two discrepancies ρ and ρ_0, the control law (20) guarantees exponential convergence of V.

$$\dot{V} = -c\left(\int_0^\infty L^3(n_d - n)dL\right)^2 = -2cV \qquad (22)$$

However, it has to mentioned that applying the discrepancy based control law (20) guarantees stability with respect to a L_p or L_∞ norm only if the zero dynamics associated with the discrepancy ρ are stable with respect to a L_p or L_∞ norm, which is in accordance with [12, 13]. As a rigorous stability analysis of the zero dynamics is difficult an heuristic approach is to study the zero dynamics of the linearized semi-discrete approximations.

The control law as depicted in Fig. 5 consists of nonlinear compensation part, which needs a measurement of the particle size distribution n_1 and n_2 (e.g. by two Parsum inline probes), and a proportional error feedback.

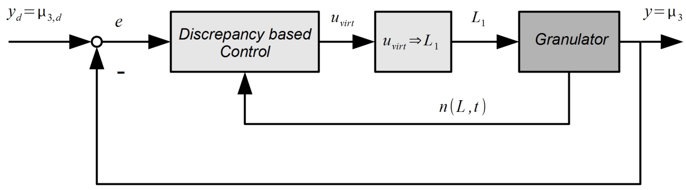

Fig. 5. Control scheme

In order to test the control law the desired set point, i.e. $\int_0^\infty L^3 n_d dL$, has been increased by 20% at $t = 0h$ and two times decreased by 20% at $t = 10h$ and $t = 15h$ as depicted in Fig. 6. As can be seen in Fig. 7 and 8 the discrepancy based control succeeds in stabilizing the desired particle size distributions n_1 and n_2 with reasonable control effort (Fig. 9).

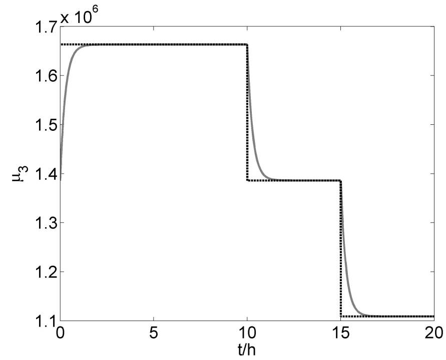

Fig. 6. Reference $\int_0^\infty L^3 n_d dL$ (dotted black) and controlled variable $\int_0^\infty L^3 n dL$ (solid gray)

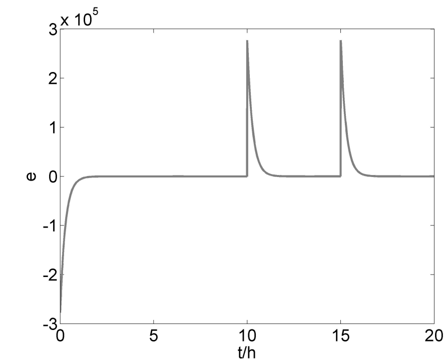

Fig. 7. Error in the particle size distribution $e = \int_0^\infty L^3\left(n_d - n\right)dL$

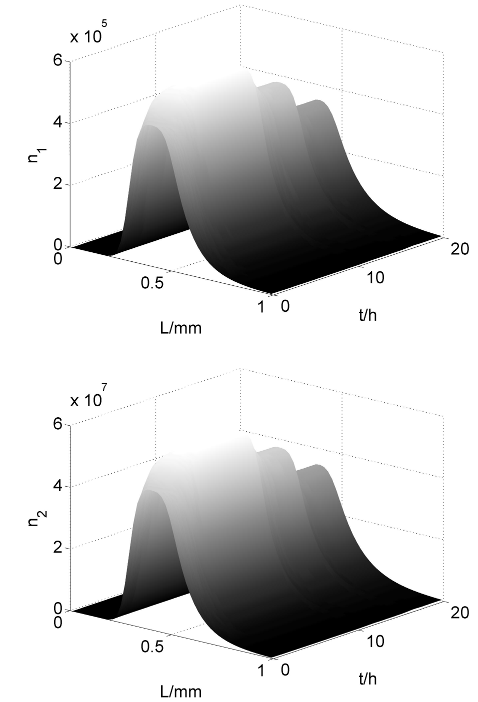

Fig. 8. Particle size distribution in the spraying zone n_1 (top) and in the drying zone n_2 (bottom)

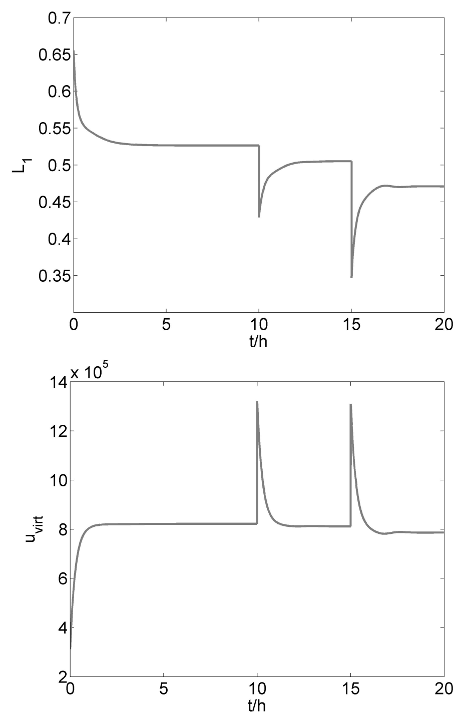

Fig. 9. Critical particle diameter L_1 (top) and virtual control u_{virt} (bottom)

5. CONCLUSION

In this contribution control of systems of population balances models has been studied using continuous pellet coating in a fluidized bed as an example. It has been shown that applying discrepancy based control stabilization and control of systems of population balances is possible. Future work will be concerned with real plant experiments, a thorough study of the zero dynamics associated with the chosen discrepancy and an extension of the linear robust control approaches proposed in [6, 7] to systems of population balances.

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