Laser-interferometer gravitational-wave optical-spring detectors

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Abstract. Using a quantum mechanical approach, we show that in a gravitational-wave interferometer composed of arm cavities and a signal recycling cavity, e.g., the LIGO-II configuration, the radiation-pressure force acting on the mirrors not only disturbs the motion of the free masses randomly due to quantum fluctuations, but also and more fundamentally, makes them respond to forces as though they were connected to an (optical) spring with a specific rigidity. This oscillatory response gives rise to a much richer dynamics than previously known, which enhances the possibilities for reshaping the LIGO-II's noise curves. However, the optical-mechanical system is dynamically unstable and an appropriate control system must be introduced to quench the instability.

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1. Introduction

A network of broadband ground-based laser interferometers, aimed to detect gravitational waves (GWs) in the frequency band $10 - 10^4\,\text{Hz}$, will begin operations next year. This network is composed of GEO, the Laser Interferometer Gravitational-wave Observatory (LIGO), TAMA and VIRGO (whose operation will begin in 2004) [1]. The LIGO Scientific Collaboration (LSC) [2] is currently planning an upgrade of LIGO starting from 2007. Besides the improvement of the seismic isolation and suspension systems, and the increase (decrease) of light power (shot noise) circulating in the arm cavities, the LIGO community has planned to introduce an extra mirror, called a signal-recycling (SR) mirror [3], at the dark-port output (see Fig. 1). The optical system composed of the SR cavity and the arm cavities forms a composite resonant cavity, whose eigenfrequencies and quality factors can be controlled by the position and reflectivity of the SR mirror. These eigenfrequencies (resonances) can be exploited to reshape the noise curves, enabling the interferometer to work either in broadband or in narrowband configurations, and improving in this way the observation of specific GW astrophysical sources [4].

The initial theoretical analyses [3] and experiments [5] of SR interferometers refer to configurations with low laser power, for which the radiation pressure on the arm-cavity mirrors is negligible and the quantum-noise spectra are dominated by shot noise. However, when the laser power is increased, the shot noise decreases while the effect of radiation-pressure fluctuation increases. LIGO-II has been planned to work at a laser power for which the two effects are comparable in the observation band 10–200 Hz [2]. Therefore, to correctly describe the quantum optical noise in LIGO-II, the results so far obtained in the literature had to be complemented by a thorough investigation of the influence of the radiation-pressure force on the mirror motion. Using a quantum-mechanical approach [6, 7] we have recently investigated [8, 9, 10] this issue. Henceforth, we shall summarize the main results of our analysis.

2. Radiation-pressure forces in conventional versus signal-recycling interferometers

In gravitational-wave interferometers composed of equal-length arms, the dynamics relevant to the output signal and the corresponding noise are described only by the antisymmetric mode of motion, $\widehat{x}$, of the four arm-cavity mirrors and by the dark-port sideband fields, which are decoupled from the other degrees of freedom [7]. In these devices, laser interferometry is used to monitor the displacement of the antisymmetric mode of the arm-cavity mirrors induced by the passage of a gravitational wave with (differential) strain h. The output of the detector can be constructed from two independent output observables, the two quadratures $\widehat{b}_1$ and $\widehat{b}_2$ [7, 9] (see Fig. 1) of the outgoing electromagnetic field immediately outside the SR mirror, which can be related to the input (noise) quadratures $\widehat{a}_1$, $\widehat{a}_2$ (see Fig. 1) and (the signal) h.
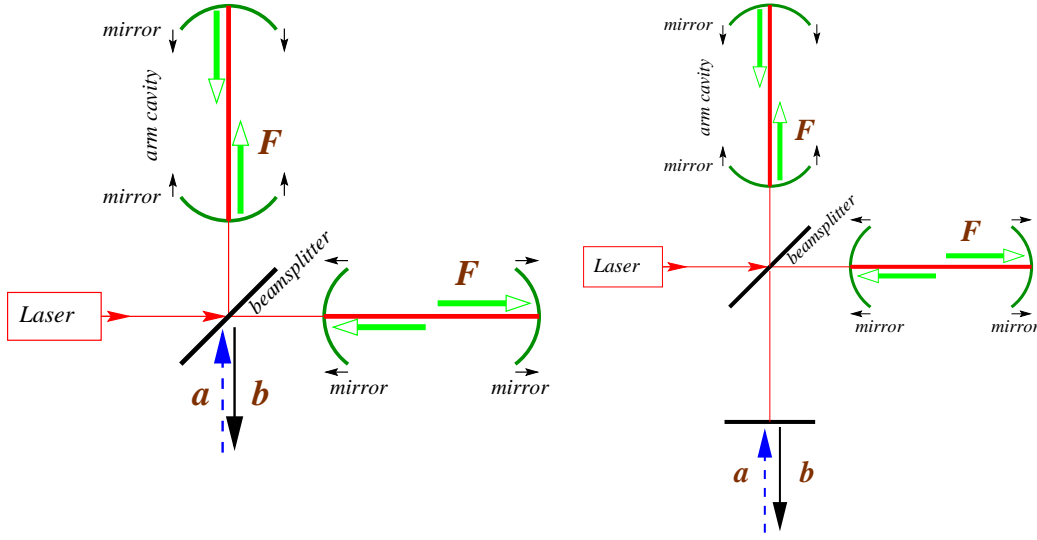
Figure 1. Schematic views of a conventional interferometer (on the left panel) and of a signal-recycling interferometer (on the right panel).

Disregarding the motion of the mirrors during the light round-trip time (quasi-static approximation), the radiation-pressure force acting on each arm-cavity mirror is $2W/c$, where W is the power circulating in each arm cavity, which is proportional to the square of the amplitude of the electric field propagating toward the mirror and c is the speed of light.

When the arm-cavity mirrors are held fixed, the radiation-pressure force can be directly related to the dark-port quadrature fields [7]. In conventional interferometers such as LIGO-I, TAMA and VIRGO (see left panel on Fig. 1) the (Fourier domain) of this radiation-pressure force $\widehat{F}_0(\Omega)$ is determined only by one of the input quadratures, say $\widehat{a}_1(\Omega)$ [7]. Since $[\widehat{a}_1(\Omega), \widehat{a}_1^\dagger(\Omega')] = 0 = [\widehat{a}_2(\Omega), \widehat{a}_2^\dagger(\Omega')]$ and $[\widehat{a}_1(\Omega), \widehat{a}_2^\dagger(\Omega')] = 2\pi i\, \delta(\Omega - \Omega')$, the response function of the *optical force* to perturbations caused by the mirror motion, which is given by $G_{FF}(t,t') \propto [\widehat{F}_0(t), \widehat{F}_0(t')]$, is zero. By contrast, in SR interferometers such as LIGO-II (see right panel in Fig. 1), the radiation pressure force depends on a linear combination, with complex coefficients, of both the input quadratures $\widehat{a}_1(\Omega)$ and $\widehat{a}_2(\Omega)$. As a consequence, the response function $G_{FF}(t,t') \neq 0$. More specifically, taking into account the mirror motion, the full radiation-pressure force in SR interferometers, is given by [10]:

$$\widehat{F}(t) = \widehat{F}_0(t) + \frac{i}{\hbar} \int_{-\infty}^{t} dt'\, G_{FF}(t,t')\, \widehat{x}(t')\,. \tag{1}$$

The second term in the RHS of the above equation can be easily explained in classical terms by noticing that the optical field fed back by the SR mirror into the arm cavities also contains the classical GW signal h. Thus the radiation-pressure force $\widehat{F}$ must depend on the history of the antisymmetric mode of motion $\widehat{x}$.

3. Dynamics, resonances and instability

In SR interferometers, the (Fourier domain) equation of motion for the antisymmetric mode of motion is [10]:

$$-\mu\,\Omega^2\,\widehat{x}(\Omega) = \text{GW Force} + \widehat{F}_0(\Omega) + R_{FF}(\Omega)\,\widehat{x}(\Omega)\,, \tag{2}$$

where $R_{FF}(\Omega)$ is the Fourier transform of the response function G_{FF} and $\mu = m/4$ is the reduced mass of the antisymmetric mode, being m the arm-cavity mirror mass. Hence, from Eq. (2) we infer that the antisymmetric mode of motion is not only buffeted by the radiation pressure force $\widehat{F}_0$, but is also subject to a harmonic restoring force with frequency-dependent spring constant [10]:

$$K(\Omega) = -R_{FF}(\Omega) \propto I_o \times (\text{SR mirror reflectivity}) \times (\text{SR detuning})\,, \tag{3}$$

where I_o is the laser light at the beamsplitter, and by SR detuning we mean the phase gained by the laser carrier frequency in the SR cavity [see Refs. [9, 10] for details]. This phenomenon, called ponderomotive rigidity, was originally discovered and analyzed in "optical-bar" GW detectors by Braginsky, Khalili and colleagues [11].

In the absence of the SR mirror the optical-mechanical system formed by the optical fields and the arm-cavity mirrors is characterized by the mechanical (double) resonant frequency $\Omega^2_{\text{mech}} = 0$, related to the free motion of the antisymmetric mode, and by the optical resonant frequency $\Re(\Omega_{\text{opt.}}) = 0$, $\Im(\Omega_{\text{opt.}}) = -1/\tau_{\text{decay}}$, where τ_{decay} is the storage time of the arm cavity. When a highly reflecting SR mirror is added and we consider configurations with low light power, the optical field (almost) purely oscillates at the eingenfrequencies $\Omega_{\pm}$ at which the total round-trip phase in the entire cavity (arm cavity + SR cavity) is $2\pi n$, with n an integer.

Since the ponderomotive rigidity $R_{FF} \propto I_o$, as we increase I_o the test masses and the optical field get coupled more and more and we have a mixing of the mechanical and pure optical resonant frequencies. More specifically, the (coupled) mechanical resonance moves from zero as $\sim I_o^{1/2}$, while the (coupled) optical resonances get shifted away from the values $\Omega_{\pm}$ as $\sim I_o$.

We have found [10] that the (coupled) mechanical resonant frequencies have always a positive imaginary part, corresponding to an instability. This instability has an origin similar to the dynamical instability induced in a detuned Fabry-Perot cavity by the radiation-pressure force acting on the mirrors [12, 11]. To suppress this instability, we proposed a feed-back control system that does not compromise the GW interferometer sensitivity. However, although the model we used to describe the servo system [10] may be realistic for an all-optical control loop, this might not be the case if an electronic servo system is implemented. Thus, a more thorough formulation should be used to fully describe this latter case [14].

4. Quantum-noise spectral density

In light of the discussion at the end of last section, let us derive the noise spectral density of a (stabilized) interferometer [10]. To identify the radiation pressure and the shot noise contributions in the total optical noise, we use the fact that they transform differently under rescaling of the reduced mass μ. Indeed, it is straightforward to show [9] that in the total optical noise there exist only two kinds of terms. There are terms that are invariant under rescaling of μ and terms that are proportional to $1/\mu$. Quite generally we can rewrite the (Fourier domain) output $\widehat{\mathcal{O}}$ as [8]:

$$\widehat{\mathcal{O}}(\Omega) = \widehat{\mathcal{Z}}(\Omega) + \mathcal{R}_{xx}(\Omega)\,\widehat{\mathcal{F}}(\Omega) + L\,h(\Omega)\,, \tag{4}$$

where by output we mean (modulo a normalization factor) one of the two (stabilized) quadratures $\widehat{b}_1$, $\widehat{b}_2$ [10] or a combination of them. In Eq. (4) $\mathcal{R}_{xx} = -1/\mu\,\Omega^2$ is the susceptibility of the antisymmetric mode of motion of the four arm-cavity mirrors and L is the arm-cavity length. The observables $\widehat{\mathcal{Z}}$ and $\widehat{\mathcal{F}}$ do not depend on the mirror masses μ [9], and we refer to them as the *effective* shot noise and *effective* radiation-pressure force, respectively. The (one-sided) noise spectral density reads [6]:

$$S_h(\Omega) = \frac{1}{L^2}\left\{S_{\widehat{\mathcal{Z}}\widehat{\mathcal{Z}}}(\Omega) + 2\mathcal{R}_{xx}(\Omega)\,\Re[S_{\widehat{\mathcal{F}}\widehat{\mathcal{Z}}}(\Omega)] + \mathcal{R}_{xx}^2(\Omega)\,S_{\widehat{\mathcal{F}}\widehat{\mathcal{F}}}(\Omega)\right\}\,, \tag{5}$$

where we defined $2\pi\,\delta\,(\Omega - \Omega')\,S_{\widehat{\mathcal{A}\mathcal{B}}}(\Omega) = \langle\widehat{\mathcal{A}}(\Omega)\widehat{\mathcal{B}}^\dagger(\Omega') + \widehat{\mathcal{B}}^\dagger(\Omega')\widehat{\mathcal{A}}(\Omega)\rangle$. Moreover, the (one-sided) spectral densities and cross correlations of $\widehat{\mathcal{Z}}$ and $\widehat{\mathcal{F}}$ satisfy the uncertainty relation [6]:

$$S_{\widehat{\mathcal{Z}}\widehat{\mathcal{Z}}}(\Omega)\,S_{\widehat{\mathcal{F}}\widehat{\mathcal{F}}}(\Omega) - S_{\widehat{\mathcal{Z}}\widehat{\mathcal{F}}}(\Omega)\,S_{\widehat{\mathcal{F}}\widehat{\mathcal{Z}}}(\Omega) \geq \hbar^2\,. \tag{6}$$

It is possible to show [10] that the ponderomotive effect discussed in Sec. 3, can be directly related to the presence of dynamical correlations between the shot-noise and radiation-pressure noise [13, 16].

In conventional interferometers such as LIGO-I, TAMA and VIRGO, the ponderomotive effect is absent, i.e. $R_{FF} = 0$. In this case, as long as squeezed-input light is not injected into the interferometer from the dark-port and/or correlations are not built up statically during the readout process [15, 7], we have $S_{\widehat{\mathcal{Z}}\widehat{\mathcal{F}}} = 0 = S_{\widehat{\mathcal{F}}\widehat{\mathcal{Z}}}$. Thus, in conventional interferometers Eq. (6) imposes the following lower bound on the noise spectral density: $S_h^{\text{conv.}}(\Omega) \geq S_h^{\text{SQL}}(\Omega) \equiv 2\hbar/\mu\,\Omega^2\,L^2$. The quantity $S_h^{\text{SQL}}(\Omega)$ is generally called the standard quantum limit (SQL) for the dimensionless GW signal $h = \Delta L/L$.

In SR interferometers, and "optical-bar" GW detectors as well [11], because of the ponderomotive effect ($R_{FF} \neq 0$) shot-noise and radiation-pressure noise are automatically correlated and Eq. (6) does no longer impose a lower bound on the noise spectral density Eq. (5). In particular, we found [9] that there exists an experimentally accessible region of the parameter space for which the quantum noise curves can beat the SQL by roughly a factor of two over a bandwidth $\Delta f \sim f$. This fact is illustrated in Fig. 2, where the square root of the noise spectral density ($h_n \equiv \sqrt{S_h}$) is plotted versus frequency, for various choices of the light power at the beamsplitter, having fixed the SR mirror reflectivity and the SR detuning. Note the two distinct valleys which go below
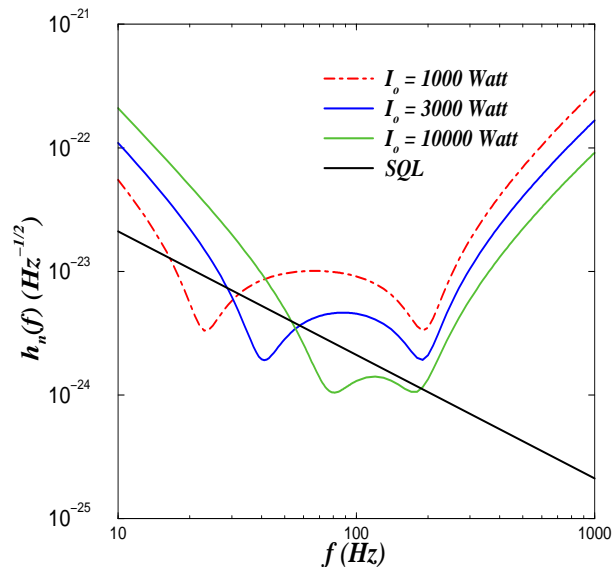
Figure 2. Plot of the square root of the quantum-noise spectral density $h_n \equiv \sqrt{S_h}$ versus frequency, for various choices of the light-power at the beamsplitter, having fixed the SR mirror reflectivity and the SR detuning. The SQL line is also shown.

the SQL line. Their position is determined by the (coupled) resonant frequencies of the optical-mechanical system discussed in Sec. 3. As anticipated in the previous section, as we increase I_o the (coupled) mechanical resonant frequency (on the left) moves from zero to the right, while the (coupled) optical resonant frequency (on the right) does not vary much, being present already as pure optical resonance in the limit of low light power.

The total noise, which includes seismic, suspension and thermal contributions, can beat the SQL only if all other noise sources can also be pushed below the SQL. These noises are not quantum limited in principle but may be technically challenging to reduce [2, 17].

5. Conclusions

Our analyses [8, 9, 10] have revealed that in SR interferometers, the dynamics of the whole optical-mechanical system, composed of the arm-cavity mirrors and the optical field, resembles that of a free test mass (mirror motion) connected to a massive spring (optical fields). When the test mass and the spring are not connected (e.g., for very low laser power) they have their own eigenmodes, namely the uniform translation mode for the free antisymmetric mode, and the longitudinal-wave mode for the spring (decoupled SR optical resonance). However, for LIGO-II laser power the test mass is connected to the massive spring and the two free modes become shifted in frequency, so the entire coupled system can resonate at two pairs of finite frequencies. Near these resonances the noise curve can beat the free mass SQL, as shown in Fig. 2. This phenomenon is

not unique to SR interferometers; it is a generic feature of detuned cavities [12, 13, 16] and was used by Braginsky, Khalili and colleagues in designing the "optical bar" GW detectors [11].

However, the optical-mechanical system is by itself dynamically unstable, and a much more careful and precise study of the control system should be carried out, including various readout schemes [14], before any practical implementation.

Acknowledgments

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