arXiv:gr-qc/9603010v1 9 Mar 1996

IFUP-TH 8/96
February 1996
gr-qc/9603010

Symmetry breaking aspects of the effective Lagrangian for quantum black holes

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Abstract. The physical excitations entering the effective Lagrangian for quantum black holes are related to a Goldstone boson which is present in the Rindler limit and is due to the spontaneous breaking of the translation symmetry of the underlying Minkowski space. This physical interpretation, which closely parallels similar well-known results for the effective stringlike description of flux tubes in QCD, gives a physical insight into the problem of describing the quantum degrees of freedom of black holes. It also suggests that the recently suggested concept of 'black hole complementarity' emerges at the effective Lagrangian level rather than at the fundamental level.

1. Introduction

The attempts to give a description of black holes consistent with the laws of quantum mechanics face well known problems. A possible approach [1, 2] assumes that at the quantum level, from the point of view of an external, static observer, the quantum degrees of freedom of a black hole are located on the horizon (see also [3]). In this approach, because of the blue-shift factor in quantities like the Hawking temperature, a static observer sufficiently close to the horizon is in a region of super-Planckian energies, where unknown physics comes into play. To describe quantitatively the horizon dynamics one can therefore resort to an effective Lagrangian approach [4, 5]. The most general effective action turns out to be of the form

$$S_{\rm eff} = -\mathcal{T}\int d^3\xi\sqrt{-h}\,\Big[1 + C_0 K + C_{I} R + C_{II} K^2 + C_{III} K_{ij}K^{ij} + \ldots\Big]\,. \quad (1)$$

The basic variables appearing in the action are the fields $\zeta^\mu(\xi)$, $\xi^i = (\tau, \sigma_1, \sigma_2)$ which define the position of the quantum, fluctuating horizon and describe a 2+1 dimensional timelike hypersurface (the world-volume) parametrized by $(\tau, \sigma_1, \sigma_2)$ and embedded in 3+1 dimensional spacetime with background metric $g_{\mu\nu}$. From these one constructs the induced metric h_{ij} and the extrinsic curvature K_{ij} which appear in eq. (1), where $h = \det h_{ij}$ and $K = K^i_i$; R is the scalar curvature of the world-volume. The coefficients of the various operators are phenomenological constants which can in principle be derived if one knows the underlying fundamental theory.

In a semiclassical expansion, the nature of the degrees of freedom entering eq. (1) is more transparent. In flat space one can write the generic fluctuation over a classical solution $\bar\zeta^\mu(\xi)$ as

$$\zeta^\mu(\xi) - \bar\zeta^\mu(\xi) = b^i(\xi)\partial_i\bar\zeta^\mu + \phi(\xi)\bar n^\mu(\xi)\,, \quad (2)$$

where $\partial_i = \partial/\partial\xi^i$ and $\bar n^\mu(\xi)$ is the normal of the classical solution $\bar\zeta^\mu(\xi)$ in the point of the world-volume labelled by ξ. One observes [6] that the fields $b^i(\xi)$ represent fluctuations along the surface and are 'pure gauge' since they can be reabsorbed with a reparametrization. The only physical quantum fluctuations are perpendicular to the surface, and are parametrized by a single scalar field $\phi(\xi)$ living in the world-volume. The definition can be generalized to curved space, where the physical fluctuations are written as

$$\zeta^\mu(\xi) = \zeta^\mu_{\rm geod}(\phi, \bar\zeta, \bar n)\,, \quad (3)$$

where, at each point ξ on the world-volume, $\zeta^\mu_{\rm geod}$ gives the geodesic parametrized by an affine parameter ϕ, which at $\phi = 0$ goes through the point $\bar{\zeta}^\mu$, with a tangent at $\phi = 0$ equal to $\bar{n}^\mu$.

Expanding at quadratic order in ϕ, in generic curved space, the action becomes [7, 5]

$$S_{\rm eff} = S_{\rm cl} - \frac{\mathcal{T}}{2} \int d^3\xi \sqrt{-\bar{h}} \left[\partial_i \phi \partial^i \phi - (\bar{K}_i^j \bar{K}_j^i + \bar{\mathcal{R}}_{\mu\nu} \bar{n}^\mu \bar{n}^\nu) \phi^2 \right] , \qquad (4)$$

where the overbar denotes the value at $\bar{\zeta}$, $S_{\rm cl} = -\mathcal{T} \int d^3\xi (-\bar{h})^{1/2}$, indices are raised and lowered with $\bar{h}_{ij}$, and $\bar{\mathcal{R}}_{\mu\nu}$ is the Ricci tensor of the embedding space evaluated at $\bar{\zeta}$.

For a planar membrane in Rindler space the term $\sim \phi^2$ vanishes and we are left with a massless scalar field living in the 2+1 dimensional world-volume. The appearance of a massless scalar particle in an effective Lagrangian leads naturally to suspect that we have to do with a Goldstone boson. Indeed, this turns out to be correct, and it is in fact well known in the context of the string description of chromoelectric flux tubes in *QCD* [8]. To our knowledge, however, this has not been properly appreciated in the studies on quantum fluctuations of domain walls or membranes, and since it gives interesting hints on the problem of quantization of black holes, we find useful to discuss it in the present context.

2. Toy Model

To understand why a Goldstone boson appears, let us see in an explicit example how an effective membrane theory emerges from a fundamental theory. As the fundamental theory we take

$$S = -\frac{1}{2} \int d^4x \left[\partial_\mu \Phi \partial^\mu \Phi + g \left(\Phi^2 - \frac{m^2}{g} \right)^2 \right] \qquad (5)$$

in flat space, $g_{\mu\nu} = (-,+,+,+)$. The theory has different sectors depending on the boundary conditions that we impose. In the sector defined by $\Phi(z \to +\infty) = m/\sqrt{g}, \Phi(z \to -\infty) = -m/\sqrt{g}$ we have a manifold of ground states given by the domain wall solutions,

$$\Phi_{\rm cl} = \frac{m}{\sqrt{g}} \tanh m(z - z_0) \, , \qquad (6)$$

labelled by a parameter z_0, which is the collective coordinate corresponding to translation invariance in the direction transverse to the domain wall. If we select a particular member of this manifold of ground states in order to perform a semiclassical expansion, we are breaking spontaneously the translation invariance along z and we expect to find a corresponding Goldstone boson. Expanding the field $\Phi(x) = \Phi_{\rm cl}(x) + \eta(x)$, the action for the fluctuations is [8]

$$S = S_0 - \frac{1}{2}\int d^4x\, \eta(x)\Delta^\Phi \eta(x) \tag{7}$$

$$\Delta^\Phi = -\partial_\mu \partial^\mu + V(z) \tag{8}$$

$$V(z) = 4m^2 - \frac{6m^2}{\cosh^2 m(z-z_0)} \tag{9}$$

Therefore the eigenfunctions have the form $\eta(x) = e^{i(k_0 t + k_1 x + k_2 y)}\psi(z)$ where $\psi(z)$ satisfies a one-dimensional Schroedinger equation

$$\left(-\partial_z^2 + V(z)\right)\psi(z) = \epsilon\psi(z) \tag{10}$$

and the eigenvalues are $-k_0^2 + k_1^2 + k_2^2 + \epsilon$. The Schroedinger equation has two bound states [8]

$$\psi_0(z) = \frac{m^2}{\sqrt{g}}\frac{1}{\cosh^2 m(z-z_0)}\,, \qquad \epsilon = 0 \tag{11}$$

$$\psi_1(z) = \frac{m^2}{\sqrt{g}}\frac{\sinh mz}{\cosh^2 m(z-z_0)}, \qquad \epsilon = 3m^2\,, \tag{12}$$

and a continuum of modes ψ_{k_3} which starts at $4m^2$, $\epsilon = 4m^2 + k_3^2$. The normalization of the modes has been chosen for later convenience.

The crucial point is the existence of a mass gap separating the mode ψ_0 from the rest of the spectrum. This means that, if we are interested in low energy physics, $E^2 < 3m^2$, we can integrate out all modes except the mode ψ_0. Expanding η in normal modes and using the notation $\xi = (t, x, y)$

$$\eta(\xi, z) = c_0(\xi)\psi_0(z) + c_1(\xi)\psi_1(z) + \int dk\, c_k(\xi)\psi_k(z) \tag{13}$$

the integration in dz in the action, eq. (7), can be performed explicitly and the action S becomes a functional of the fields $c_0(\xi), c_1(\xi), c_k(\xi)$ living in the world-volume. The effective action for the mode c_0 is obtained integrating over all massive modes,

$$\exp\left(iS_{\rm eff}[c_0]\right) = N \int Dc_1 \prod_k Dc_k \, e^{iS} \, , \tag{14}$$

where N is a normalization factor. At quadratic order in η the normal modes decouple and

$$S_{\rm eff} = -\frac{\mathcal{T}}{2} \int d^3\xi \; \partial_i c_0 \partial^i c_0 \, , \tag{15}$$

where

$$\mathcal{T} = \int_{-\infty}^{\infty} dz \, \psi_0^2(z) = \frac{4m^3}{3g} \, . \tag{16}$$

Expanding at higher order in η we obtain a coupling between the various modes, which generate higher dimension operators in the effective lagrangian for c_0. These terms, however, are irrelevant in the low energy limit.

The fact that a mass term in eq. (15) is absent can be understood noting that $\psi_0(z) = \partial_z \Phi_{\rm cl}$ and then

$$\Phi_{\rm cl}(t,x,y,z) + c_0(t,x,y)\psi_0(z) = \Phi_{\rm cl}(t,x,y,z + c_0(t,x,y)) + O(c_0^2) \, . \tag{17}$$

Therefore infinitesimal rigid translations in the z direction are realized on the world-volume field c_0 as $c_0(\xi) \to c_0(\xi) +$ const., and this symmetry of the embedding space forbids the presence of a mass term in the effective action for c_0. We see that $c_0(\xi)$ is a Goldstone boson which lives in the world-volume of the domain wall and is associated to the spontaneous breaking of translation symmetry. Note that this field propagates only along the membrane, since it has $k_3 = 0$, and the associated mode $\psi_0(z)$ is localized around the membrane, and it determines its thickness through the parameter m.

To understand the relation between $c_0(\xi)$ and the field $\phi(\xi)$ defined in eq. (2) let us see how, in the same toy model defined by eq. (5), the effective lagrangian (1) can be explicitly derived. The technique was discussed in ref. [9] and a systematic evaluation of higher order terms has been presented in ref. [10]. The idea is to separate explicitly the dependence on the transverse direction of the quantities which appear in the action (5), so that we can integrate it out. The first step is to choose an appropriate coordinate system suited to the domain wall that we are considering, which is taken to be a small fluctuation over a planar solution. Thus, in our flat space example, rather then using cartesian coordinates (t, x, y, z) we use three coordinates $\xi^i = (\tau, \sigma_1, \sigma_2)$ which parametrize the world-volume and, as a coordinate in the transverse direction, we use the affine parameter λ which parametrizes the geodesic which pass through the point of the domain wall labelled by ξ and is orthogonal to the domain wall. At least in a neighbourhood of the domain wall this coordinate system is well defined, and this is all we need when considering small fluctuations around a planar wall. In this coordinate system the normal is $n^\mu = (0, 0, 0, 1)$ even if the wall is non planar. Next one introduces the tensors $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ and $K_{\mu\nu} = n_{\mu;\nu}$ where the semicolon denotes the covariant derivative. They satisfy relations which can be obtained as follows. Computing the Lie derivative of $h_{\mu\nu}$ in this coordinate system one finds

$$\frac{\partial h_{\mu\nu}}{\partial \lambda} = 2K_{\mu\nu}\,, \tag{18}$$

which we take as our first fundamental equation. Furthermore, in flat space, the Riemann tensor is zero. We write explicitly the equation for the component $3\mu 3\nu$ in this coordinate system, where $x^\mu = (\tau, \sigma^1, \sigma^2, \lambda)$

$$0 = \mathcal{R}^3_{\mu 3\nu} = \Gamma^3_{\mu\nu,3} - \Gamma^3_{\mu 3,\nu} + \Gamma^3_{\rho 3}\Gamma^\rho_{\mu\nu} - \Gamma^3_{\rho\nu}\Gamma^\rho_{\mu 3}\,, \tag{19}$$

where Γ is the Christoffel symbol and the comma is the ordinary derivative. Observing that in this coordinate system $K_{\mu\nu} = n_{\mu,\nu} - \Gamma^\rho_{\mu\nu} n_\rho = -\Gamma^3_{\mu\nu}$ and that similarly $K^\nu_\mu = \Gamma^\nu_{\mu 3}$ and $\Gamma^3_{\mu 3} = 0$, we get

$$\frac{\partial K_{\mu\nu}}{\partial \lambda} = K^\rho_\mu K_{\rho\nu}\,, \tag{20}$$

which is the second fundamental equation. The equation of motion for Φ, written separating explicitly the transverse and longitudinal parts, reads

$$\frac{\partial^2 \Phi}{\partial \lambda^2} + K\frac{\partial \Phi}{\partial \lambda} + \nabla_i \nabla^i \Phi - 2g\Phi(\Phi^2 - \frac{m^2}{g}) = 0\,. \tag{21}$$

Eqs. (18,20,21) are now expanded in powers of the small parameter $\varepsilon = l/L$ where $l = 1/m$ is the thickness of the wall and L is the typical lengthscale over which the world-volume bends ($L = \infty$ for an exactly planar wall). The condition that the wall be a small perturbation over the planar solution is implemented requiring

$$l\frac{\partial \Phi}{\partial \lambda} = O(1)\,, \qquad l\frac{\partial \Phi}{\partial \xi^i} = O(\varepsilon)\,. \tag{22}$$

Introducing the dimensionless quantities $u = \lambda/l, v^i = \xi^i/L, \Psi = (\sqrt{g}/m)\Phi$ and $k_{\mu\nu} = LK_{\mu\nu}$ one obtains [10]

$$\frac{\partial h_{\mu\nu}}{\partial u} = 2\varepsilon k_{\mu\nu} \tag{23}$$

$$\frac{\partial k_{\mu\nu}}{\partial u} = \varepsilon k_{\mu\rho} k^{\rho}_{\nu} \tag{24}$$

$$\frac{\partial^2 \Psi}{\partial u^2} - 2\Psi(\Psi^2 - 1) + \varepsilon k \frac{\partial \Psi}{\partial u} + \varepsilon^2 \mathcal{D}_i \mathcal{D}^i \Psi = 0\,, \tag{25}$$

where $\mathcal{D}_i$ is the covariant derivative on the world-volume, with respect to the rescaled variables v^i. Each of the quantities appearing in eqs.(23-25) can now be expanded in ε, e.g.

$$\Psi = \Psi_0 + \varepsilon \Psi_1 + \frac{\varepsilon^2}{2} \Psi_2 + \ldots \tag{26}$$

and the equations can be solved order by order in ε. This allows to determine explicitly the dependence on $u = \lambda/l$. Then, writing the original action in the form

$$S = \int d^3\xi d\lambda \sqrt{-g}\, \mathcal{L}\,, \tag{27}$$

where $\sqrt{-g}$ is the Jacobian for the transformation from the cartesian coordinates (t, x, y, z) to (ξ^i, λ), the integral over λ can be performed explicitly and the result [10] is eq. (1). Of course the computation gives also an explicit expression for the constants C_0, C_I, etc. in eq. (1). In particular, one finds $C_0 = 0$ because it is given by the integral of an odd function of u from $-\infty$ to $+\infty$ (note however that $C_0 \neq 0$ if we work in a finite volume $-L_1 \leq z \leq L_2$), and C_{III} can be set to zero since in flat space the operator $K_{ij}K^{ij}$ is not independent of R and K^2 because of the Gauss-Codacci identity. The other coefficients turn out to be[1]

$$\mathcal{T} = \frac{4m^3}{3g} \tag{28}$$

$$C_I = \frac{1}{m^2} \frac{\pi^2 - 6}{24} \tag{29}$$

$$C_{II} = -\frac{1}{m^2} \frac{5}{48}\,. \tag{30}$$

[1] We have found here a numerical discrepancy in $\mathcal{T}, C_{II}$ with the result quoted in ref. [10]. For comparison, the quantities λ, η in ref. [10] are denoted here $g/2$ and $m/\sqrt{g}$ respectively.

Eq. (1) can now be expanded around a classical solution $\bar{\zeta}^\mu$ and, if we consider only the leading term $\sim \sqrt{-h}$ the result [7, 5] is given by eq. (4). For a planar domain wall in flat space the mass term in eq. (4) vanishes.

3. Implications

The conclusion from this exercise is as follows. In the toy model defined by eq. (5) all computations can be performed explicitly. We can explicitly derive the effective Lagrangian, eq. (1), including the numerical value of the phenomenological constants, we can introduce a field ϕ, eq. (2), parametrizing physical quantum fluctuations, and we can derive the action which governs its dynamics, which is just the action of a massless scalar field living in a world-volume with $\bar{h}_{\rm ij} = (-,+,+)$. On the other hand, we can solve the spectrum of the fluctuations of Φ, eqs. (8), and we see explicitly what is the reason which allows to use an effective Lagrangian approach: it is the existence of a Goldstone boson, separated by a mass gap from the rest of the spectrum. This allows to quantify explicitly what does it mean low energy in the effective Lagrangian approach: it means $E^2 < 3m^2$. In the range $3m^2 < E^2 < 4m^2$ we must take into account also the mode $c_1(\xi)$ and above $4m^2$ we have the continuum. The comparison of eqs. (4) (with $\bar{h}_{\rm ij} = (-,+,+)$, $\bar{K}_i^j \bar{K}_j^i = 0$ and $\bar{\mathcal{R}}_{\mu\nu} = 0$) and (15) shows that $\phi(\xi) = c_0(\xi) + O(c_0^2)$, i.e. in the limit of small fluctuations $\phi(\xi)$ is nothing but the Goldstone mode $c_0(\xi)$. The identification does not extend to finite fluctuations. This can be seen observing that for a planar membrane in flat space the translation symmetry $z \to z + a$ is implemented on ϕ as $\phi \to \phi + a$ exactly, as we read from eq. (2) setting $n^\mu = (0,0,0,1)$. Instead c_0 transforms as $c_0 \to c_0 + a$ only for infinitesimal values of a, as we see from eq. (17). To obtain a representation of a finite translation, all modes c_k must be taken as a basis, and they transform non-linearly between themselves.

Let us see what can we learn from the above discussion in the case of the effective Lagrangian for quantum black holes. Of course in this case we do not know the fundamental theory from which eq. (1) should emerge. In the approach of refs. [4, 5] one therefore *postulates* that, for a static observer outside the horizon, the variables $\zeta^\mu(\xi)$ which describe the position of the quantum, fluctuating horizon are the relevant degrees of freedom at low energies; the action (1) then follows from symmetry considerations. In the following we limit ourselves to the leading term in eq. (1),

$$S_{\rm memb} = -\mathcal{T} \int d^3\xi \sqrt{-h}\,. \tag{31}$$

Let us consider first the Rindler metric, which is the limit of the Schwarzschild metric for large black hole mass at a fixed distance from the horizon, and is the metric appropriate for an observer with constant acceleration g in Minkowski space. We denote Minkowski coordinates by (T, x, y, Z) and we define Rindler coordinates t, z from $T = z \sinh gt\,, Z = z \cosh gt$; this mapping only covers the wedge $|Z| \geq |T|, Z \geq 0$ (see Fig.1). The Minkowski metric becomes in Rindler coordinates $g_{\mu\nu} = (-g^2 z^2, 1, 1, 1)$ and in these coordinates the equation of motion of the action (31) has a solution [4] of the form

$$\bar\zeta^\mu(\xi) = (\tau, \sigma_1, \sigma_2, \frac{z_0}{\cosh g\tau})\,. \tag{32}$$

while, using the Minkowski coordinates, it takes the form

$$\bar\zeta^\mu(\xi) = (z_0 \tanh g\tau, \sigma_1, \sigma_2, z_0)\,. \tag{33}$$

If we now expand the action around this solution we get eq. (4) with the term $\sim \phi^2$ equal to zero [5]. In the variables $\xi^i = (\tilde\tau = z_0 \tanh g\tau, \sigma_1, \sigma_2)$ we have $\bar h_{ij} = (-1, 1, 1)$ and therefore the equation of motion is the massless Klein-Gordon equation in a flat space with boundaries in the temporal direction, since $|\tilde\tau| \leq z_0$. The action governing the field ϕ is therefore invariant under the field transformation $\phi(\xi) \to \phi(\xi) + a$. From the previous discussion, we are lead to ask whether this transformation can be interpreted as a symmetry operation in the embedding space where the (unknown) fundamental theory lives. The interesting point is that, if we limit ourselves to the Rindler wedge defined above, there is no such a symmetry. In fact, an infinitesimal transformation $\phi \to \phi + a$ generates a translation along the Z axis in Minkowski space since, $\zeta^\mu(\xi) = \bar\zeta^\mu + \phi\,\bar n^\mu$ and the normal $\bar n^\mu$ to the classical solution (33) points along the Z axis, but such transformation is not a symmetry for the Rindler wedge.

However, if we consider the full Minkowski space, rather than the Rindler wedge, then the transformation $\phi \to \phi + a$ is associated to the symmetry of translation along the Z axis in the embedding space. Thus, in analogy with the toy model, eq. (5), the field ϕ can be related to a Goldstone boson if the underlying theory lives in the full Minkowski space, i.e. the maximum analytical extension of the Rindler wedge.

This simple observation gives the following suggestion. In the spirit of black hole complementarity [2] one tries to describe a quantum black hole, from the point of view of an external, static observer, without making any reference to what happens inside the horizon. The degrees of freedom of the black hole are taken to live in a small region near the horizon, the so called 'stretched horizon' [2, 11]. The above discussion, however, suggests that if we look for a fundamental theory which in the low energy limit (i.e. at sub-Planckian energies) reproduces the effective action for quantum black holes, we cannot limit ourselves to the region outside the horizon. The fundamental theory, in the Schwarzschild case, must live in the maximum analytic extension of the Schwarzschild space (or in Minkowski space if we work in the Rindler limit).

Such a theory should be defined without reference to any particular observer. It is only when we try to derive an effective theory from this fundamental theory that a dependence on the observer appears. A static observer outside the horizon will obtain his effective action integrating over the fundamental variables in the region from where he cannot receive signals. A free falling observer, instead, can receive signals from any region and therefore he cannot define an effective action.

It is this procedure which introduces an observer dependence in the low energy theory. It appears therefore that the 'tension' between the point of views of a static observer and a free falling observer, which has been termed 'black hole complementarity' [2] is something which emerges at an effective, rather than at a fundamental level.

As a final remark we observe that the identification of ϕ with a Goldstone boson has been done in the Rindler limit; in the Schwarzschild case, instead, the field ϕ is not massless, although the effective mass term vanishes in the limit of large black hole mass [5]. However, the idea that the fundamental theory should live in the maximal analytical extension and that black hole complementarity only emerges at an effective level should be of rather general validity and should carry over from the Rindler limit to the Schwarzschild case.

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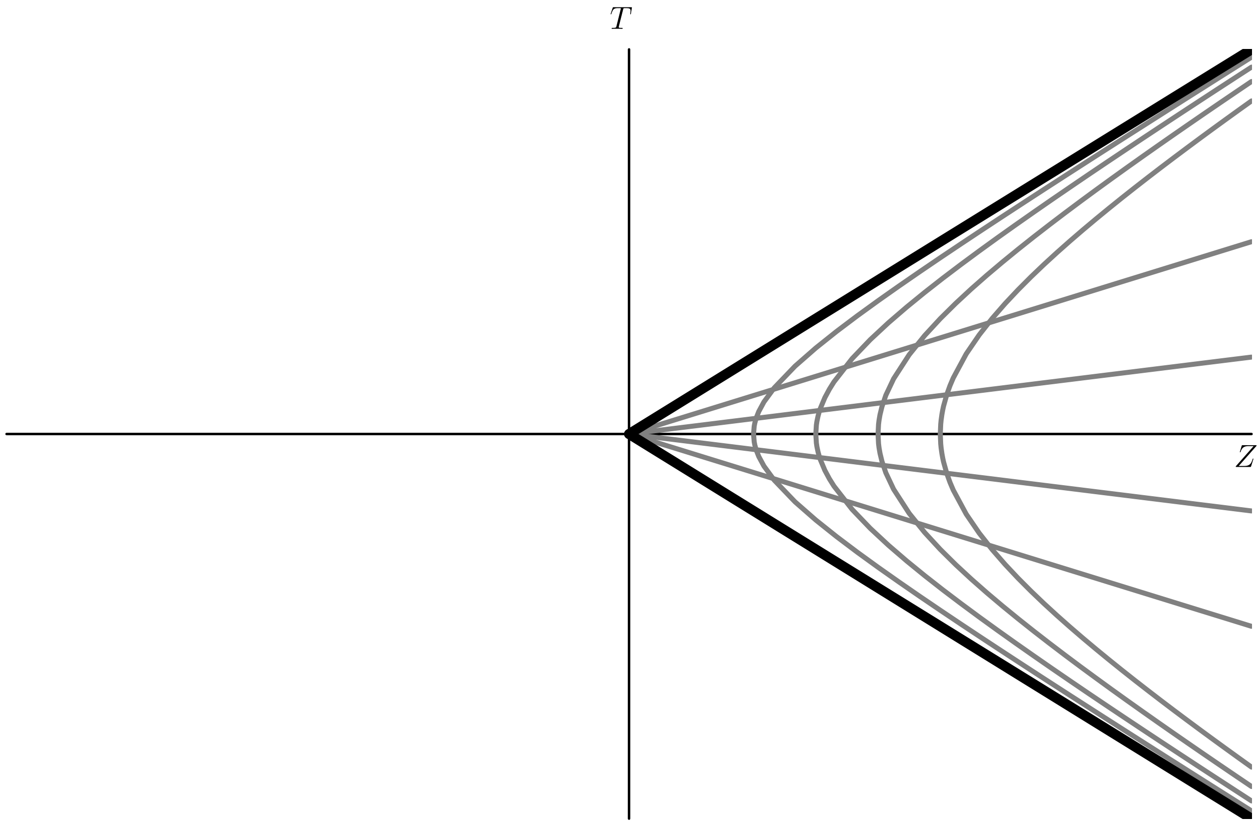

Fig.1: Rindler wedge