# An Economic Analysis of the Issues and Challenges in Climate Policy Decision Making

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*I am convinced that this challenge and what we do about it will define us, our era and ultimately our global legacy.*

Ban Ki-moon, United Nations Secretary-General

# 1

# Introduction

Anthropogenic climate change poses one of the greatest challenges for policy decision making. The climate has undergone many major changes throughout the Earth's history. However, the current climate change exhibits some features that are unusual, and is thus considered to be of the utmost policy concern. First of all, in contrast to earlier changes, the current change is attributed to human activities and is referred to as anthropogenic climate change. In fact, emissions from economic activities have been found to be the main source of the currently observed high concentrations of atmospheric greenhouse gases and are therefore held responsible for global warming. Moreover, compared to other incidents of climate change during the past 650,000 years, the atmospheric concentration of $CO_2$ - one of the most abundant greenhouse gases in the atmosphere - has never been higher, and it has risen exceptionally fast (IPCC (2007b)). According to the Intergovernmental Panel on Climate Change (IPCC), the resulting warming by the end of the century is likely to be in the range of 1.6°C to 6.9°C above preindustrial temperatures if no further emission reduction efforts are undertaken (IPCC (2007b)).[1] This magnitude of warming at such an unprecedented speed is expected to exert a significant detrimental impact on natural systems and human life in the future (IPCC (2007a)). The problem of anthropogenic climate change and its consequences could be tackled by a comprehensive climate policy,

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[1] In accordance with the guidelines of the IPCC's Fourth Assessment Report, the term "likely" is used to describe probabilities that are higher than 66%.

consisting of decisive global emission reduction efforts as well as measures to alleviate the impacts that are already locked in. Yet, up to now the design and implementation of an appropriate policy response is challenged by an unparalleled combination of obstacles to decision making.

This thesis aims to explore some of the major issues in impeding climate policy decision. The purpose of this chapter is to outline the broad context of this topic and to embed the research articles that follow this introduction in the thesis. Section 1.1 introduces the most prominent challenges to climate policy making examined through the perspective of economic theory. Section 1.2 specifies the objectives and structure of the thesis.

## 1.1 The Major Challenges to Climate Policy Decision Making

Economic theory helps us to identify and understand the challenges to climate policy decision making and their causes. Climate change is perceived to be a negative externality, describing costs that are imposed on others but are not paid for by those who cause them. As long as emitting greenhouse gases are not priced there is little or no incentive to reduce emissions. In view of the anticipated climate impacts, policy intervention is justified that internalizes these effects by establishing a market in which emissions allowances are traded, or by creating institution(s) that regulate the emissions. A multitude of negative externalities have been successfully internalized by policy intervention. Yet, the climate change problem features a coalescence of characteristics that distinguish it from others: (i) it is global, (ii) its impacts develop over a long time, (iii) the projection of its consequences are subject to ubiquitous uncertainties in the causal chain of climate change, (iv) the impacts and also the policy response are largely irreversible, and (v) it must be addressed by a diverse portfolio of climate policy measures.[2]

*Global Problem*: Climate is a global, non-excludable public good. Greenhouse gas emissions diffuse in the atmosphere and thus affect the global climate system irrespective of where they are emitted. The atmospheric greenhouse gas concentration cannot be controlled by any single nation, but requires international collective action. As a global authority that could enforce emission reductions does not exist, global action rests on the voluntary participation of sovereign nations. However, voluntary cooperation is deterred by the free-riding incentives emerging from non-excludability. While the benefits of climate policy can be enjoyed by every country, the costs of curbing emissions must be borne by the cooperating countries. The free-riding incentives are strengthened by the

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[2]For similar categorizations see IPCC (2001), Levin et al. (2012), Pillet (1999), Stern (2007) or Wagner & Zeckhauser (2012).

unequal distribution of emissions and their anticipated consequences around the world. The worst climate change impacts are expected to affect the poorest countries, which have contributed least to the accumulation of greenhouse gases in the atmosphere (e.g. Smith et al. (2001)). In contrast, most of the high emitting countries expect relatively moderate impacts, which reduces their incentives to curtail their contribution to the climate problem. In theory, free-riding incentives can be overcome if mechanisms such as transfer payments are introduced, so that every country gains from participating and bears an equitable share of the burden of reducing emissions. However, in addition to heterogeneity in the expected damages, further asymmetries, such as in economic development and historical responsibility, lead to irreconcilable notions of how these mechanisms are to be designed. Studies of game theory, for example by Fuentes-Albero & Rubio (2010), have generally found that the level of voluntary cooperation on emission reduction efforts is rather low, which mirrors the unsuccessful attempts in reality to negotiate a legally binding global climate agreement.

*Long Time Horizon*: The extent of global warming is determined by the concentration of the greenhouse gases, which is nourished by the sum of emissions over time. Furthermore, the repercussions of today's emissions will mature over decades owing to some rather slow warming processes in the climate system. Thus, climate policy needs to be optimally designed for a long time horizon that is over several generations. However, the required intertemporal welfare evaluation crucially depends on the choice of the social discount rate, which determines the weight assigned to future welfare. The question of which social discount rate is to be applied has ignited an ongoing controversy: on the one hand, there is the "positive" approach that draws on empirically observable market interest rates for quantification; on the other hand, the "normative" school relies on normative criteria to address the intergenerational trade-off.[3] This point of view is advocated by Stern (2007), who criticizes the procedure for discounting as a concept that deprives future generations of their representation in present-day decisions. A different problem of the long time horizon arises from the general incentives for political institutions to satisfy their current (voting) citizens' interests. Accordingly, the policy decision to reduce emissions, which imposes mitigation costs in the near term, will continue to be postponed unless the current society attaches a comparatively high importance to the fate of future generations (Levin et al. (2012)).

*Ubiquitous Uncertainties*: Welfare assessment encounters a myriad of uncertainties in the causal chain of climate change, which ranges from the emission of greenhouse gases to the ultimately felt impacts. Scientific understanding of the physical processes involved is

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[3]A more detailed introduction of both concepts is given, among others, by Karp & Traeger (2013).

still incomplete, which leaves unanswered questions about the strength of the greenhouse gas effect and the vulnerability of climate subsystems. Socio-ecological uncertainties surround the impact of climate change on the relationship between human societies and nature, for example, in connection with the spread of diseases. There is socio-economic uncertainty attached to the effects of climate change on human welfare and the gains from climate policy.[4] Distinguishing uncertainties according to their severity, as done by Knight (1921), has been shown by Ellsberg (1961) to be of behavioural meaningfulness. Knightian risk describes the situation in which outcomes cannot be predicted with absolute confidence, but sufficient statistical information is available to derive statements about their likelihood. Knightian uncertainty or ambiguity arises, if probability statements are rendered impossible due to missing observations. Climate policy assessment faces ambiguity in almost every component of the causal chain. How to design policy given these deep uncertainties has not yet been clarified. The policy design must also take into account that some elements of the uncertainties will be resolved due to new scientific findings and further observations, which may make it necessary to adjust policy in the future. Other elements of the uncertainties are more fundamental or intrinsic, for instance, those generated by the long time horizons under consideration, and they will remain uncertain. These intrinsic uncertainties particularly relate to the socio-economic sphere, in which damage cost assessments are conducted over future generations that may have different economies and demographics, preferences and attitudes.

*Irreversibilities*: According to Henry (1974), a decision is defined as being irreversible if it limits future possibilities of choice for a long time. The climate policy decision faces two kinds of irreversibilities: economic irreversibility and ecological irreversibility. The first one relates to the irreversibility of the sizeable investments needed to implement climate policy; for example, the phasing out coal powered plants requires huge investment in alternative energies. These sunk costs must be balanced against the benefits of avoiding ecological irreversiblity. Ecological irreversibility refers to the rather low decay rate of greenhouse gases, especially of $CO_2$. As a consequence, greenhouse gases continue to cause global warming long after they have been emitted. Ecological irreversibility is also associated to the possibility of irreparable climate damage and catastrophic events. To some extent, the climate changes gradually and slowly, but some of its impacts may occur abruptly when some climatological thresholds are transgressed. A growing body of scientific research (e.g. Rockström et al. (2009) and Steinacher et al. (2013)) strives to identify policy targets that protect the climate from crossing these thresholds and concludes that the time to preserve "the safe operating space for humanity" (as phrased

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[4]This classification of uncertainty is commonly made in the IPCC reports, e.g. IPCC (2001).

by Rockström et al. (2009)) is running out.

*Choice of Measures*: Mitigation is not the only measure that can be employed to reduce climate change impacts. Adaptation and geoengineering measures become increasingly important the longer emission reduction efforts are postponed. There are a wide array of technologies in the realm of geoengineering or climate engineering that are either designed to remove carbon from the atmosphere or to manage solar radiation. So far, insufficient technological maturity and the partially understood side-effects have ruled out the adoption of these technologies. Yet, climate engineering may become an indispensable complementary strategy if decisive emission reduction efforts are taken too late. Climate engineering can also be perceived to be a substitute for mitigation, as even the prospect of operative readiness is reported to result in the easing of emission reduction efforts (Rickels et al. (2011)). Different to climate engineering, adaptation is accepted as an integral part of effective climate policy, as emphasized by IPCC (2007a). It is vital for alleviating the impacts that are already locked in, owing to the emissions in the past and at present. However, the optimal mix of mitigation and adaptation is still far from being clear, as both measures interact with each other in a complex system of substitutions and complementarities. So far, the literature (e.g. Kane & Shogren (2000) and Lecocq & Shalizi (2007)) has pointed out that they can be perceived as being strategically complementary. Mitigation can prevent irreversible and severe damage that it is difficult or even impossible to adapt to, while adaptation can address damage that is inevitable due to past and present emissions. Ingham et al. (2005) and Tol (2005a) argue that the measures are also economic substitutes, as they compete for naturally scarce resources and employing one measure may decrease the marginal benefits of the other.[5]

The five dimensions of the decision problem are not clear-cut; rather, they overlap and affect each other. For instance, the operative readiness of techniques to remove carbon from the atmosphere alleviates ecological irreversibility. The inability to find a global agreement on emission reduction may force countries to adopt localized measures, such as adaptation and solar radiation management instead. Intrinsic uncertainty grows with the length of the time horizon being considered. Decisions about whether to take precautionary steps by reducing emissions or relying on measures that may alleviate the future impacts also depend on the representation of the future generations' welfare in the present-day considerations. One important interaction is economic irreversibility and uncertainty. If the economic irreversibility did not exist, policy could be easily and frequently adjusted as soon as new scientific findings or observations were made. However, as pointed out by economic theory, the requirement for large-scale sunk costs generates

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[5]Further effects of interaction are explained by IPCC (2007a).

incentives to wait for new information to arrive, instead of embarking on a set climate policy path (e.g. Pindyck (2000, 2002)).

## 1.2 THESIS OUTLINE

Although research on climate policy decision making has made substantial progress in recent years, numerous particular aspects are still not well understood owing to a very complex interplay of the many factors involved. The purpose of this thesis is to provide new insights into the policy response to being confronted with the different (interacting) challenges posed by climate change. More specifically, it aims to contribute to the understanding of the overall decision problem outlined in Section 1.1 by addressing selected aspects of it in each chapter. In this spirit, particular attention is paid to the effects of facing the following: economic and ecological irreversibilities, the uncertainty and ambiguity, the problem of combining mitigation and adaptation measures, and the implications of the countries' asymmetries concerning international collective cooperation on emissions reduction.

The first three chapters take a real options perspective that is developed to explicitly account for the tension between uncertainty and economic irreversibility. This approach discloses the value of waiting for new information to arrive that is incorporated in climate policy assessment. In other words, real options quantify the opportunity costs of taking action now rather than waiting for uncertainty to be reduced. It sets the stage for the investigation of how other characteristics of the climate change problem influence policy decisions.

Chapter 2, *The Race Against Time: Optimal Climate Policies and Costly Inaction*, is motivated by the latest scientific findings that the time to meet climate policy targets, which limit the risk of unacceptable environmental change, is presumably running out. How climate policy targets influence emission reduction efforts is mainly studied by utilizing the expected utility approach (e.g. Held et al. (2009) and Nordhaus (2010)). This strand of literature generally finds that focussing on climate policy targets intensifies emission reduction efforts. Unlike these studies, this article accounts for the value of waiting that is generated by uncertainty and economic irreversibility. Consequently, it allows the question to be posed about whether the knowledge of facing a closing window of opportunity could significantly counteract the incentives to wait, and thus accelerate emission reduction efforts. For this, the paper develops a non-perpetual real options framework in which the option to adopt policies that comply with the target is only available for a limited amount of time. In this framework, the effects of two kinds of uncertainty are examined: stochasticity in the climate damage costs and stochasticity in the temperature

evolution. In both cases, the closing of the window of opportunity accelerates emission reduction efforts, especially if the option to act expires soon. However, the effects are shown to be comparatively small, which indicates that climate policy inaction is likely to prevail.

Chapter 3, *Dark Clouds or Silver Linings? Knightian Uncertainty and Climate Change*, investigates how economic irreversibility and Knightian uncertainty in the climate damage costs affect the decision on when to curb emissions. A review by Stern (2007) reveals that the existing estimates of the future climate damage costs are subject to enormous ambiguity. These damage cost assessments are not only based on differing appraisals of vulnerabilities and capabilities for adaptation, but they also ignore the impact of extreme weather events or catastrophes to a great extent. The substantial degree of ambiguity is illustrated by comparing the assessments of Mendelsohn et al. (2000), Nordhaus & Boyer (2000) and Tol (2002), which vary between zero and three per cent of loss of GDP for 3°C warming. This study transfers the ideas of Nishimura & Ozaki (2007) and Trojanowska & Kort (2010) about how to enhance a real options model by Knightain uncertainty to a model that examines the decision on when to curb emissions. First, the decision by an ambiguity-averse policy maker is investigated; and then the analysis extends to a range of ambiguity preferences. This study finds that policy adoption is delayed longer when the policy maker is more optimistic about the future outcomes. Furthermore, this study also identifies that the range of optimal policy responses, which are implied by alternative preferences for ambiguity, is of a non-negligible size. This result emphasizes the difficulties in reaching an objectively justified climate policy decision, as the decision crucially depends on subjective attitudes towards ambiguity.

Chapter 4, *The Optimal Climate Policy of Mitigation and Adaptation: A Real Options Theory Perspective*, directs attention to the question of how mitigation and adaptation can be optimally combined. Different to the existing literature, which is dominated by the expected net present value approach, it analyses how the optimal mix is influenced by the different degrees of uncertainty in the climate damage costs and by the irreversibility associated with the mitigation and adaptation decision. To this end, this study develops a novel real options modelling framework in which the policy maker holds a portfolio of mitigation and adaptation options. The mitigation option gives the opportunity to decide on the optimal timing of committing to a certain emission reduction target. Exercising the adaptation option means optimally expanding the adaptation capital stock that helps to protect against damages proactively. This paper demonstrates that the dualistic approach to climate policy is impeded by the interaction of uncertainty and economic irreversibility. Compared to the expected net present value approach, it

gives more priority to adaptation as the preferred measure. If the marginal benefits of investing in adaptation are sufficiently low, mitigation is given more emphasis.

Chapter 5, *Can International Environmental Cooperation be bought: Comment*, goes beyond considering one global policy maker by taking into account the implications of the countries' heterogeneity when international environmental cooperation is negotiated. This study addresses the research by Fuentes-Albero & Rubio (2010) which analytically solves a non-linear, game-theoretical model that incorporates two types of countries and continuous strategies. In the case of heterogeneity in the damage costs, Fuentes-Albero & Rubio (2010) conclude that, even though side-payments are not made, an agreement between one high- and and one low-damage country is self-enforcing, given that the disparities are not very large. This result is proven to be incorrect by demonstrating that such a coalition is only internally stable, while external stability is not satisfied. Consequently, asymmetries in damage costs provoke countries to defect from cooperative behaviour, unless some mechanism is established to extinguish the free-riding incentives without transgressing any notions of fairness.

*The door is closing. I am very worried - if we don't change direction now on how we use energy, we will end up beyond what scientists tell us is the minimum [for safety]. The door will be closed forever.*

Fatih Birol,
Chief Economist at the International Energy Agency

# 2

# The Race Against Time: Optimal Climate Policies and Costly Inaction[1]

## 2.1 Introduction

Article 2 of the United Nations Framework Convention on Climate Change demands a "stabilisation of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system".[2] Consequently, numerous studies have attempted to identify and examine the climate targets that are supposed to guide climate policy and are thought of as safety constraints, beyond which societal and environmental disruptions and catastrophic events are considered to be more likely. The target that has become a critical part of emission reduction negotiations is the 2°C target, which allows maximum global warming of 2°C throughout the twenty-first century.[3] A more holistic approach by Rockström et al. (2009) and Steinacher et al. (2013) takes into account multiple (interlinked) climate targets that must not be missed. For both approaches, recent contributions by Meinshausen et al. (2009), Steinacher et al. (2013) and Vliet et al. (2012) show that these targets will soon move out of reach if

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[1]This chapter is co-authored by Yu-Fu Chen and Michael Funke.
[2]For the full text of the convention, see http://unfccc.int/resource/docs/convkp/conveng.pdf.
[3]See Jaeger & Jaeger (2010) and Randalls (2010) for an in-depth discussion of the 2°C target.

the emission reduction efforts continue to be delayed. Despite this increasing body of scientific evidence, global emissions are not expected to be reduced soon. Under the Cancún Accord, countries recognized that sharp reductions in emissions were required, in order to limit the increase in the global temperature to less than 2°C throughout the twenty-first century. However, the Accord stopped short of actually delivering a binding worldwide agreement.[4] This paper aims to investigate the climate policy decision in the context of a closing window of opportunity before a climate target moves out of reach. To this end, we apply and extend a methodology - namely real options analysis - that explicitly accounts for incentives to delay climate policy and thus offers an illustrative tool to investigate how the knowledge of having only a limited amount of time to act influences the policy decision.

An optimal climate policy that is focused upon a climate target is largely studied by utilizing the (expected) net present value approach. This strand of literature generally finds that the presence of a climate target tends to accelerate the mitigation efforts substantially. A full review of this literature is beyond the scope of this paper, but some studies will be mentioned. Nordhaus (2010) estimates that the 2°C target requires immediate emission cuts, implying a price of $82.05 per ton of carbon (2005 prices) in the year 2015 and steep price increases for decades afterwards. In contrast, a policy that implies warming of almost 3°C requires only half that price in 2015. Accounting for uncertainty about climate sensitivity and the climate response time scale, Held et al. (2009) apply a chance-constrained approach to show that the 75% likelihood of achieving the 2°C target makes drastic emission cuts necessary (with a maximum investment of 3% of the GDP in renewable energy sources between 2030 and 2050).[5] While the (expected) net present value approach produces quantitative results describing the optimal policy path, it is not designed to account for further mechanisms affecting the policy decision. In fact, the policy decision deserves a closer inspection to understand why scientific evidence of a climate target moving out of reach does not seem to make a difference to actual global inaction. This analysis requires an alternative methodology that looks beyond the traditional expected net present value approach and can contribute to our understanding of the decision problem.

Unlike the expected net present value approach, real options analysis explicitly quantifies the incentives to delay investments. If an investment involves at least partially sunk costs and the benefits of this investment are uncertain, delaying the investment, if possible,

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[4]For the Cancún Accord at a glance, see UNFCCC (2010).

[5]Further studies, such as those by Lorenz et al. (2012), O'Neill & Melnikov (2008), Schmidt et al. (2011), Webster et al. (2008) and Yohe (2000), deal with the possibility of learning and revising targets over time.

may have a value to the policy maker. Adopting climate policy incurs at least partially sunk costs and their implied benefits are uncertain. Of course, a delay may come at the cost of higher climate damage costs as the temperature increases even further, but this cost must be balanced against the benefits of waiting for new information to arrive. Real options analysis accounts for both the expected net present value and the opportunity costs of taking action now - the latter of which are captured by the real option value.[6] Pindyck (2000) shows that the option to wait has a positive value as long as uncertainty is not completely resolved. The policy maker thus waits longer before curbing emissions. However, it is not clear how the knowledge of having a limited amount of time left to act before a climate target moves out of reach interacts with the incentives to delay climate policy. Does it accelerate emission reduction efforts by significantly decreasing the value of waiting?

This paper features a simple real options model that reflects the most important characteristics of this decision problem. To this end, the race against time is captured by restricting the availability of the option to a given time period, the duration of which is exogenously given to the policy maker. The decision maker can only take measures to meet some target before the deadline expires.[7] Afterwards, this goal moves out of reach and the economy may have to bear higher climate damage costs.

The structure of the remainder of the paper is as follows. To illustrate the closing window of opportunity, Section 2.2 presents a basic overview of the cumulative emission trajectories that conform to the 2°C target. In Section 2.3, the design of the continuous-time modelling set-up is presented. Subsequently, in Sections 2.4 and 2.5, we illustrate the working of two model specifications - involving stochasticity in the damage function and stochasticity in the temperature evolution - through numerical exercises and examine the sensitivity of the main results with respect to the key parameters. Finally, in Section 2.6, a summary and some policy implications stemming from the previous modelling exercise are provided. Further details of some technical derivations are available in the appendices.

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[6]A very informative introduction to real options analysis and a more detailed reasoning of the importance of accounting for the real option value can be found in Dixit & Pindyck (1994). A more advanced survey of real options analysis is given by Stokey (2009). Anda et al. (2009) elaborate on the advantages of real options analysis over the conventional net present value approach by investigating processes that can be described by a heavy-tailed distribution. Further applications of real options analysis to climate policy decisions can be found in Chen et al. (2011a), Dobes (2010), Lin et al. (2007), Linquiti & Vonortas (2012), Maybee et al. (2012), Pindyck (2000, 2002), Strand (2011), Watkiss et al. (2013) and Wirl (2006). Earlier contributions stressing the importance of accounting for the tension between uncertainty and irreversibilities in the context of climate policy are made by Kolstad (1996), Narain & Fisher (1998) and Ulph & Ulph (1997).

[7]The assumption of only one global policy maker is sufficient to isolate and analyse the effects of the limited opportunity to meet some climate target. As the next step, it is possible to extend this framework by accounting for multiple decision makers having different views of the desirability of such a target.

## 2.2 Admissible Emission Trajectories under Climate Policy Targets

Temperature projections need to account for uncertainty relating several factors, e.g. the quantity of all greenhouse gases (not only $CO_2$), feedback effects, inertia in the climate system, cooling and warming effects produced by different aerosols and so forth. Emission reduction efforts that can meet some long-term temperature targets are assessed by paying special attention to $CO_2$ emissions. Due to its abundance and its remarkable longevity in the atmosphere, $CO_2$ exerts a dominant influence on the temperature evolution in contrast to rather short-lived greenhouse gases and aerosols. As pointed out by Archer (2005), half of $CO_2$ emissions are removed by the natural carbon cycle within a century, but a substantial fraction will stay in the atmosphere for several millennia.[8] Positive feedback effects will contribute to the atmospheric concentration by releasing $CO_2$ out of the present carbon sinks such as the terrestrial biosphere, by the end of the century; see Cox et al. (2000). The full extent of the consequences caused by the atmospheric carbon build-up is not yet observable, as $CO_2$-attributable global warming processes are diagnosed as rather slow.[9]

For these reasons, Allen et al. (2009), Meinshausen et al. (2009) and Zickfeld et al. (2009) find that over a period of a few decades the peak warming is remarkably insensitive to the shape of the emission trajectory and depends only on the cumulative total. Meinshausen et al. (2009) provide explicit numbers for this cumulative total that are compatible with the 2°C objective with a certain degree of probability. As a substantial part of the global carbon budget has already been used up in the first 10 years of this century, the remaining cumulative total is assessed to be 750 Gt for the time period until 2050. At this level, the probability of the global temperature rise exceeding 2°C throughout the twenty-first century is calculated as 33 per cent. Beyond this, Meinshausen et al. (2009) also point out that the total proven fossil fuel reserves are large enough to move the 2°C target out of reach with a probability of 100 per cent.

To obtain an idea about the carbon budget approach, Figure 2.2.1 sketches examples of global emission pathways admitting cumulative $CO_2$ emissions of 750 Gt during the time period 2010 - 2050.[10]

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[8]Being based on climate models of differing complexity, other studies support these findings on the whole; see for example Lenton et al. (2006) and Matthews & Caldeira (2008).

[9]For instance, the warming of the oceans lags behind considerably, so that the full effects on temperature are not yet felt. However, the slow ocean mixing that delays the warming would also be responsible for slow cooling. Hence, the benefits of a decrease in atmospheric carbon concentrations would be widely offset; see Matthews & Caldeira (2008) and Solomon et al. (2009).

[10]The past emission trajectory matches the observed data, which are taken from www.cerina.org/home and www.iwr.de/klima/ausstoss_welt.html. The computation of the future trajectories corresponds to the scientifically based equation (9) in Raupach et al. (2011), in which the business-as-usual growth rate
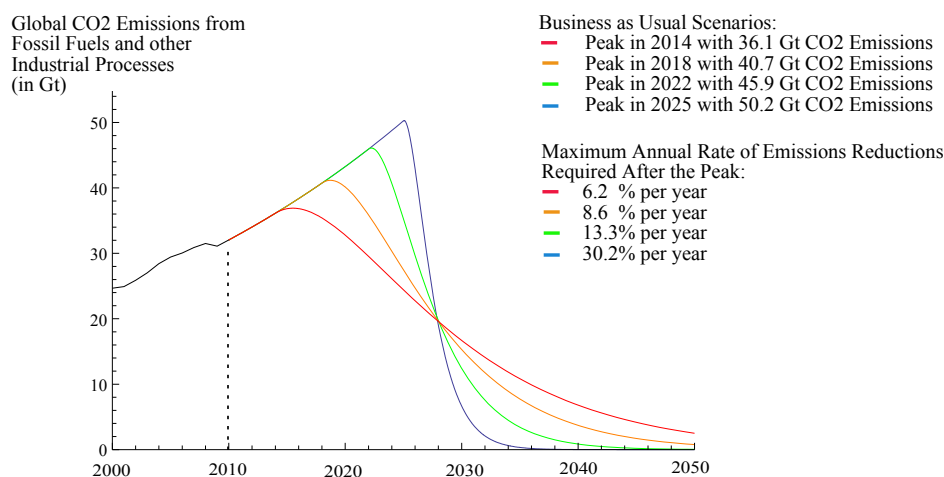
**Figure 2.2.1:** Alternative Carbon Dioxide Emission Pathways Complying with the 2°C Target with a Probability of 67 Per Cent.

The global emissions of $CO_2$ decreased slightly between 2008 and 2009 following the worldwide financial and economic crisis. Nevertheless, the global emissions again reached record levels in 2010. Each trajectory merges an initial business-as-usual phase with a subsequent mitigation phase that is assumed to be delayed until 2014 (red), 2018 (orange), 2022 (green) and 2025 (blue), respectively. Albeit stylized, this graph helps us to understand the key points that are implied by the carbon budget approach. Firstly, the window of opportunity to limit global warming to 2°C is still open, but will close soon. Secondly, the outcomes also illustrate that the longer the start of the mitigation phase is delayed, the steeper the subsequent reduction in emissions has to be to meet the 2°C target. This occurs due to the realistic assumption of increasing annual emissions in the business-as-usual scenario, so that the total carbon budget tends to be exhausted quickly. Some of the exemplified emission trajectories involve almost unachievable reduction requirements, as two-digit cuts in annual emissions seem to be technologically and economically infeasible from today's perspective.[11]  Finally, Figure 2.2.1 indicates the imperative to shut down $CO_2$ emissions almost entirely after 2050, even if the emission

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of emissions is assumed to be 3 per cent.

[11]An implicit assumption when tracing the illustrative $CO_2$ pathways in Figure 2.2.1 is the non-availability of negative emission technologies. The reason is that most decarbonization technologies are still in early stages of research and development and large-scale deployment in the timescales needed is very uncertain. We cannot be sure that all these technologies will work in practice outside the laboratory, i.e. the scalability and rollout potential on larger scales is uncertain. Furthermore, considerable cost uncertainties exist. Lemoine et al. (2012) model the optimal combination of abatement, research and development, and negative emission policies under the anticipated availability of negative emission strategies with stochastic technological change.

trend is reversed in 2014.

Other studies raise the concern that the mitigation options to reach such a climate policy target are likely to disappear much earlier due to the legacy of existing long-lived infrastructure and inertia in energy demand, which is discussed by Guivarch & Hallegatte (2011) and Ha-Duong et al. (1997), and due to upper limits on the sustained emission reduction rates, as illustrated by Stocker (2013). Accounting for the increasing energy demand that is triggered by global economic and population growth, IEA (2011) projects that the expansion of the high-carbon infrastructure in the next five years will already mark the crossing of the 450ppm threshold, which is thought to be equivalent to the 2°C target.[12]

Whether the 2°C target is the exact guardrail, which represents a minimum for safety, is not clear. Hansen (2005) criticizes such a target as unsuitable for framing climate policy, as it already commits the world to significant climate change. At the UN climate change conference in Cancún, governments agreed to review the 2°C target in the light of new scientific studies on the effects of climate change and to consider lowering the maximum to 1.5°C.

Rockström et al. (2009) look beyond one target and identify nine partly interlinked Earth-system processes and their associated thresholds, which define "the safe operating space for humanity". Their analysis suggests that three of the boundaries have already been breached. Steinacher et al. (2013) examine the permissable carbon budget by imposing limits on six climate variables and find that the allowable cumulative total of emissions is lower than that implied by the temperature target only. Whichever target or set of targets might be the most appropriate, the above-mentioned studies suggest that early action is urgently required.

How should policy makers respond to such a small window of opportunity? The answer might be less straightforward if the following reservations are considered. First of all, the policy decision would need to be made on the basis of climate damage cost assessments that are rather vague due to substantial ecological and economic uncertainties. Furthermore, enormous emission reductions imply large sunk costs, which may not be recouped before long. Moreover, the worst effects of global warming and thus the benefits of a climate policy reducing them may not be felt for decades, whereas the costs of tackling climate change will burden the economies immediately. Hence, in spite of all the warnings, policy makers may be tempted to wait instead of taking action.[13]

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[12]Currently, the $CO_2$ concentration is measured as 396.81 ppm, featuring annual growth rates of 2-3 ppm in the last years; see http://www.esrl.noaa.gov/gmd/ccgg/trends/mlo.html.

[13]Further reasons to delay mitigation efforts relate to the uncertainty about mitigation costs and effectiveness, and the potential for making use of geoengineering technologies and for being capable of adapting

Real options analysis explictly accounts for these incentives to delay climate policy. In the next section, we present a model that translates the deadline imposed by climate targets into a non-perpetual real option for climate policy.

## 2.3 The Baseline Window-of-Opportunity Modelling Set-Up

This section anchors our modelling approach in the existing real options literature. Before we begin our theoretical discussion, we believe that it would be helpful to characterize our use of real options models. Recent research documents that it is more than a guideline for decision makers. More precisely, there is ample evidence that policy makers employ a "real options heuristic" [Kogut & Kulatilaka (2001)], i.e. retain the upside potential without the downside risk of fully committing up front. That means that in a situation of substantial uncertainties about the benefits of a policy, decision makers keep the options to act alive. Afraid of committing themselves to huge expenses, they tend to wait for further information. However, as explained in Section 2.2 the option to limit global warming to 2°C will expire some time in the near future. The consequential question that arises is whether and how this affects the policy maker's decision. By incorporating the opportunity to act explicitly, the following model is set up to provide an answer.

We assume that a global social planner strives to find the optimal timing for cutting emissions by maximizing the flow of consumption over time.[14] She faces the intergenerational trade-off problem that the costs of curbing emissions burden current generations, while the benefits of doing so will be enjoyed by future generations. Moreover, bad timing will certainly lead to one of the following two irreversibility effects. Investing too early in mitigation technologies could trigger enormous sunk costs that are not recouped for many years. Waiting too long may cause irreversible damage to ecological systems that are valuable to human health or the economy. However, ubiquitous uncertainties in almost every component of projections and especially in the assessment of future climate damage render a well-informed decision about the timing almost impossible. Put differently, all plans depend decisively on the unknown sensitivity of losses to climate change. The unknown sensitivity is thus modelled as uncertain in the following. Any other lack of knowledge is assumed to be resolved for the sake of analytical tractability. Expressed

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to a good deal of the climate damages. These effects are beyond the scope of this paper and deserve separate analytic treatment.

[14]In this framework the world is treated as a single entity in the interest of simplicity. The world climate policy equilibrium can be constructed as a symmetric Nash equilibrium in mitigation strategies. The equilibrium can be determined simply by looking at the single country policy, which is defined ignoring the other countries' mitigation policy decisions [Leahy (1993)]. Other climate policy measures, such as adaptation or geoengineering, are not accounted for in the modelling framework. A worthwhile extension of this model could be to investigate the optimal mix of these measures, given that some of them buy more time while others, such as mitigation, are required to meet the climate policy target.

mathematically, the policy maker solves the following objective function, which consists of the expected net present value of future consumption levels:

$$W\left(X, \Delta T\right) = E\left[\left(1 - w(\tau)\right) \int_{t=0}^{\infty} L\left(X_t, \Delta T_t\right) C_t e^{-rt} dt\right], \qquad (2.1)$$

where $E[\cdot]$ is the expectation operator and $C_t$ is global consumption over time with the initial value normalized to 1. In its simplest form, the level of global consumption is assumed to be equivalent to the level of global GDP. Climate change is modelled to reduce this level of GDP/consumption to $L\left(X_t, \Delta T_t\right) C_t$. The function $L$ is driven by $\Delta T_t$, which describes scientifically estimated changes in temperature, and by $X_t$, which is a (positive) stochastic damage function parameter determining the sensitivity of losses to global warming. The flow of the net GDP/consumption is discounted by $r$. If the policy maker takes measures to limit the temperature increase to a certain target $\tau$, she is obliged to pay mitigation costs that amount to a certain percentage $w\left(\tau\right) > 0$ of the annual GDP. As the option to reach this target is perceived to be expiring soon, we make the simplifying assumption that the mitigation costs $w\left(\tau\right) > 0$ do not increase with time, but remain about the same within this limited time horizon.[15] In the case of no climate policy, the mitigation costs $w\left(\tau\right)$ are zero.[16]

Instead of trying to model climate impacts in any detail, we keep the problem analytically simple by assuming that damages depend only on temperature change, which is chosen as a measure of climate change. To be precise, following Pindyck (2009, 2012), we assume that the function $L$ is implied by the exponential loss function

$$L\left(X_t, \Delta T_t\right) = e^{-X_t(\Delta T_t)^2}, \qquad (2.2)$$

where $0 < L\left(X_t, \Delta T_t\right) \leq 1$, $\partial L / \partial\left(\Delta T_t\right) \leq 0$ and $\partial L / \partial X_t \leq 0$. Note that the function $L$ denotes the actual output in percentage terms, relative to the potential output without climate change. Therefore, the total damage cost ratio is equivalent to $1 - L$. The GDP at time $t$ net of damage from warming is given by $L(X_t, \Delta T_t)GDP_t$. Intuitively, a high value of $X_t$ means that damage is sharply curved in $\Delta T_t$.

Before we turn to the modelling of the uncertainty that is attached to $X_t$ in equation (2.2), we briefly introduce the other component in the loss function: the temperature increase $\Delta T_t$. For this, we adopt the commonly used climate sensitivity function in Pindyck

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[15]Please note that this assumption brings the great benefit of considerably limiting the complexity of the numerical solution.

[16]Mitigation decisions are often modelled in a stylized and abstract way. Here, we assume a one-off decision. It remains to be seen whether more realistic assumptions can be accommodated without jeopardizing the main conclusions.

(2009, 2012) and Weitzman (2009a). The single linear differential equation compresses all the complex physical processes involved by capturing climate forcings and feedbacks in a simplified manner. Hence, a direct link between the atmospheric greenhouse gas concentration $G_t$ and the temperature increase $\Delta T_t$ is obtained by

$$d\Delta T_t = m_1 \left( \frac{\ln (G_t/G_p)}{\ln 2} - m_2 \Delta T_t \right) dt, \tag{2.3}$$

where $G_p$ is the inherited pre-industrial baseline level of greenhouse gas and $m_1$ and $m_2$ are positive parameters. The first term in the brackets stands for the radiative forcing induced by doubling of the atmospheric greenhouse gases. The second term represents the net of all negative and positive feedbacks. A positive parameter for this term thus rules out a runaway greenhouse effect. The parameter $m_1$ describes the thermal inertia or the effective capacity to absorb heat by the earth system, which is exemplified by the oceanic heat uptake.

Let $H$ define the considered time horizon. In the business-as-usual scenario, the maximal temperature increase is assumed to double the warming after $H$ years. This is tantamount to $\Delta T_t \to 2\Delta T_H$ for $t \to \infty$, which implies $2\Delta T_H = 1/m_2$ as the equilibrium and $m_1 m_2$ as the adjustment speed. The change in temperature increases linearly in the logarithm of greenhouse gas concentrations and thus $m_1 m_2 = \frac{\ln 2}{H}$. Cancelling terms and rearranging gives

$$d\Delta T_t = \frac{\ln (2)}{H} (2\Delta T_H - \Delta T_t) dt \tag{2.4}$$

and

$$\Delta T_t = 2\Delta T_H \left( 1 - e^{-\frac{\ln 2}{H} t} \right), \tag{2.5}$$

if the initial value $\Delta T_0$ is set to zero. Equation (2.4) is an essential building block in the real options modelling set-up, while equation (2.5) is useful for integrating the intertemporal climate change damage function.[17]

If the policy maker reduces emissions, a certain temperature target is assumed to be met after $H$ years, i.e. $\Delta T_H \leq \tau$. In this case, equations (2.4) and (2.5) are reshaped to

$$d\Delta T_t = \frac{\ln (2)}{H} (2\tau - \Delta T_t) dt, \tag{2.6}$$

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[17]The increase in temperature is generated by an unspecified natural science climate model. Ultimately, we take $\Delta T_t$ and thus the geophysical microfoundation from climate models and impose this mathematically upon our economic model. This allows us to bypass climate and atmospheric modelling.

and

$$\Delta T_t = 2\tau \left( 1 - e^{-\frac{\ln 2}{H}t} \right), \tag{2.7}$$

respectively.

Let us now focus on the other component of equation (2.2) describing the sensitivity of losses to global warming. The sensitivity of the future society's welfare to global warming depends to a large extent on the intrinsic uncertainty caused by the lack of knowledge of future habits, tastes and economies' ability to adapt to climate change. Particularly for long timescales, which are typically considered in climate economics, this intrinsic uncertainty increases. Intrinsic uncertainty is commonly assumed to follow a stochastic process such as geometrical Brownian motion with (deterministic) drift parameter $\alpha$ and standard deviation $\sigma$.

$$dX_t = \alpha X_t dt + \sigma X_t dB_t, \tag{2.8}$$

where $B$ is a standard Wiener process; see for example Pindyck (2000). The fluctuation of $X_t$ over time complicates considerably the decision on whether to exercise the real options of adopting the climate policy. Equation (2.8) allows one to trace the uncertainty transmission to optimal policies, as social welfare $W$ thus evolves as

$$
\begin{aligned}
W(X, \Delta T) &= E\left[ (1 - w(\tau)) \int_0^\infty e^{-X_t (\Delta T_t)^2} e^{-(r-g_0)t} dt \right] \\
&\cong E\left[ [(1 - w(\tau)) \int_0^\infty \left( 1 - X_t \Delta T_t^2 + \frac{1}{2} \left( X_t \Delta T_t^2 \right)^2 \right) e^{-(r-g_0)t} dt \right],
\end{aligned}
\tag{2.9}
$$

with a constant consumption/GDP growth rate of $g_0$ and the assumption that $r$ is greater than the expected consumption growth rate $g_0$. Note that the exponential loss function of $X_t$ renders an explicit analytical solution of the Ito-integral impossible. Therefore, we use second-order Taylor's expansions approximations in the numerical analysis below.

In the following, the decision on whether to curb emissions now is derived by comparing the value of action with the value of inaction. The welfare value of implementing the environmental policy now, denoted by $W^A(X, \Delta T; \tau) \equiv W^{\text{Action}}(X, \Delta T; \tau)$, is computed by equation (2.9) with $w(\tau) > 0$ and the temperature equation (2.7). After utilizing the relationship

$$E[X_t^n] = X_0^n e^{\left( n\alpha + \frac{1}{2} n(n-1)\sigma^2 \right)t}, \tag{2.10}$$

which is derived by means of Ito's Lemma, the welfare for taking action now evolves as

$$W^{A}\left(X, \Delta T; \tau\right) = \left(1 - w\left(\tau\right)\right)\left[\frac{1}{r - g_0} - 4\tau^2\gamma_1 X + 8\tau^4\gamma_2 X^2\right], \qquad (2.11)$$

where

$$\gamma_1 = \frac{1}{\eta_1} - \frac{2}{\eta_1 + \frac{\ln 2}{H}} + \frac{1}{\eta_1 + 2\frac{\ln 2}{H}},$$

$$\gamma_2 = \frac{1}{\eta_2} - \frac{4}{\eta_2 + \frac{\ln 2}{H}} + \frac{6}{\eta_2 + 2\frac{\ln 2}{H}} - \frac{4}{\eta_2 + 3\frac{\ln 2}{H}} + \frac{1}{\eta_2 + 4\frac{\ln 2}{H}},$$

$$\eta_1 = r - g_0 - \alpha,$$

and

$$\eta_2 = r - g_0 - \left(2\alpha + \sigma^2\right).$$

Note that it is assumed that both $\eta_1$ and $\eta_2$ are positive.[18]

Alternatively, the policy maker may want to continue to emit $CO_2$ at the same level and therefore $\Delta T_t$ becomes $\Delta T_H$ at $t = H$, but no mitigation costs are incurred, i.e. $w(\tau) = 0$. Applying the Hamilton-Jacobi-Bellman principle and Ito's Lemma to equation (2.9), we obtain the inaction value $W^{N}\left(X, \Delta T; \Delta T_H\right) \equiv W^{\text{No Action}}\left(X, \Delta T; \Delta T_H\right)$, which can be described by the corresponding partial differential equation. The solutions to $W^{N}\left(X, \Delta T; \Delta T_H\right)$ and hence the partial differential equation consist of two components, a particular solution and a general solution

$$W^{N}\left(X, \Delta T; \Delta T_H\right) = W^{NP}\left(X, \Delta T; \Delta T_H\right) + W^{NG}\left(X, \Delta T; \Delta T_H, t^*\right). \qquad (2.12)$$

Both solutions have a straightforward economic meaning. The business-as-usual policy is valued by the particular solution, which is derived by solving equation (2.9) with $w(\tau) = 0$:

$$W^{NP}\left(X, \Delta T; \Delta T_H\right) = \frac{1}{r - g_0} - 4\Delta T_H^2\gamma_1 X + 8\Delta T_H^4\gamma_2 X^2, \qquad (2.13)$$

where the parameters have the same forms as in equation (2.11).[19] Let $t^*$ denote the remaining amount of time to take action so that the climate target will not be transgressed. The value of the real options $W^{NG}\left(\bar{X}, \Delta T; \Delta T_H, t^*\right)$ is obtained from the homogenous part of the partial differential equation. As discussed in Section 2.2, the limited time to act implies the availability of real options of only a few years' time. This implies that at the end of $t^*$ years, $0 < t^* < H$, the real options value approaches zero. This focus upon optimal policies over $(0, t^*)$ reflects the largely irreversible build-up of $CO_2$ in the

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[18]Please see Appendix 2.A.

[19]Please see Appendix 2.A.

atmosphere and clearly deviates from the infinite horizon assumption that is assumed in almost all the variants of real options models. Though it is not possible to pin down exactly how many years are left for the policy maker to act before it is too late, we assume a fixed time of years left for the policy maker to pursue aggressive moves to curb emissions. The exact value of $t^*$ is considered to be given and out of the control of the policy maker. While it would be more realistic to endogenize $t^*$, we simply assume $t^*$ as exogenous and constant. The effects of different values for $t^*$ are elaborated on in the next section.

After tidying up for

$$W^{\mathrm{A}}\left(\bar{X}, \Delta T; \tau\right) = W^{\mathrm{NP}}\left(\bar{X}, \Delta T; \Delta T_H\right) + W^{\mathrm{NG}}\left(\bar{X}, \Delta T; \Delta T_H, t^*\right), \qquad (2.14)$$

the value-matching condition for the optimal stopping problem for the policy maker is represented by

$$4\gamma_1 \left[\Delta T_H^2 - (1 - w(\tau))\tau^2\right]\bar{X} - 8\gamma_2 \left[\Delta T_H^4 - (1 - w(\tau))\tau^4\right]\bar{X}^2$$
$$= \frac{w(\tau)}{r - g_0} + W^{\mathrm{NG}}\left(\bar{X}, \Delta T; \Delta T_H, t^*\right), \qquad (2.15)$$

where the two terms on the left-hand side denote the benefit of policy adoption. The first term on the right-hand side quantifies the necessary up-front investment (sunk costs) and $W^{\mathrm{NG}}\left(\bar{X}, \Delta T; \Delta T_H, t^*\right)$ denotes the non-perpetual real options. The value $\bar{X}$ describes the threshold at which the policy-maker exercises the real options today in order to limit the future temperature increase to less than $\tau$ at $t = H$. This decision necessitates the payment of the annual mitigation costs $w(\tau)$ as a percentage of the GDP. The sunk cost component of equation (2.15) reflects the irreversible commitment. As soon as the option to cut emissions is exercised, the opportunity to wait and act later as more information about $X_t$ becomes available is irreversibly lost.

We have now laid out an applicable analytical approach that directly addresses the issue of the limited time to act. It is well known that closed-form solutions for non-perpetual real options models usually do not exist.[20] Therefore, we seek a numerical solution. In the remainder of this paper, we perform a series of calibrations of this model.

## 2.4  Numerical Simulations of the Baseline Model

Formal theory is essential in enabling us to organize our knowledge about climate problems in a coherent and consistent way. However, the formal theory needs to be applied to data

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[20]See, for example, Hull (2010).

if it is to enhance our understanding and have relevance for practical problems. This calibration exercise will provide new insights and may thus contribute to climate policy discussions, which are certainly influenced by the limited time to act. For this purpose, we map the theoretical framework presented above to real-world data. Where possible, the parameter values are drawn from empirical studies. However, the determination of some parameters is somewhat speculative or they are drawn from back-of-the-envelope calculations.[21] Therefore, for each parameter, a sensitivity analysis over a sufficiently wide grid is performed, while keeping an eye on robustness. The unit time length corresponds to one year and annual rates are used when applicable. Our base parameters are chosen to be $\alpha = 0$, $\sigma = 0.075$, $r = 0.025$, $g_0 = 0.0$ and $H = 100$. The temperature increase $\Delta T_H$ is assumed to be 3.4°C, which is equivalent to 4°C of warming since the pre-industrial level. The target $\tau$ is assumed to be 1.4°C, which is equivalent to 2 degrees of warming compared with the pre-industrial level. In order to assess the mitigation costs, Edenhofer et al. (2010) examine the energy-environment-economy models MERGE, REMIND, POLES, TIMER and E3MG in a model comparison exercise.[22] Despite the different structures employed in the models, four of the five models show a similar pattern in mitigation costs for achieving the first-best 400 ppm $CO_2$ concentration pathway. The mitigation costs are estimated to be approximately 2 per cent of the worldwide GDP if the policy is adopted in the near term. These costs turn out to be of a similar order of magnitude across the models. We therefore assume that $w(\tau) = 0.02$.

As the sensitivity of losses $X_t$ fluctuates over time, we have to pay special attention to the magnitude of the resulting climate damage. As an illustration and in order to gain intuition, Figure 2.4.1 shows the numerically simulated percentage of damage $(1 - L)$ implied by the loss equation (2.2). The curves are derived by assuming three alternative constant $X$ terms in the temperature equation (2.4). The considered time period ranges from $t = 0$ to $t = 200$. Two effects must be recognized. Firstly, the minimum of $L(X_t, \Delta T_t)$ and therefore the maximum of GDP net of damages, $L(X_t, \Delta T_t)GDP_t$, is obtained for the lowest value of the drift term. Secondly, as can be easily seen in the graph, $L$ spreads out considerably during the time of undertaking no mitigation. For $t = 50$ years, the damage is 3.89 per cent of GDP for constant $X_t = 0.01$, 3.12 per cent of GDP for $X_t = 0.008$ and 2.35 per cent of GDP for $X_t = 0.006$. After $t = 100$ years, the corresponding damage is

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[21]Despite the increasingly detailed understanding of climate processes from a large body of research, various parameters involved remain inevitably unanswered except in retrospect.

[22]In order to improve model comparability, the macroeconomic drivers in the five modelling frameworks employed were harmonized to represent similar economic developments. On the other hand, different views of technology diffusion and different structural assumptions regarding the underlying economic system across the models remained. This helps to shed light on how different modelling assumptions translate into differences in mitigation costs.

10.92 per cent of GDP for $X_t = 0.01$, 8.83 per cent of GDP for $X_t = 0.008$ and 6.70 per cent of GDP for $X_t = 0.006$.[23]
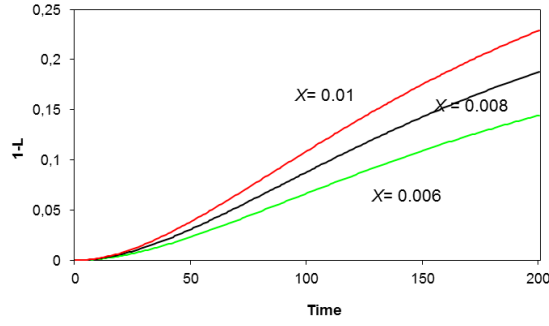


**Figure 2.4.1:** Simulated Loss Due To Global Warming as a Percentage of GDP

To simulate the full model, we have to solve the partial differential equation by utilizing an explicit finite difference method. To this end, it is transformed into a one-factor partial difference equation (see Appendix 2.B). The variable $X$ and the parameter $t$ need to be expressed as a network mesh of discrete points, $\Delta X$ and $\Delta t$. Afterwards, the partial differential equation can be displayed as a set of finite difference equations that are numerically solvable in a backward scheme and subject to corresponding discrete-time boundary conditions (see Appendix 2.C).[24] We use the following benchmark values for the explicit finite difference method: $X_{\max} = 0.05$, $\Delta t = 0.0001$, $\Delta X = 0.0002 \cdot \sigma$.[25]

We now solve for the optimal timing of mitigation. The following graphs show thresholds that split the space spanned by $X$ shocks into action and inaction areas. In the inaction area, the marginal reward for pursuing $CO_2$ reductions is insufficient and policy makers prefer to wait. The economic explanation for the thresholds $\bar{X}$ is straightforward. The index $X$ is part of the loss function. The smaller $\bar{X}$ is, the faster the policy response will be. For the sake of clarity, Figure 2.4.2 offers an isolated inspection of the impact of alternative time horizons upon the climate policy threshold for the baseline parameters. Broadly speaking, the results suggest that the limited time to act has a significant impact

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[23]These numbers are in the range of common assumptions in the literature. In Weitzman (2009b), the damage costs are calibrated to be 9 (25) per cent of the GDP for 4°C (5°C) of warming and Millner et al. (2010) consider damages of 6.5 per cent of the GDP for 5°C of warming.

[24]The first paper to recognize that option prices could be obtained with a finite difference solution to the partial differential equation was Schwartz (1977). The finite difference method proceeds by replacing differentials with differences and then solving over a grid of time and state variables subject to the boundary conditions. A thorough review of the state of the art in numerical finite difference techniques along with an exhaustive list of references is offered by Duffy (2006).

[25]The benchmark values of $\Delta t$ and $\Delta X$ are chosen to ensure a positive coefficient of equation (2.43) and convergence and stability for (2.42) in Appendix 2.C in an explicit finite difference method scheme.

upon the threshold for $t^* < 5$ years. In the case of a very small $t^*$, rational policy makers will pursue immediate measures to curb emissions. As outlined in Section 2.2, the current research indicates that the time left to reach the 2°C target would imply a rather low $t^*$. Accordingly, the results shown in Figure 2.4.2 elevate the urgency of climate change policies.[26]
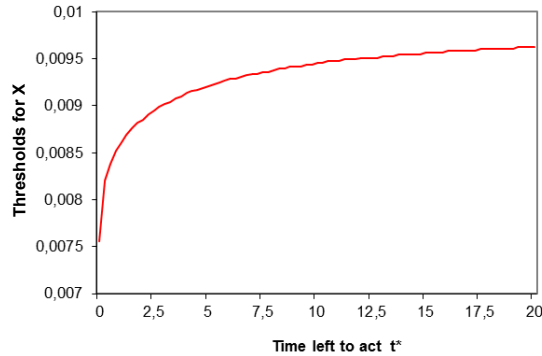


**Figure 2.4.2:** The Impact of Alternative Time Horizons $t^*$ Upon the $X$ Threshold

This result of curbing emissions aggressively contrasts the slow, incremental approach to $CO_2$ mitigation in reality and fits the urgency emphasized by Krugman (2010). He warns against relying on models that advocate delaying mitigation measures. For instance, the optimal policy in Nordhaus's cost-benefit model would stabilize the atmospheric carbon dioxide concentration at a level about twice its pre-industrial average, which is supposed to lead to a temperature of 3°C. Decreasing emissions are not required before 2045. This strategy has only modest negative effects on global welfare, according to the RICE model.[27] However, the crucial question arises of how trustworthy such a projection really is. On the one hand, the consequences of such warming are hardly predictable. On the other hand, looking back at historic experiences does not reveal information, as for most of the time span of human civilization the global climatic patterns have remained within a very narrow range. Hence, it cannot be taken for granted that such a policy will not cause a dangerous climate crisis.

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[26]However, a large caveat should accompany any use of that number because it assumes that the climate policies will be both efficient and effective. Obstacles to climate policy are exemplified by government failure, regulatory capture and the impact of rent-seeking behaviour within the policy process. Climate policy is likely to be a large source of economic rents from policy interventions. Note that this is an exploratory paper and is by no means intended to give blanket approval to any proposal for climate protection.

[27]Please see Nordhaus (2010) for more information.

The remaining problem of epistemic uncertainty can be approached in a relatively straightforward, although computationally expensive, manner. Epistemic uncertainty arises from a lack of knowledge regarding the true value of parameters and is typically specified by parameter perturbation. Since the observed data are not directly accessible, the only information about epistemic uncertainty available to the modeller is in the form of bounds of the parameter values. In the three-dimensional Figures 2.4.3a - 2.4.3c below, parameter variation is addressed using a "two-at-a-time" approach.[28] This shows how different parameter domains interact, and can indicate the parameters that have the greatest influence on climate policy responses.

In particular, the assessments of the climate damage costs exhibit a broad range of uncertainty and always lead to controversies.[29] Beyond the issue of the likely consequences of warming, it is debatable how non-market goods like human life and the intrinsic values of ecosystems are appropriately monetarized and how catastrophes that have a low probability but high impacts are included. Furthermore, the future capabilities for adapting to climate change are hardly predictable. By comparing 28 studies on marginal damages costs in different regions, Tol (2005c) emphasizes that the estimates give insights into the signs, orders of magnitude and patterns of vulnerability but remain speculative. To study the effects of uncertainty in the assessments, Figure 2.4.3a illustrates the results for different values of $\sigma$. It provides an important twist to the story by revealing the adverse effects of uncertainty on the policy makers' decision. The combination of the limited time to act and even moderate increases in uncertainty may make the rational policy response weaker, not stronger. The reason is that the benefits of waiting for uncertainty to dissipate overwhelm the cost of moving too slowly. Thus, rational policy makers will not necessarily behave prudently to keep nature from passing the 2°C threshold. Put differently, the high $\sigma$ - small $t^*$ constellation is a double-edged sword. For a high $\sigma$, the temptation to avoid tackling climate change is hard to resist, although climate science suggests that a steep near-term reduction in emissions is very likely to be needed.

Another substantial source of uncertainty is represented by the temperature increase $\Delta T_H$. The IPCC's first assessment, published back in 1990, predicted warming of 3°C by 2100, with no confidence bands. The second IPCC assessment, in 1995, suggested warm-

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[28]An alternative approach to implementing uncertainty and complexity in the model would be to use probability distributions weighting the resulting impacts accordingly. The development of such a computational demanding real options framework is beyond the scope of this paper.

[29]In a recent work, Maslin & Austin (2012) warn that up-to-date climate models offer improvements to our understanding of complex climate processes, but produce wider rather than smaller ranges of uncertainty. In other words, the understanding of climate change has become less, rather than more, clear over time.
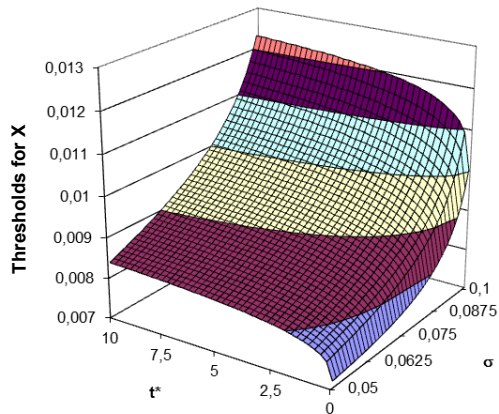
**(a)** Sensitivity to $t^*$ and $\sigma$
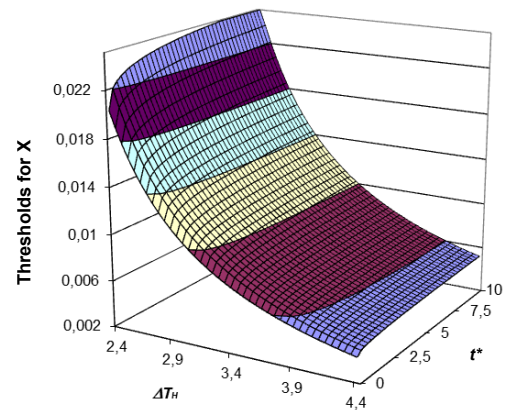


**(b)** Sensitivity to $t^*$ and $\Delta T_H$
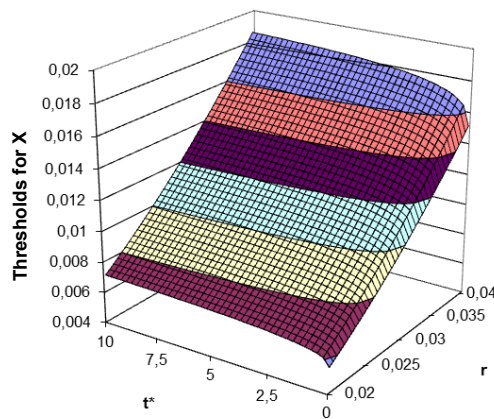


**(c)** Sensitivity to $t^*$ and $r$

**Figure 2.4.3:** Impacts on the X Threshold

ing of between 1°C and 3.5°C. The third, in 2001, widened the bands to project warming of 1.4°C to 5.8°C. The fourth assessment in 2007 restrained them again, from 1.8°C to 4.0°C. At the moment, it seems unlikely that the scientific uncertainty will be completely resolved in the near future. Quite the reverse, Kevin Trenberth (Head, National Center for Atmospheric Research in Boulder, Colorado) recently warned in a commentary in *Nature Online* (21 January 2010) headlined "More Knowledge, Less Certainty" that "the uncertainty in AR5's predictions and projections will be much greater than in previous IPCC reports."[30] The reason for this is that as "our knowledge of certain factors [responsible for global warming] does increase," he wrote, "so does our understanding of

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[30]See http://www.nature.com/climate/2010/1002/full/climate.2010.06.html.

factors we previously did not account for or even recognize."[31] In other words, there is still tremendous and in some cases even increasing uncertainty in the climate projections. Figure 2.4.3b outlines the joint impact of different temperature predictions $\Delta T_H$ and the time left to act $t^*$ to prevent the temperature from overshooting the 2°C target. For any $\Delta T_H$ the curve exhibits the same concave shape as in Figure 2.4.2. Hence, irrespective of the magnitude of the predicted temperature, a rational policy maker will take mitigation actions earlier for a small $t^*$. However, the effect of varying $\Delta T_H$ is enormous and has a greater influence on the optimal policy threshold than the limited time to act. The policy threshold $\overline{X}$ doubles in size when assuming $\Delta T_H = 2.9$°C instead of $\Delta T_H = 3.9$°C and it increases even more when taking $\Delta T_H = 2.4$°C instead of our base calibration $\Delta T_H = 3.4$°C. Hence, the decision about when to implement a climate policy is radically influenced by the projection of the temperature increase. As in reality broad uncertainty ranges of the temperature dynamics exist, this simulation highlights the huge problems involved in reaching a decision in favour of a mitigation strategy.[32] What is a reasonable estimation on which to base the climate policy decision?

Next, we take a closer look at the impact of the discount rate $r$. To explore the sensitivity to alternative discounting assumptions, we employ a range of $0.02 < r < 0.04$. As expected, the results in Figure 2.4.3c affirm the view that higher discount rates will bolster the reasons for adopting a "wait-and-see" attitude towards climate policy. This is due to the fact that for a larger value of $r$ the intertemporal damage is substantially smaller. In other words, a higher discounting factor will trigger later adoption and a lower intensity of climate policy. This highlights the importance of attaining a consensus on the discount rate before an appraisal of the optimal timing of policy implementation can be undertaken. Another important conclusion from Figure 2.4.3c is that the effects of a higher discount rate trumps the effects of the limited time to act.

To sum up, we may conclude that the knowledge of having a limited time to act should accelerate climate policy significantly, particularly if the window of opportunity will close very soon. However, ubiquitous uncertainties in the projections of the temperature increase and the future damage costs as well as the different opinions on the discount rate

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[31]Up-to-date climate models are trying to come to grips with a range of factors that have been ignored or only sketchily dealt with in the past. One troubling aspect is the role of clouds, because nobody can work out exactly whether warming will change them in a way that amplifies or moderates global warming. Another problem in understanding clouds is the role of aerosols, which dramatically influence the radiation properties of clouds. It therefore comes as no surprise that the resulting error bands are extremely wide.

[32]This raises the question of how much we can expect medium-run climate projections to improve. Can we reduce forecast errors? How much can uncertainty be reduced as models improve? Although climate models have improved and societal needs push for more accurate decadal climate projections over the next 10-30 years, decadal projections are still in their infancy and the prospect of useful decadal projections is far from assured [see Cane (2010)].

are shown to be of more importance to the decision. In particular, the uncertainties in the damage costs are demonstrated to have adverse effects. Despite the urgency to take action, this kind of uncertainty delays the implementation of a climate policy.

## 2.5 Modelling Stochastic Changes in Temperature

Analysing a complex and inherently uncertain problem such as climate change and global warming requires us to consider multiple forms of uncertainty, as well as the incomplete nature of different types of knowledge. Aside from intrinsic uncertainty in the damage function, the stochasticity and variability in the temperature evolution contribute to the complexity of the climate policy decision. We address this issue now by adding a mean-reverting stochastic term to the temperature equation while treating $X$ as constant. Ultimately, this means that the deterministic equations (2.4) and (2.6) are replaced by stochastic mean-reverting Ornstein-Uhlenbeck processes.[33] To bring clarity to the exposition that follows, we henceforth assume

$$d\Delta T_t = \frac{\ln(2)}{H}\left(2\Delta T_H - \Delta T_t\right)dt + \sigma dB_t \tag{2.16}$$

and

$$d\Delta T_t = \frac{\ln(2)}{H}\left(2\tau - \Delta T_t\right)dt + \sigma dB_t, \tag{2.17}$$

where equation (2.16) represents the stochastic process for possible paths of temperature changes in the business-as-usual scenario, and equation (2.17) describes the equivalent in the climate policy scenario. Both equations show bounded uncertainties for $\Delta T$.

As the expectation value of the exponential loss function, which comprises the mean-reverting processes (2.16) and (2.17), is not analytically obtainable, numerical procedures need to be employed to derive the solutions of the particular integrals $W^{\mathrm{NP}}(\Delta T; \Delta T_H)$ and $W^{\mathrm{A}}(\Delta T; \tau)$, respectively. The value-matching condition is then used to obtain the threshold temperature change $\overline{\Delta T}$, at which it is optimal to implement climate policy.

$$W^{\mathrm{A}}\left(\overline{\Delta T}; \tau\right) - W^{\mathrm{NP}}\left(\overline{\Delta T}; \Delta T_H\right) = W^{\mathrm{NG}}\left(t, \overline{\Delta T}; \Delta T_H, t^*\right), \tag{2.18}$$

---

[33]Alaton et al. (2002) study weather futures prices based on temperature indices using Ornstein-Uhlenbeck stochastic processes. Subsequent weather derivative pricing papers also take into consideration extended Ornstein-Uhlenbeck processes.

or explicitly

$$E\left[\int_0^\infty e^{-X_t(\Delta T_t)^2}e^{-(r-g_0)t}dt\,\middle|\,\overline{\Delta T}=\Delta T_{t=0},\tau\right]$$

$$-E\left[\int_0^\infty e^{-X_t(\Delta T_t)^2}e^{-(r-g_0)t}dt\,\middle|\,\overline{\Delta T}=\Delta T_{t=0},\Delta T_H\right]$$

$$=w(\tau)E\left[\int_0^\infty e^{-X_t(\Delta T_t)^2}e^{-(r-g_0)t}dt\,\middle|\,\overline{\Delta T}=\Delta T_{t=0},\tau\right]+W^{\mathrm{NG}}\left(t,\overline{\Delta T};\Delta T_H,t^*\right),$$

$$(2.19)$$

where

$$W^{\mathrm{A}}\left(\Delta T;\tau\right)=(1-w(\tau))\,E\left[\int_0^\infty e^{-X_t(\Delta T_t)^2}e^{-(r-g_0)t}dt\,\middle|\,\tau\right],\qquad(2.20)$$

$$W^{\mathrm{NP}}\left(\Delta T;\Delta T_H\right)=E\left[\int_0^\infty e^{-X_t(\Delta T_t)^2}e^{-(r-g_0)t}dt\,\middle|\,\Delta T_H\right],\qquad(2.21)$$

and $W^{\mathrm{NG}}\left(t,\Delta T;\Delta T_H,t^*\right)$ describes the non-perpetual real options value.[34] The left-hand side of equation (2.19) denotes the benefit of mitigation, while the right-hand side represents the present value of the sunk costs and the value of the real options sacrificed due to curbing greenhouse gas emissions.

We use the same benchmark values as in the previous section, where possible. The constant value of $X$ is now set to 0.08, as it is no longer a stochastic variable. The risk parameter $\sigma$ in equations (2.16) and (2.17) needs some consideration. The variance of the process is given by

$$\sigma_{\Delta T}^2=\mathrm{Var}\left[\Delta T_t\right]=\frac{\sigma^2}{2\frac{\ln(2)}{H}}\left(1-e^{-2\frac{\ln(2)}{H}t}\right).\qquad(2.22)$$

The accurate specification of the process that the temperature follows over time is a prerequisite for the analysis. To this end, we evaluate the stochastic process under scrutiny for a wide range of parameters. In particular, we turn our attention to the accuracy of the calibrated temperature dynamics at longer horizons. The result is that the grid $0.1<\sigma<0.25$ yields a reasonable domain of temperature variability. Therefore, as the benchmark value, we take $\sigma=0.175$.[35]

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[34]As a practical matter, though, the numerical solution of the model using finite difference methods becomes computationally very challenging. See Appendix 2.D.

[35]The parametrization is also broadly consistent with the findings by Luterbacher et al. (2004) and
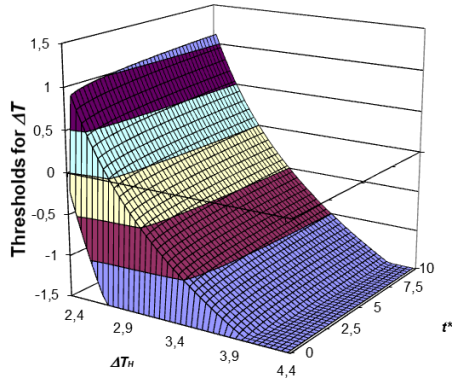
**(a)** Sensitivity to $t^*$ and $\Delta T_H$
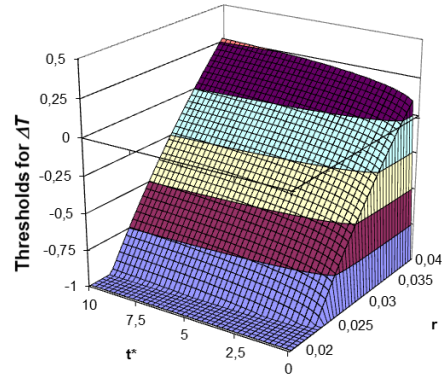


**(b)** Sensitivity to $t^*$ and $r$
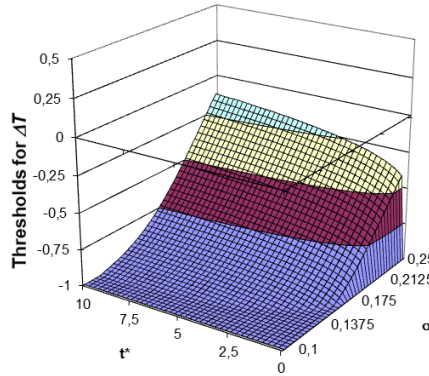


**(c)** Sensitivity to $t^*$ and $\sigma$

**Figure 2.5.1:** Impacts on the X Threshold

However, solving the stochastic model at fixed values of the model parameters results in estimates of optimal policies that do not take into account the uncertainty in the model input parameters (known as epistemic uncertainty).[36] Therefore, we again explore robustness via parameter perturbation. In keeping with Figure 2.4.3, Figure 2.5.1 illustrates the sensitivity of the optimal climate policy over a range of parameters for the predicted temperature increase $\Delta T_H$ and the discount rate $r$. Additionally, the impacts of different assumptions on the temperature variability $\sigma$ is examined. Whenever the

Swanson et al. (2009) regarding the magnitude of the temperature variability.

[36]Most economic problems involve epistemic as well as aleatoric uncertainty. In the modelling phase, sometimes it may be difficult to determine whether a particular uncertainty should be put into one category or another. This may raise the philosophical question of whether there is any aleatoric or epistemic uncertainty at all. Clearly, this question does not make sense outside the model universe. From an economic policy point of view, all uncertainties are the same as a lack of knowledge. This supports the case for a rather agnostic view of the sources of uncertainty.

threshold of taking action $\overline{\Delta T}$ assumes negative values, the policy maker adopts the climate policy instantaneously. Conversely, positive threshold values mean that the policy maker should wait for more information to arrive in the future and keep the real option alive. Figure 2.5.1 confirms that the limited time to act accelerates climate policy, and that this effect is outweighed by small changes in $\Delta T_H$, $r$ and $\sigma$.[37] Hence, despite the bounded uncertainty for $\Delta T$ in (2.16) and (2.17), both specifications of stochasticity give the same qualitative results. This can be interpreted as evidence of the robustness and structural validity of the results in Section 2.4 above.[38]

## 2.6  Conclusions

Recent scientific studies by IEA (2011), Meinshausen et al. (2009) and Steinacher et al. (2013) indicate that global greenhouse gas emissions need to be substantially reduced in upcoming years, in order to limit global warming to 2°C. This motivated us to investigate the climate policy implications of a time-limited window of opportunity from a real options perspective. Real options quantify the opportunity costs of adopting a policy now and making the involved irreversible investments rather than waiting for new information to arrive. As shown by Pindyck (2000), the option to wait has a positive value as long as the uncertainty is not completely resolved, which implies that the policy maker waits longer before undertaking emission reduction efforts. In this paper, a non-perpetual real options framework is developed to investigate whether the closing window of opportunity significantly reduces the value of waiting and thus accelerates mitigation.

In describing a highly complex picture, we focus on two sorts of uncertainty: stochasticity in the climate damage costs and in the temperature evolution. Furthermore, the robustness of the results with respect to some key model parameters is examined. A unifying message from mapping out different layers of uncertainty could be that policy makers have to take steps to cut emissions now, so that a radical, hasty and extremely costly shift towards carbon-neutral alternatives is not necessarily required. Although a global shift in energy- and carbon-intense investment patterns is required to prevent a

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[37]Note that the mean-reverting processes in (2.16) and (2.17) contribute to the abrupt decrease in the thresholds for a very small $t^*$. The term $(\ln(2)/H)\,(2\Delta T_H - \Delta T)$ increases with a negative $\Delta T$, leading to a fast rise in real options values when negative $\Delta T$ are presented. Such an effect is smaller when the thresholds for $\Delta T$ are positive, as shown in Figure 2.5.1. Furthermore, the uncertainty for geometrical Brownian motion is smallest when $X$ is small in Section 2.4, while uncertainty for equations (2.16) and (2.17), the term $dB$, does not depend on the value of $\Delta T$. Both of these reasons contribute to the big jumps in the thresholds for a very small $t^*$.

[38]This usage of "robustness" should not be confused with the concept of robustness in the econometric literature, which refers to the insensitivity of the estimated coefficients to adding or removing sample observations.

long-term high-carbon lock-in, the policy makers will probably not take drastic action in the near future. Ubiquitous uncertainties in the projections of the temperature increase and the future damage costs as well as the different opinions for discounting the future consumption flows affect the decision considerably. In particular, the uncertainties in the damage costs are shown to have adverse effects. Despite the urgency of taking action, this kind of uncertainty may lead to a range of inaction, in which the policy makers prefer to postpone emission reductions. Instead of saying "there is not much time left", we unfortunately may have to note: "time is running out". That, in a nutshell, is the dilemma of climate change.

APPENDICES

## 2.A Derivation of Equation (2.11) and Equation (2.13)

By applying Ito's Lemma to the logarithm of $X_t$ in equation (2.8), we obtain $\forall\, t \geq 0$ :

$$X_t = X_0 e^{(\alpha - \frac{1}{2}\sigma^2)t + \sigma B_t}. \tag{2.23}$$

After raising equation (2.23) to the power of $n$, the application of the expectation value yields

$$
\begin{aligned}
E[X_t^n] &= X_0^n e^{n(\alpha - \frac{1}{2}\sigma^2)t} E[e^{n\sigma B_t}] \\
&= X_0^n e^{(n\alpha - \frac{1}{2}n\sigma^2)t} e^{\frac{1}{2}n^2\sigma^2 t} \\
&= X_0^n e^{\left(n\alpha + \frac{1}{2}n(n-1)\sigma^2\right)t}.
\end{aligned}
\tag{2.24}
$$

This relationship is utilized to compute equation (2.9) for a climate policy:

$$
\begin{aligned}
&W^{\mathrm{A}}\left(X, \Delta T; \tau\right) \\
&= E\left[(1 - w(\tau)) \int_0^\infty \left(1 - X_t \Delta T_t^2 + \frac{1}{2}\left(X_t \Delta T_t^2\right)^2\right) e^{-(r - g_0)t} dt\right] \\
&= E\left[(1 - w(\tau)) \int_0^\infty \left(1 - X_t 4\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 \right.\right. \\
&\qquad\qquad\qquad\qquad \left.\left. + \frac{1}{2}X_t^2 16\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r - g_0)t} dt\right] \\
&= (1 - w(\tau)) \int_0^\infty \left(1 - 4E[X_t]\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 \right. \\
&\qquad\qquad\qquad\qquad \left. + 8E[X_t^2]\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r - g_0)t} dt \\
&= (1 - w(\tau)) \int_0^\infty \left(1 - 4X_0 e^{\alpha t}\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 \right. \\
&\qquad\qquad\qquad\qquad \left. + 8X_0^2 e^{(2\alpha + \sigma^2)t}\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r - g_0)t} dt.
\end{aligned}
\tag{2.25}
$$

The second equality holds as the conducting of a climate policy is assumed to put the temperature equation (2.7) into effect. The third equality is obtained by applying Fubini's theorem before rearranging and taking advantage of the monotonicity of the expectation value and the last equality holds due to equation (2.24). By expanding the terms

$$\left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 = 1 - 2e^{-\frac{\ln 2}{H}t} + e^{-2\frac{\ln 2}{H}t} \tag{2.26}$$

and

$$\left(1 - e^{-\frac{\ln 2}{H}t}\right)^4 = 1 - 4e^{-\frac{\ln 2}{H}t} + 6e^{-2\frac{\ln 2}{H}t} - 4e^{-3\frac{\ln 2}{H}t} + e^{-4\frac{\ln 2}{H}t}, \qquad (2.27)$$

we obtain after integrating

$$W^{\mathrm{A}}(X, \Delta T; \tau) = (1 - w(\tau)) \left[ \frac{1}{r - g_0} - 4\tau^2 X_0 \left( \frac{1}{\eta_1} - \frac{2}{\eta_1 + \frac{\ln 2}{H}} + \frac{1}{\eta_1 + 2\frac{\ln 2}{H}} \right) \right.$$
$$\left. + 8\tau^4 X_0 \left( \frac{1}{\eta_2} - \frac{4}{\eta_2 + \frac{\ln 2}{H}} + \frac{6}{\eta_2 + 2\frac{\ln 2}{H}} - \frac{4}{\eta_2 + 3\frac{\ln 2}{H}} + \frac{1}{\eta_2 + 4\frac{\ln 2}{H}} \right) \right],$$
$$(2.28)$$

where

$$\eta_1 = r - g_0 - \alpha$$

and

$$\eta_2 = r - g_0 - \left(2\alpha + \sigma^2\right),$$

which is the same as equation (2.11).

Please note that the welfare value of the business-as-usual policy $W^{\mathrm{NP}}$ evolves in an analogical way. Hence, its solution is the same but with $w(\tau) = 0$ and equation (2.5), which gives equation (2.13).

## 2.B    Derivation of the One-Factor Partial Differential Equation for Non-Perpetual Real Options

The corresponding partial differential equation to equation (2.9) for the case of business-as-usual is denoted by the following Bellman equation by Ito's Lemma:

$$(r - g_0 - \alpha) W^{\mathrm{N}} = \left(1 - X_t \Delta T_t^2 + \frac{1}{2} \left(X_t \Delta T_t^2\right)^2\right) + \left(\frac{\ln(2)}{H} \left(2\Delta T_H - \Delta T_t\right)\right) \frac{\partial W^{\mathrm{N}}}{\partial \Delta T} \quad (2.29)$$
$$+ \alpha X \frac{\partial W^{\mathrm{N}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{N}}}{\partial X^2} + \frac{\partial W^{\mathrm{N}}}{\partial t}.$$

It is the usual practice in financial derivatives that a two-factor partial differential equation (2.29) can be solved by two-dimensional finite difference methods. However, we can use the method of separation of variables to reduce (2.29) into a one factor partial differential equation, as we know that the non-perpetual real options are related to the diffusion

process $X$. Without the stochastic process in equation (2.9), the real options terms do not exist. On the contrary, the policy maker considers the process $\Delta T_t$ as an exogenous variable in the business-as-usual case. Furthermore, the particular solution to equation (2.13) implies that the solutions to equation (2.29) consist of the mathematical product of two different components: one for $X_t$ and the other for $\Delta T_t$. The discussion indicates that we can use the method of separation of variables to solve and simplify equation (2.29).

$$W^{\mathrm{NG}} = f\left(\Delta T\right) Y\left(X, t\right). \tag{2.30}$$

Substituting (2.30) back into equation (2.29) yields

$$(r - g_0 - \alpha) f\left(\Delta T\right) Y\left(X, t\right) = \frac{\ln\left(2\right)}{H}\left(2\Delta T_H - \Delta T\right) Y\left(X, t\right) \frac{df\left(\Delta T\right)}{d\Delta T} + \alpha X f\left(\Delta T\right) \frac{\partial Y\left(X, t\right)}{\partial X}$$
$$+ \frac{1}{2}\sigma^2 X^2 f\left(\Delta T\right) \frac{\partial^2 Y\left(X, t\right)}{\partial X^2} + f\left(\Delta T\right) \frac{\partial Y\left(X, t\right)}{\partial t}. \tag{2.31}$$

Dividing both sides by $f\left(\Delta T\right)$, we obtain

$$(r - g_0 - \alpha) Y\left(X, t\right) = \frac{\ln\left(2\right)}{H}\left(2\Delta T_H - \Delta T\right) \frac{Y\left(X, t\right)}{f\left(\Delta T\right)} \frac{df\left(\Delta T\right)}{d\Delta T}$$
$$+ \alpha X \frac{\partial Y\left(X, t\right)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 Y\left(X, t\right)}{\partial X^2} + \frac{\partial Y\left(X, t\right)}{\partial t}. \tag{2.32}$$

To make the partial differential equation (2.32) solvable by the separation of variables, $\frac{\ln(2)}{H}\left(2\Delta T_H - \Delta T\right) \frac{1}{f(\Delta T)} \frac{df(\Delta T)}{d\Delta T}$ has to be a constant linear term. This implies that the solutions of $f\left(\Delta T\right)$ take the form

$$f\left(\Delta T\right) = \left(2\Delta T_H - \Delta T\right)^2 \tag{2.33}$$

and

$$\frac{\ln\left(2\right)}{H}\left(2\Delta T_H - \Delta T\right) \frac{Y\left(X, t\right)}{f\left(\Delta T\right)} \frac{df\left(\Delta T\right)}{d\Delta T} = -2\frac{\ln\left(2\right)}{H} Y\left(X, t\right). \tag{2.34}$$

Equation (2.34) ensures the separation of equations and yields the following new partial differential equation for $Y\left(X, t\right)$ by substituting (2.34) back into (2.32):

$$\left(r - g_0 - \alpha + 2\frac{\ln\left(2\right)}{H}\right) Y\left(X, t\right) = \alpha X \frac{\partial Y\left(X, t\right)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 Y\left(X, t\right)}{\partial X^2} + \frac{\partial Y\left(X, t\right)}{\partial t}. \tag{2.35}$$

Therefore, we obtain the solution

$$W^{\mathrm{NG}} = \left(2\Delta T_H - \Delta T\right)^2 Y\left(X, t\right), \tag{2.36}$$

where $Y(X, t)$ follows equation (2.35). The results are similar to Chen et al. (2011a) apart from the fact that equation (2.35) has the term $\partial Y / \partial t$ due to the "limited time to act" real options. Equation (2.35) can be solved by numerical methods, such as finite difference methods. By combining equations (2.35) and (2.36), we then obtain the desired one-factor partial differential equation for non-perpetual real options:

$$\left( r - g_0 - \alpha + 2\frac{\ln(2)}{H} \right) W^{\mathrm{NG}} = \alpha X \frac{\partial W^{\mathrm{NG}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} + \frac{\partial W^{\mathrm{NG}}}{\partial t}. \quad (2.37)$$

The main difference between (2.29) and (2.37) is the transition of the term $\left( \frac{\ln(2)}{H} (2\Delta T_H - \Delta T_t) \right) \frac{\partial W^{NG}}{\partial \Delta T_t}$ in equation (2.29) into the higher effective discount rate of equation (2.37), increased by a factor of $2\frac{\ln(2)}{H} \cong 1.39\%$ for $H = 100$. The meaning is straightforward, as higher changes in temperature in the future lead to a lower intertemporal value of consumption and GDP. This is equivalent to lower real options values being caused by higher effective discount rates.

## 2.C  EXPLICIT FINITE DIFFERENCE METHOD SCHEME FOR EQUATION (2.37)

For real options with maturity $t^*$, the boundary conditions are

$$W^{\mathrm{NG}}(t, X_t = 0, \Delta T_t) = 0 \quad (2.38)$$

and

$$\lim_{x \to \infty} W^{\mathrm{NG}}(t, X_t = x, \Delta T_t)$$
$$= \max \left[ \lim_{x \to \infty} \left( W^{\mathrm{A}}(t, X_t = x, \Delta T_t; \tau) - W^{\mathrm{NP}}(t, X_t = x, \Delta T_t; \Delta T_H) \right), 0 \right], \quad (2.39)$$

where $W^{\mathrm{A}}(t, X, \Delta T; \tau)$ and $W^{\mathrm{NP}}(t, X, \Delta T; \Delta T_H)$ are from equations (2.11) and (2.13), respectively. The terminal condition is

$$W^{\mathrm{NG}}(t = t^*, X_{t^*}, \Delta T_{t^*}) = 0, \quad (2.40)$$

which is used as the starting points as the explicit finite difference method is backwards computing from $t = t^*$ to $t = 0$. The condition of

$$W^{\mathrm{NG}}(t, X_t, \Delta T_t) = \max \left[ W^{\mathrm{A}}(t, X_t, \Delta T_t; \tau) - W^{\mathrm{NP}}(t, X_t, \Delta T_t; \Delta T_H), 0 \right] \quad (2.41)$$

is checked for every $t$ since it is a free-boundary condition for real options in a sense that real options can be exercised at any time. Accordingly, equation (2.37) for real options

$W^{\mathrm{NG}}$ can be approximated by a function that is defined on a following two-dimensional grid, i.e. $W^{\mathrm{NG}}\left(i\Delta t, j\Delta X\right) \equiv v_{i,j}$. For the explicit approximation, the partial derivatives are approximated by

$$\frac{\partial W^{\mathrm{NG}}}{\partial X} = \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta X}, \tag{2.42}$$

$$\frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} = \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta X^2}, \tag{2.43}$$

$$\frac{\partial W^{\mathrm{NG}}}{\partial t} = \frac{v_{i+1,j} - v_{i,j}}{\Delta t}. \tag{2.44}$$

Substituting the above equations back into equation (2.37) yields

$$\left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right) v_{i,j} = \alpha j \Delta X \left(\frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta X}\right)$$
$$+ \frac{1}{2}\sigma^2 j^2 \Delta X^2 \left(\frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta X^2}\right) + \left(\frac{v_{i+1,j} - v_{i,j}}{\Delta t}\right). \tag{2.45}$$

Finally, rearranging and simplifying further allows us to obtain

$$v_{i,j} = a_j^* v_{i+1,j-1} + b_j^* v_{i+1,j} + c_j^* v_{i+1,j+1}, \tag{2.46}$$

where

$$a_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t} \left(-\frac{1}{2}\alpha j \Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right), \tag{2.47}$$

$$b_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t} \left(1 - \sigma^2 j^2 \Delta t\right), \tag{2.48}$$

$$c_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t} \left(\frac{1}{2}\alpha j \Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right). \tag{2.49}$$

As an analogue to equation (2.25), $W^{\mathrm{A}}\left(t, X_t, \Delta T_t; \tau\right)$ and $W^{\mathrm{NP}}\left(t, X_t, \Delta T_t; \Delta T_H\right)$ can be expressed by the following equations:

$$W^{\mathrm{A}}\left(X_t, \Delta T_t; \tau\right) = \left(1 - w\left(\tau\right)\right) \int_0^\infty e^{-(r-g_0)t} E\left[\left(1 - 4X_t \tau^2 \left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right.\right.$$
$$\left.\left. + \frac{1}{2}\left(4X_t \tau^2 \left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right)^2\right)\right] dt \quad (2.50)$$

and

$$W^{\mathrm{NP}}\left(X_t, \Delta T_t; \Delta T_H\right) = \int_0^\infty e^{-(r-g_0)t} E\left[\left(1 - 4X_t \Delta T_H^2 \left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right.\right.$$
$$\left.\left. + \frac{1}{2}\left(4X_t \Delta T_H^2 \left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right)^2\right)\right] dt, \quad (2.51)$$

where the term $(t + j\Delta t)$ reflects the temperature at time $= (t + j\Delta t)$ when computing the pay-offs for real options. Solving equations (2.51) and (2.50) is very time-consuming since we need to compute the integrals at each time step backwards. Note that equation (2.5) shows that the early temperature increase is not great for a small $t$. Furthermore, as we compute the values of $W^{\mathrm{NP}}\left(X_t, \Delta T_t; \Delta T_H\right)$ and $W^{\mathrm{A}}\left(X_t, \Delta T_t; \tau\right)$ backwards at each step of time from $t = t^*$ to $t = 0$, $(t + j\Delta t)$ approaching $(t = 0)$ for the final values of real options, which means that at $t = 0$, (2.50) and (2.51) become

$$W^{A}\left(X_t, \Delta T_t; \Delta T_H\right) \cong \left(1 - w\left(\tau\right)\right)\left[\frac{1}{r - g_0} - 4\Delta\tau^2 \gamma_1 X + 8\Delta\tau^4 \gamma_2 X^2\right], \quad (2.52)$$

$$W^{\mathrm{NP}}\left(X_t, \Delta T_t; \tau\right) \cong \frac{1}{r - g_0} - 4\Delta T_H^2 \gamma_1 X + 8\Delta T_H^4 \gamma_2 X^2, \quad (2.53)$$

which are the same as in equations (2.11) and (2.13). Numerical testing shows that using (2.52) and (2.53), time-invariant results, for the time from $t = T$ to $t = 0$ gives almost the same numerical results as using (2.50) and (2.51). The threshold for $\bar{X}_t$ at time $t = 0$ is then obtained from the above algorithm by checking numerically the points where equation (2.15) holds.

## 2.D Numerical Schemes for Solving $W^{\mathrm{NP}}$, $W^{\mathrm{A}}$ and $W^{\mathrm{NG}}$ in Section 2.5

For the stochastic processes (2.16) and (2.17) of temperature changes, $W^{\mathrm{NP}}$, $W^{\mathrm{A}}$ and $W^{\mathrm{NG}}$ satisfy the Bellman equations

$$(r - g_0) W^{\mathrm{NP}} = e^{-X\Delta T^2} + \frac{\ln(2)}{H} (2\Delta T_H - \Delta T) \frac{\partial W^{\mathrm{NP}}}{\partial \Delta T} + \frac{1}{2}\sigma^2 \frac{\partial^2 W^{\mathrm{NP}}}{\partial \Delta T^2}, \quad (2.54)$$

$$(r - g_0) W^{\mathrm{A}} = e^{-X\Delta T^2} + \frac{\ln(2)}{H} (2\tau - \Delta T) \frac{\partial W^{\mathrm{A}}}{\partial \Delta T} + \frac{1}{2}\sigma^2 \frac{\partial^2 W^{\mathrm{A}}}{\partial \Delta T^2}, \quad (2.55)$$

$$(r - g_0) W^{\mathrm{NG}} = \frac{\ln(2)}{H} (2\Delta T_H - \Delta T) \frac{\partial W^{\mathrm{NG}}}{\partial \Delta T} + \frac{1}{2}\sigma^2 \frac{\partial^2 W^{\mathrm{NG}}}{\partial \Delta T^2} + \frac{\partial W^{\mathrm{NG}}}{\partial t}. \quad (2.56)$$

Equations (2.54) and (2.55) are both second-order ordinary differential equations and can be solved by various numerical methods. However, the exponential loss function $e^{-X\Delta T^2}$ causes some difficulties, because many methods require initial (or terminal) conditions for the differential equation and its derivative. The only known initial/terminal conditions for equations (2.54) and (2.55) relate to $\Delta T$ approaching either positive or negative infinity. To make matters worse, the solutions generated by the usual methods, such as the Runge-Kutta methods, are very sensitive to the choice of the initial/terminal points and the corresponding slopes. To obtain stable numerical solutions to equations (2.54) and (2.55), we utilize a slow iterative method used in solving differential equations – a central finite difference scheme with boundary conditions. Information about the derivative of $W$ with respect to temperature changes is not needed.

The derivatives are proxied by the following central finite differences:

$$\frac{\partial W^{\mathrm{NP}}}{\partial \Delta T} = \frac{v_{j+1} - v_{j-1}}{2\Delta Z}, \quad (2.57)$$

$$\frac{\partial^2 W^{\mathrm{NP}}}{\partial \Delta T^2} = \frac{v_{j+1} + v_{j-1} - 2v_j}{\Delta Z^2}, \quad (2.58)$$

where $W^{\mathrm{NP}} (\Delta T_j) \equiv v_j$. Substituting (2.57) and (2.58) into equation (2.54) gives

$$rv_j = e^{-\beta(\Delta T_j)^2} + \frac{\ln(2)}{H} (2T_H - \Delta T_j) \left( \frac{v_{j+1} - v_{j-1}}{2\Delta Z} \right) + \frac{1}{2}\sigma^2 \left( \frac{v_{j+1} + v_{j-1} - 2v_j}{\Delta Z^2} \right). \,(2.59)$$

Rearranging and collecting the terms give

$$v_j = \frac{\Delta Z^2 e^{-\beta(\Delta T_j)^2} + \frac{\ln(2)}{H} (2T_H - \Delta T_j) \Delta Z (v_{j+1} - v_{j-1}) + \frac{1}{2}\sigma^2 (v_{j+1} + v_{j-1})}{r\Delta Z^2 + \sigma^2} \quad (2.60)$$

The procedure for solving (2.60) is as follows:

1. Step 1: The initial selection of the starting values for all $v_j$ with two boundary conditions: $v_1 = 0$ and $v_{N+1} = 0$. To raise the speed of the iterative method, we obtain the initial values of $v_j$ by setting $\sigma = 0$, i.e. $v_j = \int_0^\infty e^{-\beta \Delta T_t^2} e^{-(r-g_0)t} dt$, with $\Delta T_t = e^{-\frac{\ln 2}{H} t} \left( \Delta T_j - 2\Delta T_H \left( 1 - e^{\frac{\ln 2}{H} t} \right) \right)$, where $\Delta T_j$ is the initial value of temperature changes.

2. Step 2: The systematic running of equation (2.60) over all the initial grid points, the setting of the new values of $v_j$ as initial points and the iterative running of the process over the grid points. After iterations, the values of $v_j$ approach the approximations of the problem.

The finite difference method/scheme is slow but stable approaching the solutions. We use the following values for numerical simulations: $\Delta Z = 0.0025$, $v_1 = v(\Delta T = -12) = 0$, $v_N = v(\Delta T = 12) = 0$. The number of iterations is 500,000. Equation (2.55) can be operationalized using the same procedure.

After approximating $W^{\mathrm{NP}}$, $W^{\mathrm{A}}$, we can focus our attention on the real option $W^{\mathrm{NG}}$ and the value-matching condition, equation (2.18). The scheme is similar to the one in Appendix 2.C. Therefore, in what follows, we only show the main equations. For the explicit finite difference approximation, the partial derivatives are approximated by

$$\frac{\partial W^{\mathrm{NG}}}{\partial \Delta T} = \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta Y}, \tag{2.61}$$

$$\frac{\partial^2 W^{\mathrm{NG}}}{\partial \Delta T^2} = \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta Y^2}, \tag{2.62}$$

$$\frac{\partial W^{\mathrm{NG}}}{\partial t} = \frac{v_{i+1,j} - v_{i,j}}{\Delta t}. \tag{2.63}$$

where $Y \equiv \Delta T$. Substituting the above equations back into equation (2.56) yields

$$
\begin{aligned}
(r - g_0) v_{i,j} = & \frac{\ln(2)}{H} (2T_H - Y_j) \left( \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta Y} \right) \\
& + \frac{1}{2}\sigma^2 \left( \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta Y^2} \right) + \left( \frac{v_{i+1,j} - v_{i,j}}{\Delta t} \right).
\end{aligned} \tag{2.64}
$$

Rearranging and collecting the terms gives

$$v_{i,j} = a_j^* v_{i+1,j-1} + b_j^* v_{i+1,j} + c_j^* v_{i+1,j+1}, \tag{2.65}$$

where

$$a_j^* = \frac{1}{1 + (r - g_0)\,\Delta t} \left( \frac{\frac{1}{2}\sigma^2 \Delta t}{\Delta Y^2} - \frac{\frac{\ln(2)}{H}\,(2T_H - Y_j)\,\Delta t}{2\Delta Y} \right), \tag{2.66}$$

$$b_j^* = \frac{1}{1 + (r - g_0)\,\Delta t} \left( 1 - \frac{\sigma^2 \Delta t}{\Delta Y^2} \right), \tag{2.67}$$

$$c_j^* = \frac{1}{1 + (r - g_0)\,\Delta t} \left( \frac{\frac{\ln(2)}{H}\,(2T_H - Y_j)\,\Delta t}{2\Delta Y} + \frac{\frac{1}{2}\sigma^2 \Delta t}{\Delta Y^2} \right), \tag{2.68}$$

While the rest of the computation procedure is similar to Appendix 2.C, there is one difference in the boundary conditions. The temperature at which the real option becomes worthless needs to be very low. We use $\Delta T_{\min} = -6$ as such a boundary condition:

$$W^{\mathrm{NG}}\left(t, \Delta T_t \to \Delta T_{\min}\right) = 0. \tag{2.69}$$

This assumption proves to be adequate, as shown by the numerical simulations.

*All scientific work is incomplete - whether it be observational or experimental. All scientific work is liable to be upset or modified by advancing knowledge. That does not confer upon us a freedom to ignore the knowledge we already have, to postpone action that it appears to demand at a given time.*

Sir Austin Bradford Hill,
Epidemiologist and Statistician

# 3

# Dark Clouds or Silver Linings? Knightian Uncertainty and Climate Change[1]

## 3.1 Introduction

The future dynamics of greenhouse gas emissions, and their implications for global climate conditions in the future, will be shaped by the way in which policy makers respond to climate projections, react to model uncertainty, and derive resultant mitigation and adaptation decisions. However, assessments of the future impacts of climate change, which shall provide the basis of a climate policy decision, are far from being conclusive. A considerable lack of scientific understanding and uncertainties about the future economic development lead to enormous ambiguities in the projections. Accordingly, the question of how to design an optimal climate policy causes huge controversies. It could be reasonable to wait for new information to arrive before taking action. An example for this procedure is given by the former U.S. president G.W. Bush's strategy. The key idea was to promote climate research with the aim to close the relevant gaps in knowledge before devising and adopting policy.[2] Critics argue that the peril of serious and irreversible climate damages

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[1]This chapter is co-authored by Yu-Fu Chen and Michael Funke.

[2]See, for example, G.W. Bush's announcement on June 11 in 2001, which is available on the webpage georgewbush-whitehouse.archives.gov.

may necessitate to take instantaneous and preventive action. This approach to the climate problem resembles the precautionary principle, which is for example endorsed by the European Union and the Rio Declaration on Environment and Development. Principle 15 in the 1992 Rio Declaration on Environment and Development states that due to threats of serious and irreversible damage, "lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation."

To provide an economic foundation for this discussion, analysing the rational decision under fundamental uncertainties has gained importance over the last years. The absence of knowledge implies that the decision maker is not capable of assigning specific probabilities to events. This contrasts the situation in which sufficient statistical information is available to describe the probability of an event by the relative frequency of its occurrence over time. Knight (1921) classifies the first case as ambiguity and the latter as risk. The importance of distinguishing between both notions of uncertainty has been acknowledged ever since the seminal experiments by Ellsberg (1961). The participants were presented a situation in which they could bet on correctly predicting the color of the ball they would blindly draw from one urn of their choice. The subjects were provided with full information about how many balls of which color are inside one of the urns, while for the other urn they were given no information at all. The results show that people prefer to bet on the first urn, indicating that the awareness of missing information affects decision making.[3] This phenomenon is generally referred to as ambiguity aversion. Although further experimental evidence by subsequent studies stresses the relevance of ambiguity aversion, the attitude towards ambiguity is not necessarily negative.[4] Among other studies, Heath & Tversky (1991) give an account of an experiment in which subjects behave rather ambiguity loving than averse.[5] Whatever the attitude towards ambiguity, the experiments show that ambiguity matters for decision making. It should be noted that the case of risk, in which the decision maker knows exactly the underlying probabilities, and the case in which the absence of knowledge prevents to form any judgement about the probabilities are extreme cases. In many decisions, there is some, albeit not

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[3]Before, it was widely accepted that a rational decision maker is considered to be indifferent between the situation where the probability is clear-cut and where it is vague, as long as the mean probability is the same for both cases, see Savage (1954).

[4]Subsequently, the so-called Ellsberg paradox has been examined and verified for alternative situations, see for example Becker & Brownson (1964), Camerer & Weber (1992), Halevy (2007), Hogarth & Kunreuther (1985), Sarin & Weber (1993) and Smith (1969). Neuro-empirical evidence has been provided by Hsu et al. (2005) by proving that certain areas in the brain respond differently to situations of risk and ambiguity.

[5]Heath & Tversky (1991) provide support for the so-called competence effect, i.e. people prefer ambiguous alternatives when they consider themselves especially competent or knowledgeable about the source of uncertainty.

enough, statistical information available that allows restricting the considerations to a set of possible probability measures. This narrower concept of ambiguity – also referred to as Knightian uncertainty – has been brought into close connection with the multiple priors approach by Gilboa & Schmeidler (1989) and with the notion of kappa-ignorance by Chen & Epstein (2002), which measures the degree of ambiguity. In the context of ambiguity aversion Gilboa & Schmeidler (1989) show that the set of probability distributions reduces to the behavioral bias to extreme pessimism, i.e. the decision maker maximises welfare of the minimum / worst scenario.

Recently, this concept has been transfered to analyse the decision when to adopt an environmental policy. Asano (2010) examines the impacts of Knightian uncertainty referring to future economic developments that affect the social costs of a pollutant, e.g. the innovation of a technology could lower the costs of a climate policy adoption. Vardas & Xepapadeas (2010) apply the Knightian uncertainty concept to the evolution of species biomass to assess ecosystem management strategies. These studies assume ambiguity aversion and come to the same conclusion that the policy is to be adopted earlier than in a situation where uncertainty is described by risk. As pointed out by the authors, this approach can be considered to be a formal way to model the precautionary principle.

This paper reexamines this conclusion by directing the attention to the ambiguous assessments of the future damage costs and providing a different view onto the effects of Knightian uncertainty. A review of the existing estimates reveals enormous uncertainties, see Stern (2007). Apart from different appraisals of vulnerabilities, impacts of extreme weather events and catastrophes are often neglected and underlying assumptions about the future economies' capability to adapt are highly controversial. Highlighting the degree of ambiguity in these assessments, the three main benchmark studies by Mendelsohn et al. (2000), Nordhaus & Boyer (2000) and Tol (2002) vary between 0 and 3 per cent of GDP losses for 3°C warming.[6] Accordingly, we develop a formal decision model in which the social planner faces Knightian uncertainty in the future climate damage costs. The degree of ambiguity is captured by the concept of kappa-ignorance.

The model we develop is based on recent theoretical analyses of decisions under uncertainty, which have highlighted the effects of irreversibility in generating "real options". In these models, the interaction of time-varying uncertainty and irreversibility leads to a range of inaction where the policy maker prefers to "wait and see" rather than undertaking a costly action with uncertain consequences. We employ this recent literature and interpret climate policies as consisting of a portfolio of options. The general idea

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[6]Pindyck (2013) has convincingly argued that several climate models give a false and misleading sense of knowledge and precision although several model ingredients are in the realm of the "unknowable".

underpinning the view that climate policies are option-rights is that climate policy can be seen as analogous in its nature to the purchase of a financial call option, where the investor pays a premium price in order to get the right to buy an asset for some time at a predetermined price (exercise price), and eventually different from the spot market price of the asset. In this analogy, the policy maker, through her climate policy decision, pays a price which gives her the right to use a mitigation strategy, now or in the future, in return for lower damages. Taking into account this options-based approach, the calculus of suitability cannot be done by simply applying the net present value rule, but rather has to consider the following three salient characteristics of the environmental policy decision: (i) there is uncertainty about future payoffs from climate policies, (ii) waiting allows policy makers to gather new information on the uncertain future, and (iii) climate policies are at least partially irreversible. These characteristics are encapsulated in the concept of real option models.[7] This strand of literature now constitutes a growing branch of the climate economics literature.[8] A limited strand of literature – particularly in mathematical economics – has extended the real options approach to analyse the interplay of irreversibility and uncertainty under Knightian uncertainty. The standard real options approach rules out the situation in which policy makers are unsure about the likelihoods of future events. It typically adopts strong assumptions about policy makers' beliefs and no distinction between risk and ambiguity is made. The usual prescription for decision making under risk is then to select an action that maximizes expected utility. This is assumed although the knowledge of climate dynamics is still far from conclusive. In the more realistic Knightian uncertainty scenario decision making therefore becomes more complex. That means that the Knightian version of the real options models differs from the plain vanilla real options model by having an entire set of subjective probability distributions, see for example Nishimura & Ozaki (2007) and Trojanowska & Kort (2010). We transfer these ideas by expanding the paper by Pindyck (2012) on uncertain outcomes and climate change policy.

To gain a thorough understanding of the model, we utilize numerical simulations that illustrate the timing of policy adoption given alternative assumptions on the degree of ambiguity and on the policy maker's ambiguity preferences. In the first step, we compare

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[7]Concise surveys of the real options literature are provided by Bertola (2010), Dixit & Pindyck (1994) and Stokey (2009).

[8]There are manifold applications of real options analysis to climate economics, which deserve to be mentioned. A full review cannot be given here, but some examples shall be mentioned. The real option to mitigate is investigated in the seminal work by Pindyck (2000, 2002) or in later analyses by Anda et al. (2009), Chen et al. (2011b), Lin et al. (2007) and Wirl (2006). The real options to undertake specific adaptation projects is explored by Dobes (2010) and Linquiti & Vonortas (2012). Optimal climate policy and the real options to invest into the energy sector is examined by Fuss et al. (2009, 2011).

the decision made in the light of risk with the decision made by an ambiguity-averse policy maker. In the second step, we keep the assumption of ambiguity aversion to assess the size of the implied effects. In the third step, we expand our analysis by examining the range of optimal policy responses that are implied by ambiguity. This range is spanned by all policy responses that are optimal under alternative ambiguity preferences. So far, it is not clear, whether in general the climate policy maker is / should be averse to ambiguity. Therefore, it is of utmost relevance to examine the robustness of the optimal decision, which may contribute to a better understanding of decision making under ambiguity.

The remainder of the paper is organised as follows. In Section 3.2, the comprehensive modelling set-up is presented. The framework incorporates cross-discipline interactions in order to derive dynamically optimal policy responses to Knightian uncertainty. Subsequently, in Section 3.3 we illustrate the working of the model through numerical exercises and examine the sensitivity of the main results with respect to key parameters. The paper concludes in Section 3.4 with a brief summary and suggestions for further research. Omitted details of several derivations are provided in the appendices.

## 3.2   THE MODEL

The following modelling framework is an extended version of the one in Pindyck (2009, 2012). This framework does not only display all essential ingredients to mirror the climate change decision problem but it also allows for incorporating real options under Knightian uncertainty. Such a stochastic dynamic programming framework may contribute to the understanding of the potential implications of Knightian uncertainty for decision-making. It should be noted that the most obvious challenge along the way is to minimize complexity so that the model setup under complex uncertainty is still tractable.[9]

The model assumes that a social planner strives to find the optimal timing of climate policy adoption by maximizing the flow of consumption over time.[10] She faces the intergenerational trade-off problem that the costs of mitigation would have to be born by the present generations, but the benefits of mitigation would accrue in the future. Moreover,

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[9]The plethora of potentially significant contributions to overall atmospheric heat balance that are not treated in the simple model used here includes changes in other well-mixed greenhouse gases, ozone, snow albedo, cloud cover, solar irradiance, and aerosols. From this list, it should be clear that the objectives of the present paper are limited ones. A more complete assessment of outcome probabilities would include detailed models of the past and future of each of these effects.

[10]In our model framework we treat the world as a single entity in the interest of brevity. The world climate policy equilibrium can be constructed as a symmetric Nash equilibrium in mitigation strategies. The equilibrium can be determined by simply looking at the single country policy which is defined ignoring the other countries' abatement policy decisions [Leahy (1993)].

a bad timing will certainly lead to one of the following two irreversibility effects. Investing too early in mitigation technologies could trigger enormous sunk costs that are not recouped before long. Waiting too long may cause irreversible damages to ecological systems that contribute to welfare. However, ubiquitous uncertainties in almost every component in the projections and especially in the assessment of future climate damages render a well-informed decision about the timing almost impossible. Put differently, all plans depend decisively on the unknown sensitivity of losses to climate change. Hence, we concentrate on the uncertainty about the future climate damage costs, whereas we assume any other lack of knowledge to be resolved for the sake of analytical tractability. Expressed mathematically, the policy maker solves the following isoelastic objective function, which consists of the expected net present value of future consumption levels:

$$W = E \left[ \int_{t=0}^{\infty} \frac{(L(X_t, \Delta\mathbb{T}_t)C_t)^{1-\delta}}{1-\delta} e^{-rt} \, dt \right], \tag{3.1}$$

where $E[\cdot]$ is the expectation operator and $C_t$ is the consumption over time with the initial value normalised to 1. In the simplest form, the level of consumption $C_t$ is assumed to be equivalent to the level of GDP. The parameter $\delta \geq 0$ is the inverse of the intertemporal elasticity of substitution and $r$ is the discount rate. The term $L(X_t, \Delta\mathbb{T}_t)C_t$ describes GDP net of climate damage costs. The function $L$ depends on scientifically estimated changes in temperature $\Delta\mathbb{T}_t$ and a (positive) stochastic damage function $X_t$ determining the sensitivity of losses to global warming.

Instead of trying to model climate impacts in any detail, we keep the problem analytically simple by assuming that damages depend only on the temperature change, which is chosen as a measure of climate change. To be precise, as in Pindyck (2009, 2012), the damage costs from warming $(1 - L)$, expressed as a share of GDP, are implied by the following exponential function

$$L(X_t, \Delta\mathbb{T}_t) = e^{-X_t(\Delta\mathbb{T}_t)^2}, \tag{3.2}$$

where $0 < L(X_t, \Delta\mathbb{T}_t) \leq 1$, $\partial L/\partial(\Delta\mathbb{T}_t) \leq 0$ and $\partial L/\partial X_t \leq 0$, i.e. higher damage costs result in less GDP.[11]

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[11]Due the scarcity of empirical information about the magnitude of the damages in question, the shape of the damage function is somewhat arbitrary. Pindyck (2012) has assumed the exponential function $L(\Delta\mathbb{T}) = \exp[-\beta(\Delta\mathbb{T}^2)]$, where $\beta$ follows a gamma distribution. This implies that future damages are fully captured by the probabilistic outcomes of a given distribution. This concept can be understood as risk. However, the present uncertainty about $\beta$ also comprises the choice of the probability distribution, which will be tackled in this paper.

Before we show how to incorporate Knightian uncertainty about $X_t$, we briefly introduce the other component in the loss function: the temperature increase $\Delta\mathbb{T}_t$. For this, we adopt the commonly used climate sensitivity function in Weitzman (2009a) and Pindyck (2009, 2012). The single linear differential equation compresses the involved complex physical processes by capturing climate forcings and feedbacks in a simplified manner.[12] Hence, a direct link between the atmospheric greenhouse gas concentration $G_t$ and the temperature increase $\Delta\mathbb{T}_t$ is obtained by

$$d\Delta\mathbb{T}_t = m_1 \left( \frac{\ln\left(G_t/G_0\right)}{\ln 2} - m_2 \Delta\mathbb{T}_t \right) dt, \tag{3.3}$$

where $G_0$ is the inherited pre-industrial baseline level of greenhouse gas, and $m_1$ and $m_2$ are positive parameters. The first term in the bracket stands for the radiative forcing induced by a doubling of the atmospheric greenhouse gases. The second term represents the net of all negative and positive feedbacks. A positive parameter for this term thus counteracts a runaway greenhouse effect. The parameter $m_1$ describes the thermal inertia or the effective capacity to absorb heat by the earth system, which is exemplified by the oceanic heat uptake.

By defining $H$ as the time horizon with $\Delta\mathbb{T}_t = \Delta\mathbb{T}_H$ at $t = H$ and $\Delta\mathbb{T}_t \to 2\Delta\mathbb{T}_H$ as $t \to \infty$, we obtain equations, which are convenient to use in a real options setting, i.e.

$$d\Delta\mathbb{T}_t = \frac{\ln\left(2\right)}{H} \left(2\Delta\mathbb{T}_H - \Delta\mathbb{T}_t\right) dt, \tag{3.4}$$

and

$$\Delta\mathbb{T}_t = 2\Delta\mathbb{T}_H \left(1 - e^{-\frac{\ln 2}{H}t}\right), \tag{3.5}$$

where $\ln\left(2\right)/H$ denotes the adjustment speed of changes in temperature to the eventual changes in temperature $2\Delta\mathbb{T}_H$.[13]

Let us now focus on the other component in equation (3.2), which is the sensitivity

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[12]Factors that influence the climate are distinguished between forcings and feedbacks. A forcing is understood as a primary effect that changes directly the balance of incoming and outgoing energy in the earth-atmosphere system. Emissions of aerosols and greenhouse gases or changes in the solar radiation are examples. A secondary and indirect effect is described by a feedback that boosts (positive feedback) or dampens (negative feedback) a forcing. The blackbody radiation feedback exemplifies an important negative feedback, whereas, for example, the ice-albedo feedback accelerates warming by decreasing the earth's reflectivity.

[13]There is considerable a priori uncertainty in the probability and scale of climate change, but at least there are historical time series data available to calibrate probability distributions for parameters important in modelling climate sensitivity. On the other hand, based on current knowledge there is a large a priori uncertainty concerning when dramatic technological breakthroughs might occur and how much impact they will have, so allowing for such possibilities should increase the spread of outcomes for global carbon emissions and their consequences.

of losses to global warming. In the following, we explain the idea of how to extend the standard real options approach, which incorporates the concept of risk, to obtain a modelling framework that accounts for Knightian uncertainty. As shown by Nishimura & Ozaki (2007), it means to generate a set of probability distributions out of the one that is assumed in the standard real options model.[14] To formalize the concept, let $(B_t)_{0 \leq t \leq T}$ be a standard Brownian motion on $(\Omega, \mathscr{F}_T, P)$ that is endowed with the standard filtration $(\mathscr{F}_t)_{0 \leq t \leq T}$ for $(B_t)$. Consider the real-valued stochastic process $(X_t)_{0 \leq t \leq T}$ generated by the Brownian motion with drift $\alpha$ and standard deviation $\sigma$:

$$dX_t = \alpha X_t dt + \sigma X_t dB_t. \tag{3.6}$$

In equation (3.6) the particular probability measure $P$ is regarded as capturing the true nature of the underlying process. This, however, is highly unlikely, as this would imply that the policy maker is absolutely certain about the probability distribution that describes the future development of $(X_t)_{0 \leq t \leq T}$. Unlike this standard case, Knightian uncertainty describes how policy makers form ambiguous beliefs. Thereby a set $\mathscr{P}$ of probability measures is assumed to comprise likely candidates to map the future dynamics. Technically spoken, these measures are generated from $P$ by means of density generators, $\theta$.[15] Such a probability measure is denoted by $Q^\theta$ in the following. By restricting the density generators to a certain range like a real-valued interval $[-\kappa, \kappa]$, we are enabled to confine the range of deviations from the original measure $P$. The broader this interval is, the larger the set of probability measures, $\mathscr{P} = \left\{ Q^\theta \, | \theta \in [-\kappa, \kappa] \right\}$, and thus the higher the degree of ambiguity. This specific notion of confining the density generators to an

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[14]Alternatively, the imprecise probability concept in Reichert (1997) employs a set of probability measures describing the uncertain model parameters. The ambiguity involved in the estimation of the global mean temperature change in the 21st century is analysed in Kriegler & Held (2005) by constructing a belief function that is the lower envelope of the corresponding distributions. The model results in large imprecisions of the estimates, highlighting the key role of uncertainties in climate projections. Apart from deriving upper and lower bounds of the sets, Borsuk & Tomassini (2005) examine other representations of the probability measures and demonstrate how to use them to describe climate change uncertainties.

[15]Assume a stochastic process $(\theta)_{0 \leq t \leq T}$ that is real-valued, measurable and $(\mathscr{F}_t)$-adapted. Furthermore it is twice integrable, hence $\theta := (\theta)_{0 \leq t \leq \mathcal{T}} \in \mathcal{L}^2 \subset \mathcal{L}$. Define $(z_t^\theta)_{0 \leq t \leq T}$ by $z_t^\theta = e^{\left( -\frac{1}{2} \int_0^t \theta_s^2 ds - \int_0^t \theta_s dB_s \right)}$ $\forall t \geq 0$. Note that the stochastic integral $\int_0^t \theta_s dB_s$ is well-defined for each $t$, as $\theta \in \mathcal{L}$. A stochastic process $\theta \in \mathcal{L}$ is a density generator, if $(z_t^\theta)_{0 \leq t \leq T}$ is a $(\mathscr{F}_t)$-martingale. Using a density generator $\theta$ another probability measure $Q^\theta$ on $(\Omega, \mathscr{F}_T)$ can be generated from $P$ by

$$Q^\theta(A) = \int_A z_T^\theta dP \quad \forall A \in \mathscr{F}_T.$$

Note that any probability measure that is thus defined is called equivalent to $P$.

interval $[-\kappa, \kappa]$ is named $\kappa$-ignorance by Chen & Epstein (2002).

Endowed with this concept we can now define a stochastic processes $(B_t^\theta)_{0 \le t \le T}$ by

$$B_t^\theta = B_t + t\theta \tag{3.7}$$

for each $\theta \in [-\kappa, \kappa]$. As Girsanov's theorem shows, each process $(B_t^\theta)_{0 \le t \le T}$ defined as above is a standard Brownian motion with respect to $Q^\theta$ on $(\Omega, \mathscr{F}_T, Q^\theta)$. Inserting the definition of $(B_t^\theta)_{0 \le t \le T}$ into equation (3.6), we obtain for every $\theta \in [-\kappa, \kappa]$

$$dX_t = (\alpha - \sigma\theta) X_t dt + \sigma X_t dB_t^\theta. \tag{3.8}$$

Equation (3.8) displays all stochastic differential equations and thus all future developments of $(X_t)_{0 \le t \le T}$ that the decision maker thinks possible. If the policy maker gives equal weight to all possible developments in equation (3.8) when making a decision, i.e. if she exhibits no specific ambiguity preferences, the interval $[-\kappa, \kappa]$ would imply a continuum of optimal policies. This continuum is examined in the next section – for the moment let us focus on the optimal policy which is implied by ambiguity aversion.

Some preliminary thoughts about what measure in $\mathscr{P} = \left\{ Q^\theta \,|\, \theta \in [-\kappa, \kappa] \right\}$ is most relevant under ambiguity aversion are provided in the following. Ambiguity aversion makes the decision maker maximize the worst case scenario, as proven by Gilboa & Schmeidler (1989). As $e^{-X_t(\Delta\mathbb{T}_t)^2} GDP_t$ is calculated as GDP net of damages, the worst case scenario is described by the largest value of $X_t$. Note that the processes $X_t$ in equation (3.8) only differ in the drift but not in the volatility terms. As an illustration we have numerically simulated equation (3.2) and (3.6) for a time period of 200 years for $\Delta\mathbb{T}_H = 1.9°C$ versus $\Delta\mathbb{T}_H = 3.4°C$ (equivalent to pre-industry levels of 2.5°C versus 4°C) of warming and three alternative drift terms. The character of the impact function (3.2) for various drift terms is shown in Figure 1. The graphs indicate the forces at play in our analysis and in particular two effects must be recognised. Firstly, the function $L(X_t, \Delta\mathbb{T}_t)$ spreads out considerably for higher temperature increases. Under the assumption of $\Delta\mathbb{T}_H = 3.4°C$ the damage is 0.09154 = 9.15 per cent of the GDP after 100 years.[16] Secondly and most importantly, the highest value of the drift term generates the maximum of $1 - L(X_t, \Delta\mathbb{T}_t)$ and therefore the minimum of the $GDP_t$ net of damages.

After having introduced the basic ingredients to the model and after having gained

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[16]The calibrated damages from warming are in the range of previous estimates. Weitzman (2009b) has assumed damage costs of 1.7 percent of GDP for 2.5°C of warming. For higher temperature increases he has assumed rapidly increasing damages of 9 (25) percent of GDP for 4°C (5°C) of warming. Millner et al. (2010) have assumed damages of 1.7 (6.5) percent of GDP for 2.5°C (5°C) of warming.
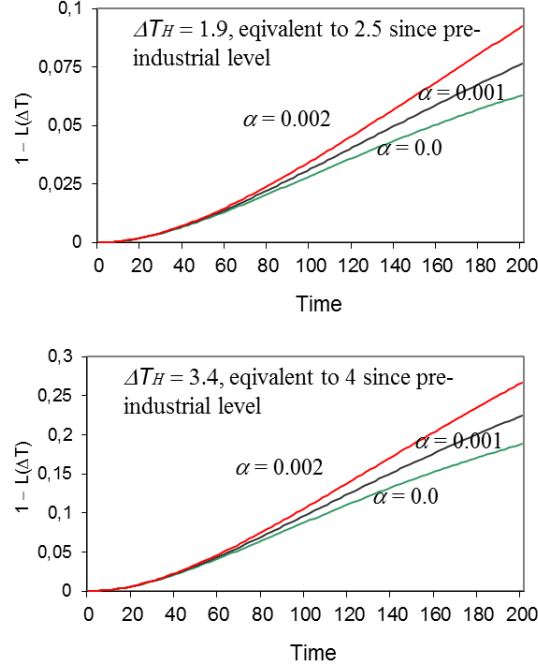
**Figure 3.2.1:** Simulated Damages $1 - L(X_t, \Delta\mathbb{T}_t)$ Due To Global Warming in Percent of GDP. The initial value for $X$ is $X_0 = 0.008$ and $H = 100$. The simulated time series are computed ignoring the uncertainty part of equation (3.6), i.e. $dX_t = \alpha X_t dt$.

some intuition about the effects of $X_t$, let us turn to the problem we must solve, which deals with "optimal stopping". The idea is that at any point in time the value of climate policy is compared with the expected value of waiting $dt$, given the available information set and the knowledge of the stochastic processes. If the ambiguity-averse decision maker conducts no climate policy – referred to as the business-as-usual approach - and faces Knightian uncertainty in equation (3.1), then the resulting intertemporal welfare, $W^{\mathrm{N}}$, with consumption growing at a rate $g_0$ and initial consumption normalised as 1 is determined as

$$
\begin{aligned}
W^{\mathrm{N}}\left(X, \Delta\mathbb{T}; \Delta\mathbb{T}_H\right) &= \min_{Q^\theta \in \mathscr{P}} E^{Q^\theta}\left[\int_{t=0}^{\infty} \frac{\left(e^{-X_s(\Delta\mathbb{T}_s)^2}C_s\right)^{1-\delta}}{1-\delta}e^{-rs}ds \,\Bigg|\, \mathscr{F}_t\right] \\
&= \frac{1}{1-\delta}\min_{Q^\theta \in \mathscr{P}} E^{Q^\theta}\left[\int_{t=0}^{\infty} e^{-X_s(1-\delta)(\Delta\mathbb{T}_s)^2}e^{-(r-(1-\delta)g_0)s}ds \,\Bigg|\, \mathscr{F}_t\right],
\end{aligned} \tag{3.9}
$$

s.t. equations (3.4) and (3.8), where "N" refers to the no-actions-taken approach, $r - (1-\delta)g_0$ is assumed to be positive, and $E^{Q^\theta}[\cdot|\mathscr{F}_t]$ represents the expectation value with

54

respect to $Q^\theta \in \mathscr{P}$ conditional on $\mathscr{F}_t$.[17] The first equation holds as ambiguity aversion implies that the policy maker reckons with the lowest expected welfare value.[18]

For the sake of analytical tractability, we apply a Taylor series expansion to $e^{-X_s(1-\delta)\Delta\mathbb{T}_s^2}$ such that

$$e^{-X_s(1-\delta)\Delta\mathbb{T}_s^2} \cong 1 - X_s\left(1-\delta\right)\Delta\mathbb{T}_s^2, \tag{3.10}$$

where $0 < L\left(\Delta\mathbb{T}_t\right) \leq 1$ and $\partial L/\partial\left(\Delta\mathbb{T}_t\right) \leq 0$ still hold.[19] By inserting (3.10) into (3.9) we thus obtain

$$W^{\mathrm{N}}\left(X, \Delta\mathbb{T}; \Delta\mathbb{T}_H\right) \tag{3.11}$$
$$= \frac{1}{1-\delta} \min_{Q^\theta \in \mathscr{P}} E^{Q^\theta}\left[\left.\int_{t=0}^{\infty} \left(1 - X_s\left(1-\delta\right)\Delta\mathbb{T}_s^2\right) e^{-(r-(1-\delta)g_0)s}ds\right| \mathscr{F}_t\right],$$

s.t. equation (3.4) and (3.8). Using Ito's Lemma and following the standard dynamic programming argument, we formulate the problem in terms of the Hamilton-Jacobi-Bellman equation[20]

$$\left(r - (1-\delta)g_0\right)W^{\mathrm{N}} = \frac{1}{1-\delta} - X^*\Delta\mathbb{T}^2 + \frac{\ln\left(2\right)}{H}\left(2\Delta\mathbb{T}_H - \Delta\mathbb{T}\right)\frac{\partial W^{\mathrm{N}}}{\partial\Delta\mathbb{T}}$$
$$+ \left(\alpha + \kappa\sigma\right)X^*\frac{\partial W^{\mathrm{N}}}{\partial X^*} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{N}}}{\partial X^{*2}}. \tag{3.12}$$

The asterisk represents the density generator $-\kappa$, meaning that $Q^*$ is generated by $-\kappa$

---

[17]For reasons of mathematical tractability we assume that the continuous Knightian uncertainty is independent of time and therefore the planning horizon is infinite. The reasoning for the perpetual assumption is that the underlying time scales in the natural climate system are much longer than those in the economic system. Technically, we consider $T \to \infty$ for $(B_t)_{0 \leq t \leq T}$ and $(B_t^\theta)_{0 \leq t \leq T}$ in the above made introduction to the concept of Knightian uncertainty.

[18]First, the ambiguity-averse policy maker takes only the probability measure into consideration that creates the worst outcomes for the welfare. Then she strives to find the policy strategy that maximizes this 'worst-case welfare function'. The maxmin nature of the problem links the analysis with contributions on robust control. See, for example, Funke & Paetz (2011).

[19]Real option models suggested in the literature seem always to make a trade-off between analytical tractability and realism. In this paper we analyse a model that combines both features into one model: the model has a rich analytical structure and nevertheless the analytical forms of the particular solutions can be obtained. As numerical simulations in Appendix 3.A show, the choice of $\theta$ minimising the welfare by the principle of the Knightian uncertainty is always $\theta = -\kappa$. The first order Taylor's expansions results display the similar qualitative results. Note that $\theta = -\kappa$ implies the worst equivalent outcome for welfare, which requests the third or higher order terms of Taylor's expansion of the welfare function to yield more accurate results. For simplicity, we opt for the first order term of Taylor's expansions to investigate the problem.

[20]For the derivation please see Appendix 3.A.

and the stochastic process $X^*$ is defined by inserting $-\kappa$ into equation (3.8):

$$dX_t^* = (\alpha + \sigma\kappa) X_t^* dt + \sigma X_t^* dB_t^{-\kappa}. \tag{3.13}$$

As indicated by Figure 1 and also proven in Appendix 3.A, the ambiguity-averse policy maker reckons with the probability measure (3.13) that exhibits the highest drift term. Given this pessimistic view that $X^*$ is perceived to be the true process, the optimal policy response is described by equation (3.12). Real options analysis specifies that the solution of (3.12) consists of the particular and general solution. The particular solution $W^{\mathrm{NP}}$ is obtained by computing the integral for $W^{\mathrm{N}}$ in equation (3.11) without considering possible policy intervention. It is straightforward to explain $W^{\mathrm{NP}}$ as the expected present value of the business-as-usual policy. The general solution, hereinafter denoted by $W^{\mathrm{NG}}$, gives the value of adopting policy in the future, which is referred to as the real options value and is obtained by

$$
\begin{aligned}
(r - (1-\delta) g_0) W^{\mathrm{NG}} = {} & \frac{\ln(2)}{H} (2\Delta\mathbb{T}_H - \Delta\mathbb{T}) \frac{\partial W^{\mathrm{NG}}}{\partial \Delta\mathbb{T}} \\
& + (\alpha + \kappa\sigma) X^* \frac{\partial W^{\mathrm{NG}}}{\partial X^*} + \frac{1}{2} \sigma^2 X^{*2} \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^{*2}}.
\end{aligned} \tag{3.14}
$$

Now, we turn our attention to the welfare value of implementing climate policy. Let us assume that the policy maker is willing to pay annual mitigation costs $w(\tau)$ as a percentage of GDP to limit the temperature increase at $t = H$ to $\tau$.[21] The temperature evolution is then described by

$$d\Delta\mathbb{T}_s = \frac{\ln(2)}{H} (2\tau - \Delta\mathbb{T}_s) \, ds \tag{3.15}$$

and

$$\Delta\mathbb{T}_t = 2\tau \left(1 - e^{-\frac{\ln 2}{H} t}\right), \tag{3.16}$$

which evolve as variants out of the equations (3.4) and (3.5) by setting $\Delta\mathbb{T}_H = \tau$.

Analogous to the derivation procedure in Appendix 3.A, the intertemporal welfare function of taking action to reduce the green house gas emission, $W^{\mathrm{A}}$, is then given by

$$
\begin{aligned}
(r - (1-\delta) g_0) W^{\mathrm{A}} = {} & (1 - w(\tau))^{1-\delta} \left(\frac{1}{1-\delta} - X^* \Delta\mathbb{T}^2\right) + \frac{\ln(2)}{H} (2\tau - \Delta\mathbb{T}) \frac{\partial W^{\mathrm{A}}}{\partial \Delta\mathbb{T}} \\
& + (\alpha + \kappa\sigma) X^* \frac{\partial W^{\mathrm{A}}}{\partial X^*} + \frac{1}{2} \sigma^2 X^{*2} \frac{\partial^2 W^{\mathrm{A}}}{\partial X^{*2}},
\end{aligned} \tag{3.17}
$$

---

[21]In practical terms, this means that the policy maker reduces $G_t$ in equation (3.3) so that the increase in temperature is limited to less than $\tau$ at $t = H$. While endogenised mitigation costs would be a more realistic modelling choice, we use a simple assumption about constant mitigation costs to focus attention on the issues of the impacts of the Knightian uncertainty.

which is derived from the following integral

$$W^A(t=0, X, \Delta \mathbb{T}; \tau)$$

$$= \frac{1}{1-\delta} E^{Q^*} \left[ (1 - w(\tau))^{1-\delta} \int\limits_{t=0}^{\infty} \left( 1 - X_s^* (1-\delta) \Delta \mathbb{T}_s^2 \right) e^{-(r-(1-\delta)g_0)s} ds \, \middle| \, \mathscr{F}_t \right], \quad (3.18)$$

s.t. equation (3.8) and equation (3.15). If climate policy is time-consistent, then the solutions to $W^A$ can be obtained by integrating equation (3.18) directly.

As mentioned earlier, the aim of this analysis is to determine the optimal timing of mitigation, which allows to limit the temperature increase to some $\tau$ at $t = H$. As long as the value of postponing policy $W^N$ is higher than the value of implementing policy $W^A$, it is optimal to continue the business-as-usual policy. As soon as both values $W^N$ and $W^A$ are identical, the optimal strategy is to take action. Accordingly, the threshold of taking action to limit global warming to $\tau$ at $t = H$ is computed from the identity

$$W(\text{taking action}) = W(\text{business} - \text{as} - \text{usual}) + \text{Real options}. \quad (3.19)$$

The threshold of taking action, denoted as $\bar{X}$, is expressed in terms of the observed values of the stochastic process $X^*$. Substituting, we have

$$W^A\left(\bar{X}, \Delta \mathbb{T}; \tau\right) = W^{NP}\left(\bar{X}, \Delta \mathbb{T}; \Delta \mathbb{T}_H\right) + W^{NG}\left(\bar{X}, \Delta \mathbb{T}; \Delta \mathbb{T}_H\right). \quad (3.20)$$

Exercising the real options $W^{NG}\left(\bar{X}, \Delta \mathbb{T}; \Delta \mathbb{T}_H\right)$ implies that the policy maker forgoes the option to wait and to act later as more information about $X_t$ becomes available. In other words, real options analysis explicitly accounts for the opportunity costs of early action.

Please note that all terms in (3.20) are affected by Knightian uncertainty. Accordingly, the impact of Knightian uncertainty is not necessarily monotonous for the policy maker.[22]

The next step is to solve the particular integrals of $W^{NP}$ and $W^A$, and the real options expression $W^{NG}$. As shown in Appendix 3.B the following particular integrals result from Ito's Lemma:

$$W^{NP}(X, \Delta \mathbb{T}; \Delta \mathbb{T}_H) = \frac{1}{1-\delta} \left[ \frac{1}{r - (1-\delta) g_0} - 4\Delta \mathbb{T}_H^2 (1-\delta) \gamma X^* \right] \quad (3.21)$$

---

[22]Real options dominate the particular integral with extreme Knightian uncertainty, while the effect of smaller Knightian uncertainty on the particular integral is prevailing.

and

$$W^{\mathrm{A}}\left(X,\Delta\mathbb{T};\tau\right)=\frac{\left(1-w\left(\tau\right)\right)^{1-\delta}}{1-\delta}\left[\frac{1}{r-\left(1-\delta\right)g_0}-4\Delta\tau^2\left(1-\delta\right)\gamma X^*\right] \tag{3.22}$$

where

$$\gamma=\frac{1}{\eta}-\frac{2}{\eta+\frac{\ln 2}{H}}+\frac{1}{\eta+2\frac{\ln 2}{H}}, \qquad \eta=r-\left(1-\delta\right)g_0-\left(\alpha+\kappa\sigma\right).$$

Note that it is assumed that $\eta$ is positive.

After obtaining the particular solutions of equations (3.21) and (3.22) analytically, we now turn our attention to the real options term $W^{\mathrm{NG}}$ in equation (3.14). In Appendix 3.C we show that the general solutions have the form:

$$W^{\mathrm{NG}}\left(t=0,X,\Delta\mathbb{T};\Delta\mathbb{T}_H\right)=A_1 X^{*\beta_1}\left(\Delta\mathbb{T}^2-4\Delta\mathbb{T}_H\Delta\mathbb{T}+4\Delta\mathbb{T}_H^2\right), \tag{3.23}$$

where $\beta_1$ is the positive root of the quadratic characteristic equation

$$\frac{1}{2}\sigma^2\beta\left(\beta+1\right)+\left(\alpha+\kappa\sigma\right)\beta-\left(r-\left(1-\delta\right)g_0+2\left(\frac{\ln\left(2\right)}{H}\right)\right)=0, \tag{3.24}$$

and $A_1$ is the unknown parameter to be determined by the value-matching and smooth-pasting conditions. The meaning of equation (3.23) is straightforward. For a small $\Delta\mathbb{T}_H$ the value of the options to take actions is small – the option of taking action is reduced for less global warming. The effective discount rate for real options is a positive function of $\ln\left(2\right)/H$. As we know from equation (3.4), $\ln\left(2\right)/H$ also denotes the adjustment speed of changes in temperature. Higher temperature adjustment speed (for example, $H=50$ years instead of $H=100$ years) means that the damage is higher and thus the option value is smaller. After obtaining the solutions to equation (3.20) by applying the value-matching condition, the smooth-pasting condition is given by equalising the derivative of (3.22) with respect to $X^*$ with the sum of the derivatives of (3.21) and (3.23) with respect to $X^*$. Substituting (3.21) – (3.23) back into the value-matching and smooth-pasting conditions yields

$$\begin{aligned} 4\gamma\left(\Delta\mathbb{T}_H^2-\Delta\tau^2\left(1-w\left(\tau\right)\right)^{1-\delta}\right)\bar{X}^* &= \frac{1-\left(1-w\left(\tau\right)\right)^{1-\delta}}{\left(r-\left(1-\delta\right)g_0\right)\left(1-\delta\right)} \\ &+ A_1\bar{X}^{*\beta_1}\left(\Delta\mathbb{T}^2-4\Delta\mathbb{T}_H\Delta\mathbb{T}+4\Delta\mathbb{T}_H^2\right), \end{aligned} \tag{3.25}$$

and

$$4\gamma \left( \Delta \mathbb{T}_H^2 - \Delta\tau^2 \left(1 - w\left(\tau\right)\right)^{1-\delta} \right)$$
$$= A_1\beta_1\bar{X}^{*\beta_1-1} \left( \Delta\mathbb{T}^2 - 4\Delta\mathbb{T}_H\Delta\mathbb{T} + 4\Delta\mathbb{T}_H^2 \right). \qquad (3.26)$$

The solution to the decision problem under ambiguity aversion is fully given by the equations (3.21) – (3.26). Likewise, the continuum of solutions implied by all processes $X_t$ in equation (3.8) can be generated by inserting all $\theta \in [-\kappa, \kappa]$. In the next section, we will examine the impacts of Knightian uncertainty by conducting a numerical simulation of this analytical solution.

## 3.3 Numerical Simulations and Results

While the preceding section has laid out the modelling framework, we now focus on a thorough numerical analysis of the model. We aim to clarify whether the more realistic assumption of ambiguity implies a timing of policy adoption that is different from the one in the usually applied approach of risk. To this end, we start by exploring the decision under ambiguity aversion. Afterwards, this assumption is relaxed in order to observe the range of optimal policy responses that are implied by alternative ambiguity preferences. Is the range rather big, we may deduce that this subjective assumption on the attitude towards ambiguity matters significantly. The magnitude of these effects is compared with the sensitivity to the other parameters which calibration is up for debate.

The baseline calibration of these parameters requires the use of judgement, i.e. they reflect a back-of-the-envelope calculation.[23] The unit time length corresponds to one year. Our base parameters are $\sigma = 0.075$, $\kappa = 0.02$, $r = 0.04$, $\alpha = 0.0$, $g_0 = 0.01$, $\delta = 0.0$, and $H = 100$. $\Delta\mathbb{T}_H$ is assumed to be 3.4°C which is equivalent to 4 degrees of warming since the pre-industrial level. The climate policy target $\tau$ is assumed to be 1.4°C, which is equivalent to 2 degrees of warming compared with the pre-industrial level. Special attention has to be paid to the calibration of $w(\tau)$. The term $w(\tau)$ represents the achievability and costs of climate targets. What are the economic costs of reaching the target of climate stabilisation at no more than 2°C above pre-industrial level by the end of this century? To assess this question, Edenhofer et al. (2010) have compared the energy-environment-economy models MERGE, REMIND, POLES, TIMER and E3MG in

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[23]Despite the increasingly detailed understanding of climate processes from a large body of research, various parameters involved inevitably remain inestimable, except in retrospect. Moreover, the calibrated model is not based on detailed time series data in the way econometric models are and does not have the projective power of the latter.

a model comparison exercise.[24] Despite different structures employed in the models, four of the five models show a similar pattern in mitigations costs for achieving the first-best 400 ppm $CO_2$ concentration pathway. These costs turned out to be of a similar order of magnitude across the models, i.e. approximately 2 per cent of the worldwide GDP. We therefore assume that $w(\tau) = 0.02$.

First, we consider the thresholds of mitigation, i.e. we calculate the optimal timing of curbing emissions that allows achieving the 2°C target. The optimal strategy is to adopt the climate policy right now if $X_t^* \geq \bar{X}^*$ and to continue waiting if $X_t^* < \bar{X}^*$, where $\bar{X}^*$ is the threshold value.[25] The first set of graphs, Figure 3.3.1 – 3.3.5, illustrates the optimal timing problem from the perspective of an ambiguity-averse policy maker. Afterwards, a broad range of preferences is accounted for by conjecturing a whole set of $\theta$-values, $\theta \in [-\kappa, \kappa]$.

To start with, in Figure 3.3.1 we focus on the sensitivity of the optimal thresholds to the degree of ambiguity $\kappa$. A higher $\kappa$-value specifies a higher level of uncertainty. The case of $\kappa = 0$ characterises the situation in which ambiguity has been resolved. Then, the set of probability measures boils down to one single measure, the same one that would be postulated in a traditional real option framework.
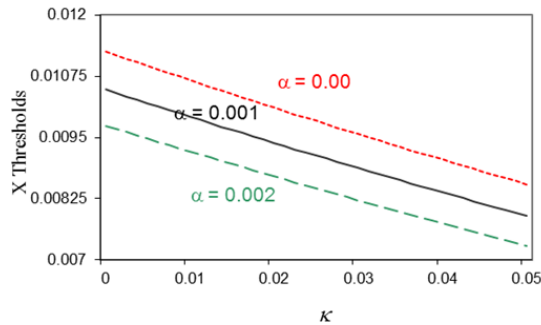


**Figure 3.3.1:** The Climate Policy Thresholds for Alternative $\kappa$'s and $\alpha$'s under Ambiguity Aversion

The numerical results indicate an acceleration of climate policy for higher degrees of

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[24]In order to improve model comparability, the macroeconomic drivers in the five modelling frameworks employed were harmonised to represent similar economic developments. On the other hand, different views of technology diffusion and different structural assumptions regarding the underlying economic system across the models remained. This helps to shed light on how different modelling assumptions translate into differences in mitigation costs. Low stabilisation crucially depends upon learning and technologies available.

[25]It is worth conjecturing that the existence of the no action area sheds light on why policy makers often deem it desirable to stay put, contrary to intuition which stems from thinking in terms of a simple cause and effect framework.

ambiguity. Increasing ambiguity has an unequivocally positive impact upon the timing of optimal climate policy and shrinks the continuation region in which exercising climate policy is suboptimal. The reason is that higher degrees of ambiguity force the ambiguity-averse policy maker to anticipate even worse future outcomes and to act sooner. This result for the special case of ambiguity in the damage costs is in line with the existing research by Asano (2010) and Vardas & Xepapadeas (2010). Figure 3.3.1 also shows that this result is insensitive to the choice of the drift parameter $\alpha$, which can be regarded as a measure of the trend in the economy's vulnerability. The higher $\alpha$, the more vulnerable the economy is over time. Therefore it is clear, that a higher trend in vulnerability implies a decrease of the threshold.
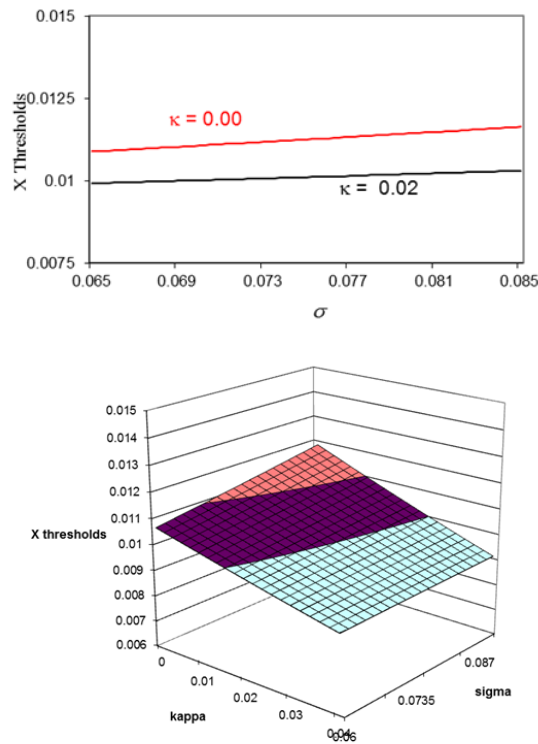


**Figure 3.3.2:** The Climate Policy Thresholds for Alternative $\sigma$'s and $\kappa$'s under Ambiguity Aversion

Figure 3.3.2 provides an analysis of the threshold effects for alternative degrees of risk/noise $\sigma$. The threshold value at which climate policy is implemented is shown to increase in the noisiness level $\sigma$. Irrespective of the degree of ambiguity, the noise makes projections of future climate damage costs less reliable, which generates the incentive to wait for new information to arrive instead of taking action. Hence, increased risk $\sigma$ leads to

a delay in policy action. In contrast, increased Knightian uncertainty in combination with ambiguity aversion tend to accelerate optimal timing. Additional observations concerning the scale of these effects emerge from a bird's eye examination of the 3-dimensional figure. It is observable that an increase in $\kappa$ has a mildly bigger impact on the climate policy threshold, meaning that the waiting incentives are counteracted. An ambiguity-averse policy maker takes the most pessimistic view on the future outcomes and therefore adopts precautionary measures. Noise in the projections are thus of a little less importance.

How to calibrate the discount rate is one of the most controversial questions in the economic literature on climate change, e.g. see Stern (2007). As this problem is still far from being resolved, the sensitivity to alternative discounting assumptions needs to be explored. Figure 3.3.3 confirms common knowledge that higher discount rates bolster the reasons for taking a "wait and see attitude" towards climate policy. The choice of the discount rate decisively determines the weight of future climate damage costs in the welfare considerations. The higher the value of $r$, the less far-sighted the policy maker becomes and the later mitigation efforts are undertaken. Figure 3.3.3 also reveals that the problematic choice of the discount rate is of more importance to the ambiguity-averse policy maker than the degree of ambiguity. Indeed, the effects caused by a marginal increase of $r$ undo the outcomes induced by a marginal increase of $\kappa$. This emphasizes the importance of reaching an agreement on the choice of the discount rate value.
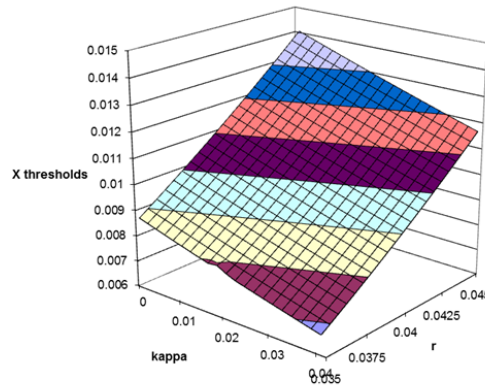


**Figure 3.3.3:** The Climate Policy Thresholds for Simultaneous Changes in the Discount Rate $r$ and $\kappa$ under Ambiguity Aversion

As already indicated at the beginning of this section, estimations of the abatement costs also face a lot of uncertainties. Figure 3.3.4 provides a sensitivity analysis of the thresholds with respect to $w(\tau)$, i.e. we illustrate the impact of alternative climate stabilisation costs upon the threshold. This simulation shows that higher climate stabilisation

costs lead to an increase of the no action area, i.e. the mitigation threshold is moved upwards. Intuitively, this makes perfect sense. Higher costs make climate policies less attractive for policy makers, so policy makers hesitate to perform them in the first place. This incentive to delay policy is again counteracted by an increase of the degree of ambiguity. More precisely, under ambiguity aversion the option value of the climate policy opportunity is again lower than in the standard model real options model. Therefore, an ambiguity-averse policy maker acts earlier. The magnitude of both effects appear to be about the same size.
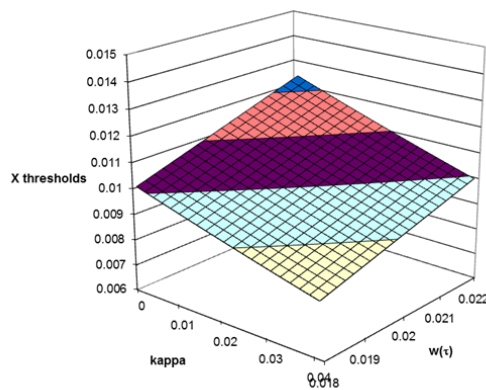


**Figure 3.3.4:** The Climate Policy Thresholds for Alternative Costs of Climate Stabilisation under Ambiguity Aversion

Finally, we analyse how different expected degrees of warming in the business-as-usual scenario, i.e. changes in $\Delta\mathbb{T}_H$, affect the threshold. Looking back on all IPCC assessments, it becomes evident that uncertainty about $\Delta\mathbb{T}_H$ could not significantly be resolved in the last years.[26] Accounting for different assumptions of $\Delta\mathbb{T}_H$, Figure 3.3.5 clearly indicates that the tactic to postpone policy adoption becomes less attractive for higher projected temperature increases. In other words, higher $\Delta\mathbb{T}_H$ values accelerate climate policies by shrinking the no action area. In contrast to the simulations above, an increase of $\Delta\mathbb{T}_H$ and $\kappa$ work in the same direction. The effects of a change in $\kappa$ is only of secondary importance.

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[26]One has to admit that despite more observations, more sophisticated coupled climate models and substantial increases in computing power, climate projections have not narrowed appreciably over the last two decades. Indeed, it has been speculated that foreseeable improvements in the understanding of the underlying physical processes will probably not lead to large reductions in climate sensitivity uncertainty. See Roe and Baker (2007).
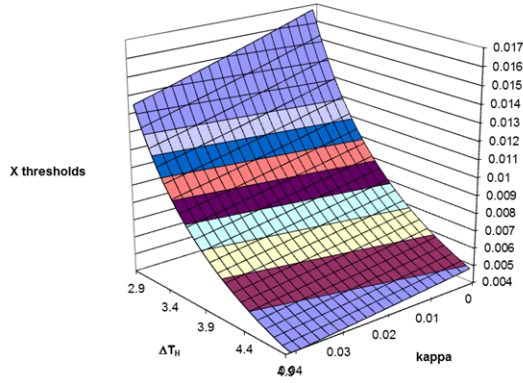
**Figure 3.3.5:** The Impact of Changes in $\Delta\mathbb{T}_H$ Upon the Threshold under Ambiguity Aversion

Now, we broaden our view on ambiguity by accounting for a range of $\theta$-values, i.e. $\theta \in [-0.1, 0.1]$. Each value in this interval leads to a different optimal policy response. A higher value is tantamount to specifying the policy maker as more ambiguity loving or optimistic. A lower value is equivalent to more ambiguity aversion or pessimism. How important is such a subjective attitude towards ambiguity for the design of an optimal policy design? How wide is the "operating space" for the policy maker before she decides how to react to ambiguity? To answer these questions, we again make use of simulations that compare the induced threshold effects with the effects implied by varying the values of $\sigma$, $r$, $w(\tau)$ and $\Delta\mathbb{T}_H$, respectively. The threshold curves in Figure 3.3.6 – Figure 3.3.9 confirm what intuition suggests: The decision maker delays policy adoption the longer the more optimistic she is about the future.
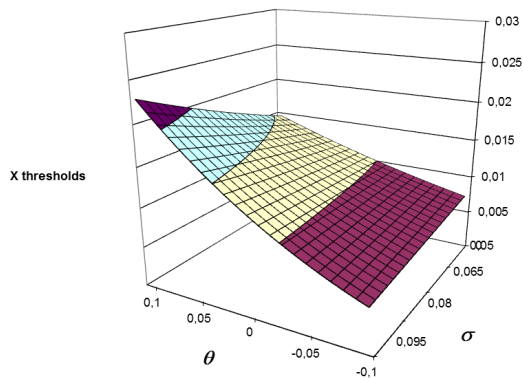


**Figure 3.3.6:** The Thresholds for Alternative Preferences $\theta$ and Noise Parameters $\sigma$

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This effect is not of negligible size, as for example displayed by Figure 3.3.6 and 3.3.7. Indeed, the attitude towards ambiguity is crucial to the timing of policy adoption, more crucial than the existence of noise (Figure 3.3.6) and the amount of the mitigation costs (Figure 3.3.7). In particular, the first mentioned finding stresses the value-added of accounting for ambiguity in the real options framework. Furthermore, we can observe that the ambiguity preferences influence the response to noise. As seen before in Figure 3.3.2, noise in the damage costs projections cannot dissuade the pessimistic policy maker from taking preventive action. In contrast, the optimistic policy maker's decisison is more susceptible to incentives to postpone mitigation efforts. A similar effect can be also observed in Figure 3.3.7, albeit it is less significant. A pessimistic policy maker is less worried about the costs of mitigation, as the avoidance of the worst outcomes, which she considers to be very likely, will certainly outweigh these costs. For the optimists an increase in the costs $w(\tau)$ could be a reason to delay mitigation efforts, because in her calculation the benefits of mitigation are lower.
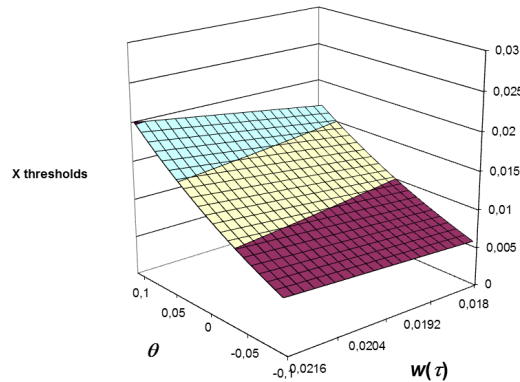


**Figure 3.3.7:** The Thresholds for Alternative Preferences $\theta$ and Mitigation Costs $w(\tau)$

Figure 3.3.8 contrasts the threshold effects of ambiguity to the effects by another subjective assumption on decision making, to wit the discount rate $r$. Evidently, a myopic and optimistic policy maker prefers to wait rather long until she commits herself to curbing emissions. In contrast, a far-sighted and pessimistic policy maker needs to take action sooner. These subjective assumptions on decision making appear to affect the threshold almost equally.

Figure 3.3.9 explores the scale of effects implied by alternative ambiguity preferences and projections of the temperature increase $\Delta \mathbb{T}_H$. The higher the projected increase is, the sooner the policy maker has to take action. Furthermore it is evident that the differences in the reponse implied by alternative preferences vanish for higher temperature projections.
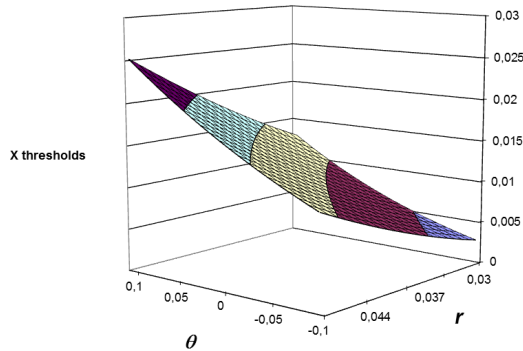
**Figure 3.3.8:** The Thresholds for Alternative Preferences $\theta$ and Discount Rates $r$

In view of bad news about the future temperature evolution even the optimist finds it difficult to delay the low-emission policy much longer. Scientifically provided evidence that the future turns out harmful thus restricts the "operating space" for the policy maker. In contrast, lower temperature projections can either be considered to be bad enough to justify early action (from the pessimist's point of view) or they can be dismissed as rather insignificant and unworthy to trigger efforts soon (from the optimist's point of view). These projections thus leave a wide "operating space". As there is enormous uncertainty about the climate sensitivity parameter, temperature projections are typically given in ranges. When considering the interval for $\Delta \mathbb{T}_H$ that is illustrated in Figure 3.3.9, we observe a vast variety of optimal policy responses, which is mainly caused by the uncertainty about the temperature increase.
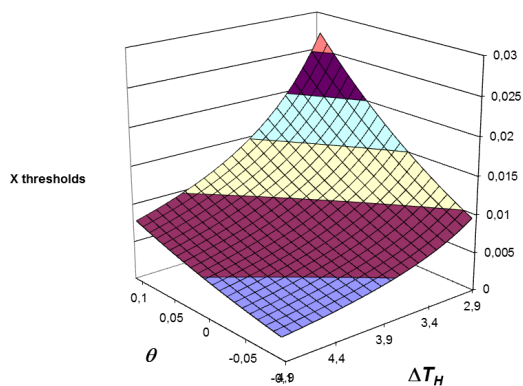


**Figure 3.3.9:** The Thresholds for Alternative Preferences $\theta$ and Projections of the Temperature Increase $\Delta \mathbb{T}_H$

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## 3.4  Conclusions

Lack of scientific knowledge and uncertainty about the future economic development lead to a considerable degree of ambiguity in the assessments of future climate damage costs. To analyse the question of how ambiguity affects the climate policy decision, we enhance a standard real options model by the Knightian uncertainty concept. This offers the opportunity to explore the effects by alternative degrees of ambiguity and ambiguity preferences.

A unifying message from our paper could be stated as follows: We have demonstrated that Knightian uncertainty affects irreversible climate policies in a way which significantly differs from the impact of risk. High degrees of ambiguity are shown to have a positive effect on the willingness to adopt mitigation efforts, if the policy maker is assumed to be averse to it. However, two caveats must be mentioned. Firstly, the simulations demonstrate that early action might be thwarted by uncertainty about other crucial parameters such as the mitigation costs or the projected temperature increase. The result reveals that Knightian uncertainty does not necessarily imply extreme policy activism. Secondly, whether a climate policy maker is / should be assumed to be ambiguity-averse is not clear. If we allow for a range of preferences in the simulations, we observe that policy adoption is delayed the longer the more optimistic she is about the future outcomes. Indeed, this effect is not of negligible size, which indicates a wide range of possible policy responses. Put differently, whatever policy response is adopted now, it will lack robustness concerning ambiguity. This must not be understood as a reason to defer the policy decision, but as evidence of how important it is to reach a political agreement on how to deal with ambiguity. This could, for example, result in the perception to follow the precautionary approach as taken by the EU.

We believe that our application of Knightian uncertainty comes with an advantage and a disadvantage. The advantage is that it allows one to recognise the difference between risk and uncertainty and its implied size of effects.[27] Thus, it provides a more realistic grounding for assessing climate policy and deriving optimal and rational policy decisions when ambiguity is involved. On the other hand, one has to admit that the comparative static results also have their limitations. First, the numerical results do not account for the fundamental dynamic nature of climate policies.[28] Second, we have focussed on Knightian uncertainty in the damage function. However, there are further

---

[27]To quote from Mastrandrea & Schneider (2004, p. 571) "we do not recommend that our quantitative results be taken literally, but we suggest that our probabilistic framework and methods be taken seriously". See also Schneider & Mastrandrea (2005).

[28]One may also follow a different strategy. Instead of tailoring policies towards one future in particular, one may find institutional arrangements, regulatory policies and technologies of adapting to many possible future climate scenarios.

layers of uncertainty in complex climate models about which we have ambiguous beliefs. Our analysis may therefore be considered as a first step and it may be refined in several ways. One future research question is the possibility of tipping points. In addition to a high level of complexity, the major challenge of this extension is the need to incorporate thresholds, discontinuities and sudden switches, which remain poorly understood on a theoretical level.[29] Another interesting direction goes towards a more detailed analysis of short- and medium-run climate projections.[30] We hope to take up some of these tasks in our future work and we consider it probable that this research agenda and the conceptual follow-up issues will continue to warrant substantial research effort in the future.

---

[29]The climate literature on tipping points is, indeed, a fast growing industry. Unfortunately, there are not any models yet incorporating such nonlinearities into micro-founded decision-making frameworks with Knightian uncertainty. It must be emphasised that the model described here is sufficiently general to study various tipping points. It is only necessary to fine-tune the framework for specific nonlinearities and to embed further stochastic processes.

[30]In the simulations in Section 3.3, the impact of Knightian uncertainty is "statically" addressed. Hence, we may next aim to study the temporal implications of Knightian uncertainty, and the impact of less medium-run ambiguity resulting from more reliable decadal projections upon optimal climate policies.

### 3.A  DERIVATION OF EQUATION (3.12)

First, we to show that the $Q^\theta \in \mathscr{P}$ that minimises the expectation value in equation (3.11) is generated by $\theta = -\kappa$.

Additionally Fubini's theorem for conditional expectations transforms $W^{\mathrm{N}}(X, \Delta \mathbb{T}; \Delta \mathbb{T}_H)$ to

$$\frac{1}{1-\delta} \min_{Q^\theta \in \mathscr{P}} \int\limits_{t=0}^{\infty} e^{-(r-(1-\delta)g_0)s} E^{Q^\theta} \left[ 1 - X_s (1-\delta) \Delta \mathbb{T}_s^2 | \mathscr{F}_t \right] ds. \qquad (3.27)$$

By applying Ito's Lemma to the logarithm of $X_s$ we obtain $\forall s \geq 0$ :

$$X_s = X_0 e^{(\alpha - \frac{1}{2}\sigma^2 - \sigma\theta)s + \sigma B_s^\theta} = X_0 e^{(\alpha - \frac{1}{2}\sigma^2 - \sigma\theta)s} e^{\sigma B_s^\theta}. \qquad (3.28)$$

Obviously it holds that

$$X_s = X_0 e^{(\alpha - \frac{1}{2}\sigma^2 - \sigma\theta)s} e^{\sigma B_s^\theta} \leq X_0 e^{(\alpha - \frac{1}{2}\sigma^2 + \sigma\kappa)s} e^{\sigma B_s^\theta} \quad \forall s \geq 0 \quad \forall \theta \in [-\kappa, \kappa]. \qquad (3.29)$$

Due to the monotonicity of the conditional expectation value, we obtain

$$\begin{aligned}
&E^{Q^\theta} \left[ 1 - X_0 e^{(\alpha - \frac{1}{2}\sigma^2 - \sigma\theta)s} e^{\sigma B_s^\theta} (1-\delta) \Delta \mathbb{T}_s^2 | \mathscr{F}_t \right] \\
&\geq E^{Q^\theta} \left[ 1 - X_0 e^{(\alpha - \frac{1}{2}\sigma^2 + \sigma\kappa)s} e^{\sigma B_s^\theta} (1-\delta) \Delta \mathbb{T}_s^2 | \mathscr{F}_t \right] \\
&= \left( 1 - X_0 e^{(\alpha - \frac{1}{2}\sigma^2 + \sigma\kappa)s} \right) (1-\delta) \Delta \mathbb{T}_s^2 E^{Q^\theta} \left[ e^{\sigma B_s^\theta} | \mathscr{F}_t \right] \\
&= \left( 1 - X_0 e^{(\alpha - \frac{1}{2}\sigma^2 + \sigma\kappa)s} \right) (1-\delta) \Delta \mathbb{T}_s^2 \, e^{\frac{1}{2}\sigma^2 s} \\
&= \left( 1 - X_0 e^{(\alpha - \frac{1}{2}\sigma^2 + \sigma\kappa)s} \right) (1-\delta) \Delta \mathbb{T}_s^2 \, E^{Q^{-\kappa}} \left[ e^{\sigma B_s^{-\kappa}} | \mathscr{F}_t \right] \\
&\qquad \forall s \geq 0, \ \forall \theta \in [-\kappa, \kappa].
\end{aligned} \qquad (3.30)$$

Thus, the measure $Q^{-\kappa} \in \mathscr{P}$ minimises the expectation value in (3.11), which we therefore denote as $Q^*$. Consequently the process $X$ that results from implementing $\theta = -\kappa$ into equation (3.8) shall be called $X^*$.

For the following considerations let $W^{\mathrm{N}}(X, \Delta \mathbb{T}; \Delta \mathbb{T}_H)$ be conveniently abbreviated by $W^{\mathrm{N}}$.

The corresponding Hamilton-Jacobi-Bellman equation to equation (3.11) is as follows:

$$
\begin{aligned}
\left(r - (1-\delta) g_0\right) W^{\mathrm{N}} &= \frac{1}{1-\delta}\left(1 - X^* (1-\delta) \Delta\mathbb{T}^2\right) + \frac{1}{dt} E^{Q^*}\left[dW^N | \mathscr{F}_t\right] \\
&= \frac{1}{1-\delta} - X^* \Delta\mathbb{T}^2 + \frac{1}{dt} E^{Q^*}\left[dW^N | \mathscr{F}_t\right].
\end{aligned}
\tag{3.31}
$$

$W^{\mathrm{N}}$ is obviously differentiable at least once in $\Delta\mathbb{T}$ and twice in $X^*$, which allows to apply Ito's Lemma:

$$
\begin{aligned}
dW^{\mathrm{N}} &= \frac{\partial W^{\mathrm{N}}}{\partial \Delta\mathbb{T}} d\Delta\mathbb{T} + \frac{\partial W^{\mathrm{N}}}{\partial X^*} dX^* + \frac{\partial^2 W^{\mathrm{N}}}{\partial X^{*2}} (dX^*)^2 \\
&= \frac{\ln(2)}{H} \left(2\Delta\mathbb{T}_H - \Delta\mathbb{T}_t\right) \frac{\partial W^{\mathrm{N}}}{\partial \Delta\mathbb{T}} dt \\
&\quad + \frac{\partial W^{\mathrm{N}}}{\partial X^*} \left[(\alpha + \sigma\kappa) X_t^* dt + \sigma X_t^* dB_t^{-\kappa}\right] + \frac{1}{2}\sigma^2 X^{*2} \frac{\partial^2 W^{\mathrm{N}}}{\partial X^{*2}} dt,
\end{aligned}
\tag{3.32}
$$

by using equation (3.4) in the text. Taking expectation of (3.32) and dividing by $dt$ we obtain

$$
\frac{E\left[dW^{\mathrm{N}}\right]}{dt} = \frac{\ln(2)}{H} \left(2\Delta\mathbb{T}_H - \Delta\mathbb{T}_t\right) \frac{\partial W^{\mathrm{N}}}{\partial \Delta\mathbb{T}} + (\alpha + \kappa\sigma) X_t^* \frac{\partial W^{\mathrm{N}}}{\partial X^*} + \frac{1}{2}\sigma^2 X^{*2} \frac{\partial^2 W^{\mathrm{N}}}{\partial X^{*2}}.
\tag{3.33}
$$

Substituting (3.33) back to the Hamilton-Jacobi-Bellman equation (3.31) gives

$$
\begin{aligned}
\left(r - (1-\delta) g_0\right) W^{\mathrm{N}} &= \frac{1}{1-\delta} - X^* \Delta\mathbb{T}^2 + \frac{\ln(2)}{H} \left(2\Delta\mathbb{T}_H - \Delta\mathbb{T}\right) \frac{\partial W^{\mathrm{N}}}{\partial \Delta\mathbb{T}} \\
&\quad + (\alpha + \kappa\sigma) X^* \frac{\partial W^{\mathrm{N}}}{\partial X^*} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{N}}}{\partial X^{*2}},
\end{aligned}
\tag{3.34}
$$

which is equation (3.12) in the text.

Note that the above proof only holds for the first order Taylor's expansion of the exponential loss function $e^{-X_s(1-\delta)\Delta T_s}$. We need to numerically confirm that the above results also hold for the exponential loss function. While it is almost not possible to obtain the direct expectation of the exponential function, we can numerically test the impact of $\theta$ on $W^{\mathrm{N}}$ by using simple Monte Carlo simulations of equation (3.6). The discrete-time approximation of equation (3.6) is shown as follows,

$$
X_{t+\Delta t} - X_t = (\alpha - \sigma\theta) X_t \Delta t + \sigma\sqrt{\Delta t} X_t \epsilon_t.
\tag{3.35}
$$

where $\epsilon_t$ is generated from a standard normal distribution $N(0,1)$ generator and $\Delta t$ is the discrete-time representation of $dt$. The value of $W^{\mathrm{N}}$ is computed from the following approximations,

$$
W^{\mathrm{N}} = \frac{1}{1-\delta} \sum_{n=0}^{n_{\max}} e^{-X_{n\Delta t}(1-\delta)(\Delta T_{n\Delta t})^2} e^{-(r-(1-\delta)g_0)n\Delta t} \Delta t,
\tag{3.36}
$$

where $\Delta T_{n\Delta t} = 2\Delta T_H \left(1 - e^{-\frac{\ln 2}{H} n\Delta t}\right)$, an discrete-time version of equation (3.5). The results of the Monte Carlo simulations are shown in Figure 3.4.1. Note that $\Delta t = 0.01$, $n_{\max} = 30,000$ with $10,000$ rounds of Monte Carlo simulations, and the rest of benchmark values are the same as ones in the text.



**(a)** Monte Carlo simulations

**(b)** First Order Taylor Expansion Result
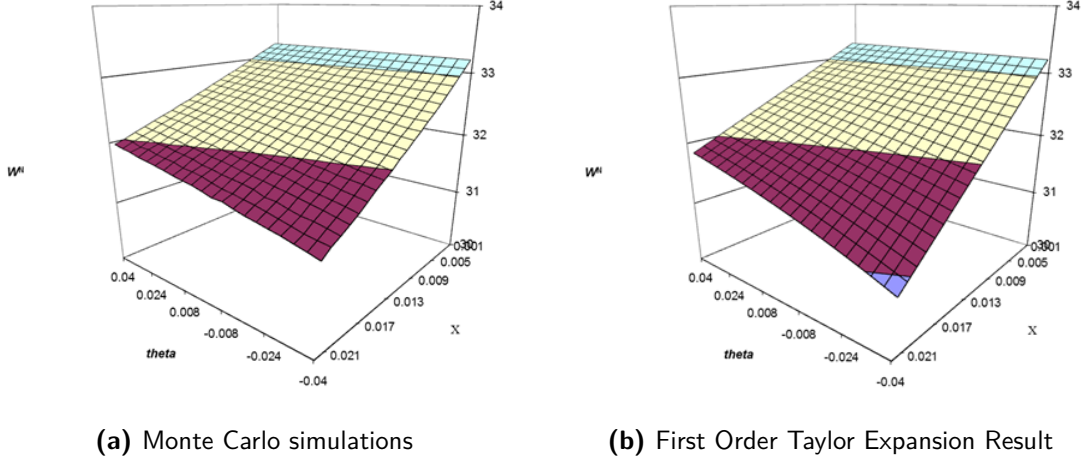
**Figure 3.4.1:** Comparison of the Results for $W^{\mathrm{N}}$

The results of Monte Carlo simulations clearly show that the nature responds with choice of $\theta = -\kappa$ to yield the minimal value of $W^{\mathrm{N}}$, while the results by the first order Taylor's expansion exhibit similar qualitative outcome, albeit over-estimating the impact of reduction in $\theta$ on $W^{\mathrm{N}}$.

## 3.B PARTICULAR SOLUTIONS TO $W^{\mathrm{NP}}$ AND $W^{\mathrm{A}}$

Using equations (3.11) and (3.5) yields the following particular integral,

$$W^{\mathrm{NP}}(X, \Delta \mathbb{T}; \Delta \mathbb{T}_H) = \frac{1}{1-\delta} \int_{t=0}^{\infty} \left(1 - X^* e^{(\alpha + \kappa \sigma)s}(1-\delta)\right.$$

$$\left.\left(2\Delta \mathbb{T}_H \left(1 - e^{-\frac{\ln 2}{H} s}\right)\right)^2\right) e^{-(r-(1-\delta)g_0)s} ds. \tag{3.37}$$

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In the same manner we employ equation (3.18) and (3.16) to derive

$$W^{\mathrm{A}}\left(X, \Delta \mathbb{T} ; \tau\right) = \frac{(1 - w\left(\tau\right))^{1-\delta}}{1-\delta} \int\limits_{t=0}^{\infty} \Big(1 - X^{*} e^{(\alpha + \kappa \sigma)s}\left(1 - \delta\right)$$

$$\left(2\tau\left(1 - e^{-\frac{\ln 2}{H}s}\right)\right)^{2}\Big) e^{-(r-(1-\delta)g_0)s} ds. \tag{3.38}$$

Equations (3.37) and (3.38) result from Ito's Lemma which means that equation (3.28) with $\theta = -\kappa$ is applied to equation (3.11) and (3.18), respectively. Furthermore please note that $E^{Q^{-\kappa}}\left[e^{\sigma B_s^{-\kappa}} | \mathscr{F}_t\right] = e^{\frac{1}{2}\sigma^2 s}$. By expanding the term

$$\left(1 - e^{-\frac{\ln 2}{H}t}\right)^{2} = 1 - 2e^{-\frac{\ln 2}{H}t} + e^{-2\frac{\ln 2}{H}t} \tag{3.39}$$

we obtain

$$\left(1 - X^{*} e^{(\alpha + \kappa \sigma)s}\left(1 - \delta\right)\left(2\Delta \mathbb{T}_H\left(1 - e^{-\frac{\ln 2}{H}s}\right)\right)^{2}\right) e^{-(r-(1-\delta)g_0)s} \tag{3.40}$$

$$= e^{-(r-(1-\delta)g_0)s} - 4\Delta \mathbb{T}_H^2\left(1 - \delta\right) X^{*} e^{(\alpha + \kappa \sigma)s}\left(1 - 2e^{-\frac{\ln 2}{H}s} + e^{-2\frac{\ln 2}{H}s}\right) e^{-(r-(1-\delta)g_0)s}$$

Substituting (3.40) back into (3.37) and integrating yields

$$W^{\mathrm{NP}}\left(X, \Delta \mathbb{T} ; \Delta \mathbb{T}_H\right) \tag{3.41}$$

$$= \frac{1}{1-\delta}\left[\frac{1}{r-(1-\delta)g_0} - 4\Delta \mathbb{T}_H^2\left(1 - \delta\right) X^{*}\left(\frac{1}{\eta} - \frac{2}{\eta + \frac{\ln 2}{H}} + \frac{1}{\eta + 2\frac{\ln 2}{H}}\right)\right]$$

where

$$\eta = r - (1-\delta)g_0 - (\alpha + \kappa \sigma)$$

Similarly, we have

$$W^{\mathrm{A}}\left(X, \Delta \mathbb{T} ; \tau\right) \tag{3.42}$$

$$= \frac{(1 - w\left(\tau\right))^{1-\delta}}{1-\delta}\left[\frac{1}{r-(1-\delta)g_0} - 4\Delta \tau^2\left(1 - \delta\right) X^{*}\left(\frac{1}{\eta} - \frac{2}{\eta + \frac{\ln 2}{H}} + \frac{1}{\eta + 2\frac{\ln 2}{H}}\right)\right]$$

which are equations (3.21) and (3.22), respectively.

## 3.C General Solution $W^{\mathrm{NG}}$ for $W^{\mathrm{N}}$

We guess the solution to equation (3.14) has the following functional form:

$$W^{\mathrm{NG}}\left(t=0, X, \Delta\mathbb{T}; \Delta\mathbb{T}_H\right) = AX^{*\beta}\left(\Delta\mathbb{T}^2 + C\Delta\mathbb{T} + D\right). \tag{3.43}$$

where $A$, $C$, $D$ are some parameters. Calculating derivatives, we obtain

$$\frac{\partial W^{\mathrm{NG}}}{\partial\Delta\mathbb{T}} = AX^{*\beta}\left(2\Delta\mathbb{T} + C\right), \tag{3.44}$$

$$X^*\frac{\partial W^{\mathrm{NG}}}{\partial X^*} = \beta AX^{*\beta}\left(\Delta\mathbb{T}^2 + C\Delta\mathbb{T} + D\right) \quad \text{and} \tag{3.45}$$

$$X^{*2}\frac{\partial^2 W^{\mathrm{NG}}}{\partial X^{*2}} = \beta\left(\beta - 1\right)AX^{*\beta}\left(\Delta\mathbb{T}^2 + C\Delta\mathbb{T} + D\right). \tag{3.46}$$

Substituting the above-outlined equations back to equation (3.14) and rearranging yields

$$2\left(\frac{\ln\left(2\right)}{H}\right)AX^{*\beta}\left(\Delta\mathbb{T}^2 - \left(2\Delta\mathbb{T}_H - \frac{C}{2}\right)\Delta\mathbb{T} - C\Delta\mathbb{T}_H\right)$$
$$= \left[-\left(r - \left(1 - \delta\right)g_0\right) + \left(\alpha + \kappa\sigma\right)\beta + \frac{1}{2}\sigma^2\beta\left(\beta - 1\right)\right]$$
$$AX^{*\beta}\left(\Delta\mathbb{T}^2 + C\Delta\mathbb{T} + D\right). \tag{3.47}$$

Solving (3.47) requires

$$\Delta\mathbb{T}^2 - \left(2\Delta\mathbb{T}_H - \frac{C}{2}\right)\Delta\mathbb{T} - C\Delta\mathbb{T}_H = \left(\Delta\mathbb{T}^2 + C\Delta\mathbb{T} + D\right). \tag{3.48}$$

Thus, we have

$$C = -4\Delta\mathbb{T}_H \tag{3.49}$$

and

$$D = -C\Delta\mathbb{T}_H = 4\Delta\mathbb{T}_H^2. \tag{3.50}$$

Plugging (3.49) and (3.50) into (3.47), we obtain

$$\left[-\left(r - \left(1 - \delta\right)g_0 + 2\left(\frac{\ln\left(2\right)}{H}\right)\right) + \left(\alpha + \kappa\sigma\right)\beta + \frac{1}{2}\sigma^2\beta\left(\beta - 1\right)\right]W^{\mathrm{NG}} = 0, \tag{3.51}$$

where $W^{\mathrm{NG}} = AX^{*\beta}\left(\Delta\mathbb{T}^2 - 4\Delta\mathbb{T}_H\Delta\mathbb{T} + 4\Delta\mathbb{T}_H^2\right)$. The solution of (3.51) requires

$$\left(\alpha + \kappa\sigma\right)\beta + \frac{1}{2}\sigma^2\beta\left(\beta - 1\right) - \left(r - \left(1 - \delta\right)g_0 + 2\left(\frac{\ln\left(2\right)}{H}\right)\right) = 0. \tag{3.52}$$

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Let $\beta_1$ and $\beta_1$ be the positive and negative roots of the above characteristic function, respectively. By some manipulations, this leads to

$$W^{\mathrm{NG}} = A_1 X^{*^{\beta_1}} \left( \Delta\mathbb{T}^2 - 4\Delta\mathbb{T}_H\Delta\mathbb{T} + 4\Delta\mathbb{T}_H^2 \right) - A_2 X^{*^{\beta_2}} \left( \Delta\mathbb{T}^2 - 4\Delta\mathbb{T}_H\Delta\mathbb{T} + 4\Delta\mathbb{T}_H^2 \right) \quad (3.53)$$

As we only consider the option to take action, we need to set the boundary condition such that $\lim\limits_{X \to 0} W^{\mathrm{NG}}(X) = 0$, which is tantamount to a zero option value of a climate policy, if climate change causes no damages that reduce the GDP. Therefore, the general solution with the negative root can be ignored. Consequently, we obtain

$$W^{\mathrm{NG}} = A_1 X^{*^{\beta_1}} \left( \Delta\mathbb{T}^2 - 4\Delta\mathbb{T}_H\Delta\mathbb{T} + 4\Delta\mathbb{T}_H^2 \right). \quad (3.54)$$

*So while we must take on the great challenge of mitigation with urgency and commitment [...] we must never lose sight of the importance of planning for and acting on adaptation now.*

Sir Nicholas Stern, Professor of Economics
and Chair of the Grantham Research Institute on
Climate Change and the Environment

# 4

# The Optimal Climate Policy of Mitigation and Adaptation: A Real Options Theory Perspective

## 4.1 INTRODUCTION

The negotiations at the past Conferences of the Parties to the UN Framework Convention on Climate Change have illustrated that the interests in and ideas about global cooperation on reducing emissions diverge considerably. At the same time, the global emission rates keep breaking new records every year and climate policy goals like the 2°C target become less likely to be achieved. Even if every country stopped emitting today, the warming trend would continue for several decades due to inertias in the climate system. Therefore, climate change is certain to happen and it will lead to changes in the environment and in the living conditions in more and more countries. Appropriately designed adaptation measures may help to gain from beneficial changes or to alleviate adverse impacts. Accordingly, climate policy can only be optimal if it factors in mitigation as well as adaptation. The best way to combine the two measures to fight climate change is, however, still far from being conclusive.

In the light of the urgency and relevance of this topic, the literature devoted to analysing the mix of the two measures is expanding rapidly . Kane & Shogren (2000) and Lecocq & Shalizi (2007) argue that mitigation and adaptation can be considered to be

strategic complements and do not stand alone if policy is optimally designed. Mitigation prevents irreversible and potentially unmanageable ramifications, whereas adaptation is necessary to alleviate the impacts that are already locked in by climate change. Ingham et al. (2005) show that mitigation and adaptation are economic substitutes on the cost as well as on the benefit side. On the cost side, the investments in these measures compete for resources that are naturally scarce. On the benefit side, the usage of one option decreases the marginal benefit of the other. More precisely, mitigating emissions will successfully avoid damage and thus less adaptation is needed. Conversely, adapting effectively to global warming and the related consequences decreases the marginal benefit of emission reductions, as for example noted by Tol (2005a). As suggested by de Zeeuw & Zemel (2012), already the prospect of adapting in the future is increasing the current emission rate.[1] Quite recently, the existence of complementary and substitution effects was confirmed by Integrated Assessment Models such as AD-WITCH by Bosello et al. (2009, 2010, 2011) and Bosello & Chen (2010), AD-DICE by de Bruin et al. (2009), Ada-BaHaMa by Bahn et al. (2012) and AD-FAIR by Hof et al. (2009).[2] Interestingly, Bosello et al. (2010) and de Bruin et al. (2009) identify the trade-off between the two measures to be asymmetric. The two measures crowd each other out, but the effect of mitigation on adaptation is found to be weaker. In the short- and medium term, the benefits of mitigation are argued to be too small to reduce significantly the need to adapt. Moreover, both studies exhibit higher expenditures on adaptation, indicating that adaptation is the preferred measure. However, this result is very sensitive to the assumption concerning the discount rate: the more far-sighted the policy maker is assumed to be, the more attractive mitigation becomes. The reason is that the time gap between the occurrence of costs and the occurrence of benefits is much longer in the case of mitigation due to slow and lagged dynamics in the climate system. In contrast, adaptation can become effective as soon as it is fully implemented.

The understanding of how uncertainty affects the optimal mix is still at a "very early stage", as pointed out by Agrawala et al. (2011b). Felgenhauer & Bruin (2009) investigate the effects of uncertainty about climate sensitivity in a two-period model with learning. This kind of uncertainty is shown to reduce both mitigation and adaptation

---

[1]Aside from the outlined structure of the strategic complementarity and trade-offs, IPCC (2007a) identifies specific examples of adaptation measures that can facilitate or exacerbate mitigation. If adaptation efforts involve an increased usage of energy, the total level of emissions that has to be mitigated increases. This is for example the case for air conditioning as a measure to adapt to heat or for seawater desalination as a measure to adapt to draughts. Other adaptation measures can facilitate mitigation, as they also decrease emissions. Buildings that are designed to reduce vulnerability to extreme weather events may also decrease the energy needs for heating and cooling.

[2]An extensive survey of this literature is provided by Agrawala et al. (2011b).

efforts. Furthermore, mitigation efforts are shown to be more sensitive to uncertainty than adaptation efforts. It is reasoned that uncertainty about climate sensitivity has long-run implications, affecting the decision about the long-run measure of mitigation more significantly. A multi-stage-decision under uncertainty about the benefits of both measures is qualitatively discussed by Felgenhauer & Webster (2013b), who suggest that the differences in the time lags between adopting a measure and learning about its benefits make adaptation and mitigation imperfect substitutes.

This paper aims to complement the research on the optimal policy mix of adaptation and mitigation under uncertainty by accounting for characteristics that cannot be fully captured by the normal net present value approach. It is generally agreed that the climate policy decision needs to take into account that (i) there is uncertainty about the future benefits of mitigation as well as of adaptation, (ii) waiting allows policy makers to gather new information about the uncertain future, (iii) the required investments in both policy measures are at least partially irreversible, which means that disinvesting cannot fully recover all the expenditures and (iv) the greenhouse gases accumulate and remain in the atmosphere long after they are emitted. On the one hand, the opportunity to wait for new information to arrive may induce the policy maker to delay costly and irreversible policy measures. On the other hand, a wait-and-see attitude may burden future generations with costs of an unknown size that are caused by irreversible climate damage. Hence, it may seem rational to adopt climate policy as soon as possible. These considerations show that the tension between uncertainty and these two types of irreversibility generates some value of delaying or accelerating investments. Differently from the above-mentioned studies, which apply a normal net present value approach, this paper explicitly accounts for this value.

This value of waiting– also referred to as the value of managerial flexibility – is considered to be a real option. This concept has its roots in the evaluation of financial options as developed by Black & Scholes (1973) and Merton (1973). On financial markets, the investor pays a premium price to obtain the right, but not the obligation, to buy an asset for some time at a predetermined price. Profit is made when the price of the underlying asset rises above the predetermined price and the option is exercised. Even then, it can be profitable to wait to exercise the option and to speculate for a further price increase in the underlying asset. Hence, holding the option is still of value due to uncertainty about the future asset price. The concept soon turned out to grant considerable insights into capital investment decisions and is thus referred to as real options analysis (ROA). Similar to exercising a financial option, most capital investment decisions are (at least partially) irreversible due to sunk costs incurred by the investment. Furthermore, the

investor often faces uncertainty about the profits the investment will generate, because the prices of inputs or outputs may vary over time. In such a situation, the flexibility to delay an investment may be of value, as more information about the involved uncertainties can be gained as time passes. ROA is designed to capture the value of waiting and thus exceeds the normal net present value approach. Early applications of ROA to investment decisions are for example given by McDonald & Siegel (1986) and Pindyck (1988, 1993). The studies by Kolstad (1996) and Ulph & Ulph (1997), published soon afterwards, focus their attention on the implications of irreversibility and uncertainty for climate policy. The ROA conception is that the policy maker has the "right" to adopt these climate policy measures in return for lower future damage costs. Accordingly, the real options value captures the opportunity costs of implementing such a policy now rather than waiting for new information to arrive. In almost all cases, ROA is conducted solely to examine either mitigation or adaptation and not both together. The mitigation option is investigated in the seminal work by Pindyck (2000, 2002) or later analyses by Anda et al. (2009), Baranzini et al. (2003), Chen et al. (2011a,b), Lin et al. (2007), Nishide & Ohyama (2009) and Wirl (2006).[3] The real option to undertake specific adaptation projects is explored by Dobes (2008, 2010), Hertzler (2007), Linquiti & Vonortas (2012), Nordvik & Lisø (2004) and Watkiss et al. (2013). In practice, however, more than one measure is available to fight climate change, and their optimal mix might be affected by uncertainty and irreversibility as well.[4] The first attempts to analyse the optimal balance of mitigation and adaptation by means of real options theory are presented by Maybee et al. (2012) and Strand (2011). As a result of a non-formal discussion, Maybee et al. (2012) anticipate that, due to the local nature of adaptation, the benefits of adaptation seem to be more guaranteed and thus greater priority is given to adaptation measures. Strand (2011) examines how the decision to mitigate is affected by adaptation, but adaptation is not treated as a real option but as an exogeneously given process.

To provide a more realistic picture of the policy maker's portfolio to fight climate change, this paper develops a new modelling framework for a portfolio of mitigation and adaptation real options. The adaptation options allow the policy maker to postpone investment or to invest the optimal portion of the GDP in projects that alleviate climate change impacts. The mitigation option gives the opportunity to choose the optimal timing for curbing emissions. Incorporating both real options into the same framework

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[3]While the above-mentioned research deals with one global decision maker, the work by Barrieu & Chesney (2003) and Ohyama & Tsujimura (2006, 2008) analyses the strategic agents' decision on when to curb emissions.

[4]Evidence of adverse effects by uncertainty and irreversibility on climate policy is found in psychology. Gifford (2011) argues that the existence of sunk costs, uncertainty and risks belongs to the barriers or "dragons of inaction" that hinder mitigation and adaptation efforts.

implies that the values of the individual options are affected by each other's presence. This paper can thus investigate the interaction of the two values of waiting. How are the decisions to design the optimal mix of mitigation and adaptation affected by uncertainty and irreversibility?[5]

The remainder of this paper is organized as follows. Section 4.2 gives an overview of the most important properties of the modelling framework. Section 4.2.1 provides all the required equations to derive the optimal adaptation policy in Section 4.2.2 and the optimal mitigation timing in Section 4.2.3. The numerical simulations are presented in Section 4.3. Section 4.4 concludes the paper. More details are available in the technical Appendices 4.A - 4.D.

## 4.2 A Real Options Model of Adaptation and Mitigation

The decision regarding when to cut emissions is complicated. Firstly, the predicted benefits of mitigation involve huge uncertainties. Secondly, exercising the option to mitigate involves large sunk costs – for example induced by a switch to $CO_2$ neutral technologies in the energy sector. As soon as this option is exercised, the decision maker gives up the possibility to wait for new information to arrive. The combination of sunk costs and uncertainty generates opportunity costs of adopting the policy now. However, as the damage is largely irreversible, exercising this option can also create sunk benefits. The net of these opportunity costs and benefits is reflected by the value of the real option and must be included in the decision model. In contrast to normal cost-benefit analysis, this approach can therefore explain how optimal policy and its timing is influenced by uncertainty and irreversibility.[6]

In practice, a decision maker also holds the option to invest in a better adjustment to the future impacts of climate change. The possibilities to adapt are manifold. The category of adaptation measures that is most relevant to the context of climate policy design is referred to as anticipatory or proactive adaptation, because it can be planned and taken in advance.[7] Additionally, these measures may be classified on the basis of the

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[5]Alternatively, one may consider only one real option that offers the opportunity to switch between different "modes" of climate policy, e.g. to do nothing, to mitigate only, to invest a certain, albeit not optimised, portion of the GDP in adaptation or to do both (in this case switching between all the modes might not be allowed). These kinds of real options models are for example applied to assess decisions to invest in the electricity sector, e.g. see Fuss et al. (2009, 2011). Obviously, this simplification cannot adequately encapsulate the interaction of the respective values of waiting, as they are not individually modelled.

[6]A more detailed introduction to the real option modelling framework is provided by Dixit & Pindyck (1994) and Stokey (2009).

[7]The measures falling into the opposite category are implemented as soon as the damage occurs (reactive adaptation); see Smit et al. (2000) for this and other categorizations.

type of damage they reduce. Dykes and early warning systems are meant to lessen the impacts of occasionally occuring climate catastrophes. Other measures, like sea water desalination, land-use zoning, air conditioning, thermal insulation, vaccination programmes or the breeding of more resilient crops, help to alleviate everyday life that has been made difficult by gradually evolving climate change. Although differing in the purpose they address, almost all adaptation efforts require investments that are largely sunk. Furthermore, it is not clear in advance whether their design is both perfectly suitable for and effective in decreasing the future damage costs. The combination of irreversible investments and the uncertainty of the resulting benefits implies that adaptation projects can be modelled as real options. Consequently, the policy maker holds a portfolio of different option types: one option to mitigate and options to invest in adaptation.

In this paper, the options are modelled to reflect certain characteristics of adaptation and mitigation. Mitigation addresses the source of the climate change problem by reducing the amount of emitted greenhouse gases (GHGs). Once abated, these emissions cannot cause future damage. Therefore, early mitigation efforts can be considered to be the best insurance against climate change damages. Adaptation addresses the outcome of the climate change problem by alleviating the present or expected damages.

The decision to mitigate is modelled as a commitment to a certain emission reduction target, but it is not meant to be a continuous investment decision that can be immediately adjusted if necessary. In this context, making this distinction is important, as the first mentioned specification resembles a one-off decision and implies less flexibility to react to shocks or to new information pouring in. This idea of modelling better reflects reality, as mitigation efforts are negotiated in terms of emission reduction targets and are stipulated by an international treaty for longer periods of time. In contrast, adaptation is not about committing to a particular target but about investing in suitable projects wherever and whenever required. Accordingly, in the model, the decision maker can switch between waiting to invest and investing the optimal amount of money. However, the implementation of adaptation projects is assumed to take time.

The model also incorporates a stylized notion of adaptation capacity. The capacity is here understood to comprise all the means that enable the adoption of adaptation measures rather quickly.[8] Climate damage is assumed to compromise these means. This is consistent with the observation, as for example indicated by Smith et al. (2001), that countries already suffering from climate damage lack the capacity for quick adaptation.

Finally, adaptation provides a local public good in most cases. Hence, economic theory suggests that adaptation should be supplied by the countries or local communities

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[8]The capacity to adapt depends on many factors, e.g. on the institutional system, economic and technological development, knowledge, values, ethics and cultures; see for example Adger et al. (2009).

that benefit from these measures in the first place. However, as outlined by Lecocq & Shalizi (2007), several reasons corroborate the idea of modelling adaptation as a strategy that requires international collective action. A large number of countries lack the institutional, technological and financial capacities to meet their adaptation needs, a fact that calls for international aid and cooperation. Moreover, while mitigation forces the polluter to pay, adaptation is required where the damage occurs and not necessarily where it is primarily caused. Hence, equity justifies the international funding of adaptation projects. Furthermore, planning adaptation internationally could be effective. For example, it is beneficial to internalize externalities that may be caused by adaptation measures. Some projects may be operated in a more cost-effective fashion if they are carried out transnationally. In fact, the United Nations negotiates on adaptation and mitigation in the same breath.[9] Accordingly, in this paper, both mitigation and adaptation are considered to concern global policy.

For simplicity, technological progress is not incorporated into the modelling framework. Accounting for further real options that allow investment in R&D of one or the other climate policy measure would be a valuable next research step. Alternatively, the technological progress in these measures could be modelled as additional sources of uncertainty. However, the implementation of exogenously defined technological progress based on some ad hoc assumptions about how the technologies to mitigate and or to adapt may develop is not considered to be a worthwhile improvement of this analysis.

The procedure for incorporating both real options into one framework is as follows. The policy maker has the choice of when to switch from the high- to the low-emission scenario. In both scenarios, adaptation efforts are undertaken optimally. The optimal timing of mitigation is then inserted back into the adaptation model to obtain the optimal adaptation policy given that the emissions are optimally reduced.

### 4.2.1 THE MODEL

In the following model, it is assumed that a forward-looking and risk-neutral policy maker strives to find the optimal policy for adaptation and mitigation by weighing the flow of consumption against the policy costs. More precisely, the policy decision is based on maximizing welfare, which can be expressed by

$$W = \mathbb{E}_0 \left[ \int_0^\infty \left[ \ Y(t) \left(1 - D(t)\right) - C_a(t) - C_m(t) \ \right] e^{-rt} dt \right], \qquad (4.1)$$

---

[9]Accordingly, the Kyoto Protocol has not only stipulated emission reductions, but also established a fund that finances adaptation projects and programmes in needy member states; see `http://www.adaptation-fund.org/`.

where $\mathbb{E}_0$ describes the expectations operator conditioned on the information given in the present period $t = 0$. Here, the level of consumption is assumed to be equivalent to the level of the GDP $Y(t)$. Climate change causes damage costs $D(t)$, which reduce the level of the GDP. The costs of adaptation and mitigation are given by $C_a(t)$ and $C_m(t)$, respectively. The discount rate is described by $r$.

In the following, $Y(t) \equiv Y$ is assumed to be constant. Hence, all the processes that drive economic growth are ignored, in particular technological change.[10]

The proportion of climate damage costs $D$ in equation (4.1) can be expressed by an exponential function:

$$D(t) = 1 - e^{-\frac{\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^\phi}}, \qquad (4.2)$$

where $\rho \in [0,1)$, $\alpha, \phi, \psi \in \mathbb{R}_+$. The exponent $\phi$ determines how quickly the effectiveness of adaptation decreases. This exponential function depends on the functions $M(t)$, which describes the accumulation of GHGs in the atmosphere, $A(t)$, which reflects the adaptation efforts, and $\theta(t)$, which causes stochasticity in the social costs of climate change. For notational ease, the exponential function is referred to:

$$\Upsilon(\theta(t), M(t), A(t)) = e^{-\frac{\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^\phi}}. \qquad (4.3)$$

The uncertainty regarding the gravity of the losses inflicted by pollution is either caused by a lack of knowledge about the values of certain key parameters or intrinsically given. Economic models exhibit a substantial degree of instrinsic uncertainty. Even if all the parameters were known, there would still be uncertainty due to random exogenous events and fluctuations in the system. This kind of uncertainty is immense over long time horizons, which need to be considered to assess climate policies. Therefore, it is important to analyse the effects caused by intrinsic uncertainty. Pindyck (2000) suggests modelling

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[10]How the GDP growth affects the optimal policy mix is not the pivotal question in this paper and it is thus ignored in the following for the sake of limiting the computational effort. It is certainly worthwhile addressing this question as well, as the implementation of these policy measures and economic growth may exhibit interesting interaction effects. Some adaptation projects are thought necessary to allow for / facilitate economic growth, especially in developing countries. Conversely, as for example pointed out by Jensen & Traeger (2013), economic growth increases the expected future wealth, which may delay mitigation, as present generations are less willing to forego consumption today. Tsur & Withagen (2013) argue that these future, richer generations could more easily afford to invest in adaptation. However, it should not be forgotten that economic growth is the main driver of emissions and thus of the climate problem. A worsening of the climate conditions limits the possibilities to adapt and requires ever more refined technologies to alleviate the impacts. How economic growth affects both policy measures is thus a question of whether these technologies will be available and how costly they will be. The ambiguity in the relationship between the GDP and the adaptation costs is emphasized by Agrawala et al. (2011a). They find that in AD-WITCH and AD-DICE contrary but valid assumptions are made about this relationship.

the intrinsic uncertainty in the damage costs by utilizing a geometric Brownian motion with drift $\mu$, variance $\sigma$ and Wiener process $z$:

$$d\theta = \mu\theta dt + \sigma\theta dz. \tag{4.4}$$

Let $\theta$ capture all the processes that cannot be controlled by the policy maker, e.g. tastes or population growth. This process reflects the above-described characteristics of intrinsic uncertainty. First of all, the present level of social costs can be observed, whereas the future costs remain uncertain. Secondly, the longer the time horizon considered, the more uncertainty increases, which makes a reasonable decision on climate policy strategy difficult.[11]

For simplicity, I assume that there is no ecological uncertainty in the accumulation of GHGs in the atmosphere. As in Nordhaus (1994), it evolves according to:

$$\frac{dM}{dt} = \beta E(t) - \delta M(t), \tag{4.5}$$

where $\beta$ is the marginal atmospheric retention of emissions $E$. The natural rate of depletion is given by $\delta$, $0 < \delta < 1$. Once emitted, a certain percentage of the GHGs will stay in the atmosphere for a long time, as described by equation (4.5). For simplicity, the emissions are assumed to be proportional to the GDP without losses:

$$E(t) = \epsilon(1 - m(t))Y, \tag{4.6}$$

which can be curbed according to an emission reduction target $m$, $0 \leq m \leq 1$. Mitigation, however, incurs costs of:

$$C_m(t) = \kappa_1 m(t)^{\kappa_2} Y, \tag{4.7}$$

with $\kappa_1 \geq 0$ and $\kappa_2 > 1$ so that $C_m(t) < Y$ for all $t$ holds.[12] The convexity of this function relates to the increased costs and efforts required when choosing a higher emission reduction rate $m$.

Proactive adaptation can be considered to be a capital stock that lowers the harm inflicted by climate change; see Bosello et al. (2009, 2010, 2011) and Bosello & Chen

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[11]Alternatively, stochasticity could be modelled by a mean reverting process. This approach would imply that the policy maker has a good idea, albeit not perfect knowledge, of how the social costs will develop over long time horizons. The uncertainty about the costs in the very distant future is thus not significantly greater than the uncertainty about the costs in the near future. This would certainly be a feasible assumption if the climate damage cost function only depends on the atmospheric pollution, which is perfectly known in this model set-up. Here, I argue that there are many more factors that influence the damage costs, in particular economic factors, which are difficult to anticipate over long time horizons.

[12]Equations (4.6) and (4.7) are versions of the corresponding functions in Nordhaus (2010) without any technological progress.

(2010). The evolution of this capital stock is given by:

$$\frac{dA}{dt} = a(t)Y - \xi A(t). \tag{4.8}$$

This stock depreciates at a rate of $\xi \in (0,1)$. The decision maker can allocate a share $a(t)$, $a(t) < 1$ for all $t \geq 0$, of the GDP to investments in adaptation capital. These investments are assumed to be irreversible, i.e. $a(t) \geq 0$ for all $t \geq 0$. The investment costs are assumed to be convex, i.e. adaptation efforts take time. To account for the adaptive capacity, I also assume that the time to adapt increases with unabated damage. In other words, the (financial, institutional, technological, etc.) means that facilitate the quick conducting of adaptation measures deteriorate due to unabated climate damage. Accordingly, the "unit costs" of adaptation are the same, but the adjustment costs increase if the climate damage worsens.[13] These cost effects are disentangled by:

$$C_a(t)) = \gamma_1 a(t)Y + \frac{1}{2}\gamma_2 \frac{(a(t)Y)^2}{\Upsilon(\theta(t), M(t), A(t))}, \tag{4.9}$$

where the parameters $\gamma_1$ and $\gamma_2$ are positive. Additionally, the calibration of these two parameters must rule out $a \geq 1$.

Accounting for all the above-mentioned equations, the model is solved by first determining the optimal flow of investments $(a(t)Y)_{t\geq 0}$ for the high- ($m = 0$) and low-emission ($m > 0$) scenarios, as outlined in Section 4.2.2.

### 4.2.2 ADAPTATION POLICY

The decision maker strives to find the optimal strategy for investing in adaptation given emission policy $E$ or $m$. Welfare is thus rephrased as:

$$W(\theta(t), M(t), A(t); m(t) \equiv m) = \max_{0 \leq a(t) \leq 1} \mathbb{E}_0 \left[ \int_0^\infty \left( Y \Big( \Upsilon(\theta(t), M(t), A(t)) \right. \right.$$

$$\left. \left. - \gamma_1 a(t) - \frac{1}{2}\frac{\gamma_2}{\Upsilon(\theta(t), M(t), A(t))}a(t)^2 Y \right) - C_m \right) e^{-rt} dt \right]. \tag{4.10}$$

---

[13]Please note, that the "unit costs" of adaptation only stay the same in the absence of technological progress.

By applying Ito's Lemma, this optimization can be expressed by a Hamilton-Jacobi-Bellman equation:

$$rW = Y\Upsilon - C_m + (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial\theta^2}$$
$$- \xi A\frac{\partial W}{\partial A} + Y\max_{0\leq a(t)\leq 1}\left\{a\frac{\partial W}{\partial A} - \gamma_1 a - \frac{1}{2}\frac{\gamma_2}{\Upsilon}a^2 Y\right\}, \tag{4.11}$$

where the functional arguments are dropped to simplify the notation. Equation (4.11) implies the first-order condition for the optimal investment:

$$a^* = \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right). \tag{4.12}$$

The optimality condition clarifies whether and how much to invest. The marginal welfare of adaptation increases with higher pollution $M$ and a higher $\theta$. Therefore, the investment efforts increase in a situation of worse climate impacts. However, these efforts are slowed down by a decrease in $\Upsilon(\theta, M, A)$, reflecting a reduced adaptive capacity. Accordingly, the optimal policy design needs to incorporate considerations about maintaining sufficient adaptive capacity so that future generations are not limited in their options to adapt to climate change. This emphasizes the importance of the assumption of $\frac{\gamma_2}{\Upsilon(\theta(t),M(t),A(t)}$ being the adjustment cost parameter in equation (4.9).

Depending on the marginal value of adaptation, the investment strategy can then be summarized as:

$$a^* = \begin{cases} 0 & \text{for} \quad 0 \leq \frac{\partial W}{\partial A} \leq \gamma_1 \\ \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right) & \text{for} \quad \gamma_1 < \frac{\partial W}{\partial A} \leq \gamma_1 + \frac{\gamma_2 Y}{\Upsilon} \\ 1 & \text{for} \quad \frac{\partial W}{\partial A} > \gamma_1 + \frac{\gamma_2 Y}{\Upsilon} \end{cases} \tag{4.13}$$

It is optimal to start investing in adaptation as soon as the marginal welfare of adaptation is higher than $\gamma_1$. Please note that $a^* = 1$ is ruled out and only serves as a upper boundary. When reinserting the optimal investment policy (4.13) into equation (4.11), the resulting Hamilton-Jacobi-Bellman equation is defined differently in the range of possible values $\mathbb{R}^3_+ = \{(\theta, M, A) : \theta, M, A \geq 0\}$. In the region $S_1 = \left\{(\theta, M, A) : 0 \leq \frac{\partial W}{\partial A} \leq \gamma_1\right\} \subset \mathbb{R}^3_+$, welfare can be expressed by:

$$rW = Y\Upsilon - C_m + (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial\theta^2} - \xi A\frac{\partial W}{\partial A}. \tag{4.14}$$

If the marginal welfare of adaptation is sufficiently low, it is optimal not to invest. Then, the decision maker receives the expected present welfare given for the scenario of never

investing in adaptation. However, the stochastic fluctuations of $\theta$ may cause less favourable conditions and increase the marginal welfare of adaptation in the future. The value of the opportunity to invest in the future is clearly influenced by these stochastic fluctuations and by the fact that the investment costs are sunk. Accordingly, this opportunity is quantified by a real options value. The welfare in the region $S_1$ is therefore given by the sum of the expected present welfare of never investing and the real options value to expand the existing adaptation capital stock in the future.

In the region $S_2 = \left\{ (\theta, M, A) : \gamma_1 < \frac{\partial W}{\partial A} \right\} \subset \mathbb{R}^3_+$, welfare can be expressed by:

$$
\begin{aligned}
rW = Y\Upsilon - C_m &+ (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial \theta} \\
&+ \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial \theta^2} - \xi A\frac{\partial W}{\partial A} + \frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W}{\partial A} - \gamma_1\right)^2 .
\end{aligned}
\tag{4.15}
$$

As soon as the marginal welfare trespasses on the value $\gamma_1$, the policy maker starts to invest at the optimal rate given by equation (4.12). However, it is possible that the stochastic fluctuations of $\theta$ may decrease the marginal welfare of adaptation in the future. Such a decrease in the social costs may render investments in adaptation unnecessary and the policy maker can stop investing without costs. Therefore, the solution to the welfare in the region $S_2$ is only given by the expected present value of investing $a^* = \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right)$.

As the threshold at which the decision maker optimally switches from one investment regime to the other as well as the rate of optimal investment are given in terms of the marginal welfare of adaptation, the system is solved by deriving the partial derivatives of equations (4.14) and (4.15). More precisely, with the abbreviations $w = \frac{\partial W}{\partial A}$, $w_\theta = \frac{\partial^2 W}{\partial \theta A}$, $w_{\theta\theta} = \frac{\partial^3 W}{\partial \theta\theta A}$, $w_A = \frac{\partial^2 W}{\partial A^2}$ and $w_M = \frac{\partial^2 W}{\partial M A}$ the marginal welfare of adaptation for $S_1$ can be expressed as:

$$
\begin{aligned}
(r+\xi)w = &Y\Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}} + (\beta\epsilon(1-m)Y - \delta M)\,w_M + \mu\theta w_\theta + \frac{1}{2}\sigma^2\theta^2 w_{\theta\theta} \\
&- \xi A w_A \quad \forall(\theta, M, A) \in S_1;
\end{aligned}
\tag{4.16}
$$

its equivalent for $S_2$ is given by:

$$
\begin{aligned}
(r+\xi)w = &Y\Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}} + (\beta\epsilon(1-m)Y - \delta M)\,w_M + \mu\theta w_\theta + \frac{1}{2}\sigma^2\theta^2 w_{\theta\theta} \\
&+ \left(\frac{\Upsilon}{\gamma_2}(w - \gamma_1) - \xi A\right)w_A + \Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}}\frac{1}{2\gamma_2}(w - \gamma_1)^2 \quad \forall(\theta, M, A) \in S_2;
\end{aligned}
\tag{4.17}
$$

By equations (4.14) - (4.17) as well as equation (4.13) describing the threshold between $S_1$ and $S_2$ in terms of the marginal welfare of adaptation, the system is fully described. However, due to the complexity, the system cannot be solved analytically but requires numerical treatment. The applied numerical routine is a fully implicit finite difference method, as explained in Appendix 4.A.

### 4.2.3 Mitigation Policy

The timing of undertaking mitigation efforts, i.e. increasing $m = 0$ to some $m > 0$, depends on the optimal adaptation policy that is conducted in these emission scenarios. Hence, the recipe in Section 4.2.2 needs to be applied to derive the welfare of adaptation for $m = 0$ and for $m > 0$, respectively. The difference in the respective welfare values $W(\theta, M, A; m > 0) - W(\theta, M, A; m = 0)$ would describe the benefits of reducing emissions, if the decision to mitigate were a now-or-never decision. This net present value consists of the direct benefits that are given by less pollution and of the indirect benefits from prescribing a different adaptation strategy. These indirect benefits can be understood as the value of the additional flexibility in adaptation investments. As opposed to a now-or-never decision, the decision on when to cut emissions involves uncertainty and irreversibility, which gives waiting to mitigate a value that is expressed by its real option $W^M(\theta, M, A; m = 0)$. Depending on the optimal adaptation activities in the no-mitigation scenario $m = 0$, the real option to mitigate is expressed as follows:

$$
\begin{aligned}
rW^M = (\beta\epsilon Y - \delta M)\frac{\partial W^M}{\partial M} + \mu\theta\frac{\partial W^M}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^M}{\partial\theta^2} \\
- \xi A\frac{\partial W^M}{\partial A} + \frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - \gamma_1\right)^2 \mathbb{1}_{\{(\theta,M,A)\in S_2\}},
\end{aligned}
\tag{4.18}
$$

where $\mathbb{1}_{\{(\theta,M,A)\in S_2\}}$ is one in the region $S_2$ and zero in the region $S_1$.

The threshold of mitigation is derived by comparing the real options value (4.18) with the benefits of switching from the high- to the low-emission scenario, $W(\theta, M, A; m > 0) - W(\theta, M, A; m = 0)$. Again, the solution cannot be found analytically but requires numerical treatment, as described in Appendix 4.B.

To obtain the optimal policy thresholds, the mitigation threshold is computed by taking the optimal adaptation policy into account and the optimal adaptation policy needs to incorporate the optimal timing of the emission reduction efforts.

To achieve a better understanding of the interaction of mitigation and adaptation, a numerical analysis of the model needs to be conducted. This analysis consists of four parts. First, the optimal policy mix is investigated. Then, the interaction of the two measures is explored. Afterwards, the contribution of the ROA to the analysis of the climate policy decision is demonstrated. It must be noted that calibrating the model is particularly challenged by the lack of estimates concerning adaptation. As emphasized by Agrawala & Fankhauser (2008) and Bosello et al. (2009), studies of adaptation have been limited to a few economic sectors, countries and measures and are thus insuffcient to provide reliable estimates. Accordingly, some caution is required when interpreting the quantitative insights of modelling exercises based on these estimates. The last part of this numerical analysis is thus devoted to a sensitivity analysis of these parameters and other parameters that are controversially discussed. Studying the model as proposed may be comprehensive enough to provide some meaningful insights into the pivotal effects.

The base calibration is as follows. Emissions $E$ assume the value of 0.033 trillion of $CO_2$ metric tonnes, as estimated by EDGAR (the Emission Database from Global Atmospheric Research) for 2011.[14] For the same year, the IMF reports the global GDP to amount to $Y = 78.97$ trillion US dollars (PPP).[15] The present concentration of almost 400 ppm translates into $M = 40$ ($\times$ 10 ppm).[16] A considerable simplification of the numerical routine is offered by assuming that $\delta = 0$. This means that the parameter $\beta$ has to be adjusted to reflect the average increase in the atmospheric $CO_2$ concentration over the time horizon of interest. According to the latest measurements from the Mauna Loa Observatory, Hawaii, the current atmospheric $CO_2$ concentration can be assumed to increase by about 3 % per year. The parameter $\beta$ is thus computed to be 9.09 ppm per trillion $CO_2$ metric tonnes.

In most integrated assessment models, as in DICE, the damage function implictly factors in optimal adaptation efforts. Therefore, de Bruin et al. (2009) recompute the damage function in DICE by disentangling the adaptation costs from the damage costs. I adapt the damage function (4.3) to their calculations for the doubling of $CO_2$ and to the rather arbitrary assumption, which is needed for the numerical routine, that an extremely high concentration of 4200 ppm $CO_2$ in the atmosphere would lead to a total loss of GDP. For the concentrations that are likely to be reached in the near future and

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[14]This database is created by European Commission and the Joint Centre (JRC)/PBL Netherlands Environmental Assessment Agency; see `http://edgar.jrc.ec.europa.eu`.

[15]The data originate from the World Economic Outlook Database, October 2012 edition.

[16]Information about the measurements can be retrieved from `http://co2now.org`.

are thus of relevance to this study, the resulting damage function is very similar to the damage function presented by de Bruin et al. (2009). The current value of the cost parameter is set to be $\theta = 10$. This choice implies an approximate value for the social cost of $CO_2$ of about 16 US dollars, which is in the range of the estimates surveyed by Tol (2005b).[17] Just as controversial and crucial as the calibration of the damage function is the assignment of a value to the discount rate $r$. Here, I settle for a 2.5% discount rate.

The mitigation costs are chosen to be slightly lower than the function estimated by Cline (2011), which is based on a large set of model results compiled by the Stanford Energy Modeling Forum study EMF 22.[18] The mitigation target $m$ is premised on the emission reduction targets that countries would have to adhere to in order to satisfy the Copenhagen Accord by 2020. As reported by Cline (2011), these efforts would mean a 9% reduction in global emissions.

As many diverse forms of capital can be considered to be adaptation capital, it is also controversial to determine the depreciation rate $\xi$ in equation (4.8). For example, Bosello et al. (2011) and de Bruin et al. (2009) choose a value of 10%, while Agrawala et al. (2011a) and Felgenhauer & Webster (2013a) settle for a depreciation rate of 5%. I choose to compromise with $\xi = 0.075$.

As pointed out by Nishide & Ohyama (2009), the stochastic path of $\theta$ should be chosen somewhat arbitrarily, since associated data are lacking. A plausible calibration is represented by $\sigma = 0.07$ and $\mu = 0$.

The other parameters, such as those describing the costs and the effectiveness of adaptation, are chosen to comply with the rather broad estimates that are also used to calibrate the AD-DICE model by de Bruin et al. (2009) and the AD-WITCH model by Bosello et al. (2009, 2010, 2011). The reference point of calibration is the doubling of atmospheric $CO_2$. Concerning this point, an extensive review of the impact assessment literature by Tol et al. (1998) values the adaptation costs at about 7% - 25% of the total damage costs. Further studies, for example by Mendelsohn (2000) and Reilly et al. (1994), give the impression that the amount of damage that is reduced by adaptation in the calibration point could lie between 30% and 80%. Consistent with these ranges, the calibration of $\gamma_1$ and $\gamma_2$ determines the adaptation costs that are incurred by reacting to a doubling of $CO_2$ to be at least 0.18% of the GDP.[19] This number is thus of the

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[17]The social cost is derived by taking the net present value of the future damages caused by an additional ton of $CO_2$. The review by Tol (2005b) illustrates the diversity of the social cost assessments. In order to obtain a vague idea about whether the calibration of $\theta$ is feasible, i.e. the implied social cost is within the range of assessments, the exponential function in equation (4.3) can be roughly approximated by its first-order Taylor expansion.

[18]An overview of the EMF scenarios caan be found in Clarke et al. (2009).

[19]This value is calculated assuming that $\theta \equiv \theta_0$.

same order of magnitude as the estimates produced by the AD-WITCH and AD-DICE models.[20] The full listing of the parametrization is given in Appendix 4.C.

In the following, the simulation results are demonstrated by three-dimensional graphs of the state variables that are assumed to be given at the point in time when the decision has to be made. For each combination of already installed adaptation capital $A$ and level of atmospheric $CO_2$ concentration $M$, the threshold of taking action is derived in terms of the observed value of $\theta$. The resulting threshold curves thus divide the space of $(\theta, M, A)$ values into regions of optimal policy. The lower region spans all the values in which it is optimal to postpone policy adoption. In all the values above the threshold, the policy maker implements the policy immediately. In the case of adaptation, it additionally holds true that the intensity of investment efforts is higher the greater the distance to the threshold. The purpose of this representation is to investigate how the curves shift under alternative assumptions and to draw conclusions concerning the implied policy decisions.

Figure 4.3.1 illustrates the optimal policy of adaptation and mitigation. In this simulation, the two climate policy measures interact with each other. The adaptation threshold shown by Figure 4.3.1a takes into consideration the optimal timing of mitigation. The mitigation threshold given by Figure 4.3.1b is obtained by incorporating the information about the optimal investment into adaptation. The optimal policy mix unfolds by considering Figure 4.3.1c, which displays both thresholds together.
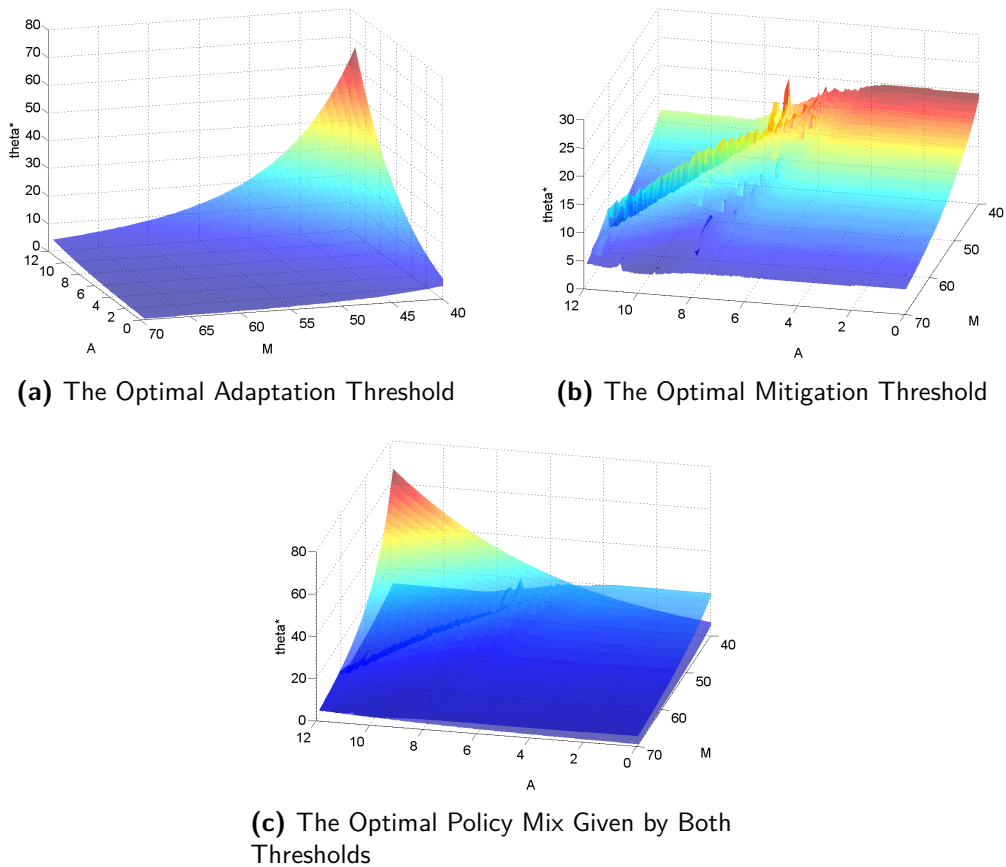
Figure 4.3.1a shows that the threshold of adaptation shifts upwards for more installed adaptation capital, i.e. investment becomes less necessary. This effect is more pronounced for lower values of $M$. Moreover, it is clear that the region of inaction shrinks for higher pollution $M$. Therefore, the results confirm what intuition tells us: investment in adaptation needs to be undertaken sooner the more the economy is exposed to climate change damage.

The mitigation threshold in Figure 4.3.1b reveals some familiar features, which have already been observed in the mitigation real options literature as well as some new characteristics. As is generally known, the mitigation threshold shifts downwards for higher pollution levels. That means that a higher atmospheric $CO_2$ concentration increases the urgency to cut emissions soon. In contrast to the hitherto existing research that focuses on mitigation as the only real option, the mitigation threshold in this paper features discontinuities or sudden jumps, which appear to be located on a curve. Figure 4.3.1c explains that the source of these discontinuities is the intersection of the two threshold curves. Indeed, at these points, the description of the mitigation real options (4.18) switches to a

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[20]These models estimate the costs to be 0.19% (AD-WITCH) and 0.28% (AD-DICE) of the GDP. For a comparison of the two models, see Agrawala et al. (2011a).

different functional form, which causes the associated threshold to drop (seen from low to high $A$-levels).[21] In other words, this drop is attributed to the phasing out of adaptation investments. Hence, adding adaptation to the model grants a new perspective on optimal mitigation, which can be discussed in more detail by considering Figure 4.3.1c.



**(a)** The Optimal Adaptation Threshold



**(b)** The Optimal Mitigation Threshold



**(c)** The Optimal Policy Mix Given by Both Thresholds

**Figure 4.3.1:** The Optimal Policy

Figure 4.3.1c discloses two different regimes of optimal policy. For low levels of adaptation capital $A$ or high pollution $M$, the mitigation threshold hovers above the adaptation threshold. In this area, the optimal policy action can be described as follows. Below the adaptation threshold, the policy maker will neither invest in adaptation nor undertake any emission reduction efforts. In between the two thresholds, the optimal

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[21]As the numerical solution procedure approximates the partial derivatives, these discontinuities cause errors in their neighbourhood, which materialize as single-point peaks. The induced errors vanish at more distant points to the intersection. For illustrational purposes, some of the single-point peaks are not displayed in Figure 4.3.1. The corresponding graphs with all the single-point peaks are available on request.

strategy is to expand only the adaptation capital stock. As soon as the upper threshold has been reached for the first time, mitigation complements adaptation. Accordingly, the policy maker is advised to invest first in adaptation before curbing emissions. The question of why adaptation is the preferred alternative is answered by the acute exposure to climate damage with low $A$ and/or high $M$ values.[22] If properly planned and managed, the adaptation projects that are undertaken first are relatively inexpensive, completed quickly and effective. Emission reduction is of less importance, because it does not help to cure the present vulnerability.

For bigger adaptation capital stocks, the adaptation threshold moves above the mitigation threshold. Below the mitigation threshold, climate policy efforts are dispensable, because the climate damage costs are very low. As soon as the mitigation threshold has been crossed for the first time, emissions need to be reduced. Only if the process moves above the adaptation threshold is investment in adaptation optimal. This area describes the optimal policy of well-adapted economies, which are less exposed to climate damage. Investing more in adaptation becomes inefficient, while mitigation becomes the preferred measure. All in all, Figure 4.3.1c thus points out the key role of being well adapted: it is optimal to reduce the current vulnerability to climate change first and then to cut emissions to reduce the future impacts.

To understand the curvature of the mitigation threshold, we have to dissect the components of adaptation, which are added to the mitigation model, and examine their effects on the timing of mitigation. Adaptation means (i) to enjoy the benefits of the already existing capital stock and (ii) to have the opportunity to expand this stock. If the existing stock is responsible for the curvature, we may speak of a complementarity effect: a sufficient build-up of adaptation capital would ensure the availability of the (financial) means to take care of the future generations' fate by curbing emissions. The better the economy is adapted, the sooner emissions are to be curbed. However, for very low and very high stocks, the mitigation threshold in Figure 4.3.1c appears to be insensitive to the $A$-levels. To clarify this issue ultimately, Figure 4.3.2 illustrates the optimal timing of mitigation under the assumption that adaptation capital exists but the opportunity to expand it is not given. First of all, it is confirmed that the timing of mitigation is rather insensitive to the existing adaptation capital stock size. The reason is that adaptation capital only grants short- or medium-term benefits, as it depreciates over time. In contrast, the benefits of mitigation are rather small in the near future and are expected to accumulate over longer time horizons. Accordingly, the current level

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[22]Here, I use the terms "exposure to climate change damage" and "vulnerability to climate change" interchangeably.

of adaptation capital cannot have a significant effect on the decision regarding whether to adopt a measure that pays in the distant future. Put differently, a high adaptation capital stock does not accelerate mitigation. As Figure 4.3.2 suggests, the current $A$-levels slightly decrease the benefits of mitigation. This small effect of substitution on the benefit side is, however, hardly visible in Figure 4.3.1b. If the adaptation capital stock is not responsible for the mitigation threshold curvature, the opportunity to expand it is. It is recognizable that for lower adaptation capital stocks $A$ the threshold in Figure 4.3.1b is much higher than its equivalent in Figure 4.3.2. This means that taking the opportunity to invest in adaptation delays the mitigation efforts, presumably due to substitution effects on the cost and benefit side. Indeed, the opportunity to invest in adaptation decreases the benefits of mitigation. In other words, the benefits of mitigation would be very high if the economy continues to be so highly exposed to climate damage. However, investing in adaptation reduces the vulnerability to climate change and thus decreases the future benefits of mitigation. In addition, investing in adaptation leave less financial means for adopting emission cuts. Consequently, the mitigation threshold shifts upwards. With higher $A$-values, the threshold in Figure 4.3.1b converges to the one displayed in Figure 4.3.2. The opportunity to expand the adaptation capital loses its value due to the decreasing effectiveness of the capital. Therefore, the effect of the opportunity to invest in adaptation on the mitigation threshold vanishes. All in all, the curvature of the mitigation threshold arises from the decreasing value of expanding the adaptation capital stock.
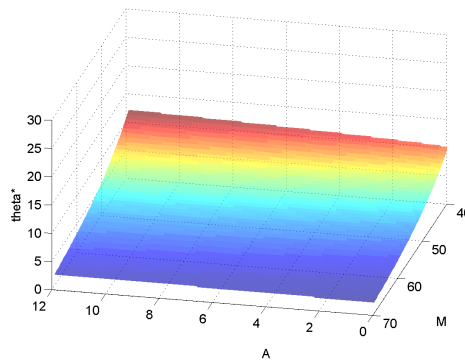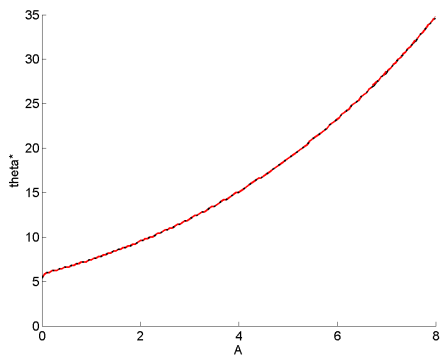


**Figure 4.3.2:** The Mitigation Threshold under the Assumption that Investing in Adaptation Is Not Possible

Next, the effects of mitigation on the adaptation option are examined. Considering Figure 4.3.1a once again, we can see that the decision to cut emissions does not lead to

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any noticeable jumps in the adaptation threshold. Therefore, the interaction between the two measures is obviously not of a symmetric nature. By analogy with Figure 4.3.2, Figure 4.3.3a demonstrates the threshold of adaptation for the scenario in which emissions cannot be curbed.
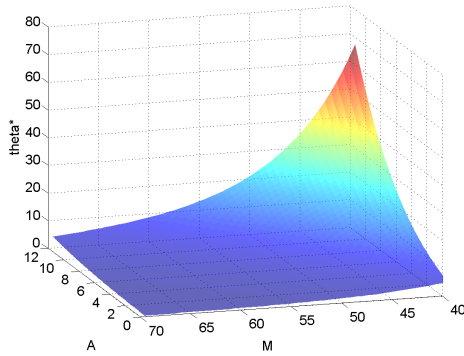


**(a)** The Adaptation Threshold in the High-Emission Scenario



**(b)** The Adaptation Threshold in the High-Emission Scenario (Black) and the Adaptation Threshold in the Optimal Mitigation Scenario (Red) Displayed in $M = 40$
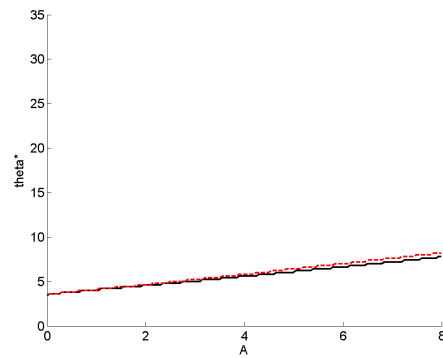


**(c)** Both Adaptation Thresholds in the Absence of Uncertainty in $M = 40$

**Figure 4.3.3:** The Adaptation Thresholds

The comparison of the thresholds in the optimal-emission scenario (Figure 4.3.1a) and the high-emission scenario shows no visible difference. Indeed, Figure 4.3.3b proves that the two thresholds are even identical at the present pollution level. The timing the investment in adaptation is thus determined by the present magnitude of atmospheric pollution $M$ but not by the future development of $M$ or the opportunity to slow down its growth. The reason is that for the decision on whether to adopt a short-term measure,

such as adaptation, the present impacts matter more than the future threats. Figure 4.3.3c illustrates the same thresholds for the deterministic case, in which $\sigma$ and the real options value of adaptation are zero. Obviously, the thresholds are much lower, which means that the policy maker is more willing to shoulder the sunk costs caused by adaptation when certain about the resulting benefits. Figure 4.3.3c also shows that there is a difference, albeit marginal, between the two thresholds in the deterministic case. This simulation thus confirms earlier findings in the literature, which describe the crowding out effect of mitigation on adaptation as rather small. It is reasoned that in the short- and medium-term the benefits of mitigation are too small to reduce significantly the current need to adapt. Comparing Figure 4.3.3b with Figure 4.3.3c leads to the conclusion that this effect of substitution with respect to timing vanishes when taking a real options perspective. The benefits of mitigation are not only too small but also too uncertain to influence the timing of the adoption of a measure that promises to improve the situation soon.

In order to conduct a more comprehensive analysis of the interaction effects, the adaptation investment levels need to be examined as well. To this end, Figure 4.3.4a provides information about the optimal adaptation efforts - specified as a percentage of the GDP - in the high-emission scenario for $M = 40$. For the present level of $\theta = 10$ the investment efforts are rather small for low levels of adaptation capital $A$ and zero for higher levels of adaptation capital.[23] Figure 4.3.4b comprises the cuts of investment efforts when emissions are curbed optimally. These cuts range from 0% up to almost 0.03% of the GDP, if all the value combinations of $A$ and $\theta$ are considered. For the $\theta$ values that can be assumed in the near future, the reduction of efforts is significantly lower than 0.01%. Figure 4.3.4c and Figure 4.3.4d illustrate the investment efforts for the deterministic version of the model. Comparing Figure 4.3.4a with Figure 4.3.4c, we can see that uncertainty makes the policy maker less willing to invest in adaptation, as shown before in Figure 4.3.3. Figure 4.3.4b and Figure 4.3.4d demonstrate that cutting emissions optimally allows the policy maker to invest less in the deterministic case than under uncertainty. In other words, if the benefits of mitigation cannot be counted on with absolute certainty, the adaptation investment efforts must not be too severely cut back.

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[23]The graph also indicates that with extremely high values of $\theta$ and extremely low values of $A$, the investment efforts may rocket upwards to approximately 0.7% of the GDP. This static analysis, however, hides the fact that this combination of very high values of $\theta$ and extremely low values of $A$ will not occur, as the policy maker expands the capital stock long before the stochastic process can fluctuate to this level. Therefore, it is not deemed necessary to implement an explicit investment budget constraint, which would add just another parameter posing calibration difficulties.
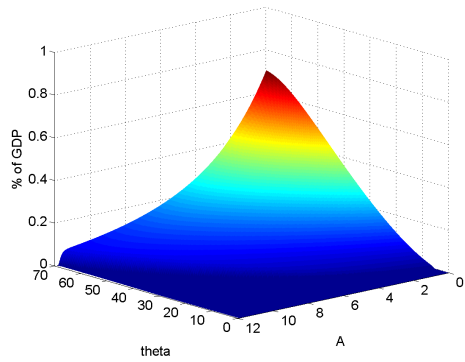
**(a)** Investment Efforts in the High-Emission Scenario under Uncertainty
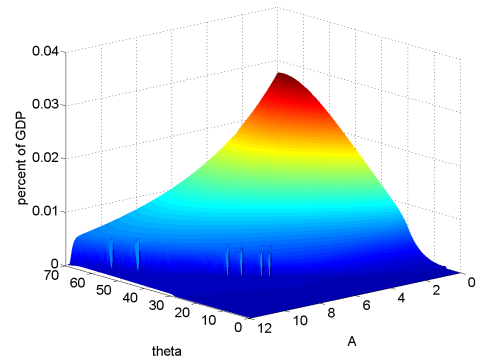
**(b)** A Reduction in the Investment Efforts due to Optimal Mitigation in the Uncertainty Scenario
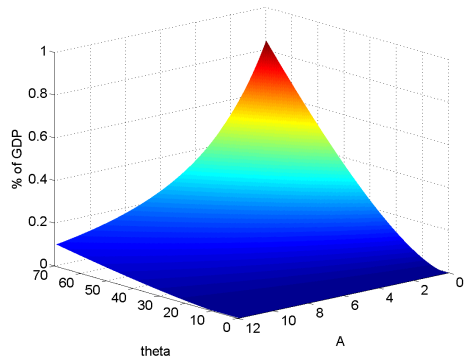
**(c)** The Investment Costs, Assuming No Uncertainty and No Mitigation

**(d)** A Reduction in the Investment Efforts due to Optimal Mitigation in the Deterministic Scenario

**Figure 4.3.4:** The Adaptation Investment Costs in $M = 40$

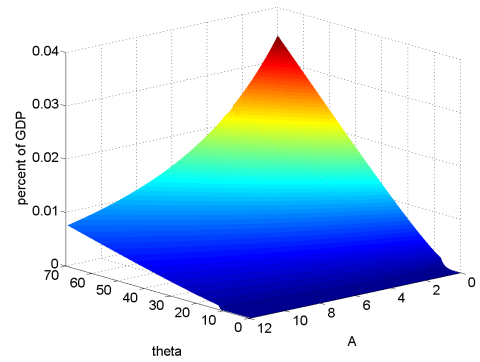Having considered the effects of interaction, we may conclude that there is considerable asymmetry in the interaction of the two real options. The timing of mitigation is not sensitive to the currently installed adaptation capital stock. However, the opportunity to expand the adaptation capital stock affects the benefits of mitigation greatly. Contrariwise, adaptation activites are only slightly influenced by the real option to mitigate.

Next, the contribution of taking the real options perspective when analysing the climate policy decision is addressed. For this, Figure 4.3.5 presents the optimal policy threshold curves under alternative assumptions. Figure 4.3.5a illustrates the case in which the uncertainty parameter $\sigma$ and the real options values are zero. A deterministic view on the optimal policy decision is for example taken by Bosello et al. (2009, 2010, 2011) and de Bruin et al. (2009). Figure 4.3.5b takes a step further by prescribing $\sigma = 0.07$ as

in the base calibration, but it postulates that only the expected net present value matters to the policy decision. The existence of any effects generated by the interaction of uncertainty and the irreversibilities are neglected. The strand of literature that accounts for uncertainty but exclusively follows the expected net present value approach to determine the optimal policy mix is represented by Felgenhauer & Bruin (2009) and Felgenhauer & Webster (2013a,b).[24]
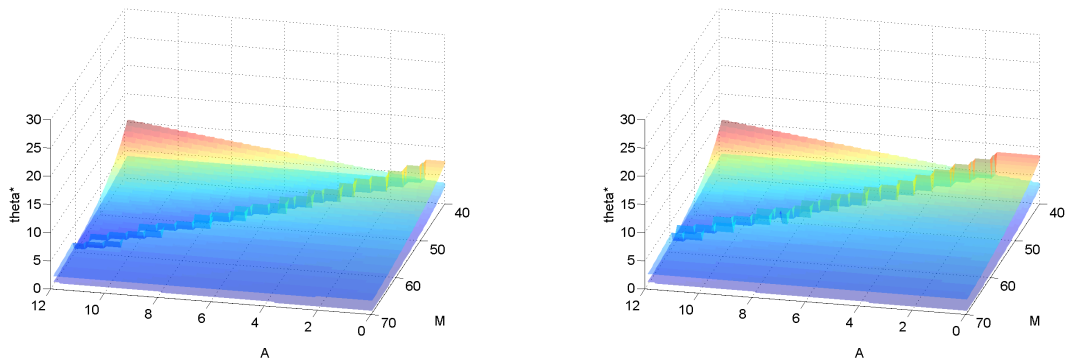


**(a)** The Optimal Climate Policy Thresholds in the Deterministic Framework

**(b)** The Optimal Climate Policy Thresholds in the Expected Net Present Value Framework

**Figure 4.3.5:** The Optimal Climate Policy Thresholds under Alternative Methodological Assumptions

Comparing the graphs in Figure 4.3.5 with each other and with Figure 4.3.1c, we can see that neglecting uncertainty and the real options values shifts the thresholds downwards to a great extent. Accounting for uncertainty but ignoring the real options approach alters the threshold curves less. For low $A$ values, the area in which emissions are not curbed is enlarged. The timing of adaptation is only slightly affected by accounting for uncertainty. What really has a big impact on the decision is the incorporation of the values of waiting or the real options values generated by the tension between uncertainty and the irreversibilities, as emphasized by Figure 4.3.1c. The area of inaction, in which neither of the climate policy measures is adopted, is shown to be significantly larger in this graph. Accordingly, this result given by the real options approach is in accordance with the existing global climate policy inaction. In contrast, Figure 4.3.5b indicates that a global climate policy of adaptation and mitigation would already have been adopted if the policy makers had not incorporated any considerations of delaying policy adoptions

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[24]It must be noted that these studies explore the effects of uncertainty attached to different components of the model. A direct comparison with these studies is thus not possible.

and waiting for more information to arrive.

Taking a closer look at the graphs, we can see that the area in which adaptation is the preferred measure is widened by the real options approach. ROA thus gives more weight to adaptation to fight acute exposure to climate change damage than the ordinary expected present value approach. As uncertainty is also accounted for in the expected net present value approach, this observation can only be explained by the interaction of uncertainty and the economic irreversibilities. Investments in adaptation are allowed to be of a small scale, which makes it possible to limit the magnitude of the sunk costs. In contrast, mitigation imposes relatively high sunk costs. The combination of comparatively low sunk costs and being less affected by uncertainty restricts the real options value of adaptation, which gives adaptation greater priority in a more vulnerable economy. On the contrary, mitigation is delayed due to its rather high sunk costs and its rather uncertain benefits. However, the marginal real options values cause the marginal benefits of adaptation to decrease much faster for high $A$ values. Accordingly, for a better-adapted economy, this approach favours the stand-alone policy of curbing emissions more than the expected net present value approach does. ROA widens the areas in which only one measure is adopted, i.e. the associated values of waiting delay the implementation of the measure that is least favoured. Consequently, the benefits of taking a real options perspective are not trivial. This perspective helps us to understand the existing reservations regarding early climate policy activities. In addition, it points out that the policy maker is rather reluctant to adopt two measures that cause sunk costs and generate more or less uncertain benefits.
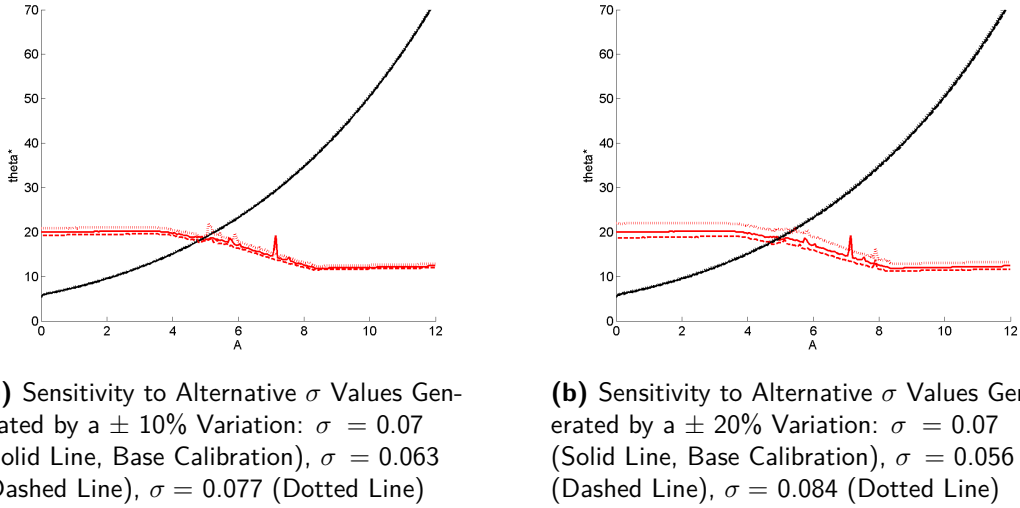
The optimal policy mix certainly depends on the above choices of the parameter values. Clarification of the involved sensitivity of the results is provided by Figure 4.3.6 - Figure 4.3.9. For the purpose of a clear visual representation, the response of the mitigation threshold (red) and the adaptation threshold (black) to alternative assumptions on the parameter values is only given for the value $M = 40$. In each case, the base calibration of the investigated parameter is varied by $\pm 10\%$. If clarity requires it, an additional graph for a $\pm 20\%$ parameter variation is presented.[25] The thresholds resulting from the new simulations are then compared with the thresholds of the base calibration.

Figure 4.3.6 indicates that mitigation is more sensitive to changes in uncertainty $\sigma$. A $\pm 10\%$ variation as shown by Figure 4.3.6a causes only small shifts in the mitigation threshold and no visible changes in the adaptation threshold. More pronounced is the result for a $\pm 20\%$ variation given by Figure 4.3.6b. The adaptation threshold appears to be almost insensitive. The benefits of mitigation, which evolve slowly over the considered time horizon, are crucially affected by intrinsic uncertainty, as it grows over time as well.

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[25]The other graphs for a $\pm 20\%$ variation are listed in Appendix 4.D.

In contrast, the adaptation decision is based on the benefits that this capital will grant in its rather short life-time. These benefits are thus more guaranteed and less affected by variations in $\sigma$. This result, however, does not imply that uncertainty is not important for the adaptation decision at all, as proven by the comparison of Figure 4.3.1c with Figure 4.3.5.



**(a)** Sensitivity to Alternative $\sigma$ Values Generated by a $\pm$ 10% Variation: $\sigma = 0.07$ (Solid Line, Base Calibration), $\sigma = 0.063$ (Dashed Line), $\sigma = 0.077$ (Dotted Line)

**(b)** Sensitivity to Alternative $\sigma$ Values Generated by a $\pm$ 20% Variation: $\sigma = 0.07$ (Solid Line, Base Calibration), $\sigma = 0.056$ (Dashed Line), $\sigma = 0.084$ (Dotted Line)

**Figure 4.3.6:** Sensitivity of the Optimal Policy Mix to Uncertainty Depicted by the Threshold of Mitigation (Red) and the Threshold of Adaptation (Black) in $M = 40$

The mix of short- and long-term policy measures may depend on the policy maker's weighting of future welfare. To this end, the effects of alternative assumptions on the discount rate value $r$ are examined. The results in Figure 4.3.7a emphasize the importance of the appropriate discount rate choice. As the lifetime of adaptation capital is relatively short compared with the effects of mitigation, the adaptation threshold is only slightly influenced by the choice of the discount rate. In contrast, small variations in the discount rate can generate huge differences in the timing of mitigation. Mitigation becomes more attractive for lower discount rates. A far-sighted policy maker cares more about the future damage costs and thus finds the long-term solution to the climate problem, mitigation, more appealing.

As mentioned in Section 4.2.1, it is debatable how GDP growth affects the measures and their technologies and therefore this is not explicitly modelled in this paper. Nonetheless, the role of alternative GDP values will be examined. Assuming a higher (lower) GDP value is tantamount to having greater (fewer) financial resources available to spend on climate policy efforts, but also to having higher (lower) emissions, higher (lower) climate

damage costs in the future and a higher (lower) total amount of emissions to reduce. The question arises of whether a higher GDP gives more priority to adaptation or to mitigation in this modelling framework. Figure 4.3.7b reports that a higher GDP level means that the adoption of both measures is accelerated. Whether one or the other option is preferred cannot be answered in general, but depends on how exposed the economy is to the climate impacts. In a well-adapted economy, the two thresholds appear to be equally sensitive to variations in the GDP level. In a badly adapted economy, adaptation is not very sensitive to the GDP, because early investment is mandatory irrespective of having a 10% higher or lower GDP level. However, a richer world can cut emissions sooner, because more financial means are left after undertaking adaptation efforts. The sensitivity of mitigation is thus higher in the area of low $A$ values.
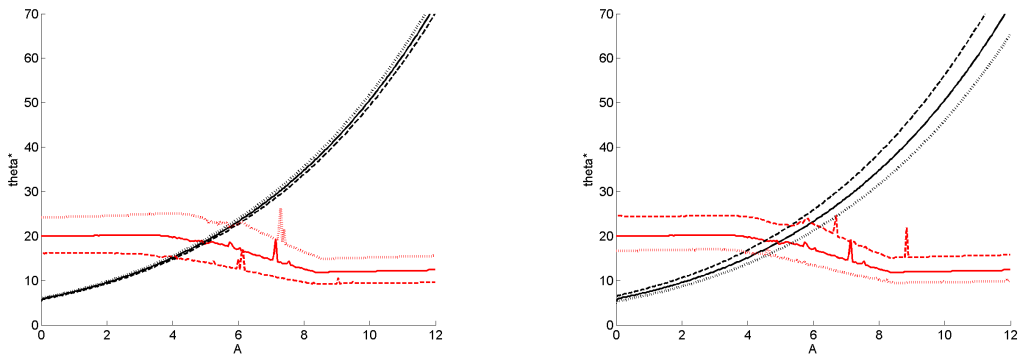


**(a)** Sensitivity to Alternative Discount Rates Generated by a $\pm$ 10% Variation: $r = 0.025$ (Solid Line, Base Calibration), $r = 0.0225$ (Dashed Line), $r = 0.0275$ (Dotted Line)
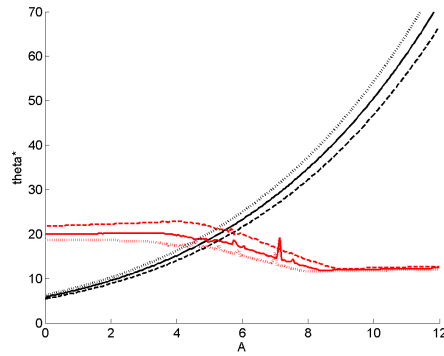
**(b)** Sensitivity to Alternative GDP Values Generated by a $\pm$ 10% Variation: $Y = 78.97$ (Solid Line, Base Calibration), $Y = 71.07$ (Dashed Line), $Y = 86.87$ (Dotted Line)

**Figure 4.3.7:** Sensitivity of the Optimal Policy Mix to Discounting and GDP

As already mentioned, it is necessary to examine the results concerning the rather vague calibration of the adaptation model. Figure 4.3.8 delivers insights into the sensitivity to the depreciation rate and the effectiveness of adaptation capital. Intuition suggests that the optimal policy mix may depend on the depreciation rate $\xi$ of adaptation. A high depreciation rate implies that the involved investments bring only short-term effects, while a lower rate makes adaptation compete with mitigation as a long-term policy. More precisely, a lower depreciation rate makes adaptation a measure that not only helps to alleviate the impacts of the current damage but that also reduces the impacts in the more distant future. Consequently, adaptation partially crowds out mitigation in the optimal policy portfolio. This is confirmed by Figure 4.3.8a, as the shifts in the thresholds

imply that the policy maker invests in adaptation sooner (later) and curbs emissions later (sooner) if the capital stock depreciates slower (faster). Thus, the effects of substitution between both policy measures is decisively affected by the durability of the adaptation projects undertaken.



(a) Sensitivity to Alternative Depreciation Rates Generated by a $\pm$ 10% Variation: $\xi = 0.075$ (Solid Line, Base Calibration), $\xi = 0.0675$ (Dashed Line), $\xi = 0.0825$ (Dotted Line)



(b) Sensitivity to Alternative Adaptation Effectiveness Parameters Generated by a $\pm$ 10% Variation: $\phi = 4.5$ (Solid Line, Base Calibration), $\phi = 4.05$ (Dashed Line), $\phi = 4.95$ (Dotted Line)
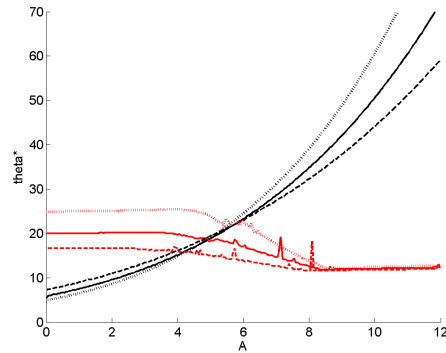
(c) Sensitivity to Alternative Adaptation Effectiveness Parameters Generated by a $\pm$ 20% Variation: $\phi = 4.5$ (Solid Line, Base Calibration), $\phi = 3.6$ (Dashed Line), $\phi = 5.4$ (Dotted Line)

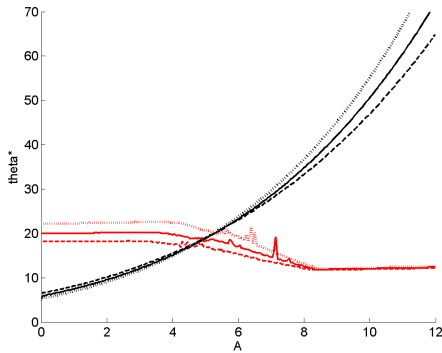**Figure 4.3.8:** Sensitivity of the Optimal Policy Mix to the Calibration of the Adaptation Parameters

The sensitivity to alternative assumptions on the effectiveness of adaptation is examined in Figure 4.3.8b and Figure 4.3.8c. How the effectiveness parameter $\phi$ affects adaptation depends on the size of the currently operating adaptation capital stock. For low values of $A$, a higher level of effectiveness of adaptation clearly incentivizes early

investment to build up a sufficient stock size. The meaning of "sufficient" also relies on the effectiveness parameter. Consequently, a higher $\phi$-value implies that investment in adaptation can be cut back sooner. The effectiveness parameter affects mitgation significantly as well. If adaptation works well to fight climate change impacts, it becomes less of an imperative to fight the root of the climate change problem. Accordingly, mitigation can be delayed.

The lack of empirical evidence requires us to test alternative values for the "unit costs" $\gamma_1$. Figure 4.3.9a indicates that higher investment costs make adaptation efforts less attractive and the threshold shifts upwards, the greater the installed adaptation capital stock is. Although adaptation efforts are more costly, the timing of mitigation is not affected. On the one hand, one may suspect that mitigation is delayed if it is undertaken after investing in adaptation, because the investment claims a bigger share of the financial resources. On the other hand, the adoption of mitigation could be accelerated in order to make the future generations less dependent on expensive investments in adaptation. At this point of our analysis, Figure 4.3.9a offers no other choice than to conjecture that the two effects balance each other out, leading to the insensitivity of the mitigation threshold.

Figure 4.3.9b investigates the influence of the other component of the adaptation costs, the adjustment costs. This parameter $\gamma_2$ is amongst the factors that determine the costs of quick capital stock expansion, which can be interpreted as evidence of the economy's adaptive capacity. As expected, the timing of adaptation is insensitive to alternative adjustment costs.[26] In contrast, the adaptive capacity has an impact on the timing of mitigation. If the capability to adapt is poor, the policy maker should not rely on adaptation as a measure to fight climate impacts. Mitigation is thus adopted sooner to reduce the need to adapt in the future.

The final point to investigate is how the mitigation costs affect the thresholds. Figures 4.3.9c and 4.3.9d show that higher mitigation costs, i.e. higher $\kappa_1$ and lower $\kappa_2$, deter the policy maker from curbing emissions. On the contrary, the necessity to invest in adaptation is not influenced by the spending on mitigation, as it only depends on the magnitude of the marginal welfare of adaptation.[27]

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[26] As derived in Section 4.2.2, the timing of adaptation is determined by the values for which the marginal welfare of adaptation is greater than the "unit costs", i.e. the adjustment costs do not affect the adaptation threshold.

[27] More precisely, when taking the partial derivative of equations (4.14) and (4.15) to obtain the marginal welfare of adaptation, the mitigation costs drop out.
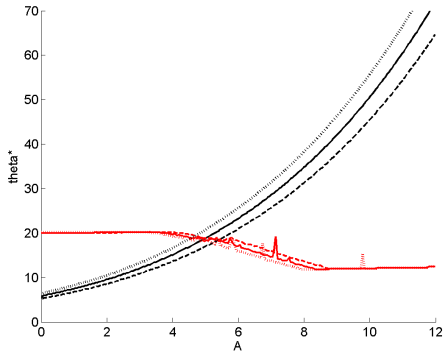
**(a)** Sensitivity to Alternative Adaptation Cost Parameters Generated by a $\pm$ 10% Variation: $\gamma_1 = 0.4$ (Solid Line, Base Calibration), $\gamma_1 = 0.36$ (Dashed Line), $\gamma_1 = 0.44$ (Dotted Line)
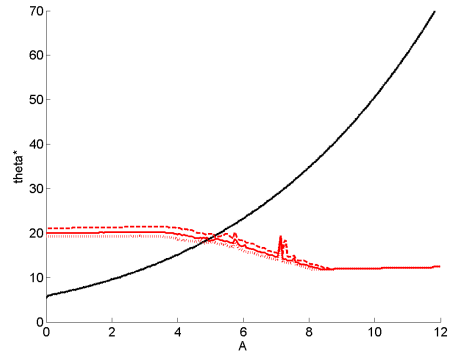


**(b)** Sensitivity to Alternative Adaptation Cost Parameters Generated by a $\pm$ 10% Variation: $\gamma_2 = 16.81$ (Solid Line, Base Calibration), $\gamma_2 = 15.13$ (Dashed Line), $\gamma_2 = 18.49$ (Dotted Line)
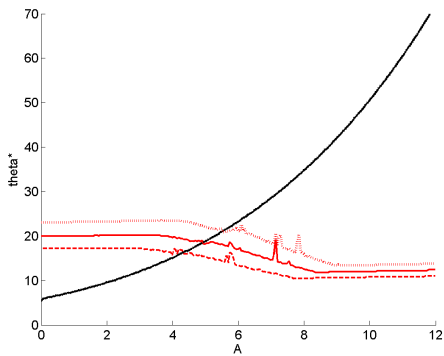


**(c)** Sensitivity to Alternative Mitigation Cost Parameters Generated by a $\pm$ 10% Variation: $\kappa_1 = 0.03$ (Solid Line, Base Calibration), $\kappa_1 = 0.027$ (Dashed Line), $\kappa_1 = 0.33$ (Dotted Line)
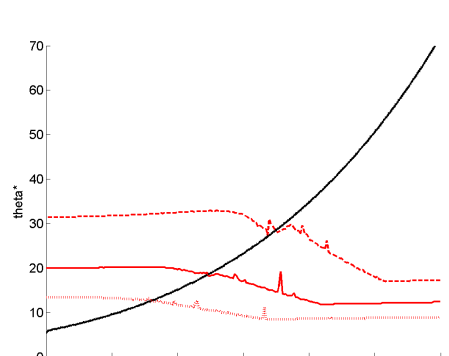


**(d)** Sensitivity to Alternative Mitigation Cost Parameters Generated by a $\pm$ 10% Variation: $\kappa_2 = 1.2$ (Solid Line, Base Calibration), $\kappa_2 = 1.08$ (Dashed Line), $\kappa_2 = 1.32$ (Dotted Line)

**Figure 4.3.9:** Sensitivity of the Optimal Policy Mix to the Calibration of the Cost Parameters

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## 4.4 Conclusion and Outlook

The optimal policy response to climate change has to account for a mix of mitigation and adaptation efforts. This paper considers this mix from the perspective of a continuous-time real options modelling framework, which allows the examination of the impacts of economic and ecological irreversibilities and intrinsic uncertainty in the future climate damage costs. To this end, a new framework for a portfolio of adaptation and mitigation options is developed. The mitigation option gives the opportunity to choose the optimal timing to commit to a certain emission reduction target. The form of adaptation that is considered can be categorized as proactive adaptation and is modelled as investments in an adaptation capital stock. Exercising the adaptation option means optimally expanding the adaptation stock. The model also features a stylized notion of adaptation capacity, which determines how quickly the adaptation proceeds and is assumed to be compromised by unabated climate damages.

The numerical simulations show the benefits of analysing the optimal climate policy decision from a real options perspective. It is not the existence of uncertainty in itself but the interaction with the irreversibilities that delays the adoption of both climate policy measures significantly. More precisely, it postpones the implementation of the first measure and it also prolongs the period until the second measure complements the policy mix. Hence, it points out that the policy maker is rather reluctant to adopt two measures that cause sunk costs and generate more or less uncertain benefits.

The optimal policy mix is determined by the differences in the characteristics of the measures. Among the most important distinguishing features are the different timescales on which the two measures work. The benefits evolve differently over time: while the investments in adaptation can pay off rather soon, the benefits of mitigation are expected to accumulate over a long time horizon. Consequently, the simulations demonstrate that adaptation is the preferred measure if the economy is currently exposed to climate change impacts. If the marginal benefits of expanding the adaptation capital stock are sufficiently low, mitigation is given a higher priority so that the root causes of climate change can be fought. Another distinguishing feature is given by the magnitude of the incurred sunk costs. Curbing emissions incurs relatively high (at least) partially irreversible costs. The tension between these costs and the uncertainty, which grows over the time horizon in which the benefits accrue, nourishes the real option to mitigate. In contrast, the benefits of the investments in adaptation are of a shorter lifetime, i.e. the benefits are less subject to uncertainty. Adaptation allows small-scale investments to be made, which means that the incurred sunk costs are not necessarily high. Consequently, compared with other

decision frameworks, the real options perspective grants adaptation more emphasis as the preferred measure in the portfolio.

The simulations also disclose significant asymmetry in the interaction of the two real options, which is again reasoned by the different timescales on which the two measures work. In particular, the simulations indicate that mitigation is delayed not only due to its own real options value but also due to the opportunity to invest in adaptation. In contrast, the timing of adaptation efforts is mainly determined by the present levels of climate change impacts and less so by the future developments of the atmospheric pollution level. Likewise, the today's investments in adaptation are only slightly affected by curbing emissions now. Hence, the real option to adapt is less affected by the presence of the opportunity to mitigate than vice versa.

An extensive sensitivity analysis reveals that the policy maker's weighting of future welfare is crucial for the optimal policy mix, because the discount rates determine the importance of emission cuts. The adaptation real option is less affected by discounting due to the above-mentioned short-term benefits of the involved investments. Further numerical simulations show that the adaptation option is exercised sooner and mitigation adopted later if adaptation depreciates less quickly. Higher "unit" costs of adaptation are demonstrated to increase the real options value of adaptation but to have no effects on mitigation. In contrast, a lower capacity to adapt accelerates mitigation.

The modelling framework is meant to be the first stepping stone towards real options models of holistic climate policy portfolios. The framework can be extended to incorporate options of Carbon Capture and Storage, options to promote technological progress or more specific adaptation options that allow the display of the manifoldness and complexity of adaptation in reality. Furthermore, it would be fruitful to account for adaptation measures that protect against catastrophic climate damage. Adaptation measures that grant different levels of flexibility are also worthwhile investigating in a real options model. Some adaptation measures have negative effects on mitigation efforts, while others have positive spillover effects, as outlined by IPCC (2007a). There are adaptation measures that are inseparable from development policies, which would represent another real option. As a result of not covering all these and many more forms of adaptation, the model is rather stylized, but it grants the advantage of having a small model to explain the interaction of two climate policy instruments under uncertainty and irreversibility.

## 4.A  SOLUTION OF THE OPTIMAL ADAPTATION POLICY

The adaptation model needs to be solved in several steps. One possible way to proceed is to compute the marginal welfare of adaptation, in order to find the threshold between the area of inaction $S_1$ and the area of action $S_2$. The information about the marginal values $w$ and the threshold location can then be used to derive the solution to the welfare function.

As already indicated in Section 4.2.2, the solution in the area of inaction consists of two parts. More precisely, the solution of $W$ for $S_1$ is given by the expected present welfare of never investing into adaptation and the real option of investing in the future. Accordingly, the marginal welfare for $S_1$ consists of the respective marginal values. Both values can be derived from equation (4.16). The marginal expected present welfare of adaptation, from now on referred to as $w^P$, is the same as the particular solution to equation (4.16). The general solution of equation (4.16) is used to find the marginal real option of adaptation. As a by-product, the location of the threshold defined in terms of the marginal values is obtained. This information about the the threshold location is then used to determine the solution to equation (4.17). For $S_2$, the real options to adapt are exercised instantaneously and thus only the expected present welfare of optimal investment needs to be computed. In an analogous manner, only the particular solution of equation (4.17) needs to be computed, which can only be derived, because its value $\gamma_1$ in the threshold is known. The information about the threshold location and the marginal welfare for $S_1$ and $S_2$ is sufficient to derive the solution to equations (4.14) and (4.15). In the following, the above-outlined steps are described in more detail.

### 4.A.1  THE PARTICULAR SOLUTION OF EQUATION (4.16)

The marginal expected present welfare of adaptation for $S_1$ equals:

$$w^P = \mathbb{E}_0\left[\int_0^\infty \left(Y\frac{\alpha\phi\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^{\phi+1}}e^{-\frac{\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^\phi}}\right)e^{-rt}dt\right], \tag{4.19}$$

with $\theta(t)$ and $M(t)$ given by equations (4.4) and (4.5), and $A(t)$ is provided by equation (4.8) with $a(t) = 0$ for all $t$. The solution of (4.19) cannot be derived analytically but can be obtained by solving (4.16) numerically. To this end, the specification of the model needs to be enriched by some more information.

$$\text{upper boundary condition for } M \to \infty: \quad w^P = 0, \tag{4.20}$$

$$\text{lower boundary condition for } \theta = 0: \quad w^P = 0, \tag{4.21}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad w^P = 0. \tag{4.22}$$

Condition (4.20) and (4.22) become clear by considering (4.19): for $M \to \infty$ as well as for $\theta \to \infty$, the exponential term converges to zero faster than its factor. Condition (4.21) explains that the integral is zero for $\theta = 0$ and it stays zero, as the geometric Brownian motion has an absorbing barrier at this point.

In the following, equation (4.14) is solved by applying the finite difference method, which gives the values of $w^P$ in terms of a discrete choice of its function arguments. This means that the continuous function $w^P$ is approximated by its discrete version $w^P(i\Delta\theta, j\Delta M, k\Delta A) = w^P_{i,j,k}$, where $0 \le i \le \mathscr{I}$, $0 \le j \le \mathscr{J}$ and $0 \le k \le \mathscr{K}$. The values are chosen so that $\mathscr{I}\Delta\theta = \theta_{\max}$, $\mathscr{J}\Delta M = M_{\max}$ and $\mathscr{K}\Delta A = A_{\max}$ with sufficiently large numbers $\theta_{\max}$, $M_{\max}$ and $A_{\max}$. The approximation of the partial derivatives by finite differences is crucial. In general, two types of finite difference schemes can be applied: the explicit and the implicit finite difference method. The explicit method has the disadvantage that the discretization must obey some constraints, which often turn out to be very restrictive. Especially for the equations at hand, the conditions for the number of steps and the length of the step sizes imply enormous computational effort. Therefore, the implicit finite difference method is applied in the following. More precisely, equation (4.16) is transformed into:

$$
\begin{aligned}
(r + \xi)w^P_{i,j-1,k} = {} & Y \frac{\alpha\phi\rho i\Delta\theta \left((j-1)\Delta M\right)^\psi}{(1 + \alpha k\Delta A)^{\phi+1}} e^{-\frac{\rho i\Delta\theta((j-1)\Delta M)^\psi}{(1+\alpha k\Delta A)^\phi}} \\
& + (\beta\epsilon(1-m)Y - \delta\left((j-1)\Delta M\right)) \frac{w^P_{i,j,k} - w^P_{i,j-1,k}}{\Delta M} \\
& + \mu i\Delta\theta \frac{w^P_{i+1,j-1,k} - w^P_{i-1,j-1,k}}{2\Delta\theta} \\
& + \frac{1}{2}\sigma^2 (i\Delta\theta)^2 \frac{w^P_{i+1,j-1,k} + w^P_{i-1,j-1,k} - 2w^P_{i,j-1,k}}{(\Delta\theta)^2} \\
& - \xi k\Delta A \frac{w^P_{i,j-1,k} - w^P_{i,j-1,k-1}}{\Delta A} \quad \forall i,j,k,
\end{aligned}
\tag{4.23}
$$

which is the same as:

$$w_{i,j,k}^P = -qY \frac{\alpha\phi\rho i \Delta\theta \left((j-1)\Delta M\right)^\psi}{(1+\alpha k \Delta A)^{\phi+1}} e^{-\frac{\rho i \Delta\theta((j-1)\Delta M)^\psi}{(1+\alpha k \Delta A)^\phi}} + w_{i-1,j-1,k}^P \; x_1 \qquad (4.24)$$
$$+ w_{i,j-1,k}^P \; x_2 + w_{i+1,j-1,k}^P \; x_3 + w_{i,j-1,k-1}^P \; x_4,$$

with

$$q = \frac{\Delta M}{\beta\epsilon(1-m)Y - \delta(j-1)\Delta M}$$
$$x_1 = q\left(\frac{1}{2}\mu i - \frac{1}{2}\sigma^2 i^2\right),$$
$$x_2 = 1 + q\left(r + \sigma^2 i^2 + \xi(k+1)\right),$$
$$x_3 = q\left(-\frac{1}{2}\mu i - \frac{1}{2}\sigma^2 i^2\right),$$
$$x_4 = -q\xi k. \qquad (4.25)$$

As the values of $w_{i,\mathscr{J},k}^P$ for all $i$ and $k$ are given by (4.20), the values of $w_{i,\mathscr{J}-1,k}^P$ can be found by using its relation to $w_{i,\mathscr{J},k}^P$ as given by equation (4.24). Accordingly, all other values $w_{i,j-1,k}^P$ can thus be computed step by step. For $A = 0$, it should be noted that the partial derivative with respect to $A$ vanishes and the 'out-of-the-grid' value $w_{i,j-1,-1}^P$ is not needed to approximate all the values in $k = 0$.

### 4.A.2 THE GENERAL SOLUTION OF EQUATION (4.16)

Consider the value of the option to invest in additional adaptation capital $W^G$, which is described by the homogeneous part of equation (4.14):

$$rW^G = (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W^G}{\partial M} + \mu\theta\frac{\partial W^G}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^G}{\partial\theta^2} - \xi A\frac{\partial W^G}{\partial A}. \qquad (4.26)$$

The real option to adapt loses value the more adaptation capital is installed. Hence, the partial derivative $\frac{\partial W^G}{\partial A}$ is negative. Defining $w^G$ as $-\frac{\partial W^G}{\partial A}$, the marginal option can be expressed as:

$$(r+\xi)w^G = (\beta\epsilon(1-m)Y - \delta M)\,w_M^G + \mu\theta w_\theta^G + \frac{1}{2}\sigma^2\theta^2 w_{\theta\theta}^G - \xi A w_A^G, \qquad (4.27)$$

which obeys the value-matching condition:

$$w^G = \max\left\{w^P - \gamma_1, 0\right\}, \qquad (4.28)$$

at the threshold of taking action. Please note that $w^P$ is the particular solution of equation (4.16), as described in Appendix 4.A.1.

To approximate the marginal option, the following additional boundary conditions are then implied:

$$\text{upper boundary condition for } M \to \infty: \quad w^G = 0, \tag{4.29}$$

$$\text{lower boundary condition for } \theta = 0: \quad w^G = 0, \tag{4.30}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad w^P = 0. \tag{4.31}$$

The conditions (4.29) and (4.31) can be explained by noting that $w^P$ is zero for $M \to \infty$ and $\theta \to \infty$. Hence, welfare cannot be increased by additional investment in adaptation capital, which makes the real option worthless, and this does not change for a slightly higher value of $A$. Condition (4.30) is due to the absorbing barrier of the geometric Brownian motion.

Equation (4.27) is approximated in a similiar way to $w^P$ in Appendix 4.A.1:

$$w^G_{i,j,k} = w^G_{i-1,j-1,k} \; x_1 + w^G_{i,j-1,k} \; x_2 + w^G_{i+1,j-1,k} \; x_3 + w^G_{i,j-1,k-1} \; x_4, \tag{4.32}$$

with $x_1$, $x_2$, $x_3$ and $x_4$ as in (4.25). The numerical procedure, however, is more complex than the one in Appendix 4.A.1. Implicit schemes for the free boundary problem given by (4.28) cannot be solved directly.[28] Therefore, the solution is derived iteratively by applying successive overrelaxation (SOR). The acceleration parameter is the value in which the spectral radius of the SOR matrix is the minimum, as explained in detail by Thomas (1999). This procedure provides the marginal real options values and as a by-product the threshold of taking action in adaptation.

### 4.A.3 The Particular Solution of Equation (4.17)

After conducting the numerical routine explained in Appendix 4.A.1 and 4.A.2, we can make use of the information about the threshold location. Denote the set of all values $(\theta, M, A)$ defining the threshold as $\mathscr{T} = \left\{ (\theta, M, A) : w^G(\theta, M, A) = w^P(\theta, M, A) - \gamma_1 \right\}$. Then, the required boundary conditions for the marginal expected present welfare for $S_2$ (henceforth referred to as $w^{P2}$) read:

$$\text{upper boundary condition for } M \to \infty: \quad w^{P2} = 0, \tag{4.33}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad w^{P2} = 0. \tag{4.34}$$

$$\text{threshold condition, for } (\theta, M, A) \in \mathscr{T}: \quad w^{P2} = \gamma_1, \tag{4.35}$$

---

[28] A more detailed explanation of this problem and further useful information about the finite difference method is given by Brandimarte (2006).

In the case of extremely high damage costs, i.e. $M \to \infty$ and/or $\theta \to \infty$, the welfare approaches zero and additional investment in adaptation will not change this.

In order to apply an implicit finite difference scheme to equation (4.17), it is necessary to deal with two troubling characteristics of this partial differential equation. The first one relates to the non-linear terms, which render the matrix manipulations required to solve the implicit schemes impossible. As stated by Thomas (1995), there is no nice way out of this problem and the easiest and most common solution is to lag parts of the non-linear term. Accordingly, I choose to lag the values of $w^{P2}$ in the non-linear terms of $\left( \frac{\Upsilon}{\gamma_2} \left( w^{P2} - \gamma_1 \right) - \xi A \right) w_A^{P2}$ and $\Upsilon \frac{\alpha \phi \rho \theta M^{\psi}}{(1 + \alpha A)^{\phi+1}} \frac{1}{2\gamma_2} \left( w^{P2} - \gamma_1 \right)^2$. The other issue relates to the changing sign of the term $\left( \frac{\Upsilon}{\gamma_2} \left( w^{P2} - \gamma_1 \right) - \xi A \right)$, which may cause instabilities in the routine. This problem can be elegantly handled by upwinding: whenever the sign is negative, $w_A^{P2}$ is approximated by the backward finite difference scheme; whenever the sign is positive, the forward finite difference scheme is used. Additionally, the scheme is made conservative by refining the discretization of the term in the $A$-direction: whenever the sign is negative, the term is discretized at the point $(i, j, k - \frac{1}{2})$ instead of $(i, j, k)$; whenever the sign is positive, the term is discretized at the point $(i, j, k + \frac{1}{2})$ instead of $(i, j, k)$, see e.g. Wilmott (1998). Denoting $\Upsilon_{i,j,k}$ as the discretized version of equation (4.3), the scheme thus reads:

$$
\begin{aligned}
w_{i,j,k}^{P} = -q \Upsilon_{i,j,k} \frac{\alpha \phi \rho i \Delta \theta \left( (j-1) \Delta M \right)^{\psi}}{(1 + \alpha k \Delta A)^{\phi+1}} \left( Y + \frac{1}{2\gamma_2} \left( w_{i,j,k}^{P} - \gamma_1 \right)^2 \right) + w_{i-1,j-1,k}^{P} \; x_1 \\
+ w_{i,j-1,k}^{P} \; x_5 + w_{i+1,j-1,k}^{P} \; x_3 + w_{i,j-1,k-1}^{P} \; x_6 + w_{i,j-1,k+1}^{P} \; x_7,
\end{aligned}
\tag{4.36}
$$

with $q$, $x_1$ and $x_3$ as in (4.25). The remaining coefficients are given by:

$$
x_5 = \begin{cases} 1 + q \left( r + \sigma^2 i^2 + \xi - \Pi_{i,j,k-\frac{1}{2}} \right) & \text{for} \quad \Pi \leq 0 \\ 1 + q \left( r + \sigma^2 i^2 + \xi + \Pi_{i,j,k+\frac{1}{2}} \right) & \text{for} \quad \Pi > 0, \end{cases}
\tag{4.37}
$$

$$
x_6 = \begin{cases} q \Pi_{i,j,k-\frac{1}{2}} & \text{for} \quad \Pi \leq 0 \\ 0 & \text{for} \quad \Pi > 0 \end{cases}
\tag{4.38}
$$

and

$$
x_7 = \begin{cases} 0 & \text{for} \quad \Pi \leq 0 \\ -q \Pi_{i,j,k+\frac{1}{2}} & \text{for} \quad \Pi > 0 \end{cases}
\tag{4.39}
$$

with $\Pi$ being:

$$\Pi_{i,j,k} = \frac{\Upsilon_{i,j,k}}{\gamma_2 \Delta A} \left(w^{P2}_{i,j,k} - \gamma_1\right) - \xi k. \qquad (4.40)$$

The values $w^{P2}_{i,j,k-\frac{1}{2}}$ and $w^{P2}_{i,j,k+\frac{1}{2}}$ are the average values of their "neighbours" $w^{P2}_{i,j,k}$ and $w^{P2}_{i,j,k-1}$, and $w^{P2}_{i,j,k}$ and $w^{P2}_{i,j,k+1}$, respectively.

Please note that defining boundary conditions for $A$ is not necessary. For $A = 0$, $\Pi$ is positive and the values in $A = 0$ can be directly derived by the scheme. Likewise, the values $w^{P2}_{i,j,\mathscr{K}}$ directly result from the scheme, because the marginal welfare of adaptation for a very high $A$ approaches zero and thus $\Pi$ is certainly negative. Hence, in the $A$ direction the scheme only uses values from inside the grid. The scheme is then iteratively solved for all the remaining values beyond the threshold of taking action.

### 4.A.4 The Particular Solution of Equation (4.14) and Equation (4.15)

Appendices 4.A.1 and 4.A.3 describe how to compute the marginal expected present welfare for $S1$ and $S2$, respectively. The idea is to insert these values for $\frac{\partial W}{\partial A}$ into the corresponding equations (4.14) and (4.15) and to apply an implicit finite difference scheme.

Equation (4.10) helps to find the boundary conditions for the particular solution of (4.14) (henceforth referred to as $W^{P1}$):

$$\text{upper boundary condition for } M \to \infty: \quad W^{P1} = -\frac{1}{r}\kappa_1 m^{\kappa_2} Y, \qquad (4.41)$$

$$\text{lower boundary condition for } \theta = 0: \quad W^{P1} = \frac{Y}{r}\left(1 - \kappa_1 m^{\kappa_2}\right), \qquad (4.42)$$

$$\text{upper boundary condition for } \theta \to \infty: \quad W^{P1} = -\frac{1}{r}\kappa_1 m^{\kappa_2} Y. \qquad (4.43)$$

For an enormous amount of pollution $M \to \infty$ or for a high value of $\theta \to \infty$, the GDP net of damage tends to zero. The mitigation costs remain as the only term in equation (4.10). In the case of $\theta = 0$, the climate damage costs remain zero and $\Upsilon \equiv 1$. Then, the integral in equation (4.10) has the analytical solution (4.42).

The same boundary conditions apply to the particular solution of (4.15) (henceforth referred to as $W^{P2}$). If the GDP net of the damage costs is close to zero, equation (4.12) shows that $a^*$ becomes zero. If the climate damage costs remain zero, there is no need to invest in adaptation. Hence, for these extreme cases, $a^*$ is zero and $W^{P2}$ behaves in the same way as $W^{P1}$.

The scheme to approximate $W^{P1}$ is then:

$$W_{i,j,k}^{P1} = -q \left( Y \Upsilon_{i,j-1,k} - \kappa_1 m^{\kappa_2} - \xi k \Delta A w_{i,j-1,k}^{P1} \right) \qquad (4.44)$$
$$+ W_{i-1,j-1,k}^{P1} \ x_1 + W_{i,j-1,k}^{P1} \ x_8 + W_{i+1,j-1,k}^{P1} \ x_3,$$

with $w^{P1}$ being given by Appendix 4.A.1, $q$, $x_1$ and $x_3$ as in (4.25) and $x_8$ is given by:

$$x_8 = 1 + q \left( r + \sigma^2 i^2 \right). \qquad (4.45)$$

The same coefficients are used for the scheme to approximate $W^{P2}$:

$$W_{i,j,k}^{P2} = -q \left( Y \Upsilon_{i,j-1,k} - \kappa_1 m(t)^{\kappa_2} - \xi k \Delta A w_{i,j-1,k}^{P2} + \frac{\Upsilon_{i,j-1,k}}{2\gamma_1} \left( w_{i,j-1,k}^{P2} - \gamma_1 \right)^2 \right) \qquad (4.46)$$
$$+ W_{i-1,j-1,k}^{P2} \ x_1 + W_{i,j-1,k}^{P2} \ x_8 + W_{i+1,j-1,k}^{P2} \ x_3,$$

with $w^{P2}$ being given by Appendix 4.A.3.

Along the same lines, the real options value as described by (4.26) can be derived. The boundary conditions are

$$\text{upper boundary condition for } M \to \infty: \quad W^G = 0, \qquad (4.47)$$
$$\text{lower boundary condition for } \theta = 0: \quad W^G = 0, \qquad (4.48)$$
$$\text{upper boundary condition for } \theta \to \infty: \quad W^G = 0. \qquad (4.49)$$

The explanation again follows the same logic. In the situation of extremely high climate damage costs, investment in adaptation would no longer be beneficial. For $\theta = 0$, there is no need to invest in adaptation. Therefore, the real options value is zero in both extreme cases.

The marginal real options value $w^G$ is then inserted into the partial differential equation (4.26), which is solved by the analogue to (4.44).

The full solution to Section 4.2.2 is then composed of the sum of the real options value $W^G$ and the expected present welfare $W^{P1}$ for $S1$ and the expected present welfare $W^{P2}$ for $S_2$.

## 4.B   The Procedure to Solve the Real Option to Mitigate

The applied solution routine to find the values of the real option to mitigate does not fundamentally differ from the finite difference method outlined in Appendix 4.A. To avoid

needless repetitions, I only outline the most important steps that need to be considered when solving equation (4.18).

As in Appendix 4.A.2, I opt to solve this free boundary problem by applying SOR. To save the computational costs of deriving the acceleration parameters, I take the ones derived in Appendix 4.A.2. Although the involved spectral radii are not equal for the two routines, they are sufficiently close to guarantee quick convergence.

As in Appendix 4.A.3, the non-linearity of the partial differential equation does not fit well with the implicit finite difference method. To solve it nonetheless, I rewrite equation (4.18) for $S_2$ as follows:

$$
\begin{aligned}
rW^M = {} & (\beta\epsilon Y - \delta M)\frac{\partial W^M}{\partial M} + \mu\theta\frac{\partial W^M}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^M}{\partial\theta^2} \\
& + \left(\frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - 2\gamma_1\right) - \xi A\right)\frac{\partial W^M}{\partial A} + \frac{\Upsilon\gamma_1^2}{2\gamma_2},
\end{aligned}
\tag{4.50}
$$

and opt to "lag" the discretized version of the partial derivative in $\left(\frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - 2\gamma_1\right) - \xi A\right)$.

Finding a boundary condition for $A = 0$ is far from being straightforward. Instead, I coarsely approximate the partial derivative $\frac{\partial W^M}{\partial A}$ in $A = 0$ by the derivative of the corresponding particular solution.

An issue of concern is caused by the switch of the functional form in equation (4.18). The resulting jump in the values may lead to errors in the approximated finite differences in the neighbourhood of the discontinuities. For instance, the real options values in the switch could drop to a suspiciously low level. With the aim of constraining the magnitude of these errors, I first solve the mitigation model that ignores the opportunity to adapt. The equation describing the real options value in that case is continuous and thus garantuees precise results. In the absence of adaptation, the urgency to mitigate is certainly higher than in the case in which the damage can be alleviated by adaptation. Accordingly, the values computed thus then serve as a lower boundary in the SOR method that derives the real options values given by equation (4.18).

## 4.C  CALIBRATION

The base calibration is as follows.

<u>Greek letters:</u>

| | | |
|---|---|---|
| adaptation parameter | $\alpha$ | 0.05 |
| atmospheric retention ratio (in ppm per trillion of $CO_2$ metric tonnes) | $\beta$ | 9.09 |
| natural rate of $CO_2$ depletion in the atmosphere | $\delta$ | 0* |

| | | |
|---|---|---|
| emission parameter (in $CO_2$ metric tonnes per US dollars PPP of GDP) | $\epsilon$ | $4.18 \times 10^{-4}$ |
| adaptation parameter | $\phi$ | 4.5 |
| adaptation cost parameter | $\gamma_1$ | 0.4 |
| adaptation cost parameter | $\gamma_2$ | 16.81 |
| mitigation cost parameter | $\kappa_1$ | 0.03 |
| mitigation cost parameter | $\kappa_2$ | 1.2 |
| drift term in the Brownian motion | $\mu$ | 0 |
| damage cost parameter | $\rho$ | $7.17 \times 10^{-12}$ |
| variance term in the Brownian motion | $\sigma$ | 0.07 |
| depreciation rate of adaptation capital | $\xi$ | 0.075 |
| damage cost parameter | $\psi$ | 4.88 |

Annotation * : This calibration represents a valuable simplication to the numerical solution routine. The parameter $\beta$ is parametrized to capture the depreciation, by making the crude assumption that the increase in atmospheric $CO_2$ follows a constant trend of 3 ppm per year.

Further parameters:

| | | |
|---|---|---|
| emissions (in trillion of $CO_2$ metric tonnes) | $E$ | 0.033 |
| Global GDP in the absence of climate damages (in trillion US dollars PPP) | $Y$ | 78.97 |
| emission reduction rate | $m$ | 0.09 |
| discount rate | $r$ | 0.025 |

"Calibration" of the implicit finite difference method:

| | | | |
|---|---|---|---|
| $\theta_{max}$ | 100 | $\Delta\theta$ | 0.2 |
| $M_{max}$ | 420 | $\Delta M$ | 0.6 |
| $A_{max}$ | 16.67 | $\Delta A$ | 0.05 |

Here, the base calibration of the investigated parameter is varied by $\pm$ 20%. The thresholds resulting from the new simulations are then compared with the thresholds of the base calibration.



**(a)** Sensitivity to Alternative Discount Rates: $r = 0.025$ (Solid Line, Base Calibration), $r = 0.02$ (Dashed Line), $r = 0.03$ (Dotted Line)

**(b)** Sensitivity to Alternative GDP Values: $Y = 78.97$ (Solid Line, Base Calibration), $Y = 63.18$ (Dashed Line), $Y = 94.76$ (Dotted Line)
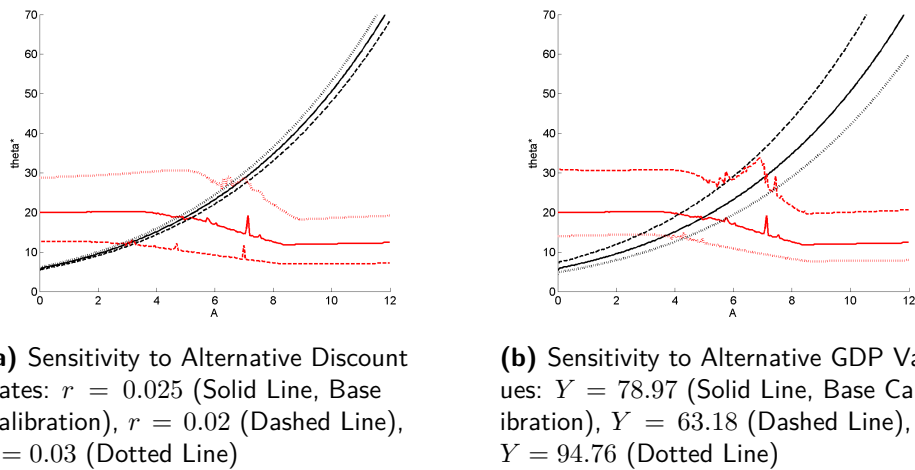
**Figure 4.4.1:** Sensitivity of the Optimal Policy Mix to Discounting and GDP, Depicted by the Threshold of Mitigation (Red) and the Threshold of Adaptation (Black) in $M = 40$
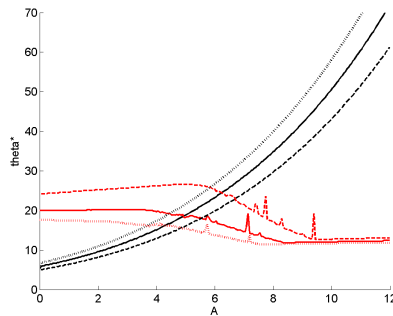


**Figure 4.4.2:** Sensitivity to Alternative Depreciation Rates Generated by a $\pm 20\%$ Variation: $\xi = 0.075$ (Solid line, Base Calibration), $\xi = 0.06$ (Dashed Line), $\xi = 0.09$ (Dotted Line)

Please note that a $-20\%$ parameter variation for $\kappa_2$ would make the mitigation cost curve concave. This case is thus ignored in the sensitivity analysis.

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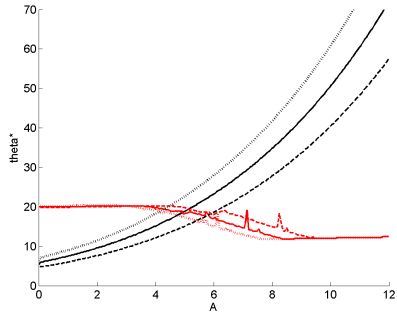
**(a)** Sensitivity to Alternative Adaptation Cost Parameters Generated by a $\pm 20\%$ Variation: $\gamma_1 = 0.4$ (Solid Line, Base Calibration), $\gamma_1 = 0.32$ (Dashed Line), $\gamma_1 = 0.48$ (Dotted Line)

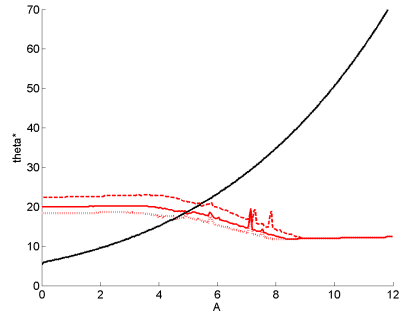**(b)** Sensitivity to Alternative Adaptation Cost Parameters Generated by a $\pm 20\%$ Variation: $\gamma_2 = 16.81$ (Solid Line, Base Calibration), $\gamma_2 = 13.45$ (Dashed Line), $\gamma_2 = 20.17$ (Dotted Line)

**Figure 4.4.3:** Sensitivity of the Optimal Policy Mix to the Calibration of the Adaptation Parameters



**(a)** Sensitivity to Alternative Mitigation Cost Parameters Generated by a $\pm 20\%$ Variation: $\kappa_1 = 0.03$ (Solid Line, Base Calibration), $\kappa_1 = 0.024$ (Dashed Line), $\kappa_1 = 0.36$ (Dotted Line)
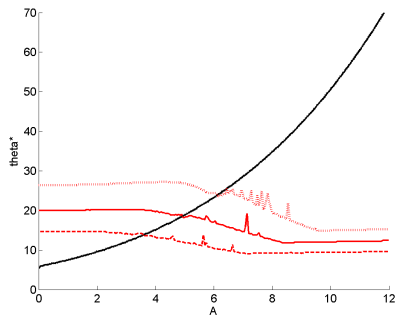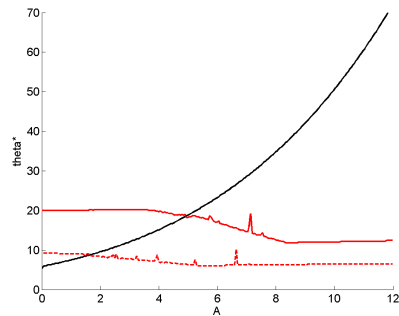
**(b)** Sensitivity to Alternative Mitigation Cost Parameters Generated by a $+20\%$ Variation: $\kappa_2 = 1.2$ (Solid Line, Base Calibration) and $\kappa_2 = 1.44$

**Figure 4.4.4:** Sensitivity of the Optimal Policy Mix to the Calibration of the Mitigation Costs

*In solidarity with my countrymen who are struggling to find food back home, I will now commence a voluntary fasting for the climate, this means I will voluntarily refrain from eating food during this COP, until a meaningful outcome is in sight.*

Yeb Sano,
Philippines Lead Negotiator at the 2013 United
Nations Climate Change Conference

# 5

# Can International Environmental Cooperation be bought: Comment[1]

## 5.1 Introduction

Most of the theoretical literature on international environmental agreements assumes that countries are homogeneous, because implementing asymmetries in benefits or costs of abatement pose great difficulties in finding analytical solutions. Thus, the effects of heterogeneity on the coalition formation have been mostly examined by means of simulation tools.

In this light, Fuentes-Albero & Rubio (2010) have made an important contribution, as they analytically solve a non-linear model allowing for two types of countries and continuous strategies. Heterogeneity in abatement and damage costs is analyzed separately, where for both cases two different institutional settings, i.e. one with and one without transfer payments, are applied.

Fuentes-Albero and Rubio show for both scenarios of asymmetry and no side-payments that the maximum level of cooperation consists of three countries of the same type. In the case of heterogeneity in abatement costs, countries of a different type are not willing to form a coalition and a transfer system contributes only to an agreement of two

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[1]This chapter is published as Glanemann (2012).

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asymmetric signatories. When the differences in environmental damages are considered, the size of the coalition even increases with the degree of heterogeneity in a self-financed payment system. Assuming that no transfers are paid, Fuentes-Albero and Rubio conclude that an agreement between one type 1 and one type 2 country is self-enforcing given that the differences in the damages are not very large.[2]

The derivation of the last mentioned result is shown to be incorrect in the following by proving that this coalition is not self-enforcing.

## 5.2   The Model and the Derivation of the new Result

At first the set-up as well as the required equations in Fuentes-Albero & Rubio (2010) are summarized.

The model is designed for two kinds of countries, of which $N_1$ are of type 1 and $N_2$ are of type 2. The types are assumed to differ only in the marginal damage costs $m_i > 0$ with $m_1 > m_2$. Hence, they bear different environmental damage costs $m_i X$, which are generated by the global pollution $X$. Implementing a national emission level $x_i$ lower than in the business-as-usual scenario $\delta$ induces abatement costs $(c/2)(\delta - x_i)^2$ with $c$ being the marginal abatement costs. The abatement and the environmental damage costs add up to the country's cost function.

The negotiation process is modeled as a two-stage game, in which in the first stage the countries decide to sign or to stay outside the agreement (membership game) and in the successive stage they determine their emissions (emission game). As this game is solved by backward induction, at first the countries' optimal emission levels are computed by minimizing their cost functions. For this the non-signatories are assumed to act non-cooperatively, whereas the signatories consider the aggregate costs of the coalition while acting non-cooperatively against the non-signatories. Assuming $n_1$ signatories of type 1 and $n_2$ of type 2, the optimal emission levels of the defecting and signing countries of type $i$ are computed to be $x_i^f = \delta - (m_i/c)$ and $x_i^s = \delta - (m_1 n_1 + m_2 n_2)/c$, respectively. The signatories' cost function is therefore:

$$C_i^s(n_1, n_2) = \frac{1}{2c}(m_1 n_1 + m_2 n_2)^2 + m_i X(n_1, n_2), \quad i = 1, 2 \tag{5.1}$$

and the corresponding function of the defecting countries is represented by:

$$C_i^f(n_1, n_2) = \frac{m_i^2}{2c} + m_i X(n_1, n_2), \quad i = 1, 2, \tag{5.2}$$

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[2]An agreement is called self-enforcing, if the countries acting in their own self-interest have incentives to sign it.

where the global emissions $X(n_1, n_2)$ add up to:

$$X(n_1, n_2) = N_1 \left(\delta - \frac{m_1}{c}\right) + N_2 \left(\delta - \frac{m_2}{c}\right)$$
$$- \frac{n_1}{c}\left(m_1(n_1 - 1) + m_2 n_2\right) - \frac{n_2}{c}\left(m_1 n_1 + m_2(n_2 - 1)\right). \qquad (5.3)$$

In the membership game the coalition $(n_1, n_2)$ is tested for stability, which means that no country has an incentive to revise its membership decision. More precisely, if the internal stability condition, i.e. $C_1^s(n_1, n_2) \leq C_1^f(n_1 - 1, n_2)$ for type 1 and $C_2^s(n_1, n_2) \leq C_2^f(n_1, n_2 - 1)$ for type 2 holds, every signatory is better off by staying in the coalition. If the external stability condition, i.e. $C_1^f(n_1, n_2) \leq C_1^s(n_1 + 1, n_2)$ for type 1 and $C_2^f(n_1, n_2) \leq C_2^s(n_1, n_2 + 1)$ for type 2 is fulfilled, the non-signatories cannot improve by acceding to the agreement. If both conditions apply, the coalition is called stable or self-enforcing. Fuentes-Albero and Rubio prove that only a coalition of one type 1 country and one type 2 country is internally stable given the differences in relative terms are not greater than 40%. They also state the countries being externally stable, but the following calculations show that the external stability condition is not satisfied. It would hold for type 1, if $C_1^f(1, 1) - C_1^s(2, 1) \leq 0$. Computing both summands separately, gives

$$C_1^f(1,1) = \frac{m_1^2}{2c} + m_1 \left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right) - \frac{m_2}{c} - \frac{m_1}{c}\right]$$
$$= \frac{m_1^2}{2c} + m_1\left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right)\right] - \frac{m_1 m_2}{c} - \frac{m_1^2}{c}$$
$$= -\frac{m_1^2}{2c} - \frac{m_1 m_2}{c} + m_1\left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right)\right] \qquad (5.4)$$

and

$$C_1^s(2,1) = \frac{1}{2c}(2m_1 + m_2)^2$$
$$+ m_1\left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right) - \frac{2}{c}(m_1 + m_2) - \frac{2m_1}{c}\right]$$
$$= \frac{2m_1^2}{c} + \frac{2m_1 m_2}{c} + \frac{m_2^2}{2c} + m_1\left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right)\right]$$
$$- \frac{2m_1^2}{c} - \frac{2m_1 m_2}{c} - \frac{2m_1^2}{c}$$
$$= -\frac{2m_1^2}{c} + \frac{m_2^2}{2c} + m_1\left[N_1\left(\delta - \frac{m_1}{c}\right) + N_2\left(\delta - \frac{m_2}{c}\right)\right]. \qquad (5.5)$$

Therefore the external stability condition evolving as

$$0 \geq C_1^f(1,1) - C_1^s(2,1) = -\frac{m_1^2}{2c} - \frac{m_1 m_2}{c} + \frac{2m_1^2}{c} - \frac{m_2^2}{2c}$$
$$= \frac{1}{2c}\left(3m_1^2 - 2m_1 m_2 - m_2^2\right)$$
$$= \frac{1}{2c}\left(m_1 - m_2\right)\left(3m_1 + m_2\right), \qquad (5.6)$$

is not satisfied, as it is assumed that $m_1 > m_2$.[3]

The assumption $m_2 > m_1$ would lead to external stability for type 1, however in this case the condition for type 2

$$C_2^f(1,1) - C_2^s(1,2) = -\frac{1}{2c}\left(m_1 - m_2\right)\left(m_1 + 3m_2\right) \leq 0 \qquad (5.7)$$

cannot hold.

Hence, the coalition consisting of one type 1 country and one type 2 country does not fulfill the external stability condition and is thus not self-enforcing.

The calculations show that a coalition of two countries differing in environmental damages attracts further highly affected countries. Consequently, the low-damage country is burdened with a too high emission reduction target, so that it annuls the agreement. This leaves behind a treaty that only includes high-damage countries - an agreement of homogeneous countries.

## 5.3 CONCLUSION AND FURTHER REMARKS

Fuentes-Albero and Rubio show that only an agreement of one type 1 and one type 2 country is internally stable if heterogeneity in environmental damages and no side-payments are considered. This comment proves that the external stability condition has however not been satisfied. Therefore heterogeneity in environmental damage costs provokes countries of different types to defect from cooperating, unless they are compensated by side-payments.

As Fuentes-Albero and Rubio draw the same conclusion for the case of different abatement costs, a system that is designed to balance asymmetries seems to be a necessary tool for the formation of a global international environmental cooperation. However, the assumed self-financed transfer system that makes the size of the coalition increase with the

---

[3]Probably as a result of calculation mistakes, Fuentes-Albero and Rubio state a different equation being $C_1^f(1,1) - C_1^s(2,1) = -(m_2^2/2c)(m^2 + 2m + 1)$ with $m = (m_1/m_2)$, which is claimed to be also valid for type 2. This equation is obviously negative for all $m$ and the external stability condition would thus hold.

degree of asymmetry in the marginal environmental damages is not generally applicable. In this scenario, one or two countries with relatively high marginal damage costs buy the cooperation of the low damage countries, in order to decrease the global emission level. The incentives to buy the others' cooperation increase the greater the asymmetries get, as otherwise the less affected countries tend to have high emissions causing comparatively high damages in the vulnerable countries. This positive result confirms Barrett (2001) that inequalities can be a vehicle to establish a high degree of international cooperation. Barrett examines the case of the Montreal Protocol, where the rich countries gaining the most from the treaty bought the poor's cooperation.[4] Hence, in this special case a transfer system can exploit the existing asymmetries, as it wipes out free-riding incentives without transgressing any notions of fairness. However, negotiations on $CO_2$ emission reduction must overcome a different distribution of inequalities. Even though assessing the countries' damages caused by atmospheric $CO_2$ is always controversial, most of the literature argues that poor countries being - on the whole - most vulnerable (see for example the review of several studies in Smith et al. (2001) or Mendelsohn et al. (2006)).[5] Hence, according to Fuentes-Albero and Rubio, a high level of cooperation could be achieved, if these countries compensate the developed world for reducing emissions. Due to aspects of equity and budget constraints the examined transfer scheme is unlikely to be ever up for debate, let alone lead to a high degree of international cooperation. A realistically enforceable system of side-payments does not only eliminate free-riding incentives but also satisfy criteria of feasibility and fairness. The difficulty of finding such a transfer system is illustrated by the failed UN climate change negotiations of the past years.

Instead of overcoming asymmetries to accede to one agreement, multiple coalitions consisting of rather homogeneous countries may be more realistic and effective. Com-

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[4]The Montreal Protocol, which has been signed by almost every nation in the world, is designed to protect the ozone layer by phasing out harmful substances. Barrett argues that the industrialized countries benefit more from this treaty than the developing ones, as depletion has been reported to be mainly in high latitude regions and light-skinned people run a higher risk of getting skin cancer.

[5]Smith et al. (2001) compare different studies, which have estimated the total monetized impacts in different regions of the world for a doubling of atmospheric $CO_2$. Although there is a substantial degree of uncertainty around the numerical results, most studies show that the developing countries expect more severe damages by climate change. The reasoning is that less developed countries typically depend on climate-sensitive sectors like agriculture and lack the financial, technical and institutional capacity to adapt to climate change. Furthermore many of the poor countries already face environmentally disadvantageous conditions. This argument is stressed by Mendelsohn et al. (2006). Investigating the distributional effects of climate change on rich and poor countries, they find that the poorest half of the world's nations is threatened by the highest damages, the second richest quarter bears a relatively small burden and the richest quarter suffers almost no net impacts. They show that the poor countries' vulnerability is mainly due to their geographical location. As low latitude regions are already exposed to very high temperatures, further warming has worrying consequences.

bined with exclusive membership rules the entry of countries being different could be rejected, which prevents the treaty from becoming unprofitable and unstable. Given the negative results concerning asymmetries, further research to develop a theoretical model that incorporates heterogeneity and multiple coalitions is thus required.[6]

Moreover, it would be interesting to examine whether the resulting destructive effects of heterogeneity on cooperation are due to the implementation of linear damage costs alongside quadratic abatement costs. This kind of modeling emphasizes the costs of abatement over the costs of pollution. The implementation of non-linear damages, however, pose problems of analytical tractability, as Fuentes-Albero and Rubio also point out.

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[6]Indeed, the theoretical papers by Finus & Rundshagen (2003) and Asheim et al. (2006) attest the co-existence of multiple coalitions a higher efficiency. However, both papers make the simplifying assumption of symmetry. In the empirical strand of literature Eyckmans & Finus (2006) confirm the welfare enhancing effects of multiple coalition structures by implementing six world regions (USA, Japan, European Union, China, Former Soviet Union and the "Rest of the World") into an integrated assessment model.

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# References to the Quotes Introducing the Chapters

The quote introducing Chapter 1 is taken from `http://www.un.org/News/Press/docs/2007/ga10607.doc.htm`, reporting about Ban Ki-Moon's speech at the General Assembly on 31 July 2007.

The quote introducing Chapter 2 is retrieved from `http://en.wikiquote.org/wiki/Austin_Bradford_Hill`.

The quote introducing Chapter 3 is published in Stern (2009).

The quote introducing Chapter 4 was reported all over the news, e.g. `http://www.bbc.co.uk/news/science-environment-24899647`.

# Erklärung

Hiermit erkläre ich, Nicole Glanemann, dass ich mich noch keiner Doktorprüfung unterzogen oder um Zulassung zu einer solchen beworben habe.

Die Dissertation mit dem Titel

"An Economic Analysis of the Issues and Challenges in Climate Policy Decision Making"

hat noch keiner Fachvertreterin, keinem Fachvertreter und keinem Prüfungsausschuss einer anderen Hochschule vorgelegen.

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# Eidesstattliche Versicherung

Hiermit erkläre ich, Nicole Glanemann, an Eides statt, dass ich die Dissertation mit dem Titel

"An Economic Analysis of the Issues and Challenges in Climate Policy Decision Making"

selbständig und ohne fremde Hilfe verfasst habe. Andere als die von mir angegebenen Quellen und Hilfsmittel habe ich nicht benutzt. Die den herangezogenen Werken wörtlich oder sinngemäß entnommenen Stellen sind als solche gekennzeichnet.

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