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PROCEEDINGS SUPPLEMENTS

The two-phase issue in the $O(n)$ non-linear σ-model: a Monte Carlo study

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We have performed a high statistics Monte Carlo simulation to investigate whether the two-dimensional $O(n)$ non-linear sigma models are asymptotically free or they show a Kosterlitz-Thouless-like phase transition. We have calculated the mass gap and the magnetic susceptibility in the $O(8)$ model with standard action and the $O(3)$ model with Symanzik action. Our results for $O(8)$ support the asymptotic freedom scenario.

1. INTRODUCTION

The 2-dimensional $O(n)$ non-linear σ-model is defined by the action

$$S = \frac{\beta}{2}\int d^2x \left(\partial_\mu\vec{\phi}\right)^2 \qquad (1)$$

together with the condition $\vec{\phi}(x)^2 = 1$ for all spacetime points x. In this equation β is the inverse of the bare coupling constant.

Perturbation theory (PT) predicts that this model is asymptotically free for $n \geq 3$. In particular the exponential correlation length ξ on the lattice must scale as

$$\xi = C_\xi \left(\frac{1}{2\pi\beta\Delta}\right)^{\Delta} e^{2\pi\beta\Delta}\left(1+\sum_{k=1}^{\infty}\frac{a_k}{\beta^k}\right) \qquad (2)$$

where a_k are the corrections to universal scaling. Here $\Delta \equiv 1/(n-2)$. C_ξ is a non-perturbative constant which for the standard action equals [1]

$$C_\xi = \left(\frac{e}{8}\right)^{\Delta}\Gamma(1+\Delta)2^{-5/2}\exp\left(-\frac{\pi\Delta}{2}\right). \qquad (3)$$

We define the magnetic susceptibility χ as the two-point correlation function at zero momentum. It scales as

$$\chi = C_\chi \left(\frac{1}{2\pi\beta\Delta}\right)^{\Delta(n+1)} e^{4\pi\beta\Delta}\left(1+\sum_{k=1}^{\infty}\frac{b_k}{\beta^k}\right) \qquad (4)$$

where again C_χ is a non-perturbative constant. From equations (2) and (4) we conclude that in PT the ratio

$$R_{PT} \equiv \frac{\chi}{\xi^2}(2\pi\beta\Delta)^{\frac{n-1}{n-2}}\left(1+\sum_{k=1}^{\infty}\frac{d_k}{\beta^k}\right) \qquad (5)$$

tends to C_χ/C_ξ^2 as we approach the continuum limit, $\beta\to\infty$. The corrections to asymptotic scaling d_k depend on $\{a_k\}$ and $\{b_k\}$.

In a series of papers [2] another scenario has been put forward for the model defined in (1). Under reasonable hypothesis the authors prove that there is no mass gap and that this model must undergo a Kosterlitz-Thouless-like (KT) phase transition at finite beta, β_{KT}. This implies that the ratio

$$R_{KT} \equiv \frac{\chi}{\xi^{2-\eta}} \qquad (6)$$

should be constant as one approaches β_{KT} from below. Here η is a critical exponent. For the $O(2)$ model this exponent is $\eta = 1/4$. In [3] the authors show that the $O(3)$ model with the standard action on the lattice and $\eta = 1/4$ gives a constant for R_{KT} while the data for R_{PT} displays a clear drop.

Here we will show a progress report from an extensive simulation performed on the $O(8)$ model with standard action and the $O(3)$ model with the tree-level improved Symanzik action [4]. If the constancy of R_{KT} for the $O(3)$ model is a genuine physical effect, then also for the Symanzik action we should see such a behaviour. The full account of our results with better statistics and using more corrections to asymptotic scaling can be found in [5].

2. SIMULATIONS

In our simulations we have used the Wolff algorithm [6] for the updatings as well as improved estimators to measure the correlation length and

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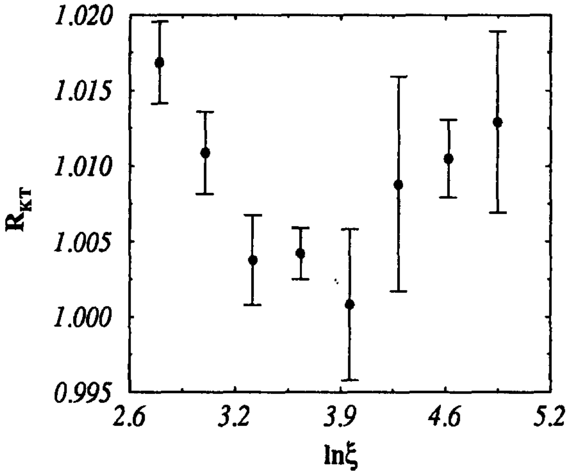

Figure 1. The ratio R_{KT} for the $O(3)$ model with Symanzik action.

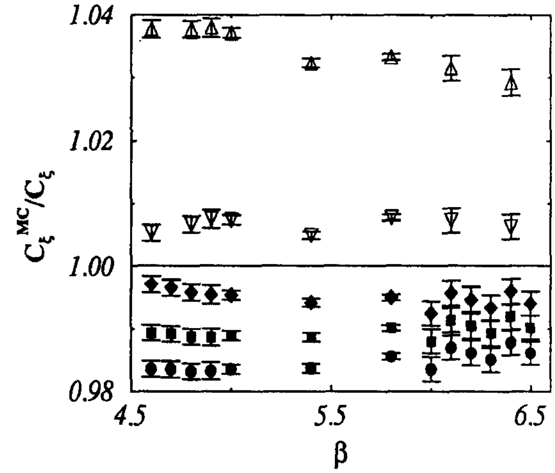

Figure 2. The non-perturbative constant for the correlation length. The 2, 3 and 4-loop results correspond to black circles, squares and diamonds respectively. The 2 and 3-loop results in the energy scheme are the up and down triangles.

magnetic susceptibility. We have performed several millions of measurements for both quantities and verified the absence of autocorrelations.

To calculate the correlation length we have measured the second moment $\xi^{(2)}$. The ratio $\xi^{(2)}/\xi$ is less than few parts per mille, so within our statistical errors, we can use the formulae (2-3).

We have chosen large enough lattice sizes L to keep finite-size effects under control. The ratio L/ξ is $7-10$. We have checked that these effects are few parts per mille.

The largest systematic error comes from the deviation from universal scaling of our data. These corrections are known up to 4 loops for the standard action and up to 3 loops for the Symanzik action [7].

3. RESULTS

3.1. The $O(3)$ model

In figure 1 we show the results for R_{KT} in the $O(3)$ model with Symanzik action. They have better statistics than those of reference [3]. In constrast with [3] our data are not constant.

We do not show here (see [5]) the data for R_{PT}. Again it is not constant although it displays better scaling than for the standard action [3]. The fits for C_ξ and C_χ agree with the prediction (3) and large-n calculations within $15-20\%$.

Assuming finite-size scaling, it has been shown that this model presents asymptotic scaling starting from $\xi \approx 10^5$ [8]

3.2. The $O(8)$ model

In figure 2 we show the ratio between the non-perturbative constant $C_\xi^{\rm MC}$ as computed from our Monte Carlo data and the prediction (3) which for the $O(8)$ model is $C_\xi = 0.10544$. If PT is correct and asymptotic scaling holds, this ratio should be equal to 1 (up to ~0.1 per mille because we measured the second moment $\xi^{(2)}$).

We show the data as obtained from the 2, 3 and 4-loop approximation in eq. (2). The data in the scheme of the energy [9] at 2 and 3-loop are also shown (we have used the energy measurements of reference [10]). All data seem to converge towards the PT prediction. A careful analysis of the next coefficients in the $1/n$ expansion suggests that further corrections should have small effects. The 4-loop data in this figure agree with (3) within 0.5%.

In figure 3 the data for R_{KT} are shown. The data are far from constant. We show the results for two values of η: $\eta = 0.25$ is the upper set of data (black circles) and $\eta = 0.22$ is the lower set (white circles).

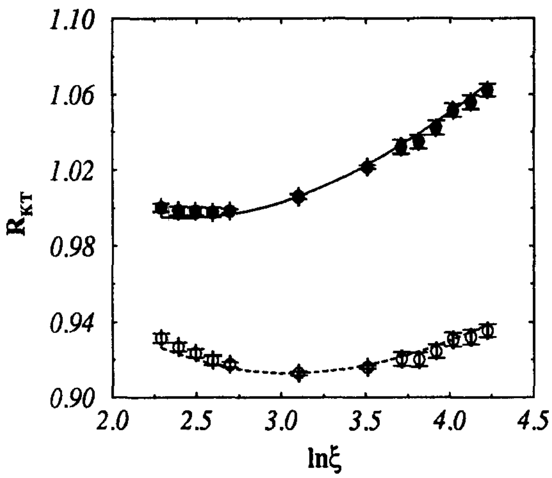

Figure 3. The ratio R_{KT} for the $O(8)$ model. Black and white circles represent $\eta = 0.25$ and $\eta = 0.22$ respectively. The lines are the PT predictions for these ratios.

The solid and dashed lines are the PT predictions for R_{KT} assuming $C_\chi = 0.103$ (this is the value obtained from a best fit performed on our data for χ; it agrees with large-n estimates within 1%) and the prediction (3) for C_ξ. We see that not only the curves are not constant but also that PT explains well its non-constancy.

In figure 4 we show the data for R_{PT}. The upper set of data is the lowest order prediction. Further corrections stabilize the result. The result clearly converges to a constant. Physical scaling is well reached at the largest correlation lengths. The corrections to universal scaling converge surprisingly well.

The solid horizontal line is the PT prediction for the constant by using $C_\chi = 0.103$ and the prediction (3) for the correlation length. We conclude that our data are in fair agreement with PT.

In conclusion our data do not support either KT or PT for the $O(3)$ model but they show clear agreement with PT for the $O(8)$ model. In the KT scenario one should explain why PT works so well for large n and large ratios L/ξ.

This work has benefited from many stimulating conversations with Andrea Pelissetto.

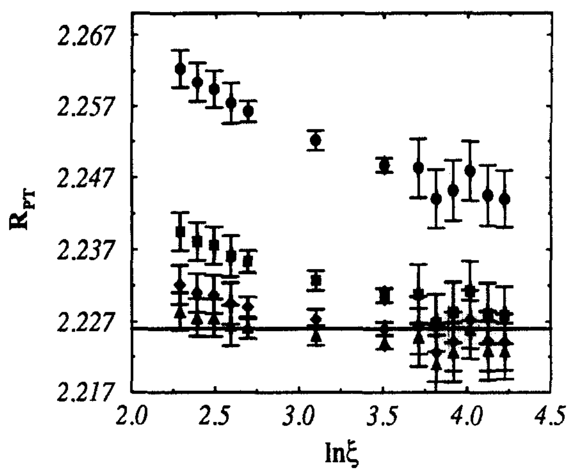

Figure 4. The ratio R_{PT} for the $O(8)$ model. The successive orders correspond to circles, squares, diamonds and triangles respectively.

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