No-Go Theorems for Unitary and Interacting Partially Massless Spin-Two Fields

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We examine the generic theory of a paratially massless (PM) spin-two field interacting with gravity in four dimensions from a bottom-up perspective. By analyzing the most general form of the Lagrangian, we first show that if such a theory exists, its de Sitter background must admit either $so(1,5)$ or $so(2,4)$ global symmetry depending on the relative sign of the kinetic terms: the former for a positive sign the latter for a negative sign. Further analysis reveals that the coupling constant of the PM cubic self-interaction must be fixed with a purely imaginary number in the case of a positive sign. We conclude that there cannot exist a unitary theory of a PM spin-two field coupled to Einstein gravity with a perturbatively local Lagrangian. In the case of a negative sign we recover conformal gravity. As a special case of our analysis, it is shown that the PM limit of massive gravity also lacks the PM gauge symmetry.

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Introduction.—In de Sitter (dS) space, unitary spin-two modes have a mass gap, as opposed to those in the flat space or anti–de Sitter space. The lightest massive spin-two modes do not correspond to the massless graviton but to a special massive field called the partially massless (PM) spin-two [1]. The PM field has one less degree of freedom (d.o.f.) than a generic massive spin-two field due to the decoupling of the scalar mode.

The PM field is gaining renewed interest in the context of the massive gravity theory of Ref. [2] and the bimetric gravity theory of Ref. [3]. With a suitable choice of parameters, these theories can be linearized around dS space and describe the propagation of massive spin-two modes. One of the natural questions is the following: When the mass is tuned to that of PM [4], can the resulting theory consistently describe the dynamics of an interacting PM field? To answer this question, one can focus on the gauge symmetry that is present in the free theory of the PM field. If the PM limit of massive or bimetric gravity is consistent, these theories should provide an extension of the linear PM gauge symmetry to the interacting level. While the emergence of such a gauge symmetry has not yet been reported, there have been many discussions on the possible (in-)consistencies of this limit; see Ref. [6] for positive results and Ref. [7] for negative results. One of the aims of the present work is to provide a definite answer to this question.

PM gauge invariance.—In the search for a theory of PM plus gravity, one may begin with the most general form of the action $S = S_{\mathrm{EH}} + S_{\mathrm{PM}}$, where the gravity sector S_{EH} is given by the Einstein-Hilbert term:

$$S_{\mathrm{EH}}[g] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R-2\Lambda), \qquad (1)$$

while $\kappa = 8\pi G_N$, and G_N is the Newton's gravitational constant. The PM part S_{PM} is not fixed for the moment, except that it is given by a perturbatively local Lagrangian $\mathcal{L}_{\mathrm{PM}}$ which is diffeomorphism invariant:

$$S_{\mathrm{PM}}[\varphi, g] = \int d^4x\sqrt{-g}\mathcal{L}_{\mathrm{PM}}(\varphi, \nabla\varphi, g, R, \ldots). \qquad (2)$$

Here, … means that there may be higher derivatives of $\varphi_{\mu\nu}$ or curvature $R_{\mu\nu\rho\sigma}$. Let us emphasize that this ansatz also covers the PM massive and bimetric gravity.

Besides the diffeomorphism symmetries, we also require the action to be invariant under PM gauge symmetries $\delta_\alpha S = 0$ where δ_α is the nonlinearly deformed PM transformation which we aim to determine together with $\mathcal{L}_{\mathrm{PM}}$. For further analysis, it is convenient to expand the action and the PM gauge transformations in powers of the PM field $\varphi_{\mu\nu}$ as

$$\begin{aligned} S_{\mathrm{EH}} &= S^{(0)}, \\ S_{\mathrm{PM}} &= S^{(2)} + S^{(3)} + \cdots, \\ \delta_\alpha &= \delta_\alpha^{(0)} + \delta_\alpha^{(1)} + \cdots, \end{aligned} \qquad (3)$$

where the superscript (n) means that the corresponding term involves the nth powers of $\varphi_{\mu\nu}$. Then, the PM gauge-invariance condition provides an infinite set of equations,

$$\delta_\alpha^{(1)} S^{(0)} = 0, \tag{4}$$

$$\delta_\alpha^{(0)} S^{(2)} + \delta_\alpha^{(2)} S^{(0)} = 0, \tag{5}$$

$$\delta_\alpha^{(0)} S^{(3)} + \delta_\alpha^{(1)} S^{(2)} + \delta_\alpha^{(3)} S^{(0)} = 0, \tag{6}$$

$$\cdots .$$

The first condition (4) simply tells us that $(\delta_\alpha g_{\mu\nu})^{(0)} = 0$, whereas the other conditions constrain possible forms of $\mathcal{L}_{\rm PM}$ and δ_α. We shall analyze below the second condition (5) and the third condition (6).

The quadratic part of the gauge-invariance condition (5) reads

$$\int d^4x\sqrt{-g}\left((\delta_\alpha\varphi_{\mu\nu})^{(0)}\left[\frac{\delta S^{(2)}}{\delta\varphi_{\mu\nu}}\right] + (\delta_\alpha g_{\mu\nu})^{(1)} G_\Lambda^{\mu\nu}\right) = 0, \tag{7}$$

where $G_\Lambda^{\mu\nu} \equiv R^{\mu\nu} - g^{\mu\nu}R/2 + \Lambda g^{\mu\nu}$ is the cosmological Einstein tensor. The lowest-order PM gauge transformation $(\delta_\alpha\varphi_{\mu\nu})^{(0)}$ is given by the covariantization of the free PM transformation around the dS background:

$$(\delta\varphi_{\mu\nu})^{(0)} = \left(\nabla_\mu\nabla_\nu + \frac{\Lambda}{3}g_{\mu\nu}\right)\alpha. \tag{8}$$

The gauge-invariance condition (5) can be solved for $S^{(2)}$ by properly covariantizing the free PM field action. Plugging the solution $S^{(2)}$ into Eq. (5), the PM gauge transformation of the metric tensor is determined to be [8]

$$(\delta_\alpha g_{\mu\nu})^{(1)} = 2\sigma\kappa(2\nabla_{(\mu}\varphi_{\nu)\rho} - \nabla_\rho\varphi_{\mu\nu})\partial^\rho\alpha, \tag{9}$$

where σ is the sign factor of the PM kinetic term in $S^{(2)}$ that we have introduced in order to keep track of its role.

Now we turn to the cubic part of the gauge-invariance condition (6):

$$\int d^4x\sqrt{-g}\left((\delta_\alpha\varphi_{\mu\nu})^{(0)}\left[\frac{\delta S^{(3)}}{\delta\varphi_{\mu\nu}}\right] + (\delta_\alpha\varphi_{\mu\nu})^{(1)}\left[\frac{\delta S^{(2)}}{\delta\varphi_{\mu\nu}}\right] + (\delta_\alpha g_{\mu\nu})^{(2)} G_\Lambda^{\mu\nu}\right) = 0. \tag{10}$$

Similar to the quadratic part, one can solve the condition (10) for $S^{(3)}$ by properly covariantizing the PM cubic self-interaction derived for the dS background [9]. Plugging the solution $S^{(3)}$ into Eq. (10), we obtain $(\delta_\alpha g_{\mu\nu})^{(2)}$ and $(\delta_\alpha\varphi_{\mu\nu})^{(1)}$. In particular, the expression of the latter

$$(\delta_\alpha\varphi_{\mu\nu})^{(1)} = 2\sigma\lambda(\nabla_{(\mu}\varphi_{\nu)\rho} - \nabla_\rho\varphi_{\mu\nu})\partial^\rho\alpha \tag{11}$$

will be important in the forthcoming analysis. Here, λ is the coupling constant of the two-derivative cubic interaction in $S^{(3)}$.

Let us remark that the expressions (8), (9), and (11) are unique up to field redefinitions, which are physically irrelevant. Notice that the cubic-order gauge-invariance condition (6) does not constrain the coupling constant λ at all. The coupling constants can be determined by the quartic or higher-order consistency conditions. Hence, in principle, we may have to proceed to higher orders to see the eventual (in-)consistency of the PM-plus-gravity theory. However, there exist other consequences of gauge invariance that cubic couplings must satisfy. In the following, we shall explain this point.

In general, when an action S involving a set of bosonic fields χ_i admits gauge symmetries, they must form an (open) algebra:

$$[\delta_\varepsilon, \delta_\eta] = \delta_{[\eta,\varepsilon]} + C_{ij}(\eta,\varepsilon)\frac{\delta S}{\delta\chi_i}\frac{\delta}{\delta\chi_j}, \tag{12}$$

where the gauge-algebra bracket $[\eta,\varepsilon]$ may depend on fields, and the arbitrary matrix $C_{ij} = -C_{ji}$ generates trivial symmetries. Let us now consider the gauge-algebra brackets of the PM-plus-gravity theory. By explicitly evaluating the commutator of two successive gauge transformations given in Eqs. (8), (9), and (11), the gauge-algebra brackets can be identified at the zeroth order in $\varphi_{\mu\nu}$. First, the diffeomorphisms give the usual Lie derivative. Next, the commutator between diffeomorphism and PM gauge transformation gives again a PM gauge transformation:

$$[\xi, \alpha] = \xi^\mu\partial_\mu\alpha + \mathcal{O}(\varphi). \tag{13}$$

Finally, the commutator of two PM transformations results in a diffeomorphism:

$$[\alpha_2, \alpha_1] = 2\sigma\kappa\partial_\rho\alpha_{[1}\nabla^\mu\partial^\rho\alpha_{2]}\partial_\mu + \mathcal{O}(\varphi). \tag{14}$$

In general, these gauge-algebra brackets do not define a Lie algebra due to the (possible) field-dependent pieces, but their restriction to the Killing fields, namely, the global-symmetry brackets, must do so. Moreover, there exists another important consistency condition, the admissibility condition, which must hold at the level of global symmetries.

Global symmetries.—In order to see this point more clearly, let us briefly move back to the general discussions presented around Eq. (12). We shall now analyze the closure of the gauge symmetries perturbatively. One considers the expansions

$$S = S^{[2]} + S^{[3]} + \cdots, \qquad \delta_\varepsilon = \delta_\varepsilon^{[0]} + \delta_\varepsilon^{[1]} + \cdots,$$
$$[\eta,\varepsilon] = [\eta,\varepsilon]^{[0]} + [\eta,\varepsilon]^{[1]} + \cdots,$$
$$C_{ij} = C_{ij}^{[0]} + C_{ij}^{[1]} + \cdots, \tag{15}$$

where the superscript $[n]$ stands for the total power of fields χ_i involved. Restricting the attention to the Killing fields $\bar{\varepsilon}$

defined by $\delta^{[0]}_{\bar\varepsilon}=0$, one can derive two important consistency conditions. First, one can show

$$\delta^{[0]}_{[\bar\eta,\bar\varepsilon]^{[0]}}=0, \tag{16}$$

meaning that the global symmetry is closed under the bracket $[\cdot,\cdot]^{[0]}$. Second, one can show the so-called admissibility condition

$$[\delta^{[1]}_{\bar\varepsilon},\delta^{[1]}_{\bar\eta}]=\delta^{[1]}_{[\bar\eta,\bar\varepsilon]^{[0]}}+\delta^{[0]}_{[\bar\eta,\bar\varepsilon]^{[1]}}+C^{[0]}_{ij}(\bar\eta,\bar\varepsilon)\frac{\delta S^{[2]}}{\delta\chi_i}\frac{\delta}{\delta\chi_j}, \tag{17}$$

which implies that $\delta^{[1]}_{\bar\varepsilon}$ provides a representation of the Lie algebra of the global symmetry on the space of fields.

Having the above general lessons in mind, let us come back to the PM-plus-gravity theory and consider the dS metric $g_{\mu\nu}=\bar g_{\mu\nu}$ and $\varphi_{\mu\nu}=0$ as the background. In this case, the χ_i fields in the above general discussion would correspond to $h_{\mu\nu}=g_{\mu\nu}-\bar g_{\mu\nu}$ and $\varphi_{\mu\nu}$. The global symmetries of this background are the subset of gauge symmetries which leave it invariant. The gauge parameters of the global transformations are defined as the solutions of the following Killing equations:

$$\bar\nabla_{(\mu}\bar\xi_{\nu)}=0,\qquad \left(\bar\nabla_\mu\bar\nabla_\nu+\frac{\Lambda}{3}\bar g_{\mu\nu}\right)\bar\alpha=0, \tag{18}$$

where $\bar\nabla$ is the dS covariant derivative. From the Killing equations (18) and the gauge-algebra brackets (13) and (14), the Lie brackets of global symmetries are readily computed as

$$[\bar\varepsilon_2,\bar\varepsilon_1]^{[0]}=2\left(\bar\xi^\nu_{[2}\partial_\nu\bar\xi^\mu_{1]}-\sigma\kappa\frac{\Lambda}{3}\bar\alpha_{[2}\partial^\mu\bar\alpha_{1]}\right)\partial_\mu + 2\bar\xi^\mu_{[2}\partial_\mu\bar\alpha_{1]}. \tag{19}$$

Here, we have conveniently packed the parameters as $\bar\varepsilon=\bar\xi^\mu\partial_\mu+\bar\alpha$. In order to identify the global symmetry given by Eq. (19) in the standard classification of Lie algebras, we need to solve the Killing equations (18). For that, it is convenient to reformulate them in the ambient-space formalism through the standard embedding:

$$\xi_\mu(x)=\ell^2\frac{\Xi_M(X)\partial_\mu X^M}{X^2},\qquad \alpha(x)=\ell\frac{A(X)}{\sqrt{X^2}}, \tag{20}$$

where the X^M's are the coordinates of the ambient space containing dS space as a hyperboloid with the radius $\ell=\sqrt{3/\Lambda}$. In terms of the ambient-space fields Ξ_M and A, the Killing equations simply read $\partial_{(M}\bar\Xi_{N)}=0$ and $\partial_M\partial_N\bar A=0$. The general solution can be presented as

$$\bar\Xi^M\partial_M=W_{AB}M^{AB},\qquad \bar A=V_AK^A, \tag{21}$$

where $W_{AB}=-W_{BA}$ and V_A are arbitrary parameters while M^{AB} and K^A are the global-symmetry generators:

$$M^{AB}=2X^{[A}\partial^{B]},\qquad K^A=X^A. \tag{22}$$

Using this explicit form of the generators and the bracket (19), one can derive the Lie brackets $[\![\cdot,\cdot]\!]\equiv[\cdot,\cdot]^{[0]}$ as

$$[\![M^{AB},M^{CD}]\!]=4\eta^{[A[C}M^{D]B]},$$
$$[\![M^{AB},K^{C}]\!]=2\eta^{C[B}K^{A]},$$
$$[\![K^{A},K^{B}]\!]=-\frac{\Lambda}{3}\sigma\kappa M^{AB}. \tag{23}$$

This Lie algebra contains the isometry algebra $\mathfrak{so}(1,4)$ generated by M^{AB} as a subalgebra. Together with the K^A-generators for $\sigma=+1$, they define $\mathfrak{so}(1,5)$, and for $\sigma=-1$, they define $\mathfrak{so}(2,4)$ depending on the relative sign σ between the graviton and PM field kinetic terms.

Admissibility condition.—We are now at the point to examine the admissibility condition (17). It plays an important role in higher-spin field theories [12] as well as in supergravities. In the case of PM plus gravity, it will also turn out to be a decisive condition. In order to examine the admissibility condition for the system under consideration, one first needs to linearize the transformations with respect to the metric perturbation $h_{\mu\nu}$ as

$$\delta_{\bar\varepsilon}h_{\mu\nu}=\delta_{\bar\varepsilon}g_{\mu\nu}=\delta^{[1]}_{\bar\varepsilon}h+\mathcal{O}(h,\varphi),$$
$$\delta_{\bar\varepsilon}\varphi_{\mu\nu}=\delta^{[1]}_{\bar\varepsilon}\varphi+\mathcal{O}(h,\varphi), \tag{24}$$

where the superscript [1] means that the corresponding terms are linear in $h_{\mu\nu}$ or $\varphi_{\mu\nu}$. First, from the diffeomorphism symmetry, we get the usual Lie derivative. Then, from Eqs. (8), (9), and (11), we obtain the PM transformation parts as

$$\delta^{[1]}_{\bar\alpha}h_{\mu\nu}=-2\sigma\kappa\partial^\rho\bar\alpha(2\bar\nabla_{(\mu}\varphi_{\nu)\rho}-\bar\nabla_\rho\varphi_{\mu\nu}), \tag{25}$$

$$\delta^{[1]}_{\bar\alpha}\varphi_{\mu\nu}=2\lambda\sigma\partial^\rho\bar\alpha(\bar\nabla_{(\mu}\varphi_{\nu)\rho}-\bar\nabla_\rho\varphi_{\mu\nu}) -\frac{1}{2}\partial^\rho\bar\alpha(2\bar\nabla_{(\mu}h_{\nu)\rho}-\bar\nabla_\rho h_{\mu\nu})+\frac{\Lambda}{3}\bar\alpha h_{\mu\nu}. \tag{26}$$

With Eqs. (25) and (26), we are ready to compute the commutator between two PM transformations, which is the lhs of Eq. (17). After straightforward calculations and imposing the global-symmetry conditions on gauge parameters, we obtain the commutator of two PM transformations as

$$(\delta^{[1]}_{\bar\alpha_1}\delta^{[1]}_{\bar\alpha_2}-\delta^{[1]}_{\bar\alpha_2}\delta^{[1]}_{\bar\alpha_1})h_{\mu\nu}=2\bar\nabla_{(\mu}\mathcal{A}^\rho h_{\nu)\rho}+\mathcal{A}^\rho\bar\nabla_\rho h_{\mu\nu}+2\bar\nabla_{(\mu}\mathcal{B}_{\nu)}, \tag{27}$$

$$(\delta^{[1]}_{\bar\alpha_1}\delta^{[1]}_{\bar\alpha_2}-\delta^{[1]}_{\bar\alpha_2}\delta^{[1]}_{\bar\alpha_1})\varphi_{\mu\nu}=2\bar\nabla_{(\mu}\mathcal{A}^\rho\varphi_{\nu)\rho}+\mathcal{A}^\rho\bar\nabla_\rho\varphi_{\mu\nu} +(\lambda^2+\sigma\kappa)\mathcal{C}_{\mu\nu}, \tag{28}$$

where $\mathcal{A}_\mu$, $\mathcal{B}_\mu$, and $\mathcal{C}_{\mu\nu}$ are given by

$$\mathcal{A}_\mu = 2\sigma\kappa\frac{\Lambda}{3}\bar{\alpha}_{[1}\partial_\mu\bar{\alpha}_{2]}, \tag{29}$$

$$\mathcal{B}_\mu = -2\sigma\kappa\partial^\rho\bar{\alpha}_{[1}\partial^\sigma\bar{\alpha}_{2]}(\bar{\nabla}_\rho h_{\sigma\mu} - 4\sigma\lambda\bar{\nabla}_\rho\varphi_{\sigma\mu}) - 4\sigma\kappa\frac{\Lambda}{3}\bar{\alpha}_{[1}\partial^\rho\bar{\alpha}_{2]}h_{\rho\mu}, \tag{30}$$

$$\mathcal{C}_{\mu\nu} = 4\partial^\rho\alpha_{[1}\partial^\sigma\alpha_{2]}\bar{\nabla}_{(\mu|}\bar{\nabla}_\sigma\varphi_{|\nu)\rho} + 4\Lambda\alpha_{[1}\partial^\rho\alpha_{2]}(\bar{\nabla}_{(\mu}\varphi_{\nu)\rho} - \bar{\nabla}_\rho\varphi_{\mu\nu}). \tag{31}$$

Let us analyze each term in Eqs. (27) and (28) to see whether they are compatible with the rhs of Eq. (17): (i) the terms involving $\mathcal{A}_\mu$ take the form of a Lie derivative, and they correspond to the $\delta^{[1]}_{[\bar{\alpha}_2\bar{\alpha}_1]^{[0]}}$ contribution in the rhs of Eq. (17); (ii) the terms involving $\mathcal{B}_\mu$ take the form of a linearized diffeomorphism; hence, they are related to the $\delta^{[0]}_{[\bar{\alpha}_2\bar{\alpha}_1]^{[1]}}$ contribution; (iii) there remains the $\mathcal{C}_{\mu\nu}$ term, which does not correspond to any of the contributions. Therefore, the admissibility condition requires that the coefficient of the $\mathcal{C}_{\mu\nu}$ term vanish:

$$\lambda^2 + \sigma\kappa = 0. \tag{32}$$

This equation determines the PM self-interaction coupling constant λ in terms of the gravitational one κ. Now, one has two options for a theory of PM plus gravity depending on the relative sign σ between the kinetic terms: for $\sigma = +1$, we obtain purely imaginary λ; for $\sigma = -1$, we obtain $\lambda = \pm\sqrt{\kappa}$. Let us stress that the inclusion of higher-derivative interactions cannot change this conclusion since they do not affect the form of the PM transformations [11] on which our analysis is based.

Conclusions.—In this Letter, we have investigated the most general form of the Lagrangian for PM field and gravity with a positive cosmological constant. By examining its gauge symmetries, we have determined the two-derivative cubic self-interaction of the PM field together with its coupling constant λ. Our results have several implications for different models of gravity. (i) Nonunitarity of the PM-plus-gravity theory. When the kinetic terms have a relatively positive sign with $\sigma = +1$, the gauge invariance of the PM-plus-gravity action requires the PM cubic self-interaction to have an imaginary coupling constant, which manifestly violates the unitarity. In particular, this implies that the PM limit of bimetric gravity cannot lead to a gauge-invariant Lagrangian theory. Consequently, the scalar d.o.f.—that is, the seventh d.o.f.—would not decouple from the theory in the PM limit. (ii) Conformal gravity. In the case where the kinetic terms have a relatively negative sign with $\sigma = -1$, the theory admits the global symmetry of $\mathfrak{so}(2,4)$, which is the conformal algebra in four dimensions. Hence, in this case, we actually recover conformal gravity (CG). In fact, CG is another playground for the PM field: among six d.o.f. of CG [13], the additional four d.o.f. organize themselves into a PM representation around dS space [14] (see also Ref. [15]). The PM gauge symmetry in CG is a disguised version of Weyl symmetry. (iii) PM massive gravity. The case of PM theory without gravity is covered by taking the limit $\kappa \to 0$ and choosing the background to be dS space. This limit effectively freezes out the dynamics of the metric tensor. In such a case, the global symmetry reduces to an Abelian one instead of $so(1,5)$. The admissibility condition then requires the PM cubic coupling constant λ to vanish; therefore, no two-derivative cubic interaction is consistent in the pure PM theory. This rules out the PM limit of massive gravity from the possible consistent theories of the PM field due to the presence of its two-derivative cubic interaction inherited from the Einstein-Hilbert term.

All the analyses of the present Letter were carried out in four dimensions. A similar analysis might be possible in higher dimensions where we expect that the admissibility condition would require the field content to include some massive modes as well [16].

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