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Stimulated photon-photon scattering of three colliding high-energy Gaussian beams

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Abstract

The scattering of light by light in a laser system with high-energy beams (not necessarily high-intensity) where three pulses collide is investigated. Two of the colliding beams (with wavelengths \mathbf{k}_1 and \mathbf{k}_2) scatter at each other and the third beam (with wavelength \mathbf{k}_3) stimulates this process because the presence of the third beam induces the emission of photons into the direction of the third beam itself. Since two photons always scatter into two photons and one of the scattered photons is propagating into the direction of the third beam, a fourth wave with wavelength $\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$ is generated and can be measured. In this thesis we investigate the detectability of this stimulated photon-photon scattering in laser systems with high-energy beams such as OMEGA EP in Rochester (USA). The three colliding pulses are modelled by focused Gaussian beams and a special geometry of these beams is chosen. An analytical approximation is derived and for the short-pulse performance of OMEGA EP a number of $N = 70.9$ scattered photons per shot is predicted which seems to be a detectable signal.

Zusammenfassung

Die Streuung von Licht an Licht in einem Lasersystem mit Hochenergie Pulsen (nicht notwendigerweise mit hoher Intensität), in dem drei Pulse kollidieren, wird untersucht. Zwei der kollidierenden Strahlen (mit Wellenlängen \mathbf{k}_1 und \mathbf{k}_2) streuen aneinander und der dritte Strahl (mit Wellenlänge \mathbf{k}_3) stimuliert diesen Prozess, weil die Anwesenheit des dritten Strahls die Emission von Photonen in seine Richtung induziert. Da zwei Photonen immer in zwei Photonen streuen und eines der gestreuten Photonen sich in die Richtung des dritten Strahls fortbewegt, wird eine vierte Welle mit Wellenlänge $\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$ erzeugt und kann gemessen werden. In dieser Arbeit soll nun untersucht werden, ob diese stimulierte Photon-Photon Streuung in Lasersystemen mit Hochenergie Pulsen wie OMEGA EP in Rochester (USA) gemessen werden kann. Die drei aufeinander treffenden Strahlen werden mit fokussierten Gauß-Pulsen modelliert und eine spezielle Geometrie der Strahlen wird gewählt. Eine analytische Näherung wird hergeleitet und für die kurzen Pulse von OMEGA EP wird eine Anzahl von $N = 70.9$ Photonen pro Kollision berechnet, was ein messbares Signal zu sein scheint.

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Units and Conventions

During the whole thesis we will use Gaussian units with natural units defined by:

$$\hbar = c = 4\pi\epsilon_0 = 1.$$

This means the finestructure constant α becomes:

$$\alpha = e^2 \approx \frac{1}{137}.$$

Another important physical quantity used in this thesis is the electron mass [MOHR08]:

$$m = 9.109 \times 10^{-31} \text{ kg}.$$

Vectors in Minkowski-space will have greek indices (μ, ν, \dots) and will take the values 0, 1, 2, 3. Further we will use Einstein's summation convention: Over repeated indices (one contravariant and one covariant) will be summed. Our conventions for important four-vectors are listed in Table 1.

For the Minkowski-metric we use the convention:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

That means that the D'Alembert operator has the form:

$$\square = \partial_\mu \partial^\mu = \partial_t^2 - \Delta.$$

Space coordinates	$x^\mu = (t, \mathbf{x})$
Partial derivative	$\partial_\mu = (\partial_t, \nabla)$
Four momentum	$p^\mu = (E, \mathbf{p})$
Four wavevector	$k^\mu = (\omega, \mathbf{k})$
Four-vector potential	$A^\mu = (\phi, \mathbf{A})$
Field strength tensor	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Scalar product	$p^2 = p^\mu p_\mu$

Table 1: Four-vector conventions

In order to distinguish between Lorentz-indices and spinorial indices we use for the latter big roman letters (A, B, \dots). The γ -matrices satisfy the Clifford algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

During the whole thesis we will use the Dirac-representation of the γ -matrices:

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$

with the Pauli-matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We will also use the notation of the Dirac conjugate spinor:

$$\bar{\psi} = \psi^\dagger \gamma^0.$$

For the Fourier transformation we use the convention:

$$\mathcal{F}(f(t))(\omega) = \tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int dt f(t) e^{-i\omega t}.$$

The inverse transformation therefore is:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \tilde{f}(\omega) e^{i\omega t}.$$

1 Introduction

Quantum electrodynamics (QED) is one of the best tested and most successful theories in physics. However there are still important predictions that have not been tested yet. The fact that photons interact with each other in vacuum is a very early prediction, investigated already in 1936 by Euler and Heisenberg [EULE36]. However so far it has eluded experimental observation.

The interaction among photons in vacuum is inherent for QED and is forbidden in classical electrodynamics due to the fact that Maxwell's equations are linear. In QED virtual electron-positron pairs can interact with the photons and therefore the photons can effectively interact with each other. The reason why this process has not been observed yet is the extremely small cross section. The cross section for the scattering of two photons $\sigma_{\gamma\gamma\rightarrow\gamma\gamma}$ is in the optical range of the order of $7 \times 10^{-66} \text{ cm}^2$ [LAND82]. When two laser beams collide a rough estimation of the number N of scattered photons per shot is $N = \sigma_{\gamma\gamma\rightarrow\gamma\gamma} \frac{I\tau}{\omega} \frac{\mathcal{E}}{\omega}$ with the intensity I , the energy \mathcal{E} , the frequency ω and the pulse length τ of one beam. For example for the parameters of OMEGA EP [OMEP06] this leads to an estimate number of $N = 1 \times 10^{-16}$ photons scattered per shot. We see that a direct detection of photon-photon scattering by just colliding two strong lasers is impossible with nowadays lasers. However, stimulated photon-photon scattering where a third laser induces the emission of a photon into the direction of itself might lead to detectable signals.

That the collision of three beams can increase the number of scattered photons significantly was already investigated by Varfolomeev in 1966 [VARF66]. Further theoretical investigations were made for example by R. L. Dewar in 1974 [DEWA74], F. Moulin et. al. in 1999 [MOUL99], A. Di Piazza et. al. in 2005 [DIPI05] or by J. Lundin et al. in 2006 [LUND06]. In addition experimental studies were made by D. Bernard et al. in 2000 [BERN00], that achieved an upper limit of $\sigma_{\text{lim}} = 1.5 \times 10^{-48} \text{ cm}^2$ for the cross section of $\gamma\gamma$ scattering using stimulated photon-photon scattering. This result is still 17 orders of magnitude under the prediction of QED. They used ultrashort laser pulses, however we will see that for nowadays lasers with long pulses that contain much more energy like OMEGA EP the detection of photon-photon scattering is much more probable.

The technique used in most of the mentioned theoretical investigations is an effective action approach with the effective action already derived by Heisenberg and Euler in 1936 [EULE36]. We will calculate it in the same manner. In this approach the electromagnetic fields of the colliding beams are treated classically and the nonlinearity of QED due to

photon-photon interactions is taken into account by adding nonlinear terms to Maxwell's equations.

J. Lundin et al. [LUND06] recieved a number of $N = 0.07$ scattered photons per collision of three beams for the beam parameters of the Astra Gemini Laser system at the Central Laser Facility (Rutherford Appleton Laboratory) [CLF], which has beams with very high intensity. Our aim will be to derive a number of scattered photons for laser systems with high-energy beams and not necessarily high intensities such as the OMEGA EP laser system at the Laboratory for Laser Energetics in Rochester [LLE]. We expect that it is important for the detection of photon-photon scattering to use beams with high energies. Furthermore we will consider a different geometry than the three dimensional geometry considered by Lundin et al. and we will model the colliding beams by Gaussian pulses, which is more realistic than a model of prisms with quadratic cross section which was used by Lundin et al. [LUND06], since Gaussian pulses satisfy Maxwell's equations in vacuum with sufficiently high accuracy.

This thesis is structured in such a way that we will at first present in section 2 the effective Lagrangian of QED, which will be the starting point of our following calculations. In section 3 we will begin the calculations by defining our considered geometry and the fields of the incoming beams. Then, an analytical approximation of the number of photons that are scattered during one shot is derived. In the following section 4 we use the beam parameters of OMEGA EP to get a numerical value for the number of scattered photons. Also a comparision with the result of Lundin et al. [LUND06] is performed.

2 Effective Lagrangian of QED

In the theory of classical electrodynamics the field equations that follow from the Lagrangian $\mathcal{L}_0 = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$ are linear. Hence electromagnetic waves can not interact with each other in vacuum in a classical theory. But in QED Feynman-diagrams like that in figure 1 lead to an interaction between photons. Therefore our aim is now to search for an effective Lagrangian for the electromagnetic field that can describe the nonlinearities of QED due to processes like that in figure 1. Before we will recall a few basic facts about QED in order to understand why processes like photon-photon scattering are possible in QED.

2.1 Basics of QED

The aim of this section is not to introduce the whole formalism of QED. Only a few results that are interesting for the following sections are presented. For derivations and further informations see for example [MAND10], [KAKU93], [WEIG13].

The theory of QED follows from the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi. \quad (2.1)$$

The aim in QED is to calculate the probability that a set of n particles $|i\rangle$ with four-momenta p_1, \dots, p_n , which are at the beginning far away from each other and then come close to each other and interact, scatter into a set of m particles $|f\rangle$ with four-momenta q_1, \dots, q_m . This probability is given by the modulus square of the so-called S-matrix $\langle f|S|i\rangle$.

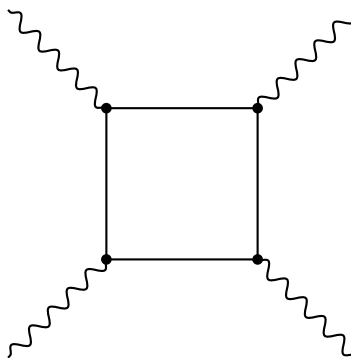


Figure 1: Feynman diagram for photon-photon scattering in the first non-vanishing order

In a simple interaction picture approach [MAND10] this matrix is given by:

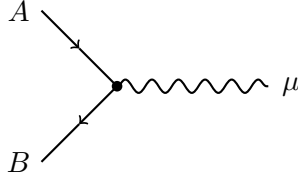
$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \dots \int d^4x_1 \dots d^4x_n T\{\mathcal{H}_I(x_1) \dots \mathcal{H}_I(x_n)\}, \quad (2.2)$$

where T denotes the time-ordering operator. The interaction Hamiltonian \mathcal{H}_I for QED is given by:

$$\mathcal{H}_I(x) = e\bar{\psi}(x)\gamma^\mu A_\mu(x)\psi(x). \quad (2.3)$$

For this Hamiltonian, rules can be formulated that simplify the evaluation of (2.2), the so-called Feynman-rules:

- Due to the cubic interaction Hamiltonian (2.3) every vertex is of the form



and carries the factor $-ie\gamma^\mu_B$. Also four-momentum conservation is imposed at each vertex.

- Each internal photon line $\mu \sim \text{wavy} \sim \nu$ carries a factor $-\frac{ig^{\mu\nu}}{p^2+i\epsilon}$ (in Feynman gauge),
- each internal fermion line $A \longrightarrow B$ carries the factor $\frac{i(\gamma^\mu_B p_\mu)}{p^2-m^2+i\epsilon}$.
- Over each undetermined internal four-momentum p we integrate with the measure $\int \frac{d^4p}{(2\pi)^4}$.
- Each incoming photon $\mu \sim \text{wavy} \bullet$ of polarization λ carries the $\epsilon^\mu(\mathbf{p}, \lambda)$,
- each outgoing photon $\bullet \sim \text{wavy} \mu$ of polarization λ carries the $\epsilon^{\mu*}(\mathbf{p}, \lambda)$,
- each incoming fermion $A \longrightarrow \bullet$ of spin r carries a factor $u_r^A(\mathbf{p})$,
- each incoming anti-fermion $A \longleftarrow \bullet$ of spin r carries a factor $\bar{v}_{rA}(\mathbf{p})$,
- each outgoing fermion $\bullet \longrightarrow A$ of spin r carries a factor $\bar{u}_{rA}(\mathbf{p})$,

- each outgoing anti-fermion $\bullet \leftarrow A$ of spin r carries a factor $v_r^A(\mathbf{p})$.

We used here the two transverse polarisation four-vectors $\epsilon(\mathbf{k}, \lambda) = (0, \boldsymbol{\epsilon}(\mathbf{k}, \lambda))$, $\lambda = 1, 2$ which are defined by $\mathbf{k} \cdot \boldsymbol{\epsilon}(\mathbf{k}, \lambda) = 0$ and $\boldsymbol{\epsilon}(\mathbf{k}, \sigma) \cdot \boldsymbol{\epsilon}(\mathbf{k}, \lambda) = \delta_{\sigma\lambda}$, and the spinors $u_r(\mathbf{p})$, $r = -\frac{1}{2}, \frac{1}{2}$ and $v_r(\mathbf{p})$, $r = -\frac{1}{2}, \frac{1}{2}$ that are solutions to the equations $(p_\mu \gamma^\mu - m)u_r(\mathbf{p}) = 0$ and $(p_\mu \gamma^\mu + m)v_r(\mathbf{p}) = 0$ and are normalized in such a way that $\bar{u}_r(\mathbf{p})u_s(\mathbf{p}) = 2m\delta_{rs}$ and $\bar{v}_r(\mathbf{p})v_s(\mathbf{p}) = -2m\delta_{rs}$.

With these rules we get a non-zero scattering amplitude for the process of photon-photon scattering (for the Feynman-diagram see figure 1). For the limit case of low frequencies ($\omega \ll m$) which is true for the optical range the total cross section is [LAND82]:

$$\sigma_{\gamma\gamma} = 0.031 \frac{\alpha^4}{m^2} \left(\frac{\omega}{m}\right)^6. \quad (2.4)$$

Now we saw why photons interact in QED and therefore lead to nonlinear behaviour of electromagnetic fields.

2.2 Effective Lagrangian in an external field

Although the formalism presented in the previous section is important to understand the background of QED we will use for the derivations in this thesis the formalism of effective Lagrangians, because in our case this formalism is much more suitable. This formalism will be presented now. The following calculations are based on the calculations by Dittrich and Reuter [DITT85].

We want to start with a discussion of the vacuum in QED.

In a classical theory the vacuum is simply defined by a vanishing field. However in a quantized field theory the vacuum still contains an energy due to vacuum fluctuations which is infinite. This infinite energy usually is subtracted, but there are still observable effects of this structure of the vacuum.

In QED we have two fields that usually are quantized: The electromagnetic vector field A_μ and the spinorial Dirac-field of the electrons/positrons ψ . In general there are vacuum fluctuations due to both fields since each of them is quantized. The case we want to study now is slightly different. We want to study an external electromagnetic field which is unquantized with a quantized Dirac-field, which is in its vacuum state. Hence we will have charge fluctuations due to virtual electron-positron-pairs, but no virtual photons and no real electron-positron pairs. That means we assume that no real electron-positron pairs are created from the vacuum. This is a good assumption if our field strengths are far below the critical value of QED ($E_{cr} = \frac{m^2}{|e|} = 1.3 \times 10^{16}$ V/cm).

If there was no Dirac-field the equations of motion were $\partial_\mu F^{\mu\nu} = 0$ which can be derived from the variational principle

$$\frac{\delta S^0[\mathbf{A}]}{\delta A^\mu} = 0, \quad (2.5)$$

with

$$S^0[\mathbf{A}] = \int d^4x \mathcal{L}^0 \quad \text{and} \quad \mathcal{L}^0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (2.6)$$

The Dirac field shall be described by

$$\mathcal{L}_\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi. \quad (2.7)$$

In order to quantize the Dirac-field one imposes the equal time anti-commutation relations

$$\{\psi^A(\mathbf{x}, t), \bar{\psi}_B(\mathbf{y}, t)\} = \gamma^0{}^A{}_B \delta(\mathbf{x} - \mathbf{y}). \quad (2.8)$$

The interaction between the external field A_μ and the Dirac-field ψ is given by

$$\mathcal{L}^W = -j^\mu A_\mu = -e\bar{\psi}\gamma^\mu\psi A_\mu. \quad (2.9)$$

Here we can easily see that the current can be derived from the action $S^W = \int d^4x \mathcal{L}^W$ via

$$\frac{\delta S^W[\mathbf{A}]}{\delta A_\mu} = -j^\mu. \quad (2.10)$$

Our aim is now to find an action

$$S_{\text{eff}}[\mathbf{A}] = S^0[\mathbf{A}] + S^1[\mathbf{A}] \quad (2.11)$$

with S^0 defined in eq. (2.6). The action S^1 should not contain the Dirac-field explicitly and shall fulfill the boundary condition $S^1[F_{\mu\nu} = 0] = 0$.

Although the action should not contain the Dirac-field explicitly we want to simulate the possibility of virtual electron-positron-pair creation. Therefore S^1 should also fulfill

$$\frac{\delta S^1[\mathbf{A}]}{\delta A_\mu} = -\langle 0|j^\mu(x)|0\rangle. \quad (2.12)$$

This request is motivated by eq. (2.10). We took the vacuum expectation value because this will lead to an expression that doesn't contain ψ explicitly anymore and since the Dirac-field is assumed to be in it's vacuum state.

The generalized Maxwell equations that follow from the variational principle $\frac{\delta S_{\text{eff}}[\mathbf{A}]}{\delta A_\mu} = 0$ then are:

$$\partial_\nu F^{\nu\mu} = -\langle 0|j^\mu(x)|0\rangle. \quad (2.13)$$

We now want to rewrite $\langle 0|j^\mu(x)|0\rangle$ such that it depends only on the field A_μ . Therefore let's redefine at first j^μ with the help of the anti-commutation relations (2.8):

$$j^\mu = e\bar{\psi}\gamma^\mu\psi - \frac{e}{2}\{\bar{\psi}, \gamma^\mu\psi\} = \frac{e}{2}[\bar{\psi}, \gamma^\mu\psi]. \quad (2.14)$$

This is possible due to the fact that the charge of the quantized Dirac-field has an infinite value in the vacuum state. The redefined current in eq. (2.14) is a current where this infinity is subtracted.

Now we can rewrite the vacuum expectation value as follows:

$$\langle 0|j^\mu|0\rangle = -e \lim_s \lim_{x' \rightarrow x} \gamma^{\mu A}_B \langle 0|T\psi^B(x')\bar{\psi}_A(x)|0\rangle. \quad (2.15)$$

Here T denotes the time ordering symbol and $\lim_{x' \rightarrow x}$ denotes a symmetrical limit defined by:

$$\lim_s = \frac{1}{2} \left(\lim_{x' \rightarrow x, x'^0 > x^0} + \lim_{x' \rightarrow x, x'^0 < x^0} \right) \quad (2.16)$$

In eq. (2.15) we can see the Green's function of the Dirac operator in an external field which is defined by:

$$(\gamma^{\mu A}_B(i\partial_\mu - eA_\mu) - m\mathbb{1}_B^A)G_C^B(x, y) = i\mathbb{1}_C^A\delta(x - y) \quad (2.17)$$

and can be written as

$$G_B^A(x, x') = \langle 0|T\psi^A(x)\bar{\psi}_B(x')|0\rangle. \quad (2.18)$$

Therefore our defining equations for S^1 are:

$$\frac{\delta S^1[\mathbf{A}]}{\delta A_\mu} = e \text{tr}(\gamma^\mu G(x, x)) \quad (2.19)$$

and

$$S^1[F_{\mu\nu} = 0] = 0. \quad (2.20)$$

These equations can be solved by:

$$S^1[\mathbf{A}] = i \text{Tr} \log \left(\frac{G[\mathbf{A}]}{G[\mathbf{0}]} \right), \quad (2.21)$$

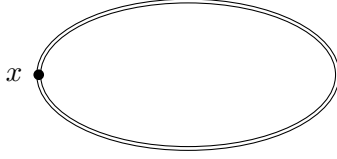


Figure 2: Feynman diagram for $G(x, x)$

with

$$G[\mathbf{A}] = \frac{i}{\gamma\pi - m + i\epsilon} \text{ and } \pi_\mu = i\partial_\mu - eA_\mu. \quad (2.22)$$

For a prove see [DITT85].

The expression Tr denotes the trace in spinor and configuration space:

$$\int d^4x \text{tr} \langle x | \dots | x \rangle. \quad (2.23)$$

The processes described by $G(x, x)$ in the presence of an external field are represented in one feynman diagram (see figure 2). The double line denotes the presence of the external field. We also can see see now that the processes we describe with this formalism are exactly the processes we are interested. We have virtual electrons/positrons but no virtual photons. Also we can see that no real electrons/positrons can appear, which is exactly what we wanted because we are interested only in interactions of photons with each other.

The formalism also works if we quantize the electromagnetic field as well. Then we would get in addition processes like in figure 3, since virtual photons can appear in this case. But the formalism is much more complicated in this case and it's not really possible to work with the results. These diagrams anyway are of higher order and since we are interested in processes like in figure 1 it is ok that we did not quantize the electromagnetic field.

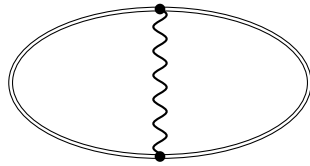


Figure 3: Feynman diagram if also the electromagnetic field is quantized

2.3 Heisenberg-Euler Lagrangian

The result (2.21) is not useful for calculations. For special cases of the external field it is possible to derive from eq. (2.21) a more useful result. The result we will need is the Heisenberg-Euler Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + \frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-ism^2} \left[e^2 s^2 \mathcal{G} \cot [es(\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F})^{\frac{1}{2}}] \right. \\ \left. \times \coth [es(\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F})^{\frac{1}{2}}] - 1 + \frac{2}{3} e^2 s^2 \mathcal{F} \right]. \quad (2.24)$$

For a derivation see for example [LAND82].

Equation (2.24) is true for constant or slowly varying fields ($\omega \ll m$). It's important that this result depends only on the two gauge and Lorentzinvariant scalars $\mathcal{F} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$ and $\mathcal{G} = \mathbf{E} \cdot \mathbf{B}$. The gauge invariance insures that $\partial_\mu \langle 0 | j^\mu(x) | 0 \rangle = 0$ (see [DITT85]).

For field strengths far below the critical value $E_{cr} = 1.3 \times 10^{16}$ V/cm this result can be simplified further:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45m^4} [(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2]. \quad (2.25)$$

This Lagrangian will be the starting point for our calculations.

The generalized Maxwell equations following from the Heisenberg-Euler Lagrangian are:

$$\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}, \quad (2.26)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad (2.27)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2.28)$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}. \quad (2.29)$$

where we defined the effective polarization and magnetization of the vacuum:

$$\mathbf{P} = \xi [2(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{E} + 7(\mathbf{E} \cdot \mathbf{B})\mathbf{B}], \quad (2.30)$$

$$\mathbf{M} = \xi [-2(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{B} + 7(\mathbf{E} \cdot \mathbf{B})\mathbf{E}], \quad (2.31)$$

with

$$\xi = \frac{4\alpha^2}{45m^4}. \quad (2.32)$$

This leads to an effective wave equation

$$\square \mathbf{E} = \nabla(\nabla \cdot \mathbf{P}) - \frac{\partial}{\partial t} \left(\frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M} \right). \quad (2.33)$$

In the following we will call the right hand side of eq. (2.33) the effective current $\mathbf{j}_{\text{eff}}(\mathbf{E}, \mathbf{B}, x)$.

This equation is non linear and therefore in general difficult to solve. But perturbatively it's possible to find solutions. When we assume we have a strong field (for example a laser field) which satisfies the usual wave equation ($\square \mathbf{E}_0 = 0$) and a small correction $\delta \mathbf{E}$ which is of order ξ , eq. (2.33) becomes:

$$\square \mathbf{E}_0 + \square \delta \mathbf{E} = \square \delta \mathbf{E} = \mathbf{j}_{\text{eff}}(\mathbf{E}_0, \mathbf{B}_0, x) \quad (2.34)$$

Here we neglected the terms of order ξ^2 and used $\square \mathbf{E}_0 = 0$.

The idea of stimulated light by light scattering is that \mathbf{E}_0 is the sum of three laser fields with frequencies ω_i , $i = 1, 2, 3$. Due to the cubic dependence of \mathbf{j}_{eff} on the fields \mathbf{E} and \mathbf{B} we have an oscillating source with the frequency $\omega_4 = \omega_1 + \omega_2 - \omega_3$ and hence we expect the generation of a fourth wave $\delta \mathbf{E}$ with $\omega_4 = \omega_1 + \omega_2 - \omega_3$. This is exactly the idea of stimulated light by light scattering. Two lasers scatter at each other and a third laser stimulates the emission of photons in the direction of the third laser. Due to the process of photon-photon scattering (figure 1) a fourth wave has to be emitted that has the four-wavelength $k_4 = k_1 + k_2 - k_3$ due to momentum conservation.

3 Induced photon-photon scattering with Gaussian laser pulses

Now we want to derive the number of scattered photons that could be detected when three laser pulses collide. At first we discuss the geometry and the electric and magnetic fields of the beams which collide. Then we use equation (2.33) to derive the number of scattered photons.

3.1 Geometry

The geometry we will consider is two dimensional. We assume we have two beams that counterpropagate and a third beam that propagates perpendicular to the other two beams (see figure 4). The pulses are assumed to be Gaussian beams. For the general form of focused Gaussian beams see for example [SALA02]. In our case it is enough to consider the form of Gaussian beams with $\epsilon = \frac{2}{kw_0} \ll 1$, because (as we will see later in eq. (3.9) and eq. (3.10)) for the calculations only the product of the three Gaussian pulses \mathbf{E}_1 , \mathbf{E}_2 and \mathbf{E}_3 will be important and the geometry ensures that this product is already extremely small for $x, y, z \sim w_0$. Therefore the fields in this region won't be important and we can assume $\epsilon \ll 1$.

At first we consider a Gaussian beam that propagates in positive z -direction and is linear polarized into the x -direction. With the above assumptions the fields now have the form:

$$\mathbf{E} = \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix}, \quad (3.1)$$

with

$$E = E_0 \frac{w_0}{w(z)} \exp \left[-\frac{(\omega t - kz)^2}{2\sigma^2} \right] \exp \left[-\frac{x^2 + y^2}{w^2(z)} \right] \sin(\omega t - kz + \psi(\mathbf{r})). \quad (3.2)$$

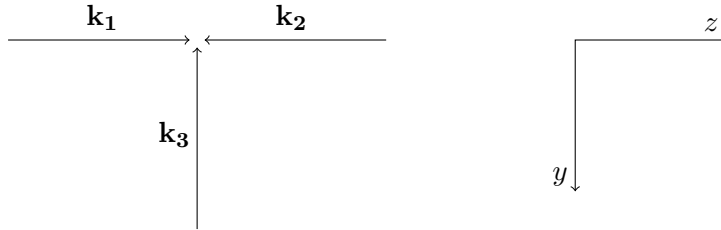


Figure 4: geometry of the beams

Here we defined:

$$w(z) = w_0 \sqrt{1 + \left(\frac{2z}{kw_0^2}\right)^2} \quad \text{and} \quad \psi(\mathbf{r}) = \psi_0 - \frac{k(x^2 + y^2)z}{2\left(z^2 + \left(\frac{kw_0^2}{2}\right)^2\right)} + \arctan\left(\frac{2z}{kw_0^2}\right). \quad (3.3)$$

ψ_0 is a constant phase, which will fall out later. For the non-constant terms of ψ one can see that for typical values of x, y and z like w_0 the terms are of order ϵ and therefore will be neglected in the following. Similarly we can write $w(z) \approx w_0$, since the term $\frac{2z}{kw_0^2}$ is of order ϵ as well.

With these approximations eq. (3.2) becomes:

$$E(x, y, z) = E_0 \exp\left[-\frac{(\omega t - kz)^2}{2\sigma^2}\right] \exp\left[-\frac{x^2 + y^2}{w_0^2}\right] \sin(\omega t - kz + \psi_0). \quad (3.4)$$

We now need the electric and magnetic fields of the three pulses with the geometry of figure 4. Also we would like to have general linear polarizations. The polarization of beam i shall be defined by the angle ϕ_i between the x -axis and the polarization vector. The angle shall be positive if it is clockwise when looking in the direction of \mathbf{k}_i .

For beam 1 (the beam with \mathbf{k}_1 in figure 4) the electric field has the form of eq. (3.4). But if we assume a general linear polarization ϕ_1 the directions of \mathbf{E} and \mathbf{B} (see eq. (3.1)) are rotated:

$$\mathbf{E}_1 = \begin{pmatrix} E_1 \cos \phi_1 \\ E_1 \sin \phi_1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B}_1 = \begin{pmatrix} -E_1 \sin \phi_1 \\ E_1 \cos \phi_1 \\ 0 \end{pmatrix}, \quad \text{with} \quad E_1(x, y, z) = E(x, y, z). \quad (3.5)$$

For the two remaining fields we get similar expressions:

$$\mathbf{E}_2 = \begin{pmatrix} E_2 \cos \phi_2 \\ -E_2 \sin \phi_2 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B}_2 = \begin{pmatrix} -E_2 \sin \phi_2 \\ -E_2 \cos \phi_2 \\ 0 \end{pmatrix}, \quad \text{with} \quad E_2(x, y, z) = E(x, y, -z) \quad (3.6)$$

and

$$E_3 = \begin{pmatrix} E_3 \cos \phi_3 \\ 0 \\ E_3 \sin \phi_3 \end{pmatrix} \quad \text{and} \quad \mathbf{B}_3 = \begin{pmatrix} -E_3 \sin \phi_3 \\ 0 \\ E_3 \cos \phi_3 \end{pmatrix}, \quad \text{with} \quad E_3(x, y, z) = E(x, z, -y). \quad (3.7)$$

Now we can substitute these results into the expressions (2.30) and (2.31) for the effective

polarization and magnetization by setting $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$ and $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3$. Hence \mathbf{P} and \mathbf{M} are sums of products, where each product consists of exactly three of the fields \mathbf{E}_i and \mathbf{B}_i . Keeping in mind that every of these fields has a factor $\sin(\omega_i t - \mathbf{k}_i \cdot \mathbf{r} + \psi_{0,i})$ and that these products therefore can be rearranged with the addition theorem

$$\sin(x) \sin(y) \sin(z) = \frac{1}{4} \left[\sin(x + y - z) + \sin(x - y + z) + \sin(-x + y + z) - \sin(x + y + z) \right] \quad (3.8)$$

we see that only terms of the form $E_1 E_2 E_3$, where the numbers of the fields are all different, can lead to a term that is proportional to $\sin(\omega_4 t - \mathbf{k}_4 \cdot \mathbf{r})$ with $\omega_4 = \omega_1 + \omega_2 - \omega_3$ and $\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$. Since we expect the emission of such a wave due to the process in figure 1 we can neglect all other terms. That means we neglect terms like $E_1 E_1 E_2$ and so on. This leads to the polarization and magnetization:

$$\mathbf{P}(\mathbf{r}, t) = 16\xi \mathbf{G}^P E_1(\mathbf{r}, t) E_2(\mathbf{r}, t) E_3(\mathbf{r}, t), \quad (3.9)$$

$$\mathbf{M}(\mathbf{r}, t) = 16\xi \mathbf{G}^M E_1(\mathbf{r}, t) E_2(\mathbf{r}, t) E_3(\mathbf{r}, t). \quad (3.10)$$

The factors \mathbf{G}^P and \mathbf{G}^M are geometric factors that depend on the polarization angles ϕ_i . In order to have geometric factors that are of the order of one we extracted the factor 16 in eq. (3.9) and eq. (3.10). The expressions for these factors are given in the appendix (A.1) and (A.2).

Now we can apply the addition theorem (3.8) and neglect the three terms on the right hand side of eq. (3.8) that lead to a different term than $\sin(\omega_4 t - \mathbf{k}_4 \cdot \mathbf{r})$. This leads to the final form of the polarization and magnetization:

$$\begin{aligned} \mathbf{P}(\mathbf{r}, t) &= 4\xi \mathbf{G}^P E_{0,1} E_{0,2} E_{0,3} \exp \left[-\frac{3x^2}{w_0^2} - \frac{2y^2}{w_0^2} - \frac{z^2}{w_0^2} \right] \\ &\times \exp \left[-\frac{(\omega_1 t - k_1 z)^2 + (\omega_2 t + k_2 z)^2 + (\omega_3 t + k_3 y)^2}{2\sigma^2} \right] \\ &\times \sin \left((\omega_1 + \omega_2 - \omega_3)t - k_1 z + k_2 z - k_3 y + \psi_{0,1} + \psi_{0,2} - \psi_{0,3} \right). \end{aligned} \quad (3.11)$$

For \mathbf{M} we get exactly the same except that \mathbf{G}^P is replaced by \mathbf{G}^M . To keep the following calculations clearer we assume here that the frequencies of the three pulses are the same:

$$\omega_0 = \omega_1 = \omega_2 = \omega_3 \Rightarrow \omega_0 = \omega_4 \quad (3.12)$$

Now we can simplify equation (3.11) and get:

$$\begin{aligned} \mathbf{P}(\mathbf{r}, t) = & 4\xi \mathbf{G}^{\mathbf{P}} E_{0,1} E_{0,2} E_{0,3} \exp \left[-\frac{3x^2}{w_0^2} - \frac{2y^2}{w_0^2} - \frac{z^2}{w_0^2} \right] \\ & \exp \left[-\frac{\omega_0^2}{2\sigma^2} ((t-z)^2 + (t+z)^2 + (t+y)^2) \right] \\ & \times \sin \left(\omega_0(t-y) + \psi_{0,1} + \psi_{0,2} - \psi_{0,3} \right). \end{aligned} \quad (3.13)$$

For later purposes we define:

$$S(\mathbf{r}) = E_{0,1} E_{0,2} E_{0,3} \exp \left[-\frac{3x^2}{w_0^2} - \frac{2y^2}{w_0^2} - \frac{z^2}{w_0^2} \right] \quad (3.14)$$

and

$$\gamma(\mathbf{r}, t) = \exp \left[-\frac{\omega_0^2}{2\sigma^2} ((t-z)^2 + (t+z)^2 + (t+y)^2) \right] \sin \left(\omega_0(t-y) + \psi_{0,1} + \psi_{0,2} - \psi_{0,3} \right). \quad (3.15)$$

This leads to:

$$\mathbf{P}(\mathbf{r}, t) = 4\xi \mathbf{G}^{\mathbf{P}} S(\mathbf{r}) \gamma(\mathbf{r}, t) \quad (3.16)$$

and equally

$$\mathbf{M}(\mathbf{r}, t) = 4\xi \mathbf{G}^{\mathbf{M}} S(\mathbf{r}) \gamma(\mathbf{r}, t). \quad (3.17)$$

The crucial point is that γ carries the whole time dependence and this will keep the following calculations clearer.

3.2 Solution of the wave equation

We now want to solve eq. (2.33) for $\delta\mathbf{E}$. We observe that eq. (2.33) is an inhomogeneous waveequation with a current \mathbf{j}_{eff} , that only depends on the fields \mathbf{E} and \mathbf{B} , but due to the perturbation ansatz \mathbf{j}_{eff} does not depend on $\delta\mathbf{E}$ any more.

Therefore we can use the Green's function method to solve the waveequation. The retarded Green's function of the D'Alembert operator is [NOLT11]:

$$G_{\text{ret}}(\mathbf{r}, t) = \frac{\delta(t - |\mathbf{r}|)}{4\pi|\mathbf{r}|} \theta(t). \quad (3.18)$$

Hence the solution to eq. (2.33) is:

$$\delta\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi} \int d\mathbf{r}' \frac{\mathbf{j}_{\text{eff}}(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|}. \quad (3.19)$$

In order to deal with the retarded time we do a Fouriertransformation in time:

$$\delta\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \frac{1}{4\pi} \int d\mathbf{r}' \frac{e^{-i\omega|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{\mathbf{j}}_{\text{eff}}(\mathbf{r}', \omega). \quad (3.20)$$

The tilde denotes the Fourier transformation. That means we need the Fouriertransformation of $\mathbf{j}_{\text{eff}}(\mathbf{r}, t)$. In order to get this transformation we use the following important property of the Fourier transformation:

$$\mathcal{F}(\partial_t f(t))(\omega) = i\omega \mathcal{F}(f(t))(\omega). \quad (3.21)$$

Using this identity the Fourier transformation of the current \mathbf{j}_{eff} becomes:

$$\tilde{\mathbf{j}}_{\text{eff}}(\mathbf{r}, \omega) = \nabla(\nabla \cdot \tilde{\mathbf{P}}) + \omega^2 \tilde{\mathbf{P}} - i\omega \nabla \times \tilde{\mathbf{M}}. \quad (3.22)$$

We will deal with the explicit forms of $\tilde{\mathbf{P}}$ and $\tilde{\mathbf{M}}$ in section 3.4.

Substituting this result into eq. (3.20) leads to:

$$\delta\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \frac{1}{4\pi} \int d\mathbf{r}' \frac{e^{-i\omega|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \left(\nabla(\nabla \cdot \tilde{\mathbf{P}}) + \omega^2 \tilde{\mathbf{P}} - i\omega \nabla \times \tilde{\mathbf{M}} \right). \quad (3.23)$$

Since we will observe the scattered photons far away from the source, we can make the common approximation:

$$\frac{e^{-i\omega|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{1}{|\mathbf{r}|} e^{-i\omega|\mathbf{r}|} e^{i\omega\hat{\mathbf{r}}\mathbf{r}'}, \quad (3.24)$$

where $\hat{\mathbf{r}}$ denotes the unit vector into the direction of \mathbf{r} . Using this approximation and performing partial integrations in order to get rid of the nabla's in eq. (3.23) we get:

$$\delta\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \frac{1}{4\pi} \frac{e^{-i\omega|\mathbf{r}|}}{|\mathbf{r}|} \int d\mathbf{r}' \omega^2 \left(\tilde{\mathbf{P}} - \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \tilde{\mathbf{P}}) - \hat{\mathbf{r}} \times \tilde{\mathbf{M}} \right) e^{i\omega\hat{\mathbf{r}}\mathbf{r}'}. \quad (3.25)$$

Now we can substitute the expressions for the polarization (3.16) and magnetization (3.17) and use that in eq. (3.16) and eq. (3.17) only the function $\gamma(\mathbf{r}, t)$ depends on time:

$$\delta\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \frac{\xi}{\pi} \frac{e^{-i\omega|\mathbf{r}|}}{|\mathbf{r}|} \int d\mathbf{r}' \omega^2 \left(\mathbf{G}^P - \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{G})^P - \hat{\mathbf{r}} \times \mathbf{G}^M \right) S(\mathbf{r}') \tilde{\gamma}(\mathbf{r}', \omega) e^{i\omega\hat{\mathbf{r}}\mathbf{r}'}. \quad (3.26)$$

That means the only Fouriertransformation we will have to perform is the fouriertransformation of $\gamma(\mathbf{r}, t)$.

3.3 Scattered energy

After having derived an expression for the electric field (3.26) we now want to calculate the energy that is produced during the collision of the three beams. Therefore we calculate at first the Poynting vector:

$$\mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{B}. \quad (3.27)$$

We see that we need at first the \mathbf{B} -field corresponding to the electric field (3.26).

In order to get the \mathbf{B} -field we look at the Fourier transformation of equation (2.27) (we use here again the property (3.21)):

$$\nabla \times \tilde{\mathbf{E}}(\mathbf{r}, \omega) = -i\omega \tilde{\mathbf{B}}(\mathbf{r}, \omega). \quad (3.28)$$

As we will measure far away from the interaction region, $\mathbf{E}(\mathbf{r}, \omega)$ given in eq. (3.26) depends on \mathbf{r} mainly due to the factor $e^{-i\omega|\mathbf{r}|}$. With this assumption the left hand side of eq. (3.28) becomes:

$$\nabla \times \tilde{\mathbf{E}}(\mathbf{r}, \omega) = -i\omega \hat{\mathbf{r}} \times \tilde{\mathbf{E}}(\mathbf{r}, \omega). \quad (3.29)$$

Hence we get for the Poynting vector:

$$\mathbf{S}(\mathbf{r}, t) = \frac{1}{4\pi} \int \frac{d\omega d\omega'}{2\pi} \tilde{\mathbf{E}}(\mathbf{r}, \omega) \times (\hat{\mathbf{r}} \times \tilde{\mathbf{E}}(\mathbf{r}, \omega')) e^{i(\omega+\omega')t}. \quad (3.30)$$

Using the Graßmann-identity we get:

$$\mathbf{S}(\mathbf{r}, t) = \frac{1}{4\pi} \int \frac{d\omega d\omega'}{2\pi} \left[\hat{\mathbf{r}} (\tilde{\mathbf{E}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}(\mathbf{r}, \omega')) - \tilde{\mathbf{E}}(\mathbf{r}, \omega') (\hat{\mathbf{r}} \cdot \tilde{\mathbf{E}}(\mathbf{r}, \omega)) \right] e^{i(\omega+\omega')t}. \quad (3.31)$$

We observe that $\hat{\mathbf{r}} \cdot \tilde{\mathbf{E}}(\mathbf{r}, \omega) = 0$ since

$$\hat{\mathbf{r}} \cdot (\mathbf{G}^P - \hat{\mathbf{r}} (\hat{\mathbf{r}} \cdot \mathbf{G}^P) - \hat{\mathbf{r}} \times \mathbf{G}^M) = 0. \quad (3.32)$$

Our aim is now to calculate the whole energy that is scattered. This energy can be calculated via:

$$\mathcal{E} = \int dt \int r^2 d\Omega (\hat{\mathbf{r}} \cdot \mathbf{S}(\mathbf{r}, t)). \quad (3.33)$$

Substituting eq. (3.31) into this expression and using that $2\pi\delta(\omega + \omega') = \int dt e^{i(\omega+\omega')t}$:

$$\mathcal{E} = \frac{1}{4\pi} \int r^2 d\Omega \int d\omega (\tilde{\mathbf{E}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}(\mathbf{r}, -\omega)). \quad (3.34)$$

We know that the electric field $\mathbf{E}(\mathbf{r}, t)$ is real. Therefore we know:

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{r}, \omega) &= \int \frac{dt}{\sqrt{2\pi}} e^{-i\omega t} \mathbf{E}(\mathbf{r}, t) = \int \frac{dt}{\sqrt{2\pi}} e^{-i\omega t} \mathbf{E}^*(\mathbf{r}, t) \\ &= \left(\int \frac{dt}{\sqrt{2\pi}} e^{i\omega t} \mathbf{E}(\mathbf{r}, t) \right)^* = \tilde{\mathbf{E}}^*(\mathbf{r}, -\omega).\end{aligned}\quad (3.35)$$

Now the total emitted energy is:

$$\mathcal{E} = \frac{1}{4\pi} \int r^2 d\Omega \int d\omega |\tilde{\mathbf{E}}(\mathbf{r}, \omega)|^2. \quad (3.36)$$

We substitute the expression we already derived for the electric field (3.26) and get:

$$\mathcal{E} = \frac{\xi^2}{4\pi^3} \int d\Omega \int d\omega \int d\mathbf{r}' \int d\mathbf{r}'' \omega^4 G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \tilde{\gamma}(\mathbf{r}', \omega) \tilde{\gamma}^*(\mathbf{r}'', \omega) e^{i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')} \quad (3.37)$$

with

$$G(\hat{\mathbf{r}}) = \left(\mathbf{G}^P - \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{G}^P) - \hat{\mathbf{r}} \times \mathbf{G}^M \right)^2. \quad (3.38)$$

The remaining task will be to solve eq. (3.37).

3.4 Fourier transformation of γ

In order to solve the integral (3.37) we need to know the form of $\tilde{\gamma}(\mathbf{r}, \omega)$. Since $\gamma(\mathbf{r}, t)$ (3.15) is mainly a Gaussian multiplied with a sinus, the transformation is easy to perform and we get as result:

$$\begin{aligned}\tilde{\gamma}(\mathbf{r}, \omega) &= \frac{i}{2} \frac{\sigma}{\sqrt{3}\omega_0} \exp\left(-\frac{\omega_0^2}{2\sigma^2}(2z^2 + y^2)\right) \\ &\times \left[\exp\left[i(\omega_0 y - \psi_{0,1} - \psi_{0,2} + \psi_{0,3}) - \frac{(\sigma^2(\omega + \omega_0) - i\omega_0^2 y)^2}{6\sigma^2\omega_0^2}\right] \right. \\ &\left. - \exp\left[-i(\omega_0 y - \psi_{0,1} - \psi_{0,2} + \psi_{0,3}) - \frac{(\sigma^2(\omega - \omega_0) - i\omega_0^2 y)^2}{6\sigma^2\omega_0^2}\right] \right].\end{aligned}\quad (3.39)$$

We observe that in ω this is a function with mainly two Gaussian peaks. One for $\omega = -\omega_0$ and one for $\omega = \omega_0$. For the integral (3.37) we need $\beta(\mathbf{r}', \mathbf{r}'', \omega) := \tilde{\gamma}(\mathbf{r}', \omega) \tilde{\gamma}^*(\mathbf{r}'', \omega)$. Hence we should take a careful look at this expression. As already mentioned $\tilde{\gamma}(\mathbf{r}', \omega)$ consists of two parts, but as these parts are very strongly peaked only one part contributes for $\omega < 0$ and the other one contributes only for $\omega > 0$. That means for $\omega < 0$

we can write:

$$\begin{aligned} \beta(\mathbf{r}', \mathbf{r}'', \omega)|_{\omega < 0} =: \beta^-(\mathbf{r}', \mathbf{r}'', \omega) &= \frac{\sigma^2}{12\omega_0^2} \exp\left(-\frac{\omega_0^2}{2\sigma^2}(2z'^2 + y'^2 + 2z''^2 + y''^2)\right) \\ &\times \exp\left[i\omega_0(y' - y'') - \frac{(\sigma^2(\omega + \omega_0) - i\omega_0^2 y')^2 + (\sigma^2(\omega + \omega_0) + i\omega_0^2 y'')^2}{6\sigma^2\omega_0^2}\right]. \end{aligned} \quad (3.40)$$

Similarly we get for $\omega > 0$:

$$\begin{aligned} \beta(\mathbf{r}', \mathbf{r}'', \omega)|_{\omega > 0} =: \beta^+(\mathbf{r}', \mathbf{r}'', \omega) &= \frac{\sigma^2}{12\omega_0^2} \exp\left(-\frac{\omega_0^2}{2\sigma^2}(2z'^2 + y'^2 + 2z''^2 + y''^2)\right) \\ &\times \exp\left[-i\omega_0(y' - y'') - \frac{(\sigma^2(\omega - \omega_0) - i\omega_0^2 y')^2 + (\sigma^2(\omega - \omega_0) + i\omega_0^2 y'')^2}{6\sigma^2\omega_0^2}\right]. \end{aligned} \quad (3.41)$$

Hence the total scattered energy (3.37) can be written as:

$$\mathcal{E} = \mathcal{E}^- + \mathcal{E}^+ \quad (3.42)$$

with

$$\mathcal{E}^- := \frac{\xi^2}{4\pi^3} \int d\Omega \int_{-\infty}^0 d\omega \int d\mathbf{r}' \int d\mathbf{r}'' \omega^4 G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \beta^-(\mathbf{r}', \mathbf{r}'', \omega) e^{i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')} \quad (3.43)$$

and

$$\mathcal{E}^+ := \frac{\xi^2}{4\pi^3} \int d\Omega \int_0^{\infty} d\omega \int d\mathbf{r}' \int d\mathbf{r}'' \omega^4 G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \beta^+(\mathbf{r}', \mathbf{r}'', \omega) e^{i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')} \quad (3.44)$$

By carefully looking at the expressions for β^- (3.40) and β^+ (3.41) we observe:

$$\beta^-(\mathbf{r}'', \mathbf{r}', -\omega) = \beta^+(\mathbf{r}', \mathbf{r}'', \omega). \quad (3.45)$$

That means that if we make the substitution $\omega \rightarrow -\omega$ in eq. (3.43) we get:

$$\mathcal{E}^- := \frac{\xi^2}{4\pi^3} \int d\Omega \int_0^{\infty} d\omega \int d\mathbf{r}' \int d\mathbf{r}'' \omega^4 G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \beta^+(\mathbf{r}'', \mathbf{r}', \omega) e^{-i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')} \quad (3.46)$$

Now we can exchange the variables $\mathbf{r}' \leftrightarrow \mathbf{r}''$ and get:

$$\mathcal{E}^- := \frac{\xi^2}{4\pi^3} \int d\Omega \int_0^{\infty} d\omega \int d\mathbf{r}' \int d\mathbf{r}'' \omega^4 G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \beta^+(\mathbf{r}', \mathbf{r}'', \omega) e^{i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')}, \quad (3.47)$$

which is identical with \mathcal{E}^+ (3.44). Hence:

$$\mathcal{E} = 2\mathcal{E}^+. \quad (3.48)$$

3.5 Solution of the integral

In this section we want to solve the remaining integrals in eq. (3.44).

At first we look at the ω -integral. The factors depending on ω are:

$$\omega^4 \exp \left[- \frac{(\sigma^2(\omega - \omega_0) - i\omega_0^2 y')^2 + (\sigma^2(\omega - \omega_0) + i\omega_0^2 y'')^2}{6\sigma^2\omega_0^2} \right] e^{i\omega \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')}. \quad (3.49)$$

As σ is of the order of 1000 (for the parameters of OMEGA EP) the function is very strongly peaked at $\omega = \omega_0$ and we can take the limit $\sigma \rightarrow \infty$. In this limit the Gaussian peak approaches a Delta-distribution, since:

$$\lim_{\sigma \rightarrow \infty} \frac{\sigma}{\sqrt{\pi}} e^{-\sigma^2 x^2} = \delta(x). \quad (3.50)$$

The integral in ω of expression (3.49) therefore becomes:

$$\frac{\sqrt{3\pi}\omega_0}{\sigma} \omega_0^4 e^{i\omega_0 \hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'')}. \quad (3.51)$$

This limit ($\sigma \rightarrow \infty$) corresponds to the assumption that the scattered wave is monochromatic. Since at the end we will take experimental parameters of laser systems with quite long pulses, this assumption will lead to a good result.

While performing the limit $\sigma \rightarrow \infty$ the factor

$$\exp \left(- \frac{\omega_0^2}{2\sigma^2} (2z'^2 + y'^2 + 2z''^2 + y''^2) \right). \quad (3.52)$$

which appears in $\tilde{\gamma}(\mathbf{r}', \omega)\tilde{\gamma}^*(\mathbf{r}'', \omega)$ (see eq. (3.39)) is approximated by 1. The remaining integral is now:

$$\mathcal{E} = C \int d\Omega \int d\mathbf{r}' \int d\mathbf{r}'' G(\hat{\mathbf{r}}) S(\mathbf{r}') S(\mathbf{r}'') \exp \left[i\omega_0 (\hat{\mathbf{r}}(\mathbf{r}' - \mathbf{r}'') - (y' - y'')) \right], \quad (3.53)$$

with

$$C = \frac{\xi^2}{\pi^3} \frac{\sigma \sqrt{3\pi}\omega_0^3}{24} E_{0,1}^2 E_{0,2}^2 E_{0,3}^2. \quad (3.54)$$

The functions $S(\mathbf{r})$ (3.14) are just Gaussians and therefore the integrals in \mathbf{r}' and \mathbf{r}'' are

easy to perform. The integral in x' for example is of the form:

$$\int dx' \exp\left(-\frac{3x'^2}{w_0^2}\right) e^{i\omega_0 \hat{\mathbf{r}}_x x'} = \frac{\sqrt{\pi} w_0}{\sqrt{3}} \exp\left[-\frac{(w_0 \omega_0)^2}{12} \hat{\mathbf{r}}_x^2\right]. \quad (3.55)$$

With $\hat{\mathbf{r}}_x$ I denoted the x -component of $\hat{\mathbf{r}}$.

The integrals in y' , z' , x'' , y'' and z'' can be performed in the same way. The total scattered energy therefore is:

$$\mathcal{E} = C \frac{w_0^6 \pi^3}{6} \int d\Omega \exp\left[-(w_0 \omega_0)^2 \left(\frac{1}{6} \hat{\mathbf{r}}_x^2 + \frac{1}{4} (\hat{\mathbf{r}}_y - 1)^2 + \frac{1}{2} \hat{\mathbf{r}}_z^2\right)\right] G(\hat{\mathbf{r}}). \quad (3.56)$$

It remains to perform the integral in Ω . The factor $\omega_0 w_0$ will be of order 60 for our experimental parameters. This means that the integrand is strongly peaked around:

$$\hat{\mathbf{r}} = \hat{\mathbf{k}}_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \quad (3.57)$$

Physically this means that the main contribution of the scattered wave propagates along the y -axis and the beam is not spread over a great solid angle Ω . Mathematically this means that we can approximate: $G(\hat{\mathbf{r}}) \approx G(\hat{\mathbf{k}}_4)$. Now we will calculate the integral (3.56) by choosing spherical coordinates:

$$\hat{\mathbf{r}} = \begin{pmatrix} \sin \theta \cos \phi \\ \cos \theta \\ -\sin \theta \sin \phi \end{pmatrix} \quad \text{with } \phi \in [0, 2\pi], \quad \theta \in [0, \pi], \quad \text{and } d\Omega = d\theta \sin \theta d\phi. \quad (3.58)$$

These are the usual spherical coordinates but rotated in such a way that for $\theta = 0$ $\hat{\mathbf{r}} = \hat{\mathbf{k}}_4$. The integral we now have to perform is:

$$\mathcal{E} = C \frac{w_0^6 \pi^3}{6} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \exp\left[-(w_0 \omega_0)^2 \left(\frac{1}{6} \sin^2 \theta \cos^2 \phi + \frac{1}{4} (\cos^2 \theta - 2 \cos \theta + 1) + \frac{1}{2} \sin^2 \theta \sin^2 \phi\right)\right] G(\hat{\mathbf{k}}_4). \quad (3.59)$$

We already discussed that due to the fact that $\omega_0 w_0$ is of the order of 60 the main contribution of the integral comes from a very small solid angle around $\hat{\mathbf{r}} = \hat{\mathbf{k}}_4$. That's

why we can approximate $\sin \theta \approx \theta$ and $\cos \theta \approx 1$. Now the integral in θ becomes:

$$\begin{aligned}
& \int_0^\pi d\theta \theta \exp \left[- (w_0 \omega_0)^2 \theta^2 \left(\frac{1}{6} \cos^2 \phi + \frac{1}{2} \sin^2 \phi \right) \right] \\
& \quad = \frac{1}{2(\omega_0 w_0)^2 \left(\frac{1}{6} \cos^2 \phi + \frac{1}{2} \sin^2 \phi \right)} \\
& \quad \times \left[1 - \exp \left(- (\omega_0 w_0)^2 \pi^2 \left(\frac{1}{6} \cos^2 \phi + \frac{1}{2} \sin^2 \phi \right) \right) \right] \\
& \quad \approx \frac{1}{2(\omega_0 w_0)^2 \left(\frac{1}{6} \cos^2 \phi + \frac{1}{2} \sin^2 \phi \right)}.
\end{aligned} \tag{3.60}$$

It remains to solve the ϕ -integral:

$$\int_0^{2\pi} d\phi \frac{1}{2(\omega_0 w_0)^2 \left(\frac{1}{6} \cos^2 \phi + \frac{1}{2} \sin^2 \phi \right)} = \frac{6\pi}{\sqrt{3}(\omega_0 w_0)^2}. \tag{3.61}$$

In the end we get:

$$\mathcal{E} = C \frac{\pi^4}{\sqrt{3}} \frac{w_0^4}{\omega_0^2} G(\hat{\mathbf{k}}_4). \tag{3.62}$$

Dividing this result by ω_0 we get the number of scattered photons:

$$N = \xi^2 \frac{\sqrt{\pi^3}}{24} \sigma w_0^4 E_{0,1}^2 E_{0,2}^2 E_{0,3}^2 G(\hat{\mathbf{k}}_4). \tag{3.63}$$

4 Experimental parameters and results

In this section we want to look at the beam parameters of OMEGA EP and get actual numbers for the scattered photons when three beams collide. This is followed by a discussion and comparison of the results.

4.1 OMEGA EP parameters

OMEGA EP has a short-pulse performance and a long-pulse performance [OMEP06]. There are two beams that are capable of the short-pulse performance. That means when the short pulse performance is used, one beam has to be split into two beams. That brings the intensity of two of the beams down by a factor of $\frac{1}{2}$. The long-pulse performance is possible for four beams and hence this reduction is only a problem of the short-pulse performance.

In order to evaluate equation (3.63) we need the values of w_0 , σ , and the field strengths of the three beams: $E_{0,1}$, $E_{0,2}$ and $E_{0,3}$. For σ we actually need ω_0 and the Gaussian pulse width τ , because σ is defined by equation (3.2) and therefore can be calculated by $\sigma = \tau\omega_0$.

The parameters of the short- and long-pulse performance are listed in Table 2.

4.2 Electric field strengths

As we can see in Table 2 we have to derive the electric field strengths from the averaged intensities.

In order to derive the electric field strengths we consider again the field of a Gaussian beam that propagates in positive z -direction as in eq. (3.1) and eq. (3.2). At $z = 0$ the fields are:

$$\mathbf{E} = \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix}, \quad (4.1)$$

parameters	short-pulse	long-pulse
Pulse width τ (ps)	0.5	5×10^3
Focal spot radius w_0 (μm)	10	50
Averaged intensity (W/cm^2)	2×10^{20}	6×10^{15}
Wavelength (nm)	1053	351

Table 2: Parameters of OMEGA EP for the short-pulse performance and the long-pulse performance. Averaged intensity means averaged over the focal spot.

with

$$E(x, y, t) = E_0 \sin(\omega t + \psi_0) \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) \exp\left(-\frac{(\omega t)^2}{2\sigma^2}\right). \quad (4.2)$$

We will now calculate the average intensity of this field in time and over the focal spot πw_0^2 in the x - y -plane. The intensity is given by the Poynting vector:

$$\mathbf{S} = \frac{1}{4\pi}(\mathbf{E} \times \mathbf{B}) = \frac{1}{4\pi} \begin{pmatrix} 0 \\ 0 \\ E^2 \end{pmatrix}. \quad (4.3)$$

As σ is of the order of 1000 the Gaussian in time won't be important for the averaging. The average in time therefore is the average of $\sin^2(\omega t + \psi)$ over a period T ($T\omega = 2\pi$):

$$\frac{1}{T} \int_0^T dt \sin^2(\omega t + \psi) = \frac{1}{2}. \quad (4.4)$$

The intensity averaged in time therefore is:

$$\frac{E_0^2}{8\pi} \exp\left(-\frac{2r^2}{w_0^2}\right), \quad \text{with } r^2 = x^2 + y^2. \quad (4.5)$$

Now we average over the focal spot by using polar coordinates:

$$\begin{aligned} |\bar{\mathbf{S}}| &= \frac{1}{\pi w_0^2} \int_0^{2\pi} d\phi \int_0^{w_0} dr \frac{E_0^2}{8\pi} r \exp\left(-\frac{2r^2}{w_0^2}\right) \\ &= \frac{E_0^2}{16\pi} \int_0^2 dx e^{-x} = \frac{E_0^2}{16\pi} [1 - e^{-2}]. \end{aligned} \quad (4.6)$$

As result we have:

$$E_0^2 = \frac{16\pi |\bar{\mathbf{S}}|}{1 - e^{-2}}. \quad (4.7)$$

Now we can rewrite our result (3.63) for the number of scattered photons in terms of the averaged intensities:

$$N = \xi^2 \frac{\sqrt{\pi^9} 16^3}{24(1 - e^{-2})^3} |\bar{\mathbf{S}}_1| |\bar{\mathbf{S}}_2| |\bar{\mathbf{S}}_3| \omega_0 \tau w_0^4 G(\hat{\mathbf{k}}_4). \quad (4.8)$$

4.3 Numerical results

Using that $\xi = \frac{4\alpha^2}{45m^4}$ we rewrite eq. (4.8):

$$N = 10.84 \left(\frac{I_1 I_2 I_3}{(10^{20} \text{ W/cm}^2)^3} \right) \left(\frac{1 \mu\text{m}}{\lambda} \right) \left(\frac{\tau}{1 \text{ ps}} \right) \left(\frac{w_0}{10 \mu\text{m}} \right)^4 G(\hat{\mathbf{k}}_4). \quad (4.9)$$

I_i denotes here the intensity of beam i averaged over the focal spot. These intensities are given in Table 2. For the short-pulse performance two of the beams have only half of the intensity that is given in Table 2. The geometry factor given in the appendix (A.3) is maximal for polarization angles $\phi_1 = \phi_2 = \phi_3 = \frac{\pi}{4}$. For these angles we have $G(\hat{\mathbf{k}}_4) = 6.89$. For the short-pulse performance this leads to a number of $N = 70.9$ photons per shot. For the long-pulse performance we get $N = 0.14 \times 10^{-3}$. The much smaller intensities for the long-pulse performance lead to a much smaller result although the much longer pulse duration τ and the bigger focal spot w_0 of the long-pulse performance lead to greater energies per beam. Hence the long-pulse performance is not suitable for a measurement. But the result of $N = 70.9$ photons per shot for the short-pulse performance seems to be a detectable number.

4.4 Comparison with the results of Lundin et al.

We now want to compare our result with results presented in other papers in particular with the result of [LUND06], where a number of $N = 0.07$ photons was derived for a three dimensional setup and for parameters of the Astra Gemini system. Their result was proportional to: $N \sim \lambda^{-1} L P_1 P_2 P_3 b^{-2} G$, where L is the length of a pulse, P_i is the power of beam i , b^2 is the focal spot size and G is a geometry factor that is defined in the same way as our geometric factor. Due to the fact that the power of a laser is proportional to the intensity times the focal spot size, we can see that their result has the same scalings as our result. This is a first evidence that our result is compatible with their result.

Nevertheless their result is three orders of magnitude smaller than our result. We observe that the beam parameters of OMEGA EP are more suitable, since the beams contain much more energy, but only a factor of 4 can be explained with the different parameters. But there is an explanation for the rest of the factor.

At first we should consider that the geometry they used is completely different. The geometric factor G , which they defined in the same way as we did, is for their three dimensional geometry one order of magnitude smaller than the geometric factor for our two dimensional geometry. Further they had to use frequency doubling to realise the

geometry they considered and therefore they considered a power loss of 60% for two of the beams. All this explains already that our result has to be of a factor 250 bigger than their result.

The remaining unexplained factor of 4 could have various reasons. For example, the interaction region of the two different geometries will be different. Also the model for the colliding beams is different.

5 Conclusions

The purpose of this thesis was to investigate the possibility of detecting photon-photon scattering in modern laser systems with high-energy pulses. Therefore, we considered the collision of three Gaussian beams. We chose a special geometry of the three incoming beams and with the effective action of Heisenberg and Euler we calculated the energy of the generated fourth beam. The result was an analytical approximation and a comparison with the calculations by Lundin et al. [LUND06] proofed that our result gives the right order of magnitude. The difference between our results and the results of Lundin et al. [LUND06] are mainly the different beam parameters of OMEGA EP which are more suitable here than the parameters of Astra Gemini, that were chosen by Lundin et al., since the beams of OMEGA EP contain much more energy. Also the geometry and the more realistic Gaussian beam model for the incoming three pulses led to differences.

It is important to mention here that the above calculations could easily be adapted for other geometries of the wavevectors \mathbf{k}_1 , \mathbf{k}_2 and \mathbf{k}_3 . The special geometry was only chosen to keep the calculations more transparent.

In order to get actual results we took parameters of the OMEGA EP laser system and got a number of $N = 70.9$ photons scattered per shot for the short-pulse performance. This number seems to be detectable and hence the detection of photon-photon scattering appears to be in principle possible with OMEGA EP. However the chosen geometry might have the drawback that the scattered photons propagate in exactly the opposite direction as the third laser beam. Therefore the detection of the scattered photons might be a difficult experimental task. Of course, other geometries could be chosen, where the scattered photons are well separated of the three beams, but then the number of scattered photons might decrease.

We have also observed that the long-pulse performance of OMEGA EP is unsuitable for the detection of light by light scattering although the beams contain much more energy and therefore the total number of photons that can scatter at each other is much higher. This comes from the fact that we consider stimulated photon-photon scattering and therefore the field strength of the third beam, which induces the scattering of a photon in the direction of beam three, is important as well.

However, we saw that the parameters of OMEGA EP are more suitable than the parameters of Astra Gemini, although Astra Gemini reaches higher intensities. So just increasing the intensities of lasers further and further won't lead to an improvement for the detection of photon-photon scattering. It is rather important to have pulses with

high energy that don't have too small intensities. For the detection of photon-photon scattering it seems to be essential to have a laser system with the right combination of high intensity and high energy beams.

In conclusion, the order of magnitude calculated here gives hope that the very important prediction of light by light scattering in QED can be measured in near future.

Appendices

We present here the formulas for the geometric factors in dependence of the polarization angles ϕ_1 , ϕ_2 and ϕ_3 :

$$\begin{aligned}
 \mathbf{G}^P = & \left[\frac{1}{16} \left(16 \cos(\phi_1) \cos(\phi_2) \cos(\phi_3) + 6 \cos(\phi_3) \sin(\phi_1) \sin(\phi_2) \right. \right. \\
 & \left. \left. + 17 \cos(\phi_2) \sin(\phi_1) \sin(\phi_3) + 17 \cos(\phi_1) \sin(\phi_2) \sin(\phi_3) \right) \right] \hat{\mathbf{x}} \\
 & + \left[\frac{11}{16} \cos(\phi_3) \sin(\phi_1 - \phi_2) \right] \hat{\mathbf{y}} \\
 & + \left[\frac{1}{16} (-11 \sin(\phi_1 + \phi_2 - \phi_3) - 3 \sin(\phi_1 + \phi_2 + \phi_3)) \right] \hat{\mathbf{z}}
 \end{aligned} \tag{A.1}$$

$$\begin{aligned}
 \mathbf{G}^M = & \left[\frac{1}{16} \left(-2 \left(3 \cos(\phi_1) \cos(\phi_2) + 8 \sin(\phi_1) \sin(\phi_2) \right) \sin(\phi_3) \right. \right. \\
 & \left. \left. - 17 \cos(\phi_3) \sin(\phi_1 + \phi_2) \right) \right] \hat{\mathbf{x}} \\
 & - \left[\frac{11}{16} \sin(\phi_1 - \phi_2) \sin(\phi_3) \right] \hat{\mathbf{y}} \\
 & + \left[\frac{1}{16} \left(-11 \cos(\phi_1 + \phi_2 - \phi_3) + 3 \cos(\phi_1 + \phi_2 + \phi_3) \right) \right] \hat{\mathbf{z}}
 \end{aligned} \tag{A.2}$$

$$\begin{aligned}
 G(\hat{\mathbf{k}}_4) = & \frac{1}{256} \left(24 \cos(\phi_1) \cos(\phi_2) \cos(\phi_3) - 2 \cos(\phi_3) \sin(\phi_1) \sin(\phi_2) \right. \\
 & \left. + 31 \cos(\phi_2) \sin(\phi_1) \sin(\phi_3) + 31 \cos(\phi_1) \sin(\phi_2) \sin(\phi_3) \right)^2 \\
 & + \frac{1}{256} \left(-31 \cos(\phi_2) \cos(\phi_3) \sin(\phi_1) - 31 \cos(\phi_1) \cos(\phi_3) \sin(\phi_2) \right. \\
 & \left. + 2 \cos(\phi_1) \cos(\phi_2) \sin(\phi_3) - 24 \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \right)^2
 \end{aligned} \tag{A.3}$$

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Erklärung

Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 12.05.2014,