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# LOGICO-COGNITIVE STRUCTURE IN THE LEXICON 

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This study is a prolegomenon to a formal theory of the natural growth of conceptual and lexical fields. Negation, in the various forms in which it occurs in language, is found to be a powerful indicator. Other than in standard logic, natural language negation selects its complement within universes of discourse that are, for practical and functional reasons, restricted in various ways and to different degrees. It is hypothesized that a system of cognitive principles drives recursive processes of universe restriction, which in turn affects logical relations within the restricted universes. This approach provides a new perspective in which to view the well-known clashes between standard logic and natural logical intuitions. Lexicalization in language, especially the morphological incorporation of negation, is limited to highly restricted universes, which explains, for example, why a dog can be said not to be a Catholic, but also not to be a non-Catholic. Cognition is taken to restrict the universe of discourse to contrary pairs, splitting up one or both of the contraries into further subuniverses as a result of further cognitive activity. It is shown how a logically sound square of opposition, expanded to a hexagon (Jacoby 1950, 1960, Sesmat 1951, Blanché 1952, 1953, 1966), is generated by a hierarchy of universe restrictions, defining the notion 'natural' for logical systems. The logical hexagon contains two additional vertices, one for 'some but not all' (the Y-type) and one for 'either all or none' (the U-type), and incorporates both the classic square and the Hamiltonian triangle of contraries. Some is thus considered semantically ambiguous, representing two distinct quantifiers. The pragmaticist claim that the language system contains only the standard logical 'some perhaps all' and that the 'some but not all' meaning is pragmatically derived from the use of the system is rejected. Four principles are proposed according to which negation selects a complement from the subuniverses at hand. On the basis of these principles and of the logico-cognitive system proposed, the well-known nonlexicalization not only of *nall and *nand but also of many other nonlogical cases found throughout the lexicons of languages is analyzed and explained.*

Keywords: complement selection, contrariety, conceptual fields, entailment, lexical gaps, logical hexagon, square of opposition, universe of discourse

1. Introduction. The present study is a continuation of investigations carried out by both authors over the past ten or so years into the relation between language and logic and the nature of the logical system operative in cognition and manifest in language. It also continues work done by the first author over the past four decades in presupposition theory (Seuren 1972, 1985, 1988, 2010:311-77, 2013:290-93), though this aspect is not further elaborated here. Primed by Horn 1990 and by a number of papers presented at the First World Congress on the Square of Opposition, held in Montreux, Switzerland, June 1-3, 2007 (Béziau \& Payette 2012), the authors recently found that some of the work reported on in the present study was anticipated by Jacoby (1950, 1960), Sesmat (1951), and Blanché (1952, 1953, 1966), who caught glimpses of what was to be discovered and were accordingly inspired. ${ }^{1}$ Blanché 1953 , in particular, is foundational for the analysis and the results reported on in the present study. The authors respectfully acknowledge the merit of these early harbingers. At the same time they recognize that the work presented here is again still an early, largely exploratory step toward what will hopefully develop into a richer, formally precise logico-cognitive charting of the lexicon and its dynamics and thus into a solid general theory of the lexi-

[^0]con. They recognize likewise that there are limits to the applicability of logical analysis to the lexicon, even if the logic involved is of the natural, cognitive kind.

The main theses defended in the present study are as follows:
(i) What is generally called the 'pragmatic ambiguity' of natural-language (inclusive versus exclusive) some, or, possible, and analogous logical operators is, in fact, a systematic logico-semantic ambiguity between cognate but distinct logical operators. Similar ambiguities are found all over the lexicons of languages, as in words like animal, which may either exclude or include humans, or actor, which may either exclude or include actresses.
(ii) Logical operators are lexical predicates differing from nonlogical predicates only in that they take propositions or propositional functions as terms, have the capacity of occupying a position in a scope hierarchy, and have mathematically definable meanings.
(iii) The much discussed nonlexicalizability of 'not all', 'not and', and other logical operators also applies to 'not (exclusive) some/or' and is found in analogous forms all over the lexicons of natural languages, which makes a pragmatic explanation look unrealistic.
(iv) This lexically widespread nonlexicalizability is best accounted for in terms of a system of logical relations holding within a lexical field, augmented with a highly functional cognitive principle by virtue of which the universe of discourse Un is systematically restricted to the sum of the situations in which a given $n$-tuple of contraries is true, turning binary contraries into contradictories within the thus restricted Un, which makes for a logic with ever more narrowly focused restricted Uns. This recursively progressive universe restriction leads to a logical structure we call KITE, which accounts for the nonlexicalizability of 'not all' and analogous cases all over the lexicon.
(v) The previous tenets establish logic as a designated subpart of lexicology and open the way for a theory of the cognitive ontogenesis of predicate logic (Seuren 2014).
A few examples will provide an introduction to the central issue of the present study. Consider some cases where a wider domain covered by an overarching lexical predicate has been split up into two or more separate subdomains.
(1) a. horse [mare, stallion]
b. pig [sow, boar]
c. sheep [ewe, ram]
d. chicken [hen, rooster]
e. house [cottage, villa, detached house, terraced house, ... ]
f. both [(two) together, (two) separately]

One notes that the items listed within square brackets are conceptual contraries in that their definitions are mutually exclusive. Moreover, there is a one-way entailMENT from each item within the square brackets to the overarching term: whatever is a mare is a horse, whatever is a cottage is a house, and so forth, but not vice versa. Section 2 shows that the fact of these entailment relations has wide implications for the logical structure of the conceptual fields concerned.

Often, one of the subdomains under the superordinate domain is denoted by a term that is homophonous with the superordinate term. Cruse (1986:256) speaks of a 'colexemic superordinate'. His examples are DOG [bitch, dog] and lion [lioness, lion]. In those cases, we use the notation o[......], where $o$, in small capitals, stands for the
overarching term, and lowercase $o$ for the homophonous subterm. This phenomenon is mostly, but not exclusively, limited to binary subdomains. Examples are given in 2.
(2) a. SOME [all, some]
b. OR [and, or]
c. CAT [tomcat, cat]
d. COW [bull, cow]
e. ANIMAL [human, animal]
f. SCHOOL [university, school]
g. TOOTH [molar, tooth]
h. RECTANGLE [square, rectangle] (Horn 2007:165)

This type of ambiguity is a central issue in the present study. Pragmaticists have argued that some and or (and other similar logical term pairs) are pragmatic derivates of SOME and $O R$. We argue in $\S 4$ that these attempts are unsuccessful mainly because we have here a type of truth-conditional and thus semantic ambiguity that is widely found in the lexicons of all languages, for which the proposed pragmatic principles do not provide a general solution.

Then there is the question of what complement is selected by natural-language negation over the bracketed terms. As already observed by Aristotle (384-322 BC), when the negation is expressed by means of a morphological element and thus lexically incorporated (for English mainly un-, in-, non-), the complement is always selected within the nearest conceptual subdomain: my neighbor's dog is not a Catholic, but neither is he a non-Catholic, because under morphological negation the complement of Catholic is selected within the narrower category of those entities that are possible religious believers, and dogs do not fall into that category. This has direct consequences for any hypothetical lexicalized form for 'not all'.

The matter becomes more complex when the negation is expressed by a full negation word (English not). Here we encounter great variability. Often, the complement in such cases is formed by the (sum of the) remaining bracketed term(s). An incomplete sentence like This is not a mare but a ... is typically completed by the word stallion, not by, for example, motor car, and when completing an unfinished sentence like This is not a villa but a ... , one will primarily look for a word denoting another kind of house, rather than for a totally unrelated word like elm tree or shoe. ${ }^{2}$ This can be interpreted as a form of universe restriction, in that the universe of discourse Un for sentences with a main predicate standing between the square brackets is often defined by the overarching term. The default complement of any lower or subordinate term is then defined within this restricted $\mathrm{Un}^{\mathrm{R}}$. Thus, the negation word over all in 2a naturally and by default selects the complement within the restricted $\mathrm{Un}^{\mathrm{R}}$, with the result that (nonlexicalized) not all is naturally understood as 'some but not all'. Likewise, This is not a tomcat is naturally interpreted as 'this is a cat', This is not a bull as 'this is a cow', This is not a human as 'this is an animal', and so forth. In these cases, the complement is selected within the restricted subuniverse defined by the overarching term. ${ }^{3}$

There thus seems to be a process active in cognition through which concept formation and low-level, implicit thinking are recursively restricted to ever smaller universes

[^1]of discourse $\mathrm{Un}^{\mathrm{R}}$, whereby the degree to which the recursion is actually realized is determined by the amount of cognitive activity taking place in a particular area. This process of PRogressive universe restriction (PUR) can be overruled in specific contexts or by intellectual reflection, both of which are capable of stopping or overruling PUR and defining the complement of a class in terms of a higher Un. This often happens with homophonous superordinates, Cruse's 'co-lexemic superordinates'. Here, the negation normally selects the less restricted Un of the capitalized superordinate term, exceeding the frame of the more restricted $\mathrm{Un}^{\mathrm{R}}$. Thus, not some is preferably interpreted as 'none', seeking the complement of SOME the way logic wants it, not of some, which would amount to all. Not or is interpreted as 'neither nor', following standard propositional logic, not as 'and'. Likewise for cat in 2c and cow in 2d: This is not a cat but $a$... is more naturally completed, outside of any specific context, by means of a word like rabbit, rather than tomcat. And This is not a cow but a ... is more naturally completed with a word like buffalo, rather than bull. But this rule is, again, not absolute. In 2e,f, for example, the complement is preferably sought within the smaller subdomain: This is not an animal but a ... is more naturally completed with human than with, say, plant. And This is not a school but a ... suggests university as a suppletion rather than, for example, museum. In 2 g the authors' intuition is ambivalent: This is not a tooth but a molar seems as natural as This is not a tooth but a piece of bone. It is not known what determines such differences. Context and situation no doubt play a role here, and possibly history and culture, and perhaps also the degree to which the superordinate term has been integrated into the basic vernacular (see Gruber 1976:280-83). Below, in $\S \S 6$ and 7 , we go into this difficult question in greater detail. It then becomes clear that scalarity and positive or negative directionality play a role.
2. Some logical background. Complement selection by negation in natural language is thus far from the simple, straightforward matter it is taken to be by the majority of logicians. One reason is, in our view, that language works with variable Uns (in Seuren 1972 the term used is flexible universe of interpretation), whereas logic always assumes a constant and stable Un. One can see the transition from traditional Aristotelian logic to Russellian logic and from there to the theory of generalized quantifiers as a sustained attempt to develop a logical system that is valid regardless of any contingent factors, depending only on meaning, in a universe of discourse Un ${ }^{\omega}$ allowing for any situation whatsoever, including even a situation without any entities (the null universe). That human cognition does not feel happy in such a Un ${ }^{\omega}$ and seeks to develop a logic valid in ever more restricted Uns so as to enhance its practical value will not be too hard to understand. As is shown below, the theory of generalized quantifiers is indeed based on a 'pure' Un ${ }^{\omega}$ allowing even for the null universe (Russellian quantificational logic does not quite make it, as it requires a nonnull universe with at least one entity). Traditional Aristotelian predicate logic, the so-called square of opposiTION (or simply the square), imposes a further restriction on its Un, in that it requires that the set of things quantified over, the R-set, be nonnull. Thus, while language has the tendency to make the universe of discourse ever more restricted both for lexical items and in running discourse, the discipline of logic embodies the opposite attempt, as it strives for an 'absolute' logic that is totally free from contingent conditions of any kind. There is thus an inherent tension between professional logic and the linguistic study of semantics. The present study attempts to help resolve that tension.

We first show that once a relation of entailment is posited, a triangle, a square, and a hexagon of logical relations follow automatically, regardless of any specific logical sys-
tem, simply by virtue of the axiom of Strict bivalence (there are just two truth values for statements of past or present facts, true and false, with nothing outside and nothing in between), the relations of entailment, contrariety, subcontrariety, contradiction, equivalence, and logical independence as standardly defined, and the operators $\neg$ (not), $\wedge($ and $)$, and $\vee(o r)$ of propositional logic. This fact, though hardly known, is important since it shows the necessity of the square and the hexagon once the (subaltern) relation of entailment from A-type to I-type sentences (see 3 below) has been posited.

Contrariety, entailment, equivalence, subcontrariety, contradiction, and, if you wish, logical independence are binary relations holding between two (sets of) propositions, not by virtue of what happens to be the case in the world but could have been otherwise, but, ideally, by CONCEPTUAL (ANALYTICAL) NECESSITY-that is, by virtue of the MEANINGS of the words used, not of physical, inductive, or social necessity. Since, curiously, all or most modern textbooks on logic (even McCawley 1981, which was written especially for linguists) fail to explain these relations adequately if they mention them at all, they are defined here, so that the analysis and the argument will be easier to follow. ${ }^{4}$

A proposition $p$ entails a proposition $q$, or $q$ Logically follows from $p$, just in case the class of situations in which $p$ is true is, by conceptual necessity, included in the class of situations in which $q$ is true. When there is PROPER inclusion, we speak of oneway entailment. Thus, when it is true that Alexander has been murdered, it is also true, by conceptual necessity, that Alexander is dead: one cannot have been murdered (in the resultative or perfective sense) and not be dead. Therefore, Alexander has been murdered entails Alexander is dead.

Entailment is sometimes confused with (material) implication. A proposition $p$ (materially) implies a proposition $q(\mathrm{p} \rightarrow \mathrm{q})$ just in case it happens to be so that either $p$ is false or $p$ and $q$ are both true. A sentence like If the light is on in Alexander's home, his car is in the garage, interpreted as a material implication, simply says that either the light is not on in Alexander's home or else the light is on and his car is in the garage. If that is so, the statement may be based on frequent observation but there is nothing conceptually necessary about it. Therefore, this $i f$-clause does not entail the consequent clause. The truth of a (material) implication is, normally speaking, a CONTINGENT rather than a CONCEPTUALLY necessary truth. The intuitively felt similarity between the two notions rests upon the fact that the grammatical if ... then construction does not differentiate between implication and entailment. And indeed, the two are parallel in that saying that $p$ entails $q$ amounts to saying that $p$ (materially) implies $q$ in all conceptually possible situations. Thus, when an entailment holds (is valid), the corresponding implication is necessarily true (McCawley 1981:73). In this study we are concerned with entailment, not with implication, since we are dealing with conceptual structures, not with contingent truth in the world.

Two propositions $p$ and $q$ are contraries just in case the class of situations in which $p$ is true and the class of situations in which $q$ is true are totally disjoint by conceptual necessity, so that $p$ and $q$ cannot both be true at the same time. Alternatively, we say that $p$ and $q$ are incompatible or mutually exclusive. For example, the propositions underlying Alexander is alive and Alexander has been murdered are contraries, since it is

[^2]conceptually (analytically) impossible for anyone or anything both to be alive and to have been murdered at the same time.

Logical equivalence of $p$ and $q$ is defined by saying that $p$ entails $q$ and $q$ entails $p$. Equivalence is thus two-way entailment. In this study, when we speak of entailment, what is meant is one-way entailment. For two-way entailment we use the term (logical) equivalence, standardly expressed by the symbol ' $\equiv$ '.

As regards subcontrariety, whereas two propositions $p$ and $q$ are contraries just in case they cannot both be true at the same time, they are subcontraries just in case they cannot both be false at the same time. This relation is much harder for intuition to process. The easiest way to do so is to consider that two propositions $p$ and $q$ are subcontraries in any given universe of discourse Un just in case the class of situations in which $p$ is true and the class of situations in which $q$ is true together exhaust all possible situations in Un, although there may be situations where both are true. This means that when $p$ happens to be false, then necessarily $q$ must be true because there are no possible situations in Un beyond those denoted by $p$ and $q$. For example, let $p$ be Alexander is dead and $q$ be Alexander has not been murdered. These can both be true at the same time, but they cannot both be false, since it is impossible for Alexander to be alive and at the same time to have been murdered.

Two propositions $p$ and $q$ are CONTRADICTORIES when there is no possible situation where they are both true or both false. Contradiction is thus the combination of contrariety and subcontrariety as defined above. Contradiction divides any given Un into two mutually exclusive halves, the one being the complement of the other in Un. The standard negation operator not $(\neg)$ has the function of turning a proposition $p$ into its contradictory $\neg p$ in Un ${ }^{\omega}$.

Finally, we define the relation of logical independence (which may also be considered to be not a logical relation but the absence of any). Two propositions $p$ and $q$ are logically independent just in case there are possible situations where both are true, both are false, or the one is true while the other is false. Thus, when $p$ and $q$ are logically independent, neither entails the other, nor are they contraries, subcontraries, or contradictories.

Let us now construct a triangle, a square, and a hexagon of logical relations on the basis of one single entailment relation from an arbitrary proposition $p$ to a proposition $q$ (Figure 1). When $p$ entails $q, p$ is incompatible with $\neg q$ : the latter two form a pair of contraries, in that they cannot both be true at the same time. $q$ and $\neg q$ are contradictories by virtue of the definition of the bivalent negation $\neg$. This gives the triangle of opposition of Fig. 1a (Seuren 2010:29). In this and the figures that follow, entailment is symbolized as ' $>$ ', contrariety as ' C ', contradictoriness as a cross or a star in the middle, subcontrariety as ' SC ', and logical equivalence as ' $=$ '. (The same construction is possible when one starts from a contrary pair $p$ and $q$, leading to entailment from $p$ to $\neg q$. This is, in fact, how Aristotle built up his square.)

The triangle of opposition of Fig. 1a can be expanded into the sQuare of opposition of Fig. 1b by virtue of the operation of contraposition, valid in bivalent systems: when $p$ entails $q$, then $\neg q$ entails $\neg p$ : Alexander has been murdered entails Alexander is dead; so, when Alexander is not dead, he cannot have been murdered. One will note that the expansion of the triangle into the square has brought along the new logical relation of SUBCONTRARIETY of $q$ and $\neg p: q$ and $\neg p$ are subcontraries because when $q$ is false, $\neg q$ is true, and $\neg q$ entails the truth of $\neg p$, and when $\neg p$ is false, $p$ is true and then also $q$, as $p$ entails $q$.


Figure 1. The construction of the hexagon from one entailment relation.
Jacoby, Sesmat, and Blanché found that the square of opposition can, in its turn, be extended to the hexagon of opposition shown in Fig. 1c when the operators and ( $\wedge$ ) and or $(\mathrm{v})$, taken from propositional logic, are added to the propositional operator $\operatorname{not}(\neg)$ already used in the triangle and the square. The $r$ vertex of the hexagon stands for ' $q \wedge \neg p$ ', while the $s$ vertex stands for ' $\mathrm{p} \vee \neg \mathrm{q}$ '. The beauty of the hexagon is that it incorporates not only the square but also a triangle of contraries $<\mathrm{p}, \mathrm{r}, \neg \mathrm{q}>$ and a triangle of subcontraries $<\mathrm{q}, \mathrm{s}, \neg \mathrm{p}>.^{5}$ This shows how these two triangles, the square, and the hexagon of Fig. 1 follow from a single relation of entailment: wherever there is an entailment relation, there are two triangles (of contraries and subcontraries), a square, and a hexagon of logical relations, given the operators of propositional logic and the relations of entailment, contrariety, subcontrariety, equivalence, and contradiction.

Now to the traditional system of logic, the (Aristotelian) square of opposition, or simply the square (Figure 2). The square, shown in Fig. 2b, uses the quantifiers all and some, plus the external negation $\neg$ and the internal negation * (the notation for the two negations is due to Seuren 2002 and later publications), giving the following eight sentence types, where the variables $R$ (restrictor term) and $M$ (matrix term) stand for predicates.

$$
\begin{array}{cll}
\text { (3) } & \text { A } & \text { All R is M. } \\
\text { I } & \text { Some R is M. } & \text { (All Romans are mortal.) } \\
\neg \mathrm{A} & \text { Not all R is M. } & \text { (Some Romans are mortal.) } \\
\neg \mathrm{I} & \text { No R is M. } & \text { (No Romans are mortal.) } \\
\text { A* }^{*} & \text { All R is not M. } & \text { (All Romans are not mortal.) } \\
\mathrm{I}^{*} & \text { Some R is not M. } & \text { (Some Romans are not mortal.) } \\
\neg \mathrm{A}^{*} & \text { Not all R is not M. } & \text { (Not all Romans are not mortal.) } \\
\neg \mathrm{I}^{*} & \text { No R is not M. } & \text { (No Romans are not mortal.) }
\end{array}
$$

Given the entailment from A to I, the triangle, the square, and the hexagon of Fig. 2 follow automatically by virtue of the construction shown in Fig. 1.

One difference between logical systems and most nonlogical systems based merely on a relation of (one-way) entailment is that the former allow for internal negation (*), which is lacking in most nonlogical cases of entailment. The combination of external and internal negation in the sentence types at issue gives rise to the equivalences stated

[^3]

Figure 2. The triangle, square, and hexagon applied to traditional logic.
in Figure 3. ${ }^{6}$ Of particular interest are the equivalences of $\neg$ A and $I^{*}$ and of $\neg I$ and $A^{*}$ : the internal and the external negations are interchangeable provided A is changed into I and vice versa. The term duality is commonly used for this property (see, for example, Blanché 1966:125, Löbner 1990). ${ }^{7}$


Figure 3. The equivalences in the logical hexagon.
Before the days of Jacoby, Sesmat, and Blanché, when there was still only the classic square, the four vertices of the square were traditionally named $\mathrm{A}, \mathrm{I}, \mathrm{E}$, and O . This convention had been introduced by the early sixth-century Aristotle commentator Boethius as a didactic device, A and I being the first two vowels of the Latin word AffIrmo 'I affirm', and E and O of the Latin word $n E g O$ ' $I$ deny'. This notation, however, is viable

[^4]only for systems where the operators in A and I are each other's duals. In systems where the dual equivalences do not hold, as in the Aristotelian-Abelardian predicate logic discussed below, there are not four but eight unified vertices, which is of particular relevance in the case of the E and O vertices. This makes this Boethian notation less useful in discussions of those systems. Below, we use the E and O notation only to denote the positions in the logical or lexical hexagons under discussion.

As is well known, the square requires that the R-set, in the cases at hand the set of Romans, be nonnull: for the square to be applicable as a logical system to the sentences cited above, there must be at least one Roman around, and analogously for any other possible value for $R$. When that condition is not fulfilled, the square collapses as a logical system. For philosophically minded logicians this is a serious fault, known as undue existential import or UEI, in that a logical system should be valid regardless of any contingent condition, such as the condition that there be at least one Roman around. But outside of the philosophical frame of mind it is perfectly possible to envisage a logical system that depends on the fulfillment of some well-chosen contingent condition, such as the condition that the R -set be nonnull. The system will be the richer and the more functional for it (see Seuren 2010:Chs. $4 \& 6$ and 2013:Ch. 8).
3. Some historical background. A few historical notes are in order at this point, to show the tension that has existed in the history of logic from the very beginning between the ideal of a philosophically 'pure' logic defined by just the meanings of the operators concerned, on the one hand, and more intuitive logics operating within universes subject to contingent restrictions, on the other. The logic or logics naturally inherent in language and cognition belong to the latter class, but professional logicians have always striven to realize the former. The question has now come to the fore because pragmaticists, along with the rest of the world, accept the ideal 'pure' logic as universally applicable even to language and cognition, neglecting or ignoring the realistic possibility that human cognition, with language in its wake, operates with logical systems that are valid within universes subject to contingent restrictions. The study of such applied logical systems is primarily of an empirical nature, contrary to 'pure' logic, which is of an a priori philosophico-mathematical nature.

As has been said, the classic square of opposition, shown in Fig. 2b, suffers from UEI. The French medieval philosopher Peter Abelard (1079-1142) was the first to diagnose this defect. He observed that when the R-set is null, both I and I* are false, which destroys their relation of subcontrariety and thus the square. He also noted, however, that Aristotle himself was not responsible for this logical error, which was, in fact, introduced into Aristotelian predicate logic, as far as we know, by his later commentators Apuleius (c. 125-180), Ammonius (c. 440-520), and Boethius (c. 480-524). Boethius, being a Christian who wrote in Latin, was extremely influential throughout the Middle Ages and later. It was mainly his enormous influence that created the widespread misconception that his version of the square was in fact the square as devised by Aristotle, complete with UEI. Yet this is not true. Aristotle himself probably saw the logical danger of restricting Un to cases where the R-set is nonnull. He was in any case not guilty of UEI. Since he never mentions the dual equivalences and only speaks of one-way entailment from A* to $\neg \mathrm{I}$ (On interpretation 20.a.19-20), he must have considered an A-type sentence false when the R -set is null-that is, he added the condition that the R-set be nonnull to the satisfaction conditions of the universal quantifier. As Abelard saw, this gives a square where A still entails I, with all that follows from it, but the duality of the two quantifiers is lost: A is no longer equivalent with $\neg I^{*}$ and $I$ is no longer equivalent with $\neg A^{*}$, but $A$ (one-way) en-
tails $\neg I^{*}$ and A* (one-way) entails $\neg \mathrm{I}$. It follows that in this original Aristotelian logic there is neither an $O$ nor an $E$ vertex, as the former has been split up into $\neg A$ and $I^{*}$, and the latter into $\neg \mathrm{I}$ and A*. This is the predicate logic found in Abelard's Dialectica (c. 1128), and which is called aristotelian-abelardian predicate logic (AAPL) in Seuren 2010, 2013. But Abelard's correction of the logical tradition went unnoticed, owing, no doubt, to the fact that he was excommunicated twice by the Holy Church for his 'heretical' views on the Trinity and the subsequent ban on his writings (Seuren 2010:172-80). The classic square of opposition thus kept being taught in its Boethian version as the standard system of predicate logic until the advent of mathematics-based standard modern predicate logic (SMPL), devised mainly by Charles Peirce, Gottlob Frege, Alfred North Whitehead, and Bertrand Russell around 1900 and now standardly accepted all over the world and in all disciplines.

What is traditionally called the 'Aristotelian square', taught for a millennium and a half in the civilized world, is thus, in fact, not Aristotelian. It is remarkable that the vast majority of the philologists and logicians have failed to see this. The reason is probably that, in the past, the philologists and historians, who actually read Aristotle's and Abelard's texts, were not logicians, whereas the logicians did not bother too much about the texts. As regards contemporary authors writing about Aristotelian logic, the vast majority of them just restrict themselves to Aristotle's syllogistic, either forgetting about Aristotle's predicate logic (the square) or dealing with it in an offhand way. Parsons (2012) is the only well-known contemporary author we know of who deals in detail with the square. Contrary to textual evidence, Parsons reads Aristotle as saying that all and some form a pair of duals while considering an I*-type sentence like Some Romans are not mortal true when there are no Romans. This looks as if it 'saves' the square, but it in fact blocks a unitary semantic definition of the existential quantifier some. Kneale \& Kneale 1962 is the only publication we know of where it is recognized that Aristotle did not consider all and some a dual pair. These authors write (1962:57): 'Aristotle ... allowed that Every man is not white could be said to entail No man is white but rejected the converse entailment' (see Seuren 2010:149-70 for extensive comment).

As a consequence of this deficient insight into the history of Western logic, it has, to our knowledge, never been noted, other than by Abelard, that the ideal of a 'pure' logic is also achieved by declaring A-type sentences 'false' when the R-set is null, with the result that duality is lost and the equivalences of $A$ and $\neg I^{*}$ and of $I$ and $\neg A^{*}$ are replaced with the one-way entailments from A to $\neg I^{*}$ and from I to $\neg A^{*}$. This is not only logically faultless within $U^{\omega}$, but also more useful for an understanding of the logic operative in language and cognition, because it saves the intuitively natural subaltern entailments from A to I , and thus from $\neg \mathrm{I}$ to $\neg \mathrm{A}$ (Seuren 2010:136-38).

SMPL, by contrast, declares A-type sentences 'true' when the R-set is null. This destroys the traditional square of opposition, which was thus replaced with SMPL. In fact, the old square was wiped out by SMPL within decades of the appearance of Whitehead and Russell's Principia mathematica (1910-1913). Only isolated pockets of resistance remained, which were largely concentrated in Catholic centers. This was because traditional logic had been introduced, together with a great deal of Aristotelian philosophy, into Catholic theology by St. Thomas Aquinas and had become part of a complex fabric of theology and metaphysics. Giving up traditional logic thus amounted to a direct attack on Catholic ideology (for extensive comment, see Jaspers \& Seuren 2014). Jacoby, Sesmat, and Blanché, who stuck to the square, were ardent Catholics. The Polish logicians Jan Łukasiewicz and Józef Maria Bocheński, who kept some distance from modern logic and did not reject the square outright, were also Catholics (the latter a Dominican priest). In his History of Western philosophy of 1946, Russell wrote (p. 206): ‘Even at the pres-
ent day, all Catholic teachers of philosophy and many others still obstinately reject the discoveries of modern logic, and adhere with a strange tenacity to a system which is as definitely antiquated as Ptolemaic astronomy'. Apart from Catholicism, however, Russell and his followers were contemptuous of the logical tradition in general: 'I conclude that the Aristotelian doctrines with which we have been concerned in this chapter [on Aristotelian logic-PS/DJ] are wholly false, with the exception of the formal theory of the syllogism, which is unimportant. Any person in the present day who wishes to learn logic will be wasting his time if he reads Aristotle or any of his disciples' (Russell 1946:212). Russell's close follower, the Frenchman Louis Couturat, is even more outspokenly scornful as regards Hamilton's triangle of contraries (Couturat 1913): 'one should not regard Hamilton as a precursor of logistic [ $=$ modern formal logic-PS/DJ]; his position is antipodal to logistic, as is abundantly proved by his total incomprehension of the works of De Morgan and his well-known polemic with the latter' (our translation).

The new Russellian logic prided itself mainly on three counts. First, it had overcome the fault of UEI by declaring A-type sentences true when the R-set is null (it was not seen that one could equally well overcome UEI by declaring A-type sentences false when the R-set is null, which would yield the logically sound system of AAPL). Then, it could claim that the semantics of the universal and existential quantifiers corresponds directly with standard set theory in that the universal quantifier expresses set-theoretic inclusion $(\subseteq)$ and the existential quantifier expresses nonnull intersection, with the result that only equivalence and contradictoriness remain as logical relations in predicate logic. This is, though true, of questionable value for an analysis of the logic of language, since cognition does not recognize the null set as a set, and other restrictions hold as well with regard to standard set theory (Seuren 2010:Ch. 3). Finally, SMPL claimed an advantage over the square in that its formal language was now capable of dealing with syntactically more complex forms of quantification, as in sentences of the type All children admire some football players. This claim is stripped of its relevance once it is realized that the formal language used for Russellian logic can equally well be used for other logical systems such as the square or AAPL. All that is needed is a judicious modification of the semantic definitions of the quantifiers. In fact, the language used for generalized quantification, where quantifiers are treated as binary higher-order predicates stating a relation between two sets of elements, is logically superior to the Russellian language for the expression of predicate logic formulae, since it allows for the null set to play its full part and thus no longer requires a nonnull universe. In reality, the new logic owed its reputation of uniqueness and inviolability, apart from the powerful rhetoric with which it was promoted, mainly to its usefulness in physics and mathematics. The usefulness of adopting it wholesale in the human sciences, however, we believe to be questionable.

In the present study, all of this is viewed from the perspective of a long-term development, spanning centuries, whereby logic gradually liberated itself from cognitively induced universe restrictions and tried to achieve the ideal of a logic that is indeed entirely independent of contingent conditions and based only on analytical meaning criteria. In this perspective, it is worth observing that SMPL as formulated in the Russellian formal quantifier language still does not quite make it, since it still requires a nonnull model, containing at least one entity. This is because this language needs the substitution of bound variables by definite terms referring to specific individuals in the model to achieve a truth value, which rules out a null model. For if the model contains no entities at all, there is no specific individual available for any term to refer to and thus no possibility for the substitution of a variable by a definite referring term. This is why, in serious logical publications of the mid-twentieth century, models were invariably de-
fined as 'nonnull' or 'nonempty'. It was not until the advent of the theory of generalized quantifiers (Mostowski 1957) that this last remnant of a contingent condition was removed from logic. In this theory, quantifiers are taken to express binary relations between sets, so that, for example, All Romans are mortal is analyzed as 'the set of Romans is included in the set of mortals'. This removes the last obstacle of SMPL, since variable substitution is no longer required and the null set is well defined in standard mathematical set theory (see Seuren 2013:266-67 for more detailed discussion).
4. The pragmatic Q- and i-principles. Meanwhile, SMPL, mainly in the language of generalized quantifiers, has remained unquestioned in formal semantics and in the philosophy of language generally, while one seeks to neutralize the obvious clash with natural logical intuitions with the help of pragmatic principles of linguistic communica-tion-that is, the USE, not the SYSTEM, of language. The American philosopher Robert Fogelin introduced a pragmatic rule of STRength: ‘Make the strongest possible claim that you can legitimately defend' (1967:20). He immediately applied this to the square, saying:

> Now by stipulation-a stipulation that squares with our intuitive sense of the matter-we shall say that the A and E propositions are stronger than the I and O propositions. Thus, with respect to the propositions that form a square of opposition the rule of strength takes this special and perfectly rigorous form: Do not employ an I or O proposition in a context where you can legitimately employ an A or an E proposition. And this special form of the rule of strength has a corollary that will become of central importance later in this section: Do not affirm one subcontrary if you are willing to deny the other. The claim I am making is this: the rule of strength (and from now on we shall be concerned only with its special form) is operative in everyday discourse, and its influence accounts for certain features of everyday discourse. (Fogelin 1967:21)

In the same year 1967, the British-American philosopher H. Paul Grice held his William James lectures at Harvard University, in which he introduced his 'conversational maxims', like Fogelin's 'rule of strength' meant to bridge the gap between 'official' logic and natural intuitions. These lectures remained unpublished for a relatively long time, a partial publication being realized in 1975, where Grice's 'maxim of quantity', taken as a principle the hearer expects the speaker to conform to, is stated as follows (Grice 1975:45; repeated in Levinson 1983:101): ‘(1) Make your contribution as informative as is required (for the current purposes of the exchange). (2) Do not make your contribution more informative than is required'. Soon, however, it was realized that this formulation left too many empirical gaps, and neo-Gricean elaborations were proposed, mainly by Horn $(1984,2004,2007)$ and Levinson (2000), the two being largely in agreement.

For pragmaticists, SOME and some are not two distinct though related logical operators, as they are for us, but instead the latter is considered a pragmatic restriction of the former, on the basis of the Gricean maxim of quantity, expanded by Levinson into his Q- and I-principles (Levinson's I-principle corresponds with Horn's R-principle). ${ }^{8}$ These principles are divided into a 'speaker's maxim' and a 'recipient's corollary'. The corollaries are taken to reflect the listener's expectation that the speaker has followed the 'speaker's maxim'. Given the length and involvedness of the texts in question, we

[^5]limit ourselves to quoting in full only the speaker's maxims. The speaker's maxim of the Q-principle is given as follows (Levinson 2000:76).
(4) Q-PRINCIPLE: Speaker's maxim: Do not provide a statement that is informationally weaker than your knowledge of the world allows, unless providing an informationally stronger statement would contravene the I-principle. Specifically, select the informationally strongest paradigmatic alternate that is consistent with the facts.
The I-principle's 'speaker's maxim' is given as follows (Levinson 2000:114).
(5) I-principle: Speaker's maxim: the maxim of Minimization. 'Say as little as necessary'; that is, produce the minimal linguistic information sufficient to achieve your communicational ends (bearing Q in mind).
Levinson $(2000: 31,35)$ attaches a 'heuristic' to the Q-principle, his 'Heuristic I', which says: 'What isn't said, isn't'. But it is not immediately clear how general the domain of application of this heuristic is. Suppose I say Kevin left yesterday: my interlocutor will then infer that Kevin did not leave yesterday morning, because I did not say so, and yesterday morning provides 'stronger' information, in Levinson's sense, than just yesterday. Likewise, however, my interlocutor will infer that Kevin did not leave yesterday afternoon, and again that Kevin did not leave yesterday evening or last night, all because I did not say so. This means that, by saying Kevin left yesterday, I would apparently create the implicature that Kevin did not leave yesterday.

This does not mean that we deny the role of pragmatics in cases of ambiguous expressions, whether of a quantificational or other nature. On the contrary, as normally in the disambiguation, in actual language use, of ambiguous expressions, pragmatics steps in to help select the intended reading. As regards the disambiguation of SOME from some and similar ambiguous pairs in the area of quantification (other conceptual fields will have different disambiguating principles), we propose that the main pragmatic disambiguating factor lies in the listener's PRESUMPTION OF SPEAKER's ADEQUATE KNOWLedge of the relevant domain, or PSAK (Seuren 2013:287-89). When it is presumed that the speaker has full and adequate knowledge of the domain quantified over, the listener will take the speaker to be committed to the narrower of the two readings. But when there is no such presumption (as, notably, in questions), or when domain knowledge plays no role (as in general rules), the wider interpretation will prevail. And in situations where pragmatic clues do not suffice, there is room for equivocation and puns, just as in other cases of ambiguity.

The same applies to the ambiguity of not all. In one, informal, reading, not all means 'some and some not'. Yet in another, more sophisticated, reading, not all leaves open the possibility of none. When I have looked at the exam results of a group of students and have seen that nobody failed, but I report on this by saying that not everybody failed, then this is, in Fogelin's words, a bare-faced lie. Yet (as observed by a referee), when the exam regulations state that students who do not answer all of the questions will fail, a failed student who claims in his appeal that he should pass because he answered none of the questions will lose his appeal. Again, the disambiguating pragmatic factor seems to be PSAK: when it is known that I have seen all of the results and I report that not everybody failed, my listener(s) will rightly consider me to have committed myself to the truth of some students did and some did not fail. But no such presumption can possibly prevail with regard to the exam regulations, which, being the result of generalized, analytical thinking, are interpreted at a less situation-dependent level of logical precision.

Likewise for lower-bounded and upper-bounded most. The informed discussion in Ariel 2008:91-100 shows that in all cases mentioned by this author the crucial disambiguating factor is again PSAK. With one exception: when reference is made to regulations or statutes. In, for example, Most delegates have voted in favor, so the proposal is accepted, the possibility of all delegates having voted in favor remains open: what counts is that the statutory condition that a majority must be in favor for a proposal to be passed be fulfilled.

The theory of conversational implicatures is also often extended to the cardinal numerals. Here our answer is that in many cases PSAK again provides the crucial disambiguating factor. Yet, as argued in Seuren 1993, there are also many instances where the numeral is the comment providing the value of a parameter that forms the topic. Thus, in How many children does Harold have? Harold has six children, the first sentence asks for the value on the parameter 'the number of Harold's children' and the second sentence provides the value 'six'. Here, six does not mean 'at least six' (that is, 'six') but 'precisely six', just as in the expressions at least six and precisely six themselves. This value-assigning interpretation does not occur with some, not all, or most, because these expressions do not function as value assignments to quantity parameters (though they may be used to specify values on other parameters).

We are thus faced with a systematic and universal truth-conditional ambiguity of a class of quantifying expressions that all imply a lower bound but may or may not imply an upper bound. Since this difference is a crisp truth-conditional one, we speak of semantic and not of pragmatic ambiguity, even if a pragmatic principle may play a role in the genesis of the ambiguity. We posit the general principle that the double-bounded meaning is basic and original, whereas the meaning requiring only a lower bound is the result of more advanced, generalizing reflection.

There is currently a widespread resistance to assuming semantic ambiguities and a corresponding preference for pragmatic extensions, going back to the Gricean dictum that 'meanings should not be multiplied beyond necessity' (Grice 1989:47). Yet Occam's parsimony principle holds throughout science, not just in the population of meanings. As it is, parsimony of meanings has been bought at the expense of a multiplication of pragmatic entities: we are now faced not only with Grice's already rich gamut of implicatures, but also with explicatures (Carston 1988), implicitures (Bach 1994), and who knows what else there is to come. But there seems to be little awareness that there is a price to be paid.

It should be clear that we are not claiming that the Q- and I- (or R-)principles are fictitious. On the contrary, as argued, for example, in Horn 2007, they may well have to be counted among the forces active in processes of linguistic change and of language use, but we deny that they have the explanatory force attributed to them in the context of the relation between language and logic. Specifically, we posit that these principles fail to account for most of those other cases of ambiguity, such as between ANIMAL and animal (where, as discussed below, we find a further splitting up of animal into (land) animal versus bird and fish), systematically found in the lexicons of the world's languages. Moreover, these principles also fail to provide an explanation for the general unlexicalizability of the $U$ vertex in the hexagon of logical relations, whereas the analysis presented here explains the unlexicalizability of both the $O$ and the $U$ vertices on the grounds of a single principle.
5. PUR, CONCEPT FORMATION, AND LEXICALIZABILITY. Our analysis is based on the principle of progressive universe restriction introduced above. This is not a logical but
a cognitive principle applied to concept formation. It embodies the hypothesis that concept formation proceeds by successive, mostly binary, hierarchical divisions, as shown in Figure 4. PUR is responsible for the natural selection of complements through the use of negation and the systematic exclusion of certain classes of complements that are legitimate in logics with a static universe of discourse.

One notes that the O and U positions of the hexagon, which are never lexicalized (see below), are not represented in Fig. 4. The reason is that there is no way of giving a place to the complement of the A position (the O vertex), namely Y plus E, or to the complement of the Y position (the U vertex), namely A plus E .


Figure 4. Hierarchical tree structure of the logical and lexical hexagons.
The core of our explanation for these lexical gaps is the causal link that we posit between the nonoccurrence of the O and U positions in Fig. 4a and their nonlexicalizability, in that, in principle, natural complement selection is restricted to the nearest subdivision, so that I selects E , but A selects Y . This explanation is of a cognitive, not a logical, nature, since it is cognition that decides on the universe of discourse in terms of which complements are selected. We say that each further subdivision creates a more restricted $\mathrm{Un}^{\mathrm{R}+1}$, separate from an already restricted larger $\mathrm{Un}^{\mathrm{R}}$ or from an unrestricted initial Un ${ }^{\omega}$.

When the O and U vertices are eliminated from the hexagon, what remains is a quadrilateral figure composed of the vertices A, I, Y, and E, as shown in Figs. 5 and 6b below. Such a structure we call a кITE. The kite form expresses its hexagonal origin in that what remains after the removal of the $U$ and $O$ vertices has the shape of a kite. We henceforth use the linear notation [I [A, Y], E] for kite structures, where the capitals A, I, Y, and E stand for the positions they occupy in the logical hexagon.

PUR amounts to the principle that a cognitive division exists within kite structures between the I and the E positions in the sense that each may give rise to a further subdivision defining a further subuniverse. In the figures below, such subdivisions are indicated by a line that we call a median, crossing the pivotal I-E axis. Medians divide any current Un into two halves, one or both of which may be up for further elaboration, depending on cognitive activity. Such subdivisions may, when culturally relevant, be reflected in specific lexical predicates.

Given the PUR principle, we thus posit a recursive generative rule for kite structures in the sense that a kite of the form [a $[\mathrm{b}, \mathrm{c}], \mathrm{d}]$ in any $\mathrm{Un}^{\mathrm{R}}$ (starting from Un ${ }^{\text {w }}$ ) can, not unlike biological cell division, give birth to $[\mathrm{b}[\mathrm{e}, \mathrm{f}], \mathrm{c}]$ or $[\mathrm{b}, \mathrm{c}[\mathrm{g}, \mathrm{h}]]$ in a $\mathrm{Un}{ }^{\mathrm{R}+1}$. That is, a shoulder of a kite ${ }^{1}$ may become the top of a new kite ${ }^{2}$, taking as its contradictory the other shoulder, which was its contrary in kite ${ }^{1}$. An example of such 'cell division' (suggested by a referee) is [ANIMAL [human, animal], plant], where animal can be split up into land animal and nonland animal: [animal [land animal, nonland animal], human]. This again can give rise to [nonland animal [bird, fish], (land) animal], mani-
fest in the frequent locution birds, fish, and animals. Another example is shown in Figure 5. Given a Un ${ }^{\mathrm{R}}$-keyed kite ${ }^{1}$ [EI [I, E], EE], where 'EI' ('existential import') embodies the condition that the R -set is nonnull and ' EE ' ('existential exclusion') the condition that the R -set is null, the rule generates a new kite $^{2}[\mathrm{I}[\mathrm{A}, \mathrm{Y}], \mathrm{E}]$ from the I shoulder of kite ${ }^{1}$, further dividing I into A and Y. This is relevant insofar as it shows the conceptual genesis of the (logically sound) Aristotelian-Abelardian logical system (for full comment see Seuren 2014). Note that standard modern Russellian logic cannot be generated this way.


Figure 5. Aristotelian-Abelardian logic as kite ${ }^{2}$ conceptually generated from kite ${ }^{1}$.
Characteristically, the E position shows signs of negativity. This is clearly so with the logical hexagon, and also with pairs like $<$ married, single $>$ (as in 8 g below), where $\sin$ gle is negative with respect to married (we say unmarried, but not *unsingle, and the concept 'single' can only be defined by reference to the concept 'married', as was observed by a referee). See also the discussion below in $\S 6$ with regard to the pairs <easy, difficult> and <old, young>. Further research will have to show the extent to which this feature can be generalized.

PUR plays a crucial role in the logic of language and cognition. Not only do entailment relations abound in the lexicons of all languages, each giving rise to a hexagon (and thus to a kite), but new logical relations also come about, or existing ones are modified, when the philosophically or mathematically hypostatized universe of discourse $\mathrm{Un}^{\omega}$ is curtailed to a restricted $\mathrm{Un}^{\mathrm{R}}$ by the elimination of classes of situations not taken into account in cognitive operations within a given ambit. Thus, when $p$ and $q$ are contraries in a given Un, they become contradictories in a $\mathrm{Un}^{\mathrm{R}}$ restricted to just the situations where either $p$ or $q$ is true. And logically independent $p$ and $q$ become subcontraries in a $U n^{\mathrm{R}}$ restricted to just $p$ and $q$.

Restriction and hierarchical structure are hallmarks of cognition. They probably enhance on-line functioning but are not directly conducive to mathematical or philosophical abstraction of the kind found in science or philosophy. There is a great deal of truth in saying that mathematics and philosophy, and science in general, are achievements attained DESPITE certain natural, functionally motivated, tendencies of human cognition. Reflection often has to undo the effects of practical thinking.

Let us now revert to the question of the nonlexicalizability of the O and U vertices. As regards the O vertex in the logical hexagon, we note that while there are monomorphemic lexicalizations for the quantifiers defining the vertices $\mathrm{A}, \mathrm{I}$, and E of the square-such as English all, some, and no, respectively-no lexicalizations have so far been found in the
languages of the world for the quantifier in the O (or $\neg \mathrm{A}$ ) vertex. ${ }^{9}$ ( $\mathrm{I}^{*}$, and internal negation generally, cannot be united with the quantifier into one word because the quantifier and the internal negation do not form one constituent.) This asymmetry was first observed for non omnis in the O vertex in Latin by Thomas Aquinas (c. 1224-1274) in his Expositio Peryermeneias lib. 1.1.10, n. 13 (see Horn 1972:Ch. 4, 1989:253, Jaspers 2005:15). Aquinas's observation was generalized by Blanché (1953:95-96, 1966), Horn (1972:Ch. 4, 1989:252-62), Löbner (1990:95), Levinson (2000:69-71), Jaspers (2005), and Seuren (2010:114-21), who treat this phenomenon as a language universal. More recently it was observed (Jaspers 2011, 2012) that the U vertex in the hexagons for predicate and propositional logic is likewise universally unlexicalized. Although we do, when practicing logic, have the intellectual ability to construct logical squares and hexagons, in daily practice we make do with (for predicate logic) the kite formed by all, SOME, some, and no (plus their synonyms and hyponyms, which are of no concern here).

Horn and Levinson explain the lexical gap for 'not all' by saying that some, defining the Y vertex, is the pragmatically motivated version of SOME and therefore, they say, the primary candidate for lexicalization. Since some implies the SOME-not of the O vertex, besides the SOME of the I vertex, and since lexicalization generally favors positive rather than negative concepts, there is no need for a lexicalization of the O vertex. This answer is defective if only because it fails to apply to the $U$ vertex and also to most of those other nonlogical but merely lexical cases where analogous lexical gaps are found.

Seuren's (2010:114-21) answer is that the three vertices of the arguably more primitive triangle of contraries are lexicalized as all, some, and no. Since, in this triangle, no is not a lexicalization of 'not some', which would yield the meaning 'all or no' (i.e. the U vertex), there is no reason to expect any lexicalization for 'not all' in the O vertex. The problem thus does not arise: negated quantifiers are just not lexicalized. This again fails to explain the corresponding lexical gaps systematically found outside the various logical systems.

The answer given in Jaspers 2005 and Larson \& Jaspers 2011 amounts to the logicocognitive hypothesis of PUR. With regard to the square, PUR allows for the postulation of a cognitively induced 'pivotal' distinction between the vertices I and E. Within I there is a secondary division between A and Y ('some but not all'), both forming a more restricted universe, so that both A and Y entail I. Jaspers points out that E, with the quantifier no (now analyzed as 'not some'), expresses the negative half of the primary division, but that O , read as 'Not all R is M ', covers not only the complement Y of A in the I-restricted Un, but also $\neg \mathrm{I}$, the complement of I in the higher Un. He then posits the principle that lexicalizations of complements cannot cross a border into complements of a higher rank but have to stay within their own (sub)division. The authors have now found that this principle applies to similar gaps systematically occurring all over the lexicons of languages, which strongly supports the position taken by Jaspers with regard to the square. The present study builds on that generalization, seeking a general ex-

[^6]planation, in terms of logical principles, in a postulated hierarchically organized conceptual organization. This has as a consequence that for any predicate entailing another predicate, that is, for any entailer, the general principle is posited that CONTRADICTORIES of lexicalized entailers in a higher Un are not lexicalized. ${ }^{10}$

PUR is again demonstrated in Figure 6, where Figs. 6a and 6c show PUR in terms of extensional fields (Seuren's 'valuation spaces') and Figs. 6b and 6d in terms of the corresponding hexagonal representations. Nonlexicalizable vertices are placed in a circle. The parallel of Figs. 6c and 6d with Figs. 6a and 6b, respectively, is obvious: *nonhusband, meaning 'either wife or single', and *nonwife, meaning 'either husband or single', are not lexicalizable, as they transgress the boundary set by the median.


Figure 6. Lexical gaps in the O and U positions for the logical hexagon and for 'nonhusband' and 'nonwife' in the corresponding lexical hexagon.

Figure 6a shows that the complements of both A and Y in $\mathrm{Un}^{\mathrm{R}}$ include E , which is on the other side of the median. In fact, however, (unlexicalized) $\neg$ A selects Y: Not all Romans are mortal is naturally understood as Some but not all Romans are mortal. Conversely, one would expect $\neg \mathrm{Y}$ to select A , so that not some would amount to 'all', but it clearly does not. A tentative answer to this question is given below in principle 4 , which says that when a word in Y position has a positive orientation, as some does, it selects E as its complement.

[^7]A further question is why 'not all' is not lexicalized in its natural meaning of 'some but not all', which should be allowed if the only condition were that no median should be crossed. The answer to this question is proposed below in principle 3, which says that a negative morpheme attached to a predicate P requires a positive directionality of P. Since all is neutral as regards positive or negative directionality, a negative morpheme is excluded.

From now on, we represent kite structures in an upright fashion, placing the I-E axis vertically in the middle, as in Figure 7. The A and Y positions we call the shoulders of the kite. As a general principle, the contradictories of the shoulders in the total hexagon are not lexicalizable, as they cross the median restricting their subuniverse.


Figure 7. The kite structure in upright position.
One notes that it makes no difference whether the A and Y positions are interchanged, as the left- and right-hand sides of the kite/hexagon are symmetrical. Some suggestions regarding this question are made below, but further research on a general rationale for assigning a specific position to the A and Y positions for the nonlogical cases is needed. One also notes that the internal negation has been silently dropped from the script. We know on independent grounds that the quantifiers in the A- and I-type sentences are duals, but that fact no longer plays a role, since there is no room for the internal negation in the argument frames of the majority of predicate groups forming a kite. Finally, one notes that the cutting out of the O and U positions from the hexagon effectively removes the triad of subcontraries $<\mathrm{I}, \mathrm{O}, \mathrm{U}>$ from this structure. This is no doubt the reason why, as every logic teacher knows, subcontrariety is very hard to grasp for logically untrained persons, whereas contrariety is an everyday notion, represented by a variety of lexical items and expressions.

Let us have a look at some further instances. Figure 8a shows the kite structure formed by the operators of propositional logic. It is well known that there is no lexicalized contradictory of and: *nand is found only in computer science. As a corollary, we note that logical iff, which is the negation of (exclusive) or and fills the $U$ position, has no counterpart in natural language either, as noted by Richard Larson in Larson \& Jaspers 2011. We may add that the nonlexicalizability of 'not and', as against the lexicalizability of 'not OR', may well be taken to be related to the fact that example 6 a is immediately felt to be equivalent to 6 b , whereas the logical equivalence between 7 a and 7 b is almost impossible to grasp in an intuitive way, despite the fact that both sentence pairs instantiate the well-known De Morgan's laws, which say that $\neg(\mathrm{p} \vee \mathrm{q}) \equiv \neg \mathrm{p} \wedge \neg \mathrm{q}$ and $\neg(\mathrm{p} \wedge \mathrm{q}) \equiv \neg \mathrm{p} \vee \neg \mathrm{q}$.
(6) a. She doesn't like trains or planes./She likes neither trains nor planes.
b. She doesn't like trains and she doesn't like planes.
(7) a. She doesn't like trains and planes. $/$ *She likes trains nand planes.
b. She doesn't like trains or she doesn't like planes.

In Figure 8 b we find the kite [allowed [obligatory, optional], forbidden] of deontic logic. An isomorphic kite [possible [necessary, possible], impossible] exists for epistemic modal logic, with POSSIBLE as 'possible perhaps necessary' and possible as 'possible but not necessary', analogously to SOME and some, or OR and or (though contingent is also used for possible). Again, as already noted in Horn 1972, *unnecessary, though a common word with the meaning 'not needed', does not occur in the epistemic sense of 'not necessary', and impossible is the opposite of POSSIBLE, not of possible.


Figure 8. The kite structures for [or [and, or], nor] and [allowed [obligatory, optional], forbidden].

To have an idea of the lay of the land, let us have a look at some further instances of lexical kites.
(8) a. [male [man, boy], female]
b. [female [woman, girl], male]
c. [adult [man, woman], minor]
d. [minor [boy, girl], adult]
e. [male [husband, bachelor], female]
f. [female [wife, spinster], male]
g. [married [husband, wife], single] ${ }^{11}$
h. [single [bachelor, spinster], married]
i. [female [mother, daughter], male]
j. [male [father, son], female]
k. [parent [father, mother], offspring]

1. [offspring [son, daughter], parent]
m . [sibling [brother, sister], parent]
n. [animate [male, female], inanimate]
o. [GAy [lesbian, gay], straight]
p. [feasible [easy, difficult], unfeasible]
q. [moving [forward, backward], stationary]
r. [lateral [left, right], center]
s. [sloping [up, down], flat]
t. [real [past, present], future]
u. [surface [land, sea], air]

Un ${ }^{\text {R: humans }}$
$\mathrm{Un}^{\mathrm{R}}$ : humans
Un ${ }^{\mathrm{R}}$ : humans
Un ${ }^{\mathrm{R}}$ : humans
$\mathrm{Un}^{\mathrm{R}}$ : human adults
$\mathrm{Un}^{\mathrm{R}}$ : human adults
$\mathrm{Un}^{\mathrm{R}}$ : human adults
$\mathrm{Un}^{\mathrm{R}}$ : human adults
$U n^{R}$ : nuclear families
$\mathrm{Un}^{\mathrm{R}}$ : nuclear families
Un ${ }^{\mathrm{R}}$ : nuclear families
$U n^{R}$ : nuclear families
Un ${ }^{\mathrm{R}}$ : nuclear families
$\mathrm{Un}^{\mathrm{R}}$ : world entities
$\mathrm{Un}^{\mathrm{R}}$ : human adults
$\mathrm{Un}^{\mathrm{R}}$ : tasks
Un ${ }^{\mathrm{R}}$ : 1D-movement
$U_{n}{ }^{R}$ : 1D-position
Un ${ }^{R}$ : terrain, roads
Un ${ }^{\mathrm{R}}$ : events in time
Un ${ }^{\mathrm{R}}$ : transport

[^8]v. [liftable [light, heavy], unliftable] Un ${ }^{\text {R }}$ : material objects
w. [impart [say, imply], uncommitted] Un ${ }^{\mathrm{R}}$ : speech situations
x. [have a belief wrt $p$ [believe $p$, believe not $p$ ], have no belief wrt $p] \quad \mathrm{Un}^{\mathrm{R}}$ : (lack of) opinion ${ }^{12}$
A few observations are in order. First, parent-offspring relations, as in $8 \mathrm{i}-\mathrm{m}$, are formulated modulo the restriction that the of-object is kept constant. Thus, if $x$ is a son of $y$, then $y$ cannot be a child of $x$, even though $y$ is, of course, the child of two other people.

More importantly, combined kite structures may form a matrix that can be unpacked into two distinct but complementary kites. In all cases, the O and U positions are unlexicalizable. Thus, 8a-d form the matrix of Fig. 9a, 8e-h of Fig. 9b, 8i-1 of Fig. 9c, and 8 m can be expanded into the matrix of Fig. 9d. Figure 9 shows that the categories male versus female crosscut a number of other (mostly binary) distinctions in the world of concepts and thus of words. In all cases one sees that the four parameters are set by the I-E axis of any kite structure, while the four values in the boxes correspond to the items in $\mathrm{Un}^{\mathrm{R}+1}$. Also, for none of the items in the boxes is there a lexicalized negative covering the remaining three items in the same matrix.


Figure 9. Matrix structures as combined kite structures.
Such clusters of complementary or otherwise related kite structures within the same subuniverse ('humans', for example) give rise to further empirical questions. How do kites relate to each other; what kite types are there; if new kites are generated from existing ones, is there a conceptual 'bottom' from which they arise (Seuren 2014); which logico-cognitive configurations other than kite structures are there; do kites intersect or do they touch on only a single vertex; and so on. Such questions cannot be properly answered now. Their answers will crystallize out as the new ideas mature and find their place in our analytical thinking. But the very fact that they present themselves suggests that a new area of empirical research into concept formation and conceptual structure is being opened up.

Then, as regards 8 x , Bartsch (1973) noted that within a $\mathrm{Un}^{\mathrm{R}+1}$ under 'have a belief with regard to $p$ ', believelthink not $p$ is equivalent with not believelthink $p$. Hence, Bartsch concludes, there is no need for a rule of negative raising (NEG-raising),

[^9]since the facts that led to the assumption of NEG-raising are explained by the universe restriction separating the cases where the subject has no belief with regard to $p$ (the E position) from those where the subject actually has a belief with regard to $p$ (the I position). For Bartsch, this kind of universe restriction is a pragmatic process; for us, however, it is semantic, as it is part of the language system. Though this analysis is correct, the conclusion that NEG-raising is dispensable still does not follow, because (i) languages differ considerably as regards the predicates that induce NEG-raising (for example, English hope does not NEG-raise but Dutch hopen does), which shows that specifically linguistic factors are involved; (ii) some negative polarity items (NPIs), such as until tomorrow with point event predicates, occur in the object clause of I don't think that $p$ but not in, for example, the object clause of *I haven't been told that $p$.
(9) a. I don't think that he will arrive until tomorrow.
b. *I haven't been told that he will arrive until tomorrow.

There must thus be a certain degree of linguistic conventionalization of NEG-raising not accounted for by the Un ${ }^{\mathrm{R}+1}$ analysis (see the discussion in Horn 1989:319-20 and Collins \& Postal 2014:Ch. 1). We let this matter rest here, but it is both interesting and gratifying to see a representative of formal, model-theoretic semantics seek access to the world of cognitive logic and cognitive semantics, albeit in pragmatic rather than in semantic terms, as early as 1973.

In general, the items in the $\mathrm{A}, \mathrm{Y}$, and E positions tend to be more firmly established as words in a language than items in the I position at the top of the kite. In many cases, the I terms require a certain degree of abstractness or cultural sophistication. When one goes through $8 \mathrm{a}-\mathrm{x}$, one finds the terms male, female, single, parent, sibling, animate, lateral, real (in a philosophical sense), surface, impart, and have a belief with regard to $p$, all of which are not part of the down-to-earth register of everyday usage but belong, in different degrees, to a more elevated level of speech. In fact, have a belief with regard to $p$ in 8 x is not even a lexical item. Nor is it clear that say whether $p$ or leave open if $p$ in [say whether $p$ [deny that $p$, assert that $p$ ], leave open if $p$ ] are proper cases of lexicalization. To the extent that this tendency is real, it confirms the hypothesis (Seuren 2010:88-114) that the triangle of contraries reflects basic, unsophisticated cognition, whereas the logical square is already, to some extent, the product of culture-driven reflection and sophistication (and SMPL of specifically mathematical sophistication).

As a further example we may mention the triangle of contraries formed by the three indexical categories of the speech situation, speaker (I), hearer (you), and others (they) (Figure 10). We, which is, in a peculiar way, regarded as the plural of $I$, is conceptually divisible into so-called 'exclusive' we ('I and they') and 'inclusive' we ('I and you'). Many languages have different forms for the two meanings (Filimonova 2005). The triangle formed by the mutually exclusive $I$, you, and they is thus extended with either inclusive we or exclusive we as a fourth term. The combination of you and they, giving a presumed 'exclusive you' as opposed to an 'inclusive you' (which would be identical with inclusive we, meaning 'you and I'), has not been attested in any language (Simon 2005). When the median separates inclusive we from they, as in Fig. 10a, exclusive we is unlexicalizable. When it separates exclusive we from you, as in Fig. 10b, inclusive we is unlexicalizable. Allowance for both kite structures leaves the 'you and they' position unlexicalizable. That position would be lexicalizable if a median could be placed between 'you and they' and 'I', but this does not seem to occur, which may be due to a possible trait of human cognition placing 'the action' in speech situations predominantly around the speaker and less around the addressee(s) or external others. Interestingly, the combination 'I and you' is expressed as a form of we, not of (inclusive) you.

Likewise, 'I and others' is again we, not (inclusive) they. The speaker thus seems to be central to the speech situation. Note that in the two cases of Fig. 10, as in the cases shown in Figs. 8 b and 8 c above, the A and Y positions (and thus the corresponding O and $U$ positions) are interchangeable: as far as we can make out given the present state of our knowledge, it makes no difference whether $I$ or $y o u$, or $I$ or they, are in A or in Y position.


Figure 10. Kite structures for the indexical triangle with inclusive and exclusive we.
We can thus formulate a first generalizing principle, which, of course, restates the old Aristotelian principle of lexical oppositions. ${ }^{13}$
(10) Principle 1: No natural negative lexicalization occurs across medians.

That is, when cognition creates a triad of contrary concepts $<\mathrm{A}, \mathrm{Y}, \mathrm{E}>$ and A and Y form a cognitive subclass distinct from E, the contrary concepts A, Y, and E of the triad, as well as the overarching concept I of the subclass formed by A plus Y , are primary candidates for lexicalization, but complements of term extensions combining A or Y with E-that is, taking elements from both sides of the median-are illicit as input for lexicalization in the common usage register of the language. Nothing is said about cases where there is no triad of contraries, as with LANDING applied to aircraft. This term covers a mini-universe $\mathrm{Un}^{\mathrm{R}}$ split up into 'landing at sea' and the prototypical 'landing on land', both lacking a specific term, without a third option. If, in a distant future, moon landings were to become a normal part of life, that would create a third contrary concept, standing, one would guess, in primary opposition to earth landing, which comprises landing on land and landing at sea.
6. Complement selection. The rationale for principle 1 seems to lie in PUR, in particular in the conceptual hierarchy shown in Fig. 4. Just as concepts are placed at a given depth of hierarchical organization, their corresponding complements can be (natural) concepts themselves, and thus be lexicalizable, only if they stay at the same hierarchical level. This gives rise to principle 2, to be taken as an overridable default principle for clausal negation.
(11) Principle 2: Natural complement selection stays primarily within the proximate $\mathrm{Un}^{\mathrm{R}}$, but there are overriding factors.

[^10]Complement selection is the main function of NEGATION, whether morphological or clausal. As has been noted by many authors and in a large variety of terminologies and theoretical frameworks (for a survey, see Horn 1989:273-96), morphological negation (un-, in-, $a$-, non-, dis-, etc.) per se stays within the smallest $\mathrm{Un}^{\mathrm{R}}$, unless the negative word has assumed a meaning of its own, as in unrest or indifferent. Thus, non-Catholics are to be sought among humans and their institutions, not, say, among animals or plants. By contrast, nonmorphological clausal negation is allowed to override principle 2. Thus, when someone says Julius is not a Catholic, one's immediate inference will be that Julius is a human being and not, for example, a dog. But it is easy to think up contexts where this inference is not made.

A striking instance of complement selection according to principle 2 is found in Fillmore 1982:121, where the author says:

> To illustrate the point with items from everyday language, we can consider the words LAND and GROUND ... The difference between these two words appears to be best expressed by saying that LAND designates the dry surface of the earth as it is distinct from the SEA, whereas GROUND designates the dry surface of the earth as it is distinct from the AIR above it. The words 'land' and 'ground', then, differ not so much in what it is that they can be used to identify, but in how they situate that thing in a larger frame. It is by our recognition of this frame contrast that we are able to understand that a bird that 'spends its life on the land' is being described negatively as a bird that does not spend any time in water; a bird that 'spends its life on the ground' is being described negatively as a bird that does not fly.

Indeed, a bird that does not live on the land lives on the water, and a bird that does not live on the water lives on the land, but a bird that lives on the ground does not fly. Yet, if planes are to be kept on the ground (are grounded), they do not leave land or water (for water planes). And a ship that runs aground has gotten stuck on the bottom of the water it has been navigating. Ground, therefore, appears to designate the surface of the earth, whether covered by water or not, especially as the provider of support for solid bodies, and is primarily opposed to the air, or the sky. In this respect, Fillmore seems to err when he says that the words land and ground 'differ not so much in what it is that they can be used to identify' as in 'how they situate that thing in a larger frame', though the latter distinction is also real. The corresponding kite structure is shown in Figure 11.


Figure 11. Kite structure for [ground [water, land], air].
We have placed land in Y position and water in A position, though the two positions are interchangeable. Our reason for doing so is that, despite their semantic differences, land and ground are semantically close. They are homophonous in Italian (terra) and in French (terre). Placing land in Y position and water in A position, as we have done, makes the kite conform to a regular pattern of close semantic affinity between terms in

I and in $Y$ position. Sometimes, as we have seen, there is even phonological identity, as with (only) some versus (at least) SOME, (exactly) two versus (at least) Two, animal versus ANIMAL, gay versus GAY, tooth (not including molars) versus TООTн (including molars: a toothache is usually a molar ache), plant (not including trees) versus PLANT (including trees), and many other similar cases. For some of the cases mentioned above, the (restricted) contradictory of the I position either is absent or needs a specific context, there not being, as far as we know, a conceptually defined opposite. For тоотн it may be nail or bone or whatever; for PLANT it may be ANIMAL (further subdivided into human and animal).

The two principles stated so far do not suffice to rule out lexicalizations of the form *nall or *un-all, since a lexical item *nall or *un-all would now, just as not all is, be free to denote the Y space within I , the $\mathrm{Un}^{\mathrm{R}+1}$ of all, with the meaning 'some but not all'. But no such lexical item exists (though see n. 9 above). A further principle is thus needed to explain the nonoccurrence of * nall or *un-all in this restricted meaning.

This further principle is not far to seek, as it is well known among lexicologists: lexically fixed morphological negation is possible only if there is an inherent positivenegative opposition between the two concepts concerned. In such cases the positive member attracts the negative morpheme to denote the restricted complement. ${ }^{14}$ This is clearly so in the case of certain versus uncertain or fit versus unfit, and a multitude of similar cases. A positive-negative opposition is found even in cases where there is no overt negative morpheme, as with easy versus difficult (but difficult is indirectly derived from Latin dis-facilis 'un-doable', the opposite of facilis 'doable'), or with even versus odd within the $\mathrm{Un}^{\mathrm{R}}$ of integers (many languages have an equivalent of 'uneven' for 'odd' but no language has an equivalent of 'unodd' for 'even'), and so on. The negative character of difficult versus the positive easy appears not only from the absence of words for *undifficult in the world's languages, and from the cognitively real negative 'feel' of difficult as opposed to easy, but is also suggested by the fact that we can say 12a, with the NPI ever, but not 12 b .
(12) a. It is difficult for some linguists to admit that they have ever been wrong.
b. *It is easy for some linguists to admit that they have ever been wrong.

Also, neutralization in how + adjective questions often provides a clue. Thus, 13a is a neutral question as to the age of the new director in question, but 13 b is not a neutral question since it presupposes that the new director is, in fact, young for his post. This makes us decide that old is positive and young is negative.
(13) a. How old is the new director?
b. How young is the new director?

In some cases, however, there is no positive-negative opposition between two related predicates, the one just denoting the other's complement within the $\mathrm{Un}^{\mathrm{R}}$ at hand. This is the case, for example, with transport by land or by sea, or with breakfast and lunch. In these cases, no negative affixation is possible, which gives rise to principle 3.

[^11](14) Principle 3: A negative affix can only be attached to the positive member of a positive-negative pair. ${ }^{15}$
Since all and its counterpart (exclusive) some do not seem to form a positive-negative pair in their $\mathrm{Un}^{\mathrm{R}+1}$, the attachment of a negative affix to all or some (* $u$ )nall or *unsome) is ruled out on general grounds (which also provide a reason for not considering the quantifier no as a lexical composite of not and some but as a quantifier in its own right).

The analysis provided so far constitutes a considerable gain. We are, however, not there yet. Our analysis does not explain why (unlexicalized) not some in Y position in no way entails, suggests, implies, or implicates all, which it should do according to the analysis as presented so far, since the complement of Y within $\mathrm{Un}^{\mathrm{R}+1}$ is A . The answer to this question is not simple, and we certainly cannot claim that we have said the last word on this issue.

Pragmaticists will probably say that the answer lies in the theory of scalarity, developed mainly by Jespersen (1917:85-86), Blanché (1953:122-26, 1966:110-19), and Horn (1972, 1989:231-45). Linguistic expressions, especially adjectives, can sometimes be ordered in scales, often called implicational scales, of an ascending or descending order, according to the degree to which a property is said to adhere. That this notion of scalarity is cognitively real appears from certain test criteria such as the use of if not, even, or absolutely (Horn 1989:239) ('!' stands for 'inappropriate').
(15) a. He likes you and he may even love you.
b. !He loves you and he may even like you.
c. He absolutely adores!!likes you.
(16) a. It's warm if not hot out today.
b. !It's hot if not warm out today.
c. The soup is absolutely boiling/??hot/!warm.

Yet the notion is, though real, not optimally clear, mainly because the members of a scalar 'family of concepts' tend to differ in many other ways than just scalarity. Some, such as most, several, or exclusive some (see Fig. 12) are positive polarity items (PPIs), or they are NPIs; some, such as many or few, are gradables, whereas others, such as most or half, are absolutes, and other cross-classifications (such as temperatures for weather, cooking, or other 'environments') may disturb the picture, with the result that no clear empirical criteria are available and no clear predictions are possible on the basis of scalarity alone.

Horn (1989:236-37) plots the quantifiers onto the logical square as in Figure 12. Yet it is not clear, for example, why the gradable many and its intensified version very many should be placed below the absolute half. If two thousand out of two million people have bird flu, then indeed one can say that many people have bird flu, but if two thousand out of two million people voted for me in an election, then I can only say that very few people voted for me and that very many or the vast majority did not.

Jespersen (1917) is unclear yet suggestive. He states:
There may even be a whole long string of words with shades of meaning running into one another and partially overlapping, as in hot (sweltering)-warm-tepid-lukewarm-mild-fresh-cool-chilly-cold-frosty-icy. If one of these is negatived, the result is generally analogous to the negativing of a numeral:

[^12]

Figure 12. Horn's (1989:237) plotting of the quantificational scale onto the logical square.
not lukewarm, for instance, in most cases means less than lukewarm, i.e. cold or something between cold and lukewarm. (p. 85)
But this fails to explain why not cold tends to be interpreted as 'less than cold' or, more precisely, tepid, and not as 'frosty' or 'icy', which are very cold rather than not cold.

Then, however, he makes a new distinction, applied to scales of three contraries, two of which are extremes and one intermediate:

If we lengthen the series given above (much-a little-little) in both directions, we get on the one hand all (everything), on the other hand nothing. These are contrary terms, even in a higher degree than good and bad are, as both are absolute. Whatever comes in between them (thus all the three quantities mentioned above) is comprised in the term something, and we may now arrange these terms in this way, denoting by A and C the two absolutes, and by B the intermediate relative[.] (p. 85)

Having thus replicated Hamilton's triangle of contraries, Jespersen (1917) then proceeds:
This amounts to saying that in negativing an A it is the absolute element of A that is negatived. (p. 86)
If we now examine what results when a word belonging to the $C$-class [i.e. the E position- $P S / D J$ ] is negatived, we shall see corresponding effects, only that immediate combinations are not frequent except in Latin, where non-nemo, non-nulli means 'some', non-nihil 'something', non-numquam 'sometimes'. Here thus the result belongs to class B [i.e. the Y position- $P S / D J$ ]. (p. 90)

Whether Jespersen's observations are all correct is open to doubt. For example, his temperature scale hot (sweltering)-warm-tepid-lukewarm-mild-fresh-cool-chilly-cold-frosty-icy appears to mix weather temperatures with liquid temperatures. Additionally, most observers will disagree with his contention that not lukewarm 'in most cases means less than lukewarm, i.e. cold or something between cold and lukewarm'. It would seem more correct to say that not lukewarm means 'either warmer or colder than lukewarm'. (In the metaphorical sense, as in The proposal did not meet with a lukewarm reception, not lukewarm clearly means 'warm, cordial'.) Consider a kite structure for temperatures in the $\mathrm{Un}^{\mathrm{R}}$ of cooking and eating (warm should be read here as 'having a temperature above unheated', entailed by both hot and lukewarm): ${ }^{16}$ [warm [hot, lukewarm], cold]. Principle 4, formulated below, predicts that not lukewarm can go either way, since lukewarm has neither a positive nor a negative orientation and is not gradable (one cannot say * very lukewarm in the literal sense, only metaphorically).

[^13]Jespersen sets apart, in the lexicon, the class of scalar triads of the form A-Y-E, which consist of two extremes and one middle term, with an ascending order as regards the degree to which a property adheres, starting from a negative and climbing up to a positive extreme, with the intermediate term in between. For such triads he claims or implies that the negation over terms in both A and E positions selects the Y position. But he remains silent on the question of what complement is selected when the term in Y position is negated.

Moreover, as regards Jespersen's overall claim that when the E position is negated, as in Latin nonnulli, nonnihil, nonnusquam, and so forth, the denotation naturally falls on the intermediate term in Y position, excluding the A position, here again doubts arise. It seems more appropriate to say that these Latin forms, if they are not instances of conventionalized litotes, denote the I position, which is what principle 2 predicts. Similarly, when we consider the kite structure [allowed [obligatory, optional], forbidden], we see that the negation over forbidden, in E position, gives allowed, which occupies the I position, not optional, in Y position. In general, the negation over the E term yields the I term, which is in conformity with principle 2 formulated above.

Further scalar triads (kites), besides sомE $[$ all - some $]$ - no, are given in 17 , where A and $Y$ form a group I opposed to E, since I is where the cognitive action is.
(17) Examples of scalar triads (kites): ${ }_{\mathrm{I}}[\mathbf{A}-\mathbf{Y}]-\mathbf{E}$
a. ${ }_{\text {Let }}[$ make - let $]-$ prevent $^{17}$
b. impart[say that $p$-imply that $p]$ - uncommitted wrt $p$
c. feasible $[$ easy - difficult] - unfeasible
d. moving[fast - slow] - stationary
e. real $[$ past - present] - future

The class of scalar triads is interesting because it shows specific asymmetries, especially in the way the complement of Y is selected in the most natural way. Consider, for example, the cases in 18 where the Y position is questioned (the arrow stands for 'immediate natural inference'; A and B are speakers in dialogue).
(18) a. A: Did he let you go?

B: No. $\rightarrow$ prevent (E)
b. A: Did he imply that $p$ ?

B: No. $\rightarrow$ uncommitted with regard to $p(\mathrm{E})$
c A: Was it difficult?
B: No. $\rightarrow$ easy (A)
d. A: Is it moving slowly?

B: No. $\rightarrow$ fast (A)
e. A: Is this happening now?

B: No. $\rightarrow$ past or future (U)
Examples 18a,b naturally select the E position, but 18c,d,e do not. This seems to be due to the general fact that the negation of the middle term $(\mathrm{Y})$ selects different complements according to the orientation of the property negated. If intensification of the property P in the Y position (e.g. by means of very (much), quite, or more than) moves P to a position closer to A , then P is a positive property and the negation of P -that is, the negation of Y-selects E. Thus, some is positively oriented, since quite some moves P to a position closer to A. (Note that let, like allow, has a positive orientation: when someone makes you go, you have so much freedom to go that there is no freedom left not to go.) But if intensification of P moves P further away from A and in the direction

[^14]of E , as in the case of difficult or slow-very difficult is closer to unfeasible than to easy and very slow is closer to stationary than to fast-then P is a negative property and the negation of P in Y position selects A , as is clearly the case in 18c,d. In 18e, intuition says that the preferred complement is also the logical complement, namely U (that is, A or E), which seems to imply that the Y term now lacks any positive or negative orientation. It must be admitted, however, that this entire territory is still largely uncharted. On the basis of this analysis, we can now tentatively formulate principle 4.
(19) Principle 4: Position Y in scalar triads selects its complement according to the orientation $o$ of the Y-predicate. If $o$ is positive, the complement is E; if $o$ is negative, the complement is A; if $o$ is neutral, the complement is (unlexicalized) U.
A rationale for principle 4 may perhaps be provided in terms of the scalar triads of the form A-Y-E. When Y has a positive orientation-that is, when intensified it moves toward A-its negation turns the other way toward E. But when Y has a negative orienta-tion-when intensified it moves toward E-its negation again turns the other way, this time toward A. And when Y has no orientation, its negation selects its logical complement U . The negation of A , as we have seen, always selects Y .
7. Polarity and negation. In Blanché 1966:64 one finds the flawless hexagon of Figure 13a, which we have remodeled in Figure 13b into the kite shape adopted here, with the E-I axis in middle upright position. This hexagon applies to all integers N, with 'at least N' as I, 'more than N' as A, 'exactly N' as Y, and 'less than N' as E. Note that both 'no less than N ' ( $=$ not E) and 'no more than N ' ( $=$ not A) naturally land at 'exactly $\mathrm{N}^{\prime}(=\mathrm{Y})$, while 'not (exactly) N ' lands at $\neq \mathrm{N}(=\mathrm{U})$, following our principle 4.


Figure 13. Blanché's hexagon of arithmetical relations in two guises.

A relevant question, in this regard, is why 20a allows for a reading that implies that John is either smaller or taller than his brother, whereas 20 b can only mean that John is smaller than his brother.
(20) a. John is not as tall as his brother.
b. John is not so tall as his brother.

When we translate Blanché's logical hexagon in the kite arrangement of Fig. 13b into the terms of linguistic equatives and comparatives, it comes out as in Figure 14a. (Horn (2012:408) does the same but keeps the arrangement of Fig. 13a.) Yet there are complications. As tall as can be used as a PPI, since it allows for a reading with an echo effect
under negation, suggesting that a previous speaker has proposed the sentence without the negation (for 20a: John is as tall as his brother), which the present speaker wishes to deny by echoing it and placing it under radical metalinguistic not, which rejects the previous utterance as a whole. Since negation over a PPI clearly does not belong to default basic-natural usage but is the product of some metalinguistic reflection (Horn 1985), one may state that PPIs accept a higher negation only at a nonbasic, more intellectually developed level, which by itself suffices to rule out natural lexicalization in the U position. ${ }^{18}$ Placing the PPI as tall as in Y position and thus not as tall as in U position creates a further obstacle to lexicalization.
In order to account for the nonecho reading of 20a, according to which John is smaller than his brother, just as in 20b, we posit a distinction between as tall as and $A S$ TALL $A S$, the former being a PPI meaning 'equally tall as', and the latter a nonpolar expression meaning 'at least as tall as' or 'having reached the degree of tallness of', and subject to an optional lexical change into the NPI so tall as under negation. This allows for the reading common to 20 a and 20 b that says that John is smaller than his brother. One notes that $A S$ TALL AS, just like SOME, OR, POSSIBLE, and similar cases of 'co-lexemic superordinates', lacks an upper bound and is only lower-bounded, whereas as tall as is double-bounded.


Figure 14. Lexical kites for equative-comparative of tall, some/SOME, or/or, and not/NOT.
In this respect, the kite for as tall as/AS TALL $A S$ shows great similarity to that for some/SOME shown in Figure 14b. In both cases, there is a 'co-lexemic superordinate' (to use Cruse's term) in I position, and the term in Y position is a PPI. Moreover, there is an

[^15]NPI alternative for the superordinate term when that is placed under negation, optional for AS TALL AS (not as tall as and not so tall as coexist in the same meaning) but obligatory for SOME (not any but *not some). The kites in Figures 14c and 14d likewise have a homophonous superordinate term and a PPI in Y position, but they lack the NPI alternative in I position that is available under negation for $A S$ TALL $A S$ and de rigueur for SOME. To what extent we have to do here with a systematic pattern following from underlying principles is unclear at the moment and any speculation seems premature, but the similarities mentioned are to be kept in mind. In addition, the resulting asymmetry between the Y and the A positions may be taken to provide a further criterion for assigning items to the one or the other.

Negation itself, as shown in Fig. 14d, also follows this pattern. It is argued in Seuren 1985:239, 2010:355 that negation in language is triply ambiguous between default pre-supposition-preserving minimal not ( $\sim$ ), as in 21a, presupposition-canceling radical not $(\simeq)$, as in 21b, and standard logical NOT ( $\neg$ ), as in 21c ('>>': 'presupposes'). ${ }^{19}$
(21) a. John has not come back. >> John went away. $\vdash$ John is still there.
b. John has not come back: he never went away. (presupposition canceling)
c. Is it true that John has come back? No, it is NOT. $\vdash$ John is still there or he never went away.
Their truth tables are given in Table 1, where ' $T$ ' stands for 'true', ' $F 1$ ' for 'minimally false' (all presuppositions are satisfied but one or more assertive truth conditions are not), and 'F2' for 'radically false' (some presupposition is not satisfied). Minimal not $(\sim)$ does not affect presuppositions and only signals that an assertive truth condition is not satisfied. Radical not ( $\simeq$ ) signals that the sentence in its scope suffers from presupposition failure. Radical not thus yields truth only if there is presupposition failure, and minimal falsity (F1) otherwise. Standard logical NOT merely signals that some truth condition, whether assertive or presuppositional, is not satisfied. It thus forms the union of minimal not and radical not. Minimal not is the most natural, or default, form of negation in natural language. Radical not is restricted in its use to assertive sentences, where it takes the entire sentence in its scope; it does not occur in lower scope positions, questions, or imperatives.

| p | minimal not <br> $\sim \mathrm{p}$ | radical not <br> $\simeq \mathrm{p}$ | standard logical NOT <br> $\neg \mathrm{p}$ |
| :--- | :---: | :---: | :---: |
| T | F 1 | F 1 | F 1 |
| F 1 | T | F 1 | T |
| F 2 | F 2 | T | T |

Table 1. Truth tables for minimal $\operatorname{not}(\sim)$, radical $\underline{\operatorname{not}(~}(\sim)$, and standard logical $\operatorname{NOT}(\neg)$.
Logical SOME or $O R$ do not normally occur in assertive sentences, but they are normal in yes-no questions and imperatives, as appears from cases like 22a-d.

[^16](22) a. A: Did the patient take some food?
b. A: Did you see John or Harry?

B: Yes, she ate all of it.
c. A: Try to sell some shares.
d. A: Ask John or Harry.

B: Yes, I saw them both.
B: Thanks, I'll try to sell them all.
B: Thanks, I'll ask both.
Table 1 shows that both minimal not and radical not entail standard logical NOT, which occupies the I position in Fig. 14d. It also shows that $p, \sim p$, and $\simeq p$ form a triangle of contraries, as they cannot be true together. The natural negation of standard logical NOT- $p$ is $p$ (in E position), completing the kite structure of Fig. 14d. The natural negations of not and not cannot be tested, both being PPIs. Regardless, they are not lexicalized.
8. Discussion. The claims we have made so far reduce in the main to the four principles stated earlier.

- Principle 1: No natural lexicalization across medians.
- Principle 2: Natural complement selection stays primarily within the proximate Un ${ }^{R}$.
- Principle 3: A negative affix can only be attached to the positive member of a positive-negative pair.
- Principle 4: Position Y in scalar triads selects its complement according to the orientation $o$ of the Y-predicate: if $o$ is positive, the complement is E ; if $o$ is negative, the complement is A ; if $o$ is neutral, the complement is (unlexicalized) U .

We have shown that when there is an entailment relation between two lexical items, a LOGICAL HEXAGON can be set up, which is reduced to a KIte structure of four positions that are in principle lexicalizable. Three of the four positions, $\mathrm{A}, \mathrm{Y}$, and E , form a triangle of contraries, whereas the I position unites A and Y . The remaining two positions $(\mathrm{O}$ and U$)$ are unlexicalizable owing to the fact that cognition makes medians create restricted universes of discourse $\left(\mathrm{Un}^{\mathrm{R}} \mathrm{S}\right)$, within which logical relations, in particular complement selection by means of the negation, are primarily taken to hold. Principle 1 blocks lexicalization across median boundaries.

Language communities are thus not totally free in creating lexical items for any given conceptual complex. It is true that lexical items arise in language communities because of the socially felt need for a word for any given simple or complex notion standing out in a prototypical form. When a society feels the need for a word denoting a well-known device for verbal communication out of earshot, a word is created: telephone. Sometimes, such new words are to some extent semantically transparent but need extra information to be properly understood. For example, in the compound word tennis racket, the semantic relation between the two component elements differs radically from that between the two elements in tennis elbow. Such compounds are thus not fully transparent (they are not compositional in the technical sense), but the limited degree of transparency they have makes them easier to remember and to interpret.

While this is in itself uncontroversial, we still maintain that there are limits to lexicalizability. McCawley argued $(1972: 255,346)$ that the likelihood of any language having a word, say *plute, for 'have expert knowledge of wine and', so that one would be able to say John plutes peanuts, meaning 'John has expert knowledge of wine and peanuts', is close to zero. The same goes for items like, say, *flonk, meaning 'like the company of women who own', as in John flonks bakeries, meaning 'John likes the company of women who own bakeries'. ${ }^{20}$ According to McCawley, the absence of such

[^17]items is due to their violating universal constraints holding in syntax, the Coordinate Structure constraint for the former, and the complex NP constraint for the latter (Ross 1967). Whether this is true is immaterial here-though it will be very interesting if it does turn out to be true, as it will then show that syntactic constraints also apply to the lexicon. But conceptual complexes like 'have expert knowledge of wine and' or 'like the company of women who own' are nevertheless highly unlikely to become functional conceptual units in societies, which in itself diminishes the likelihood of their occurrence. Apart from that, however, it is far from absurd to posit that there are universal limits to possible lexicalization, whatever the reason or reasons behind them. The present study is an attempt to show that, at a basic-natural level of culture and sophistication, lexicalizability is constrained by cognition-driven restrictions that make use of logical opposition relations.

Inevitably, the question arises of the falsifiability of the claims we make, in particular with regard to principle 1 and the concomitant notion of medians creating $U n^{R_{S}}$. First and foremost, it may be objected that the claim embodied in principle 1 has not been tested out on any languages other than English and perhaps a few neighboring languages. No systematic investigation has been carried out, by the authors or anyone else, of possible exceptions to the kinds of lexical gap described in the present article in the languages of the world. Skeptics may claim that this makes the entire exercise futile. Yet a fairly large number of linguists or practitioners of related disciplines have looked at the lexical gap constituted by the absence of single lexical items for 'not all', 'not and', and two or three additional cases in a large variety of languages, and it has been found that this gap is systematic, with only one or two possible counterexamples (see n . 9). But no widespread search for possible counterexamples has taken place with respect to other item groups, beyond the few mentioned. Here, our answer can only be: let us wait and see, or perhaps better, let us go and search. We posit the universal, and so the hunt is on for counterexamples. When they arise, we will see what is to be done. They may, for example, be of the cuckold type (see n. 20).

Meanwhile, the occurrence of possible real or apparent counterexamples raises the question of how strictly the principles formulated above are to be taken. This is a question of a general nature in that it can be asked with regard to any proposed language universal. Language universals embody what is natural about language. But we have seen that humans, owing to their talent for culture and analytical thinking, tend to rise above the level of basic-natural thought and thus create less natural and more artificial concepts, which may then appear as less natural and more artificial lexical items or grammatical constructions. On the whole, however, such more 'advanced' concepts tend not to become lexicalized parts of the common language-but there is no absolute barrier to that happening. Societies may reach such a high level of sophistication that they cut through natural predispositions, making speakers adapt to less natural ways of expressing themselves. An example is the use of nand for 'not and' in the society of computer scientists. In such cases, the burden of proof will be with us: we will have to demonstrate that there is a significant correlation between what is regarded as 'nonnatural' on the one hand and advanced, socially shared sophistication on the other, as is obviously the case with nand in the social environment in which it is used. This way, we

[^18]hope to be able to home in on the notion of naturalness with regard to language and cognition.

A further, perhaps more pressing, problem is: what will count as an actual counterexample to our claim of the unlexicalizability of the $O$ and $U$ vertices? In principle, the answer is simple. We know that whenever there is an entailment relation between two lexical items, a lexical hexagon can be constructed. Our claim is that the O and U positions of such lexical hexagons are unlexicalizable: contradictories of lexicalized entailers in a higher Un are not lexicalized. Therefore, when it is found that some $O$ or $U$ position has been lexicalized, there is a counterexample. But this is too simple. We have encountered cases where the same lexical item fits into more than one kite structure, so that what is a forbidden position in one kite may be a licensed position in another. Examples are the matrices of Fig. 9 or the lexicalizations of inclusive and exclusive we (Fig. 10 above). Here, the 'ticket principle' works: whoever has a valid ticket may pass. Lexicalization is blocked only when there is no license at all. Again, in such cases, the burden of proof rests with us: any alleged violation of our theory must be countered by a demonstration that there is a kite or other structure of oppositions licensing the item in question.

This method of countering alleged counterexamples seems to us to be sufficient for the moment. What is needed for further proof is detailed sociological and sociopsychological research, based on solid theory, determining the degree to which certain conceptual configurations are default and others are not, and what exactly it means, in terms of socio-psychological theory, to say that they are. No or hardly any work of this nature, however, has been carried out to date.

We are, therefore, very much in the position of linguists providing grammaticality judgments based on introspection and intuitive calibration of what sounds like a natural expression for a given meaning and what does not. Just as lack of experimental proof of the correctness of linguistic grammaticality judgments has not, on the whole, been an obstacle to serious work in grammar, in the same way we feel that the lack of experimental confirmation of judgments on the default value, in given societies, of conceptual structures should not stand in the way of progress. We see no other way to gain insight into these matters than by taking an advance loan on expected results of socio-psychological theory building and empirical investigations, just as linguistics has been doing since its inception in antiquity. Science is to a large extent a bootstrapping process.
9. Conclusion. For the study of language, the strict separation of logic and cognition, imposed by standard modern logic since the beginning of the twentieth century, is an obstacle to proper insight. To see that, and how, logic is operative in cognition and consequently in the SYSTEM as well as the USE of language, it is necessary to impose cognitive restrictions on the mathematically defined standard modern logical system. When that is done properly, an 'odyssey of discovery' opens itself up to the investigator. ${ }^{21}$ Part of that 'odyssey' is our discovery that the Blanché hexagon is found all over the lexicons of languages and that systematically the O and U positions are unlexicalizable at the basic-natural level: they can be lexicalized only at higher levels of intellectual activity.

We found that the explanation of this remarkable fact is best taken to lie in what we have called PUR, a natural cognitive process of progressive universe restriction, creat-

[^19]ing mini-universes of discourse that determine the space for logical operators to be operative in. We thus envisage a generative system producing new restricted universes of discourse as one or both members of a pair of contraries are expanded into a further subpair of contraries forming a new, more restricted universe of discourse. It is shown how Aristotelian-Abelardian predicate logic is thus generated from a primary contrary pair 'there is at least one instance of a given concept $R$ ' (the R -set is nonnull) versus 'there is no instance of a given concept $R$ ' (the R -set is null). Standard modern predicate logic is not that way derivable, which qualifies it as 'nonnatural'.

This explains not only why not all and not (exclusive) some are unlexicalizable at the basic-natural level, but also why unlexicalized not all naturally means 'some but not all'-something that cannot be achieved in any consistent logical system operating in terms of the philosophically or mathematically hypostatized, but psychologically unrealistic, unrestricted universe of discourse Un ${ }^{\omega}$ used in standard logic. A new problem then arose to the effect that it must now be explained why not (exclusive) some does not normally mean 'all'. A broad analysis of complement selection in natural language made us see that complement selection by elements in the Y position of the kite structure for gradable predicates is a complex issue decided, at least in part, by the positive or negative orientation of the Y-predicate (principle 4). If the orientation is positive, the E position is selected for complement; if the orientation is negative, the A position is selected; if there is no orientation, the (unlexicalized) U position is selected. Since (exclusive) some is both gradable and positively oriented, its negation does not select the A position but lands at the E position.

Finally, we have been able to show that the various logically distinct forms of negation in natural language - presupposition-preserving MINIMAL NEGATION ( $\sim$ ), presuppo-sition-canceling Radical negation ( $\simeq$ ), and standard logical negation ( $\neg$ ) -also fit into a kite structure, which throws a new explanatory light on the nature of negation in natural language.

Although we feel entitled to claim that the findings reported on in the present study represent a step forward in our understanding of the lexicon in natural language, they also bring to light new, hitherto unsuspected, problems that we have not, on the whole, been able to answer in anything but a provisional way. We take this as a good sign.

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    ${ }^{1}$ See Jaspers \& Seuren 2014 for a detailed account of this intriguing episode in the history of logic.

[^1]:    ${ }^{2}$ See Clark 1970 and Noordman 1979. Both authors conducted experiments showing hierarchically organized conceptual relations. Clark (1970) showed that immediate word associations stay within the closest common restricted (sub)domain; Noordman (1979) showed the same for complement selection by the use of negation words.
    ${ }^{3}$ This rule applies to not and (see 2b) only if and takes heavy accent and is used metalinguistically (Horn 1985), as in Not geography and history, but geography or history.

[^2]:    ${ }^{4}$ A proposition is defined as the token mental act of assigning a property to one or more entities (Seuren 2010:85). Logical relations hold between propositional types, also called 'proposition' here for ease of reference (the term sentence is also used). They hold likewise for propositional functions, which are predicates over one or more variables. The source of logical relations thus lies in the meanings of the predicates concerned.

[^3]:    ${ }^{5}$ The triangle of contraries was first proposed in Hamilton 1866 and then, probably independently, in Jespersen 1917, 1924.

[^4]:    ${ }^{6}$ For a simple formal proof of these equivalences one may use the method of valuation space analysis (VS-analysis), as presented in Seuren 2006, 2010, 2013. The valuation space of a proposition $p$ is the set of situations in which $p$ is true (van Fraassen 1971).
    ${ }^{7}$ Most predicate pairs syntactically allowing for internal negation are not duals. The predicates regret and know, for example, allow for both an internal and an external negation since we have John [regrets/knows] that I am not at home, with internal, and John doesn't [regret/know] that I am at home, with external negation. Yet regret and know are not duals, as one can easily check, even though a hexagon can be set up for them because regret entails know. Duality is thus not a necessary property of a square or hexagon (Smessaert 2011:§3). Most lexical squares or hexagons lack duality but still form squares and hexagons as described above. It is the independent property of duality, together with the fact that the quantifying predicates all and some have mathematically definable meanings, that makes all and some prime candidates for a logical system. But this does not mean that other, not primarily logical, (pairs of) predicates do not form logical structures.

[^5]:    ${ }^{8}$ The pragmaticists' attempt at deriving some from SOME on conversational grounds was supported by the sCalarity principle, first formulated in Jespersen 1917:85-86 and Blanché 1953:122-26, 1966:110-19, then further elaborated in Horn 1972, 1989:231-45. This principle says: ' $[I]$ n negativing an A [i.e. an A-type sentence- $P S / D J$ ] it is the absolute element of A that is negatived' (Jespersen 1917:86). We delay a discussion of the scalarity principle till $\S 6$, where we attempt to incorporate it into our analysis of complement selection.

[^6]:    ${ }^{9}$ A possible counterexample is referred to in Seuren 2010:114, where Aimable-André Dufatanye, who is a native speaker of Kinyarwanda and a near-native speaker of Kirundi (both African languages of the Bantu group), is mentioned as having informed the author that in his language(s) there are fully lexicalized lexemes for 'not all'. Thus, he gave the forms (subject to the nominal classification system in the language) sibose 'not all people', sihose 'not everywhere', siyose 'not the whole house', sizose 'not all houses', as opposed to ntanzu 'no house', ntamuntu 'no person', and so on. What did not become clear from Dufatanye's account is whether the 'not all' forms in his two languages mean 'not all' in the logical sense or 'some but not all', excluding 'no'. Further research is called for.

[^7]:    ${ }^{10}$ Jaspers has meanwhile extended his analysis to color terms. He found that in the languages of the world, if red, yellow, green, and blue are taken to occupy the A, I, Y, and E positions, respectively, the terms for O and U are either missing or highly artificial (English has cyan for O and magenta for U ). Moreover, <red-cyan>, <yellow-blue>, and <green-magenta> are pairs of complementary colors (Jaspers 2012). This analogy between predicate logic and color terms is not further discussed here, as it seems to be situated in a deeper, perhaps physiological, stratum of cognition than is at issue here, more to do with perception than with concepts.

[^8]:    ${ }^{11}$ Note that gay marriages fit into the same kite structure.

[^9]:    ${ }^{12}$ In Blanché 1953:92, one identifies the following kite structures: [spending [prodigal, thrifty], not-spending], [courageous [reckless, prudent], cowardly]. The kite proposed in Blanché 1953:117, 1966:89-[plausible [established, undecided], excluded]-seems semantically unmotivated: for one thing, established and undecided do not entail plausible.

[^10]:    ${ }^{13}$ Principle 1 has a direct counterpart in presupposition theory, where lexically incorporated negation necessarily preserves presuppositions and does not allow for presupposition canceling: The king of France is unwise presupposes the existence of the king of France as much as The king of France is wise does. The common element is that presuppositions likewise restrict Un, but now in running discourse (Seuren 2010:311).

[^11]:    ${ }^{14}$ One often finds the clausal negation (not) over a negative gradable adjective (and occasionally also over a positive one): not bad, not impolite, not dissatisfied, not unusual, not unpleasant, not happy, not many, not very good, and so forth, meaning 'rather good', 'rather polite', 'rather satisfied', 'rather usual', 'rather pleasant', 'rather unhappy', 'rather few', 'rather bad', respectively. This is known as the stylistic or rhetorical figure of UNDERSTATEMENT or Litotes, used ironically to make it clear that the literal expression of the opposite would be an overstatement and suggesting that the proper interpretation should be as far removed, on the other side from the positive-negative divide, as the nonnegated expression is on its side. These are special cases of complement selection that are not further discussed here, as a discussion of litotes would exceed the frame of the present study (see van der Wouden 1994:122-38 for informed discussion).

[^12]:    ${ }^{15}$ The only exceptions known are cases such as the Latin formations nonnulli 'not no' (i.e. 'quite some'), noпnияquam 'not nowhere' (i.e. 'in quite a few places'), noпnитquam 'not never' (i.e. 'quite often'), and a few more of this nature. These are probably best treated as conventionalized instances of litotes or understatement (see n. 14).

[^13]:    ${ }^{16}$ There are also scales that do not form a scalar triad but are best considered to be one dimensional. An example is the musical scale fortissimo-forte-mezzo forte-piano-pianissimo. Scalar triads or kite structures do not arise here because there is no middle I term entailed by both an A and a Y term. The terms on this scale merely exclude each other, though with gradual transitions. It is often said in pragmatics that higher values entail lower values, but for many scales that is not true, since the predicates in question are mutually exclusive. The widely used term 'implicational scales' thus seems less appropriate.

[^14]:    ${ }^{17}$ The synonymous verbs let and allow display the same ambiguity between Y and I as some versus SOME. In the normal, default reading, a sentence like I let him go excludes an interpretation 'I made him go', but, on reflection, one will have to admit that if I made him go, I also let him go. The kites for let and allow will thus be spelled out as follows.
    (i) [LET [make, let], prevent]
    (ii) [ALLow [cause, allow], prohibit]

[^15]:    ${ }^{18}$ But see Baker 1970 for the curious and as yet unexplained fact that in a sentence like There is nobody here who would not rather be in Montpelier, with the PPI rather, no echo effect is observed. This sentence simply means 'everybody here would rather be in Montpelier'.

[^16]:    ${ }^{19}$ Seuren (1985:239 and 2010:355) suggests that standard logical $\neg$ (i.e. NOT) probably does not occur in natural language. It does, however, seem to occur in yes-no questions and in imperatives, as illustrated in 21 c . Horn's important (1985) study of metalinguistic (use of) negation contains the claim that metalinguistic negation forms a category on its own. Seuren (1988, 2010:259-60, 361-63) argues that Horn's distinction cuts across the truth-functional distinctions that separate minimal and radical negation and is itself not truthfunctionally definable in any straightforward way, so that it falls outside the analysis given here. Radical not is always metalinguistic, as it implies discourse correction, but other metalinguistic uses of not appear to fall under minimal not, though with an implicitly added predicate meaning something like 'be the correct expression'.

[^17]:    ${ }^{20}$ It has been objected that the English verb cuckold is a counterexample, as it would mean 'have intercourse with the wife of'. This, however, is incorrect. The verb cuckold can also be used for a married woman

[^18]:    who has intercourse with a man who is not her husband. In that case, the woman cuckolds her husband. The verb cuckold is simply the causative of the noun cuckold meaning 'cause to become a cuckold', where a cuckold is a married man whose wife is unfaithful.

[^19]:    ${ }^{21}$ The expression is taken from notes by Paul Jacoby made available to the authors by Leo Jacoby, son of Paul Jacoby (private correspondence).

