LETTER TO THE EDITOR

Modelling the influence of temperature anisotropies on poloidal asymmetries of density in the core of rotating plasmas

R. Bilato, O. Maj, C. Angioni

Max Planck Institute for Plasma Physics, EURATOM Association, Boltzmannstr. 2, 85748 Garching, Germany

E-mail: `roberto(dot)bilato(at)ipp(dot)mpg(dot)de`

Abstract. A consistent set of equations is derived to model poloidal density asymmetries induced by temperature anisotropies in tokamak rotating plasmas. The model can be applied to compute poloidal density asymmetry of highly-charged impurities due to additional plasma heating.

PACS numbers: 52.25.Dg,52.25.Vy,52.35.Hr,52.50.Gj,52.50.Qt,52.55.Fa,52.65.Ff

Submitted to: *This is a preprint of a paper published in Nuclear Fusion (2014) vol* **24***,072003. The final version is available at http://iopscience.iop.org/0029-5515/54/7/072003/*

In tokamaks, additional heating can affect the poloidal asymmetry of impurity densities, as recently measured in the core of Alcator C-Mod plasmas [1], and already proposed in [2]. The possibility of influencing the poloidal potential by heating with waves in the ion-cyclotron (ICRF) and in the electron-cyclotron range of frequencies was already investigated in [3] (and therein citations), and a simplified model of the ICRF effects has been recently suggested in [4]. Since plasmas generally rotate at toroidal speeds large enough to affect the density of heavy ion, it is necessary to simultaneously account for plasma rotation and for temperature anisotropies in the calculation of poloidal density asymmetries of highly charged impurities.

Since the time scale of parallel equilibration is smaller than the characteristic time of cross-field transport for impurities, it is justified to analyze the parallel dynamics separately on each flux surface, by taking the limit of negligible Larmor radius and neglecting the drift terms which are connected with the calculation of the neoclassical transport, which is not the purpose of this work. Precisely, we consider here the zeroth-order equation of the usual expansion in the small parameter $\delta = \rho_i/L_\perp$ of the neoclassical transport theory [5], where ρ_i is the (poloidal) Larmor radius and $L_\perp$ is the characteristic macroscopic gradient length, in the presence of a plasma flow $\mathbf{V}_0$ comparable to thermal ion velocity $v_{\rm thi} = \sqrt{2T_i/m_i}$ [6]. However, besides the collision operator, we also take into account the effect of additional operators which describe the impact of auxiliary heating systems. Finally, we consider here only axisymmetric geometry.
Closely following the derivation in [6] and working in the velocity coordinate system shifted by the flow velocity $\mathbf{V}_0$, the zeroth-order Fokker-Planck (FP) equation for the equilibrium distribution function f_0 is

$$\begin{aligned}\frac{\partial f_0}{\partial t_0} + \left(v_\parallel \hat{b} + \mathbf{V}_0\right)\cdot\nabla f_0 - \left[\frac{q}{m}\nabla\Phi_0 + \left(\frac{\partial \mathbf{V}_0}{\partial t_0} + (v_\parallel \hat{b} + \mathbf{V}_0)\cdot\nabla\mathbf{V}_0\right)\right]\cdot\hat{b}\,\frac{\partial f_0}{\partial v_\parallel} \\ - \frac{v_\perp^2}{2}\left(\hat{b}\cdot\nabla\ln B\right)\left(\frac{\partial f_0}{\partial v_\parallel} - \frac{v_\parallel}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right) - \frac{v_\perp}{2}\left[\nabla\cdot\mathbf{V}_0 - \hat{b}\cdot\nabla\mathbf{V}_0\cdot\hat{b}\right]\frac{\partial f_0}{\partial v_\perp} = \\ \mathcal{C}_{\rm coll}(f_0) + \mathcal{Q}_{\rm rf}(f_0) + S_{\rm NBI} - L_{\rm part}\ ,\end{aligned} \tag{1}$$

where $\mathbf{B}$ is the confining magnetic field, $\hat{b} = \mathbf{B}/B$, and the subscripts $\parallel$ and $\perp$ refer to the direction of $\mathbf{B}$. On the rhs (right-hand side), $\mathcal{C}_{\rm coll}$ is the collisional operator, $\mathcal{Q}_{\rm rf}$ is the quasilinear operator describing radio-frequency (rf) heating, $S_{\rm NBI}$ is the neutral-beam-injection (NBI) source, and $L_{\rm part}$ is the particle loss term necessary to guarantee the constancy of the average densities in the presence of NBI particle sources [7]. As demonstrated in [6], in order to have a gyrotropic f_0 in rotating plasmas and in the limit of small ion gyroradius, the flow velocity is made of two components, $\mathbf{V}_0 = \Omega_\varphi\, R\,\hat{e}_\varphi + K\,\mathbf{B}$, where Ω_φ is the angular frequency of the plasma toroidal rotation, R is the distance from the torus axis, and $K\,\mathbf{B}$ is a degree of freedom on the parallel flow, used to impose the periodicity of the solution [8]. Since we are interested in the steady-state solution of (1), we set the derivatives $\partial/\partial t_0$ equal to zero. Next, we approximate f_0 with F_0 that cancels the bounce-averaged rhs, and approximate F_0 with a bi-Maxwellian, as proposed in [3],

$$F_0 \approx n_0 \left(\frac{m}{2\pi}\right)^{3/2} \frac{1}{T_\perp T_\parallel^{1/2}} \exp\left\{-\frac{m v_\perp^2}{2 T_\perp}\right\} \exp\left\{-\frac{m v_\parallel^2}{2 T_\parallel}\right\}, \tag{2}$$

where $T_\perp$ and $T_\parallel$ are the perpendicular and parallel temperatures of F_0. In absence of external heating, such as NBI and ICRF, the collision operator ensures that F_0 is Maxwellian, i.e. $T_\parallel = T_\perp$. We use (2) in the lfs (left-hand side) of (1), and by requiring the lhs to be zero the equations of our problem follow from imposing to be zero the coefficients of $(v_\perp^m v_\parallel^n)$ as reported at the beginning of each equation

$$(v_\parallel): \quad \hat{b}\cdot\left(\frac{\nabla n_0}{n_0} - \frac{1}{2}\frac{\nabla T_\parallel}{T_\parallel} - \frac{\nabla T_\perp}{T_\perp} + q\frac{\nabla\Phi_0}{T_\parallel} - \frac{m}{T_\parallel}\left(\nabla\mathbf{V}_0\right)\cdot\mathbf{V}_0\right) = 0 \tag{3}$$

$$\nabla\cdot(n_0\,\mathbf{V}_0) = 0 \tag{4}$$

$$(v_\parallel^3): \quad \hat{b}\cdot\nabla T_\parallel = 0 \tag{5}$$

$$(v_\parallel v_\perp^2): \quad \hat{b}\cdot\left[\left(\frac{T_\perp}{T_\parallel} - 1\right)\frac{\nabla B}{B} + \frac{\nabla T_\perp}{T_\perp}\right] = 0 \tag{6}$$

$$(v_\perp^2): \quad \mathbf{V}_0\cdot\frac{\nabla T_\perp}{T_\perp} + \nabla\cdot\mathbf{V}_0 - \hat{b}\cdot(\nabla\mathbf{V}_0)\cdot\hat{b} = 0 \tag{7}$$

$$(v_\parallel^2): \quad \frac{1}{2}\,\mathbf{V}_0\cdot\frac{\nabla T_\parallel}{T_\parallel} + \hat{b}\cdot(\nabla\mathbf{V}_0)\cdot\hat{b} = 0 \tag{8}$$

Equation (3) is the parallel momentum equation, (4) is the particle conservation in steady state (obtained by combining (7) and (8) with the equation for the term with no velocity dependence), (5) states that no parallel gradient of $T_\parallel$ is allowed, and (6) constrains the parallel gradient of $T_\perp$ to be controlled by the mirror force. The choice $K = 0$ satisfies (7) and (8) together with particle conservation (4). However, in the more general description which includes also neoclassical drift terms, an additional poloidal flow ($K \neq 0$) is necessary to guarantee particle conservation in presence of the neoclassical parallel return flow [8]: in other words, this term guarantees the conservation of the number of ions of a given species in a flux-tube whose cross-section varies according to the conservation of the magnetic flux. Using (5) in (3), the parallel momentum equation becomes

$$\frac{\nabla_\parallel n_0}{n_0} = -q\frac{\nabla_\parallel\Phi_0}{T_\parallel} + \frac{\nabla_\parallel T_\perp}{T_\perp} + \frac{\nabla_\parallel(mV_0^2)}{2T_\parallel}, \tag{9}$$

with $\nabla_\parallel := \hat{b}\cdot\nabla$. According to (5), $T_\parallel$ along the magnetic field line is constant, and the integration of (9) becomes straightforward

$$n_0(\psi,\vartheta) = n_{0*}(\psi)\,\frac{T_\perp(\psi,\vartheta)}{T_{\perp*}(\psi)}\,\exp\left\{-\frac{q\Phi_0(\psi,\vartheta) - m\left[V_0^2(\psi,\vartheta) - V_{0*}^2(\psi)\right]/2}{T_\parallel(\psi)}\right\}, \tag{10}$$

where star stands for the initial values defined at a poloidal angle ϑ_* (the poloidal angle ϑ is used as coordinate along the magnetic field), and ψ is the radial coordinate labelling the magnetic surfaces. The poloidal potential Φ_0 is defined up to an additive constant, which is set in such a way that $\Phi_{0*}(\psi) := \Phi_0(\psi, \vartheta_*) = 0$.

Alternatively, we can directly extend the bi-Maxwellian approximation (2) by observing that the solution of the bounce-averaged Fokker-Planck equation is always given at the point of minimum B, through which all the particles pass. We use the subscript "lfs" for the point where the confining magnetic field is minimum, and in tokamaks this point is in the low-field side. In particular, we can re-write (2) on the lfs point as

$$F_0(\psi, \vartheta_{\rm lfs}; \mathcal{E}, \mu) = \left(n_{\rm lfs}\ \exp\left\{ -\frac{\Phi_{\rm eff\ lfs}}{T_\parallel} \right\} \right) \frac{(m/2\pi)^{3/2}}{T_{\parallel,\rm lfs}^{1/2}\, T_{\perp,\rm lfs}}\ \exp\left\{ -\frac{\mu B_{\rm lfs}}{T_{\perp,\rm lfs}} \right\}\ \exp\left\{ -\frac{\mathcal{E} - \mu B_{\rm lfs}}{T_{\parallel,\rm lfs}} \right\} ,$$

where $\mu = mv_\perp^2/2B$ is the magnetic moment, and $\mathcal{E} = mv^2/2 + \Phi_{\rm eff}$ is the particle energy in the shifted-velocity coordinate system, with $\Phi_{\rm eff} = q\Phi_0 - mV_0^2/2$ the effective potential [6]. If we map F_0 along the magnetic field line while keeping constant μ and $\mathcal{E}$, F_0 remains bi-Maxwellian

$$F_0(\psi, \vartheta; v_\perp, v_\parallel) = n_0(\psi, \vartheta)\ \frac{(m/2\pi)^{3/2}}{T_\parallel^{1/2}(\psi)\, T_\perp(\psi, \vartheta)}\ \exp\left\{ -\frac{mv_\perp^2}{2T_\perp(\psi, \vartheta)} \right\}\ \exp\left\{ -\frac{mv_\parallel^2}{2T_\parallel(\psi)} \right\} , \tag{11}$$

where $T_\parallel$ is constant along the magnetic field lines in agreement with (5), n_0 is exactly (10) with the subscript $*$ replaced by lfs, and $T_\perp$ varies according to

$$\frac{T_\parallel(\psi)}{T_\perp(\psi, \vartheta)} = 1 - \left(1 - \frac{T_\parallel(\psi)}{T_{\perp\rm lfs}(\psi)} \right) \frac{B_{\rm lfs}(\psi)}{B(\psi, \vartheta)} . \tag{12}$$

The local perpendicular temperature (12) satisfies (6). This model of f_0 is consistent with the requirements on n_0, $T_\perp$ and $T_\parallel$ of the lfs of the starting equation (1). In practice, $T_\perp$ and $T_\parallel$ are completely defined by the rhs of (1) and estimated from the solution of the bounce-averaged Fokker-Planck solver, whereas the density n_0 is controlled by (3). Since in a tokamak $B \geq B_{\rm lfs}$, the mirror force reduces the temperature anisotropy when moving from the lfs to the hfs.

In passing, it is easily shown that F_0 satisfies the collisionless drift kinetic equation $v_\parallel \nabla_\parallel F_0 = 0$, valid when guiding-center drifts across the magnetic field as well as the time derivative are neglected [8] (cfr. equation (34) in [9]). In the case of ICRF heating, the distribution, when mapped on the point where the resonance crosses the magnetic surface, is indeed well approximated by a bi-Maxwellian [10]. Therefore, applying the same procedure for (11) we obtain the same expression (12) with $T_{\perp\rm lfs}$ and $B_{\rm lfs}$ replaced by the corresponding values at the IC resonance point. We note that the model proposed in [9] (precisely, equation (40)) is derived by assuming only the conservation of μ and of the kinetic energy in an inhomogeneous plasma (i.e. only with mirror force), which implies that $\nabla_\parallel n_0/n_0 = (1 - T_\perp/T_\parallel)\nabla_\parallel B/B$. Thus, n_0 is monotonic if the sign of $\nabla_\parallel B$ does not change in going from the lfs to the hfs (high-field side), as it is usually the case in tokamaks. Hence internal local maxima of density at the crossings between the IC resonance and the magnetic surface in [4] are fictitious consequence of the absolute value artificially introduced in equation (40) of [9] to mimic the typical "rabbit ears" in the contour plot of the distribution function at the outer midplane point. A similar shortcoming of the model proposed in [9] arises from the detailed analysis done in [11].

The problem is fully determined when Φ_0 and $\mathbf{V}_0$ are known. For the latter, one typically uses the experimental measurements of the plasma toroidal rotation, whereas the former is determined from equation (9) applied to electrons. Since for electrons the centrifugal force can be neglected because of the smallness of their mass, and the electron distribution function can be considered isotropic, it holds $e\,\tilde{\Phi}_0/T_e = \ln(n_e/n_{e,\mathrm{lfs}})$, where the tilde symbol stands for the poloidally varying part. In turn, the electron density is determined via the quasi-neutrality condition, namely $n_e = \sum_i Z_i n_i$, with $q = Z\,e$, and e the elementary charge. Finally, the set of normalized algebraic equations describing the poloidal variation of the plasma densities in the presence of plasma rotation and temperature anisotropies is:

$$
\begin{aligned}
\frac{n_j}{n_{j,\mathrm{lfs}}} &= \frac{T_{\perp j}}{T_{\perp,j,\mathrm{lfs}}}\ \exp\left\{-Z_j\frac{e\,\tilde{\Phi}_0}{T_{\|j}} + A_j\frac{\mathrm{M}_\varphi^2\ \left[(R/R_{\mathrm{lfs}})^2-1\right]}{T_{\|,j}/T_e}\right\}\\
n_e &= \sum_j Z_j\,n_j\\
\frac{e\tilde{\Phi}_0}{T_e} &= \ln\frac{n_e}{n_{e,\mathrm{lfs}}}\\
\frac{T_{\perp j}}{T_{\perp,j,\mathrm{lfs}}} &= \left[\frac{T_{\perp j\mathrm{lfs}}}{T_{\|j}} + \left(1-\frac{T_{\perp j\mathrm{lfs}}}{T_{\|j}}\right)\frac{B_{\mathrm{lfs}}}{B}\right]^{-1}
\end{aligned}
\qquad (13)
$$

where $\mathrm{M}_\varphi = \Omega_\varphi R_{\mathrm{lfs}}/\sqrt{2T_e/m_p}$ is the toroidal Mach number of thermal protons in thermal equilibrium with electrons, $A = m/m_p$, and m_p the proton mass.

It is instructive to perturbatively solve (13) and to generalize formulas of [12]. In this way, we have explicit expressions of the poloidal asymmetry of impurity density. For convenience, here we normalize densities and temperatures to the corresponding electronic values, the potential to e/T_e, and the radial distance from the torus axis R and the magnetic field to their values on the outer midplane point, R_{lfs} and B_{lfs}, respectively. To address both NBI and ICRF heating, we consider a plasma made of two main ion species, (m_M, Z_M) and (m_m, Z_m), with the concentration of the minority much smaller than the concentration of the majority, $Z_m\,n_m \ll Z_M\,n_M$. In absence of the minority species ($n_m = 0$), the quasi-neutrality simplifies, $n_e = Z_M\,n_M$, and the neutralizing poloidal potential becomes

$$
\tilde{\Phi}_0^{(0)} = \frac{T_{\|M}}{T_{\|M}+Z_M}\ \left[A_M\frac{\mathrm{M}_\varphi^2\ (R^2-1)}{T_{\|M}} + \ln\left(Z_M\frac{T_{\perp M}}{T_{\perp M,\mathrm{lfs}}}\right)\right]\ . \qquad (14)
$$

The presence of a minority species perturbs the potential, $\tilde{\Phi}_0 \approx \tilde{\Phi}_0^{(0)} + \delta\tilde{\Phi}$. If $|\delta\tilde{\Phi}| \ll |\tilde{\Phi}_0^{(0)}|$ as one expects if $Z_m n_m \ll Z_M n_M$, we keep only terms up to the first-order correction in $\delta\tilde{\Phi}$, namely $\exp\{-Z\tilde{\Phi}_0/T_\parallel\} \approx (1 - Z\delta\tilde{\Phi}/T_\parallel)\exp\{-Z\tilde{\Phi}_0^{(0)}/T_\parallel\}$. By exploiting that the electron density perturbation δn_e can be determined as $\delta n_e \approx \delta\Phi\exp\{\tilde{\Phi}_0^{(0)}\}$ and as $\delta n_e = Z_m n_m^{(0)} + Z_M \delta n_M$, we approximate

$$\delta\tilde{\Phi} \approx Z_m \left(n_m^{(0)} - n_{m,\mathrm{lfs}}\, n_M^{(0)}\right) \left[\exp\{\tilde{\Phi}_0^{(0)}\} + \frac{Z_M}{T_{\parallel M}} n_M^{(0)}\right]^{-1}, \tag{15}$$

with $n_m^{(0)}$ and $n_M^{(0)}$ given by the first of (13) with (14) as potential. It follows that the density of the majority species is

$$n_M \approx \left(1 - Z_m\, n_{m,\mathrm{lfs}} - Z_M \frac{\delta\tilde{\Phi}}{T_{\parallel M}}\right) n_M^{(0)},$$

whereas the density of traces of other ion species is

$$\frac{n_Z}{n_{Z\mathrm{lfs}}} \approx \frac{T_{\perp Z}}{T_{\perp Z,\mathrm{lfs}}} \left(1 - Z_Z \frac{\delta\tilde{\Phi}}{T_{\parallel Z}}\right) \exp\left\{-\frac{Z_Z}{T_{\parallel Z}}\tilde{\Phi}_0^{(0)} + A_Z\, \frac{\mathrm{M}_\varphi^2\left(R^2 - 1\right)}{T_{\parallel Z}}\right\}. \tag{16}$$

Equations (14), (16), and (15) give an explicit expression for the poloidal asymmetry of traces species in the case of ICRF heating, with temperature anisotropies inferred from the solution of the FP equations.

Equation (16) simplifies further if we consider the case of one-species plasma externally heated, like in the case of NBI heating,

$$\frac{n_Z}{n_{Z\mathrm{lfs}}} \approx \left(Z_M\, \frac{T_{\perp M}}{T_{\perp M,\mathrm{lfs}}}\right)^{-Z_Z \dfrac{T_{\parallel M}/T_{\parallel Z}}{T_{\parallel M} + Z_M}} \exp\left\{\left(1 - \frac{A_M}{A_Z}\frac{Z_Z}{T_{\parallel M} + Z_M}\right) A_Z\, \frac{\mathrm{M}_\varphi^2\left(R^2 - 1\right)}{T_{\parallel Z}}\right\}. \tag{17}$$

Because of the centrifugal force the majority density increases on the lfs with the creation of a poloidal potential which partially compensates the centrifugal force on the impurities. In general this is not enough to produce a localization of impurities on the hfs, since it is typically $A_M Z_Z / A_Z(T_{\parallel M} + Z_M) < 1$. When $T_{\perp M} > T_{\parallel M}$, however, the density of high-Z impurities can be indeed inverted up to a localization of impurities on the hfs, and this is stronger, the higher Z_Z is.

As illustrative cases, we consider a circular large-aspect-ratio tokamak, with the magnetic field approximated as $B(\psi, \vartheta) = B_0(1 - \psi\,\varepsilon_a\, \cos\vartheta)$, where ε_a is the tokamak inverse aspect ratio. In this geometry it holds $(1-\varepsilon_a)/(1+\varepsilon_a) \leq R \leq 1$. With reference to the magnetic surface characterized by $\psi\,\epsilon_a \approx 0.17$, figures (1) show the dependence of the in-out asymmetry of tungsten impurities on the temperature anisotropy of one of the ion plasma species, and this is shown for few values of M_φ. In figure (1.a) it is considered the case of pure deuterium plasma with traces of tungsten W^{+31}: the dashed lines are the values according to (17) and the dotted lines are solution of (13). When the poloidal inhomogeneity of the impurities increases, formula (17) (dashed lines) departs from the solution (13) (dotted lines) since local impurity build-ups start to modify the poloidal potential $\tilde{\Phi}_0$ via the quasi-neutrality constraint. In general, since NBI heating does not create large temperature anisotropy of the main species (roughly not larger than 1.1), an inversion of the out-in impurity asymmetry is possible only for moderate Mach numbers. The case considered in figure (1.b) refers to the conventional hydrogen (minority) IC fundamental heating in deuterium (majority) plasmas. The dashed and dotted lines correspond to the cases of fixed hydrogen concentration of 5% on the lfs. In (1.b) the W^{+31} density in-out asymmetry is plotted as function of the temperature anisotropy of the minority species in a range of values expected during ICRF heating [4]. The difference between approximated solutions (dashed lines) and numerical solution of (13) becomes larger when $T_{\perp H}/T_{\parallel H}$ is increased, as already observed in the case of figure (1.a). Note that with increasing $T_{\perp}/T_{\parallel}$ the average content of the minority decreases. Thus, we have also considered the more realistic case where the average hydrogen concentration is forced to be equal to 5% (solid lines). In this case an inversion of the out-in build-up is more easily set, as already found experimentally [1].

In conclusion, the set of equations (13) has been derived and can be applied when Ω_φ and the plasma densities and temperatures are known on the outer midplane point. The parallel and perpendicular temperatures of the heated ion species are estimated with bounce-averaged Fokker-Planck solvers, whereas the angular toroidal frequency of the plasma is typically measured. Explicit formulas, obtained perturbatively, give a correct qualitative behaviour of the in-out density asymmetry, but they fail to quantitatively reproduce the solution of (13) in the presence of strong temperature anisotropies. Despite its apparent simplicity, the solution of (13) involves the zero search of coupled transcendental equations. It is simpler and more advantageous to formulate (13) as a set of ordinary differential plus algebraic equations by replacing the first equation with (9).

Acknowledgments

One of the authors (R.B.) is grateful to P. Helander, F. J. Casson, E. Fable, M.L. Reinke, and Y.O. Kazakov for useful discussions, and M. Brambilla for his careful reading of the manuscript and his kind encouragement.

References

[1] REINKE, M. L., HUTCHINSON, I. H., RICE, J. E., et al., Plasma Physics and Controlled Fusion **54** (2012) 045004.
[2] INGESSON, L. C., CHEN, H., HELANDER, P., and MANTSINEN, M. J., Plasma Physics and Controlled Fusion **42** (2000) 161.
[3] CHOE, W., CHANG, C. S., and ONO, M., Physics of Plasmas **2** (1995) 2044.
[4] KAZAKOV, Y. O., PUSZTAI, I., FÜLÖP, T., and JOHNSON, T., Plasma Physics and Controlled Fusion **54** (2012) 105010.

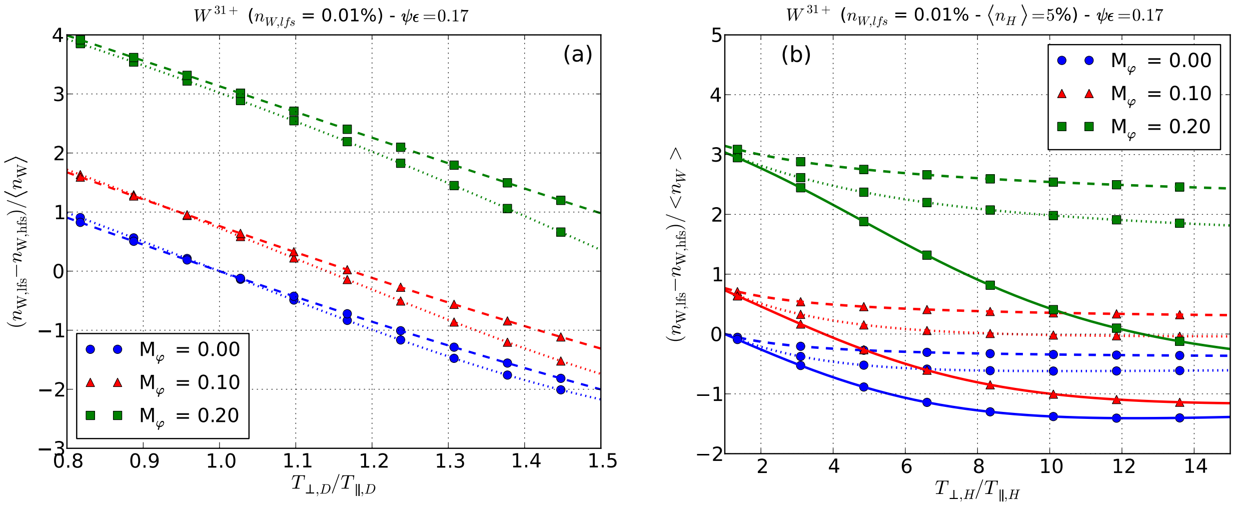

Figure 1. In-out asymmetry of W^{+31} (a) in a pure deuterium plasma (b) in a deuterium plasma with 5% of hydrogen. In (a) the temperature anisotropy of deuterium is varied (NBI heating), whereas in (b) the temperature anisotropy of the minority species changes (ICRF minority heating). Dotted lines are solutions of (13), whereas the dashed lines are solutions of the approximate formulas. Solid lines in (b) refer to runs with the average density of the minority kept constant, namely $\langle n_H \rangle = 5\%$. In both cases the parallel temperatures are set equal to the electronic one.

[5] HINTON, F. L. and HAZELTINE, R. D., Rev. Mod. Phys. **48** (1976) 239.
[6] HINTON, F. L. and WONG, S. K., Physics of Fluids **28** (1985) 3082.
[7] BILATO, R., BRAMBILLA, M., MAJ, O., et al., Nuclear Fusion **51** (2011) 103034.
[8] HELANDER, P., Physics of Plasmas **5** (1998) 3999.
[9] DENDY, R. O., HASTIE, R. J., MCCLEMENTS, K. G., and MARTIN, T. J., Physics of Plasmas **2** (1995) 1623.
[10] COOPER, W., GRAVES, J., HIRSHMAN, S., et al., Nuclear Fusion **46** (2006) 683.
[11] Di Troia, C., Plasma Physics and Controlled Fusion **54** (2012) 105017.
[12] WESSON, J., Nuclear Fusion **37** (1997) 577.