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Standard Model Vacuum Stability

and some Implications for

Higgs Inflation

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Standardmodell Vakuumstabilität und einige Implikationen für Higgs-Inflation

Die Entdeckung des Higgs Bosons vervollständigt das Standardmodell der Teilchenphysik. Erweiterungen des Standardmodells, die die Probleme nicht durch neue Physik zwischen der elektroschwachen Skala und der Planck Skala lösen, sind interessant, da so das Standardmodell im wesentlichen bis zur Planck Skala extrapoliert werden könnte, was auch durch die Werte für Higgs- und Top-Masse gestützt wird. Die Analyse der Vakuumstabilität des Higgs-Feldes sowie die Konsequenzen im Hinblick auf neue Physik, die durch nicht-renormierbare Operatoren parametrisiert wird, sind Themen dieser Arbeit. Ebenfalls wird die Rolle der Messung der Top Quark Masse an Hadronen-Beschleunigern in Zusammenhang mit der Vakuumstabilität beleuchtet. Die Matching-Bedingungen für das Renormierungsgruppenlaufen werden gegeben und die Diskussion in Richtung Metastabilität eröffnet. Mögliche Implikationen im Hinblick auf die BICEP2-Behauptung werden behandelt und die Konsequenzen von laufenden Kopplungen in Zusammenhang mit Higgs-Inflation-Szenarien werden gegeben.

Standard Model Vacuum Stability and some Implications for Higgs Inflation

The discovery of the Higgs particle has completed the SM. With the current measured Higgs and Top masses, the SM could survive up to the Planck scale. The SM cannot be a complete picture as some experimental and observational facts require the extension of the SM, however some of the extensions do not require the embedding of the SM into high energy scale, prompting the possibility that there is no new physics up to the Planck scale. We will tackle this issue and investigate the vacuum stability of the SM Higgs field and its interplay with possible new physics, parametrized by higher dimensional operators. Also the role of the measurement of the top mass at hadron colliders in the context of vacuum stability is discussed. Detailed matching conditions for the RGE and the metastability issue will be given. Possible implications of the BICEP2 claim are given and the effect of running couplings for the Higgs inflation will be reexamined, which is crucial to determine whether the Higgs inflation is still a viable scenario.

Contents

CHAPTER 1

INTRODUCTION

The question of how nature works and how we can understand the world around us has always been the motivation for physicists to work, think, observe and speculate. Taking the ingredients of special relativity and quantum mechanics and cluster decomposition, the result we got is quantum field theory (QFT), which is the tool that helps us to describe all known forces in nature except gravity. Through fruitful input from both experimentalists and theorists we were able to establish the Standard Model of Particle Physics (SM) as a very precise and extremely well working theory.

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge structure of the SM, together with spontaneous symmetry breaking, is a triumph of many generations of physicists. The SM consisting of the quantum chromodynamics (QCD) [1–3] and the Electroweak theory first proposed by Weinberg, Glashow and Salam [4, 5], describes almost all experimental data with a very high accuracy. In 2012 the Higgs boson as the last piece of the SM was discovered through both working groups, namely ATLAS [6] and CMS [7] at the Large Hadron Collider (LHC) in Geneva. Furthermore, Peter Higgs [8–10] and François Englert [11] were rewarded with the Nobel price 2013 for their contribution to "our understanding of the origin of mass of subatomic particles" which shows the great value of their work.

The last piece of the SM – the Higgs boson – has been discovered. This is the good news that we can take away from the last measurements of the LHC. However, we see no sign of new physics at the LHC. All deviations between experiment and theoretical prediction went away by increasing the amount of data taken. One might get the impression that particle physics is stuck and that there is nothing left to be done.

Fortunately, this is not the case. There are several observational facts that show us that we need physics beyond the Standard Model (BSM). Neutrino oscillations are the first evidence for BSM since neutrinos are massless in the SM [12]. Oscillations can only occur if neutrinos are massive and there is no way to obtain neutrino masses with the particle content of the SM at the renormalizable level.

Other observational evidence comes from cosmological considerations. To fit the cosmic microwave background we need a sizable amount of dark matter within the ΛCDM-model as well as a cosmological constant which is interpreted as dark energy. Also the rotation curves of galaxies and gravitational lensing indicate that the Universe should consist of 26% of dark matter.

Successful baryogenesis is also not possible within the SM because the CP-violation is to small. Baryogenesis accounts for the fact that there is an asymmetry between matter and anti-matter in the Universe [13].

A clear sign on theoretical grounds that the SM cannot be the ultimate theory of nature is that the $U(1)_Y$ coupling suffers from a Landau pole at very high energies far above the Planck scale rendering it problematic. However, a UV-completion of the SM should already operate before the Landau pole healing this deficit.

Even with all these observations which call for BSM physics, it is tempting to think about the SM to be valid up to the Planck scale and study the implications.[1] Renormalization group (RG) running of the parameters of the theory provides a possibility to extrapolate the SM from low energies to high energies. The analysis of the one-loop RG-improved effective potential shows that with the current values of the SM an instability of the potential at high scales occurs. Recently the impact of non-renormalizable operators in the Higgs potential has been studied parameterizing our ignorance of a UV-completion of the SM. This procedure works as long as all the couplings remain finite (perturbatively small) and guarantee a stable vacuum. While all the SM Landau poles are far above the Planck scale ($M_P \approx 1.2 \times 10^{19}\,\mathrm{GeV}$), the Higgs potential is apparently on the edge of being unbounded at high energies. It is thus of great importance to study in more detail

[1] We augment the Standard Model with 3 right-handed neutrinos ν_R to account for neutrino masses in exactly the same way as in the SM, namely through Yukawa couplings with the Higgs.

the issue of an unstable SM vacuum and whether new physics is necessary to stabilize it.

The question of vacuum stability of the SM will be reexamined in this thesis in the context of additional non-renormalizable operators as well as Weyl consistency relations. Vacuum stability poses a test of the self-consistency of the theory. Also the problems coming from the gauge dependence of the effective potential will be discussed and analyzed. However, this problem is not entirely theoretical since one important limiting fact is the actual value of the top mass. This question will also be discussed. In this work we will tackle the question of vacuum stability and go beyond in the direction of Higgs inflation and the consequences of a possible detection of a high tensor-to-scalar ratio through the BICEP2 collaboration [14].[2] If the one-loop RG-improved potential develops an instability at a scale below the scale of inflation which is indicated to be of $\mathcal{O}(10^{16}\,\mathrm{GeV})$ if one believes in the BICEP2 measurement this may pose serious problems for the SM vacuum in terms of Higgs fluctuations in the early universe.

After introducing standard techniques of QFT and presenting the SM we will proceed to review the standard results of the stability analysis of the SM with certain extensions. Then we will go on and present Higgs inflation as an economic idea of inflation as well as standard results from inflation. We will go beyond and tackle the question: what are the implications of a correct BICEP2 measurement would be for Higgs inflation. Tree-level analysis of Higgs inflation poses problems in the context of a high tensor-to-scalar ratio and vacuum stability which we will explore throughout this thesis. However, going beyond tree-level analysis poses serious theoretical problems since the coupling to gravity renders the theory to be non-renormalizable and without further assumptions on the underlying UV-completion, one cannot proceed. Furthermore, another problem arises in terms of a transformation between two reference frames namely Jordan frame and Einstein frame. All these topics will be covered and we will give an outlook on the theory of Higgs nflation independent of the fate of the BICEP2 claim.

[2]Higgs inflation here is the SM Higgs playing the role of the inflaton augmented with a non-minimal coupling to gravity.

CHAPTER 2

RENORMALIZATION

2.1 The Effective Action

In this chapter we want to set the stage for this work. We will present useful knowledge on QFT which is important and valuable for the understanding of this thesis. We present the effective potential and the effective action as objects of fundamental importance in QFT and go further by pointing out their physical importance beyond textbook knowledge.[1]

2.1.1 Green's Functions and Generating Functional in a QFT

Let us first consider

$$Z[J] \equiv \int \mathcal{D}\varphi \exp\left[i \int d^4x \left[\mathcal{L}(\varphi, \partial\varphi) + J(x)\varphi(x)\right]\right] \tag{2.1}$$

$Z[J]$ is the generating functional of correlation functions; correlation functions are the quantities where physical information is stored in. Here, the term $J(x)\varphi(x)$ represents a source term, which allows us to extract correlation functions by taking functional derivatives of the generating functional with respect to the source $J(x)$.

[1] Note that in section 4.4 there are several comments on physical quantities extracted from the effective potential.

$Z[J]$ can formally also be written as

$$Z[J] \equiv \exp\left[iW\left[J\right]\right] = \int \mathcal{D}\varphi \exp\left[i(S\left[\varphi\right] + \int J\varphi)\right], \qquad (2.2)$$

where $\int J\varphi$ denotes a short-hand notation for the integration over d^4x. For example the two-point function is given as

$$\langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle = \frac{1}{Z_0}\left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right) Z[J]\bigg|_{J=0}, \qquad (2.3)$$

with $Z_0 = Z[0]$.

A general Green's function can be extracted from the generating functional which is given as

$$G^n(x_1,\dots,x_n) = \langle 0|T\varphi(x_1)\dots\varphi(x_n)|0\rangle = \frac{(-i)^n}{Z_0}\frac{\delta^n Z[J]}{\delta J(x_1)\dots\delta J(x_n)}\bigg|_{J=0}. \qquad (2.4)$$

In equation (2.3) and (2.4), T stands for the time ordering operator. Both expressions do not distinguish between topologically connected and disconnected diagrams. In most of the cases one is only interested in the connected part of the Green's functions. This is achieved by the *Schwinger functional* W which is defined as

$$W[J] = -i\log\left[Z\left[J\right]\right]. \qquad (2.5)$$

The connected Green's functions are then given as

$$G^n_c(x_1,\dots,x_n) = (-i)^n\frac{\delta^n W[J]}{\delta J(x_1)\dots\delta J(x_n)}\bigg|_{J=0}. \qquad (2.6)$$

Note that W[J] does not generate vacuum graphs. With this at hand we can define the *effective action* as the Legendre transformation of (2.5)

$$\Gamma[\phi_c] = \sup_J\left(W[J] - \int d^4x\; J(x)\phi_c(x)\right), \qquad (2.7)$$

which therefore has to be convex. The meaning of ϕ_c is not clear yet and will be clarified now. At the supremum, J is given as a function of ϕ_c. Therefore, we have

$J = J_{sup} = J[\phi_c]$. With this in mind the meaning of ϕ_c can be deduced as follows:

$$0 = \frac{\delta}{\delta J(x)} \left(W[J] - \int J\phi_c \right) \Big|_{sup} \tag{2.8}$$

$$\Rightarrow \phi_c(x) = \frac{\delta W}{\delta J(x)} = \frac{1}{Z[J]} \frac{-i\delta Z[J]}{\delta J(x)} = \langle \varphi \rangle_{J[\phi_c]} \,. \tag{2.9}$$

This means that the classical field ϕ_c is given by the expectation value of the quantum field φ in presence of the source J. Performing the functional derivative of the effective action leads to

$$\frac{\delta \Gamma[\phi_c]}{\delta \phi_c(x)}\Big|_{sup} = \int_y \underbrace{\frac{\delta W[J]}{\delta J(y)}\Big|_{sup}}_{\phi_c(y)|_{sup}} \frac{\delta J(y)}{\delta \phi_c(x)}\Big|_{sup} - J(x)\Big|_{sup} - \int_y \frac{\delta J(y)}{\delta \phi_c(x)}\Big|_{sup} \phi_c(y)\Big|_{sup} \tag{2.10}$$

$$= -J(x)[\phi_c]. \tag{2.11}$$

This is an important property of Γ which is called *quantum effective equation of motion* and it shows us that if external sources J are absent, i.e. $J = 0$, (2.11) reduces to

$$\frac{\delta \Gamma[\phi_c]}{\delta \phi_c} = 0. \tag{2.12}$$

This shows why we talk about an effective action. Just as the classical physical field configurations are obtained as extrema of the action, the physical quantum field configurations arise as extrema of the effective action. The big difference between an ordinary action of classical field theory and the effective action is that the effective action already includes all quantum effects at tree-level. This will become more clear in the next sections.

2.1.2 Tree-level Evaluation of the Effective Action

To see that Γ really generates the full quantum theory already at tree-level, we will first define

$$\tilde{W}[J,\hbar] \text{ via } \exp i\tilde{W}[J,\hbar] \equiv \int \mathcal{D}\phi_c \exp \frac{i}{\hbar}(\Gamma[\phi_c] + J\phi_c) \tag{2.13}$$

where we have reintroduced $\hbar$ explicitly again. So, we just replace the classical action $S[\phi]$ by the effective action $\Gamma[\phi_c]$. Now we want to observe what happens in the classical limit, i.e. $\hbar \to 0$, which corresponds to the evaluation of the right-hand side at tree-level. In order to see what happens we first have to expand

$\tilde{W}[J,\hbar]$ in loops or equivalently in powers of $\hbar$:

$$\tilde{W}[J,\hbar] = \sum_L \hbar^{L-1} W^L[J]\,. \tag{2.14}$$

The classical limit means that the integral in (2.13) is dominated by a stationary phase which minimizes the exponent:

$$\frac{\delta}{\delta\phi_c}(\Gamma[\phi_c] + J\phi_c)\bigg|_{\phi_c=\hat{\phi}} = 0, \text{ i.e. } \frac{\delta\Gamma}{\delta\phi_c}\bigg|_{\hat{\phi}} = -J\,. \tag{2.15}$$

Evidently, we get back the quantum effective equation of motion already derived in equation (2.11). This also implies a relation between $W^0[J]$ and Γ, namely

$$W^0[J] = \Gamma[\hat{\phi}] + \hat{\phi}J. \tag{2.16}$$

But this is just the inverse Legendre transformation for Γ, so we have $W^0[J] = W[J]$ and end up with

$$\exp\left(\frac{i}{\hbar}(W[J])\right) = \frac{1}{Z[0]}\int \mathcal{D}\phi \exp\left(\frac{i}{\hbar}(S[\phi] + J\phi)\right) = \exp\left(\frac{i}{\hbar}(\Gamma[\phi] + J\phi)\right). \tag{2.17}$$

The interpretation of (2.17) is as follows: The functional integration $\mathcal{D}\phi$ is responsible for the quantum fluctuation; if we replace the classical action $S[\phi]$ with the quantum effective action $\Gamma[\phi_c]$, such that (2.11) holds, the path integral is obsolete and we recover the full quantum theory if we work at tree-level. This also shows that at tree-level, Γ and S are the same functionals, i.e.

$$\Gamma[\varphi] = S[\varphi] + \hbar K[\varphi], \tag{2.18}$$

where $K[\varphi]$ encodes the loop contribution to the effective action.

It should not be underestimated that the computation of the effective action is non-trivial. The evaluation of $\Gamma[\phi_c]$ is a very hard job. To obtain a quantity which is a little bit easier to access we define the effective potential from the effective action which can be evaluated easier in some cases.

2.2 The Effective Potential

The effective action in (2.7) gives the one particle irreducible (1PI) correlation functions by taking functional derivatives:

$$\Gamma^n(x_1, \ldots, x_n) = (-i)^n \frac{\delta^n \Gamma[\phi_c]}{\delta\phi_c(x_1)\ldots\delta\phi_c(x_n)}\,. \tag{2.19}$$

A 1PI diagram is a diagram that cannot be cut into two pieces by cutting a single internal line, so it cannot be subdivided into two disconnected diagrams. The Feynman diagrams to physical processes are built from connected Green's functions and 1PI vertices. This is why connected Green's functions are so important, since they enter physical processes in experiments. The 1PI diagrams are generators of the effective action, so we can express $\Gamma[\phi_c]$ through the 1PI vertices if we perform a Fourier transformation. The result is

$$\Gamma[\phi_c] = \sum_{n=0}^{\infty} \frac{1}{n!} \int dp_1 \ldots dp_n \delta^4(p_1+\cdots+p_n)\Gamma^n(p_1,\ldots,p_n)\tilde{\phi}_c(p_1)\ldots\tilde{\phi}_c(p_n). \tag{2.20}$$

Expanding the effective action in terms of derivatives, one ends up with:

$$\Gamma[\phi_c] = \int d^4x \left(-V_{\text{eff}}(\phi_c) + \frac{1}{2}(\partial\phi_c)^2 Z(\phi_c)\right)\,. \tag{2.21}$$

If one sets the classical field ϕ_c to a constant value the only quantity entering the effective action is the effective potential V_{eff}. This makes sense since we want to study electro-weak symmetry breaking with the help of the effective action. The vacuum expectation value (VEV) of the field is non-zero and constant since otherwise we would spontaneously break momentum conservation.

Combining (2.20) and (2.21); one obtains an expression for V_{eff} in the limit of vanishing external momenta

$$V_{\text{eff}}(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!}\phi_c^n \Gamma^n(p_i = 0)\,. \tag{2.22}$$

Taking (2.11) we can write it in the following way:

$$\begin{aligned}\exp\left(\frac{i}{\hbar}\Gamma[\phi_c]\right) &= \int \mathcal{D}\varphi \exp\frac{i}{\hbar}\left[S\left[\varphi\right] + \int \varphi J[\phi_c] - \int J[\phi_c]\phi_c\right] \\ &\overset{\varphi=\phi_c+\varphi'}{=} \int \mathcal{D}\varphi' \exp\frac{i}{\hbar}\left[S[\phi_c+\varphi'] + \int J[\phi_c]\phi_c + J[\phi_c]\varphi' - \int J[\phi_c]\phi_c\right] \\ &= \int \mathcal{D}\varphi' \exp\frac{i}{\hbar}\left[S[\phi_c+\varphi'] - \int \frac{\delta\Gamma}{\delta\phi_c}\varphi'\right] \end{aligned} \tag{2.23}$$

This is an integro-differential equation for the functional $\Gamma[\phi_c]$ and shows how hard it is to compute Γ for realistic theories: We have to integrate out the quantum fluctuations to compute the effective action Γ. To evaluate the one-loop correction to the effective potential we go back to (2.23) and follow the presentation in [15]. The loop expansion of the effective potential corresponds to an expansion in powers of $\hbar$. We start with an expansion of the action around the expectation value ϕ_c.

$$S[\phi_c+\varphi'] = S[\phi_c] + \int d^4x\ S'(x)\varphi'(x) + \frac{1}{2}\int d^4x d^4y \varphi'(x) S''(x,y)\varphi'(y) + \mathcal{O}(\varphi'^3) \tag{2.24}$$

where $S'(x) = \frac{\delta S[\phi_c]}{\delta\varphi'(x)}$ denotes functional differentiation with respect to $\varphi'(x)$. Using (2.18) and taking the functional derivative with respect to ϕ_c (2.23) reduces to

$$e^{\frac{i}{\hbar}\Gamma[\phi_c]} = \int \mathcal{D}\varphi' \exp\frac{i}{\hbar}\left[S[\phi_c] + \frac{1}{2}\int d^4x d^4y \varphi'(x) S''(x,y)\varphi'(y) - \hbar\frac{\delta K}{\delta\phi_c}\varphi'\right]. \tag{2.25}$$

Taking (2.25) and (2.18) together we can perturbatively solve for an expression for the loop correction $K[\varphi]$. Since $K[\varphi]$ is already 1-loop order and thus $\mathcal{O}(\hbar)$ the term linear in φ' does not contribute to the integral at 1-loop order, so in order to evaluate the right-hand side of (2.25) one has to perform a Gaussian integral. After that one solves for $\Gamma[\phi_c]$ and ends up with

$$\Gamma[\phi_c] = S[\phi_c] - i\ln\left(\det(-S'')^{-\frac{1}{2}}\right) \tag{2.26}$$

$$= S[\phi_c] + \frac{i}{2}\ln\left(\det(-S'')\right), \tag{2.27}$$

where field independent constants have been dropped. Taking (2.21) and combining it with (2.27), one ends up with

$$V_{\text{eff}}(\phi_c) = V(\phi_c) - \frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\det i\frac{\delta^2 S(\phi)}{\delta\phi(x)\phi(y)}\bigg|_{\phi=\phi_c} + \mathcal{O}(\hbar^2), \tag{2.28}$$

where we have omitted to display explicitly the diagonalization and dropped field independent constants.

This means that the effective action is given as a tree-level term and a loop contribution, i.e. $V_{\text{eff}} \approx V + V_{1-\text{loop}}$. Specifying the result to ϕ^4-theory, which is an important toy-model since the SM Higgs sector is similar, one ends up with the following result:

$$V_{\text{eff}} = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4 i} \ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2 - k^2}{m^2 - k^2}\right) . \tag{2.29}$$

One sees that the integral is quadratically divergent, which reflects the fact that the 1-loop effective potential needs to be renormalized. The first step to renormalization of a theory is a regularization of the divergent integrals, the topic of the next section.

2.3 Regularization

The way to proceed in the calculations of QFT is always very similar on the conceptual side but hard if one actually wants to compute something. One writes down the loop diagrams to the propagators and the couplings; these diagrams can be translated into integrals which are in general – to put in in cold words – divergent. The way to proceed now is to choose a regulator for the integral in order to deal with the divergences. The divergent part of the integrals are subtracted through the counterterms which are introduced through the concept of renormalization after specifying the renormalization conditions. We will give two different types of regulators here but, there are of course more [16].

2.3.1 Cut-off Regularization

The cut-off regulator has the advantage of a physical meaning: One sets an upper bound for the momenta in the loops. The momenta of particles are not allowed to exceed the value of the UV cut-off usually denoted by Λ.

The main disadvantages of a hard cut-off regulator are that it violates both gauge symmetry and Lorentz symmetry. As a consequence of violating gauge symmetry, the Ward identity does not hold any longer. This immediately shows that a hard cut-off is not well suited for gauge symmetries which appear in particle physics. However, in condensed matter systems the cut-off actually gets a physical meaning since gauge symmetry and Lorentz symmetry are not symmetries imposed by the Lagrangian.

2.3.2 Dimensional Regularization

Here we will introduce the idea of dimensional regularization which goes back to the work in reference [17]. We have seen that some loop integrals are quadratically divergent when we use a sharp cut-off as a regulator. One can blame the dimension of space-time for this divergence, since if the dimension of space-time was small enough, no divergence of the integral would occur. This is the idea of dimensional regularization. The space-time dimension is promoted to an arbitrary number d. If d is sufficiently small the integrals should converge and in the end we take the limit $d \to 4$ for physical quantities, which should be finite. We start with some technical details of dimensional regularization: After performing a Wick rotation with imaginary time, a typical integral to solve is of the form

$$\int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^2} = \int \frac{d\Omega_d}{(2\pi)^d} \cdot \int_0^\infty dl_E \frac{l_E^{d-1}}{(l_E^2 + \Delta)^2} \,. \tag{2.30}$$

The first factor accounts for the integration of a sphere in arbitrary dimensions. Details to this calculation are given in appendix A.2. We present the important result here:

$$\int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^2} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2 - \frac{d}{2})}{2} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \,. \tag{2.31}$$

One sees that the integral has isolated poles at negative integers for the Γ-function which corresponds to integers for $d > 4$. The approximation for $d = 4$ dimensions can be found using

$$\Gamma(2 - \frac{d}{2}) = \Gamma(\epsilon/2) = \frac{2}{\epsilon} - \gamma + \mathcal{O}(\epsilon) \,, \tag{2.32}$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. When we put everything together we obtain the result for the sample calculation as

$$\int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^2} \xrightarrow[d \to 4]{} \frac{1}{(4\pi)^2} \left(\frac{2}{\epsilon} - \log \Delta - \gamma + \log(4\pi) + \mathcal{O}(\epsilon)\right) . \tag{2.33}$$

The big advantage of dimensional regularization is that we break neither Lorentz invariance nor gauge invariance. This is why this regulator is very well suited for particle physics problems.[2] We note further that the $\frac{1}{\epsilon}$-pole is accompanied by two constant terms, namely $\log(4\pi)-\gamma$. This is a generic property of dimensional regularization and gives rise to the $\overline{\text{MS}}$ prescription if one performs the renormalization of the theory.

2.4 Renormalization of ϕ^4-Theory

To illustrate the concept of renormalization of a theory we will now show the important example of ϕ^4-theory which is more than only a toy model. It is very closely related to the Standard Model Higgs sector since we deal there in principle also with a ϕ^4-theory. However, in the realistic case more fields and gauge symmetries are involved.

The Lagrangian of ϕ^4-theory is [16]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4 \tag{2.34}$$

with a real scalar field $\phi(x)$, and λ_0 and m_0 are the bare self-coupling and the bare mass of the theory, respectively. λ_0 and m_0 are not accessible in experiments, but merely tools for calculations. To describe physics one has to perform the renormalization of the Lagrangian which has further consequences for the parameters of the theory. If one wants to formulate the Lagrangian in terms of physical and so renormalized fields, one rewrites

$$\mathcal{L} = \frac{1}{2}Z(\partial_\mu \phi_r)^2 - \frac{1}{2}m_0^2 Z\phi_r^2 - \frac{\lambda_0}{4!}Z^2\phi_r^4\,. \tag{2.35}$$

A rescaling of the field $\phi \to Z^{1/2}\phi_r$ due to wavefunction renormalization occurred to keep the residue of the propagator at 1, so that Lehmann-Symanzik-Zimmermann-formalism for computing S-matrix elements still works. Up to now the bare coupling constants still appear in the Lagrangian. We define

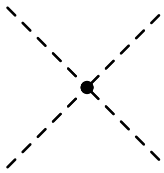
$$\delta_Z = Z - 1, \qquad \delta_m = m_0^2 Z - m^2, \qquad \delta_\lambda = \lambda_0 Z^2 - \lambda, \tag{2.36}$$

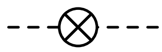
[2]The choice of the regulator should of course not have any impact on physical quantities. What we mean here is that loop calculations become easier to handle because useful relations such as the Ward identities still hold.

with the physical coupling constant λ, the physical mass m, is and the counterterms δ_i, the Lagrangian reads

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{4!}\phi_r^4 + \underbrace{\frac{1}{2}\delta_Z(\partial_\mu \phi_r)^2 - \frac{1}{2}\delta_m\phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4}_{\text{Counterterms}}. \tag{2.37}$$

$= \frac{1}{p^2 - m^2}$


$= -i\lambda$


$= i(p^2\delta_Z - \delta_m)$

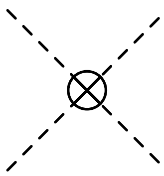
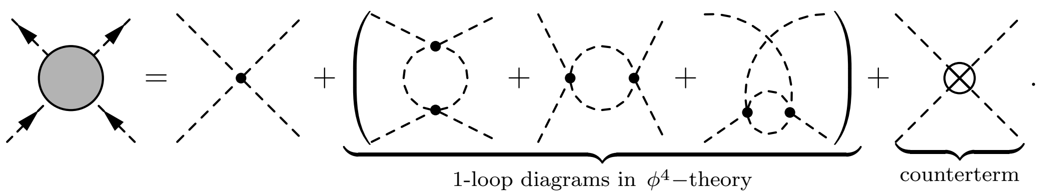

$= -i\delta_\lambda$

TABLE 2.1: Feynman rules for ϕ^4-theory in renormalized perturbation theory

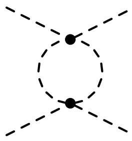
The constants m and λ are to be experimentally determined, but the counterterms δ_i have to be calculated order-by-order in perturbation theory. We compute the 1 loop contributions to the propagator and to the 4-point vertex to adapt the counterterms δ_Z, δ_m and δ_λ. We begin with the contribution of the 4-point vertex.

The 1 loop 4-vertex reads as



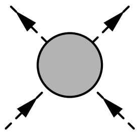
(2.38)

The 1-loop contributions in (2.38) can be calculated using the Feynman rules presented in Table 2.1. The result is



$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \frac{i}{(k+p)^2 - m^2} \equiv (-i\lambda)^2 \cdot V(p^2). \qquad (2.39)$$

The other diagrams in (2.38) can be obtained by interchanging momenta and forming kinematic invariants. One ends up with:

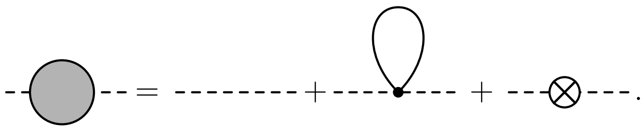


$$= -i\lambda + (-i\lambda)^2 i \left[V(s) + V(t) + V(u)\right] - i\delta_\lambda \,. \qquad (2.40)$$

Here s, t, u are the Mandelstam variables. The task now is to evaluate $V(p^2)$ introduced in (2.39). This can be done using dimensional regularization introduced in chapter 2.3.2. The result is

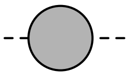
$$\begin{aligned} V(p^2) &= \frac{i}{2} \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{1}{\left[k^2 + 2xk \cdot p + xp^2 - m^2\right]^2} \\ &= \frac{i}{2} \int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{1}{\left[l^2 + x(1-x)p^2 - m^2\right]^2} \qquad (l = k + xp) \\ &= -\frac{1}{2} \int_0^1 dx \int \frac{d^dl_E}{(2\pi)^d} \frac{1}{\left[l_E^2 - x(1-x)p^2 + m^2\right]^2} \qquad (l_E^0 = -il^0) \\ &= -\frac{1}{2} \int_0^1 dx \frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}} \frac{1}{\left[m^2 - x(1-x)p^2\right]^{2-d/2}}. \end{aligned} \qquad (2.41)$$

To evaluate the counterterms δ_m and δ_Z the 1-loop evaluation of the 2-point function is still missing. The diagrams to calculate are



(2.42)

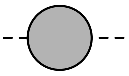
To put it in equations one uses again the Feynman rules from Table 2.1 and ends up with[3]



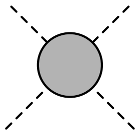
$$= \frac{1}{p^2 - m^2} + \frac{-i\lambda}{2} \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m^2} + i(p^2 \delta_Z - \delta_m) \tag{2.43}$$

$$= \frac{1}{p^2 - m^2} - \frac{i\lambda}{2} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(1 - \frac{d}{2})}{(m^2)^{1-d/2}} + i(p^2 \delta_Z - \delta_m)\,. \tag{2.44}$$

With a suitable choice of renormalization conditions, namely



$$= \frac{i}{p^2 - m^2} + \text{terms regular at } p^2 = m^2\,, \tag{2.45}$$



$$= -i\lambda \quad \text{at } s = 4m^2,\ t = u = 0\,, \tag{2.46}$$

we are in the position to determine the counterterms δ_λ, δ_Z and δ_m. One finds via the combination of (2.40), (2.41) and (2.46)

$$\delta_\lambda = -\lambda^2 [V(4m^2) + 2V(0)] \tag{2.47}$$

$$= \frac{\lambda^2}{2} \frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}} \int_0^1 dx \left(\frac{1}{[m^2 - x(1-x)4m^2]^{2-d/2}} + \frac{2}{[m^2]^{2-d/2}} \right)\,. \tag{2.48}$$

To determine the counterterms δ_Z and δ_m one observes that in (2.42) the relevant loop diagram which is calculated in (2.44) is independent of p^2. So one is able to set the counterterms as follows

$$\delta_Z = 0, \qquad \delta_m = -\frac{\lambda}{2} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(1 - \frac{d}{2})}{(m^2)^{1-d/2}}\,. \tag{2.49}$$

Note that while some of the counterterms diverge in the limit $d \to 4$, the physical parameters λ and m remain finite. The interested reader is referred to [18] where most of the presented material is taken from.

[3] The factor $\frac{1}{2}$ in (2.44) comes from symmetry considerations.

2.5 Running Couplings

In the previous section we have chosen a set of renormalization conditions in equations (2.45) and (2.46) which seemed to be physical but in principle can be chosen arbitrarily. We chose the mass m as the renormalization point but we could have chosen an arbitrary scale μ which has no physical meaning at all. However, physical theories should not depend on unphysical parameters. This is somehow strange since this scale which entered the calculations due to the choice of the renormalization conditions cannot have any physical meaning. This is exactly the idea one applies to the physical quantities of the theory, namely n-point Green's functions. Physical quantities must not depend on the unphysical renormalization scale μ. To give this ambiguity a physical meaning one introduces the renormalization group equation.

We first start with the scaling property of the 1PI. We have for an arbitrary renormalization scale μ

$$G_0^n(p_i, \lambda_0, m_0) = Z^{-n/2} G^n(p_i, \lambda(\mu), m(\mu), \mu)\,, \tag{2.50}$$

where we denote the external momenta with p_i. Bare quantities do not know anything about a renormalization scale. For this reason the derivative with respect to the renormalization scale vanishes

$$\begin{aligned} 0 &= \mu \frac{d}{d\mu} G_0^n(p_i, \lambda_0, m_0) \\ &= \mu \frac{d}{d\mu} \left(Z^{-n/2} G^n(p_i, \lambda(\mu), m(\mu), \mu) \right). \end{aligned} \tag{2.51}$$

If one applies the chain rule, one ends up with

$$0 = \left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} - \frac{n}{2}\gamma + \gamma_n \frac{\partial}{\partial m^2} \right] G^n(p_i, \lambda(\mu), m(\mu), \mu)\,, \tag{2.52}$$

where β is the beta-function and γ_m and γ are the anomalous dimension of mass and field respectively. These quantities are defined as

$$\beta(\lambda, \frac{m}{\mu}) = \mu \frac{d\lambda}{d\mu}\,, \tag{2.53a}$$

$$\gamma(\lambda, \frac{m}{\mu}) = \frac{1}{Z} \mu \frac{dZ}{d\mu}\,, \tag{2.53b}$$

$$\gamma_m(\lambda, \frac{m}{\mu}) = \mu \frac{dm^2}{d\mu}\,. \tag{2.53c}$$

The consequence is that coupling constants in a QFT are not constants at all. The renormalized, i.e. physical, couplings depend on the energy scale under consideration and are therefore running couplings. Taking the equation from the previous section one finds the beta-function for ϕ^4-theory to be at 1-loop order

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2}\,. \tag{2.54}$$

The solution to equation (2.54) with the definition in equation (2.53b) is therefore given as:

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - \frac{3\lambda(\mu_0)}{16\pi^2} \log\left(\frac{\mu}{\mu_0}\right)}\,, \tag{2.55}$$

which describes the running of the self-coupling λ. μ_0 is the chosen renormalization point while μ is the energy scale under consideration. Note that this equation for the running coupling is only applicable in the vicinity of the renormalization point μ_0, which reflects the idea of perturbation theory. The corresponding β-functions for the SM are given in the appendix B in the context of Weyl consistency relations.

With all these ingredients we now proceed to the SM.

CHAPTER 3

THE STANDARD MODEL OF PARTICLE PHYSICS

3.1 $SU(3)_C \times SU(2)_L \times U(1)_Y$ Gauge Theory

The Standard Model (SM) is a gauge theory with 18 free parameters to be determined experimentally. They are:

- 3 gauge couplings
- 6 Yukawa couplings for quarks (corresponding to 6 quark masses)
- 3 Yukawa couplings for charged leptons (corresponding to 3 lepton masses)
- 3 angles of the Cabibbo–Kobayashi–Masakawa (CKM) matrix
- 1 CP-phase of the CKM matrix
- 1 Higgs mass
- 1 Higgs self-coupling

We will give the Lagrangian for the SM and see how these parameters act within it. Here we follow [19] and [20]. The SM Lagrangian can be divided into four different parts:

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{f}} + \mathcal{L}_{\phi} + \mathcal{L}_{\text{Yukawa}} \tag{3.1}$$

Before spontaneous symmetry breaking all gauge bosons are massless, because if they were massive they would violate gauge invariance. Mass terms for the fermions are also forbidden, because of the chiral structure which we will explore when we see the particle content and the representations under the gauge groups. In addition to the terms in (3.1) there are also ghost and gauge-fixing terms which enter the Lagrangian due to quantization, but are not of relevance here. First we start with the gauge part of the Standard Model Lagrangian.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{3.2}$$

Here $G^i_{\mu\nu}$, $W^i_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength tensors for $SU(3)_C$, $SU(2)_Y$ and $U(1)_Y$ respectively. They are given as

$$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk} G^j_\mu G^j_\nu, \qquad i,j,k = 1,\dots,8 \tag{3.3a}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu, \qquad i,j,k = 1,\dots,3 \tag{3.3b}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{3.3c}$$

(3.3a) and (3.3b) show the non-abelian gauge structure of $SU(3)_C$ and $SU(2)_L$ due to the self-interactions represented by the last terms, respectively. Linear combinations of W^i_ν and B_ν describe the weak bosons $W^\pm$, Z^0 and the photon A_μ. Through the interaction with the Higgs field the weak gauge bosons become massive after spontaneous symmetry breaking which we will see when we have a closer look at the Higgs sector later.

Fermions of the Standard Model are organized in 3 families for quarks and leptons each. Only quarks transform under the $SU(3)_C$ gauge group, leptons are color singlets. All left-handed fields are doublets under $SU(2)_L$. The $U(1)_Y$-charge is assigned in the way that after spontaneous symmetry breaking the electromagnetic charge of the physical particles matches with the electrical charge observed in experiment. This means $Q = T^3_L + Y$, where Q is the electric charge T^3_L the generator of $SU(2)_L$ and Y the generator of $U(1)_Y$. The meaning of handedness in terms of representations of the Lorentz group is explained in more detail in appendix A and must not be confused with handedness in the sense of a $SU(2)_L$-charge.

In Table 3.1 the fermion content of the Standard Model is listed together with the representation under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. To go on

Names		transformation	$Q = Y + T_L^3$	particle content
Quarks	$Q_{mL} = \begin{pmatrix} u_{mL} \\ d_{mL} \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, \mathbf{\frac{1}{6}})$	$+2/3$ $-1/3$	$u_{1,2,3} = u, c, t$
	u_{mR}	$(\mathbf{3}, \mathbf{1}, \mathbf{\frac{2}{3}})$	$+2/3$	$d_{1,2,3} = d, s, b$
	d_{mR}	$(\mathbf{3}, \mathbf{1}, -\mathbf{\frac{1}{3}})$	$-1/3$	
Leptons	$L_{mL} = \begin{pmatrix} \nu_{mL} \\ e^-_{mL} \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -\mathbf{\frac{1}{2}})$	0 -1	$e_{1,2,3} = e, \mu, \tau$
	e^-_{mR}	$(\mathbf{1}, \mathbf{1}, \mathbf{-1})$	-1	$\nu_{1,2,3} = \nu_{e,\mu,\tau}$

TABLE 3.1: Transformation properties of the different families under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Q denotes the electromagnetic charge. The corresponding covariant derivatives are given in the appendix A.

we can give now the fermionic part of the SM Lagrangian. It is given as [1]

$$\mathcal{L}_{\mathrm{f}} = i\bar{Q}_{mL}\not{D}Q_{mL} + i\bar{L}_{mL}\not{D}L_{mL} + i\bar{u}_{mR}\not{D}u_{mR} + i\bar{d}_{mR}\not{D}d_{mR} + i\bar{e}^-_{mR}\not{D}e^-_{mR}. \quad (3.4)$$

D denotes the covariant derivatives of the SM particles which are defined in appendix A.4. The slash takes care of the fermionic structure through the contraction of γ-matrices with the covariant derivative $\not{D} = \gamma^\mu D_\mu$. Further information is given in appendix A.3. All fermions are massless due to the chiral structure of the electroweak sector: No Dirac-mass term for fermions can be obtained since it would not be a singlet under gauge transformations. This means that all particles in the SM are massless before spontaneous symmetry breaking (SSB). Let us proceed to the Higgs part of the SM Lagrangian.

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)\,, \quad (3.5)$$

where $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is a complex Higgs scalar. Its transformation properties under the SM gauge group are $(1, 2, \frac{1}{2})_\phi$. The covariant derivative is consequently given by

$$D_\mu\phi = (\partial_\mu + \frac{ig}{2}\vec{\sigma}\cdot\vec{W}_\mu + \frac{ig'}{2}B_\mu)\phi\,. \quad (3.6)$$

The vector $\vec{\sigma}$ denotes the 2×2-matrices which generate the $SU(2)$-group, the Pauli matrices. The square of the covariant derivative in (3.5) induces interactions between the gauge fields and the Higgs field. Taking gauge invariance and

[1] The role of the right-handed neutrino concerning a possible Majorana mass will be briefly discussed at the end of this chapter.

renormalizability as the building principles of the SM Lagrangian the Higgs potential is restricted to

$$V(\phi) = +\frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2. \tag{3.7}$$

If $\mu^2 < 0$ spontaneous symmetry breaking occurs which we will discuss in 3.2. Furthermore, the gauge bosons $W^\pm$ and Z^0 will become massive. The λ term denotes the Higgs self-interaction; an important point for our later discussion is the observation that we need $\lambda > 0$ due to vacuum stability. If $\lambda < 0$ held, the potential would be unbounded from below which is not acceptable for a well-defined theory. However, when quantum corrections to the potential are taken into account this relation will change, since λ receives loop corrections.

Now we will give the Yukawa part of the Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = - Y^e_{mn}\bar{L}_{mL}\phi e^-_{nR} - Y^D_{mn}\bar{Q}_{mL}\phi d_{nR} - Y^U_{mn}\bar{Q}_{mL}(i\sigma\phi^\dagger)u_{nR} + \text{h.c.}. \tag{3.8}$$

The SM offers no explanation on the renormalizable level for neutrino masses. But since we want to argue that the SM might be valid up to the Planck scale we need some explanation for neutrino masses which we will give in section 3.5.

3.2 Spontaneous Symmetry Breaking

The consequence of SSB is a non-vanishing VEV for the Higgs field ϕ which generates the masses of all elementary particles through the Yukawa couplings which we will discuss now.

The Higgs VEV

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{3.9}$$

can be obtained by minimizing the potential in (3.7). One obtains

$$v^2 = \frac{|\mu^2|}{\lambda}. \tag{3.10}$$

After SSB the Higgs doublet can be expanded around its VEV as

$$\phi = \begin{pmatrix} i\omega^+ \\ \frac{1}{\sqrt{2}}(v + H - iz) \end{pmatrix}, \tag{3.11}$$

where ω^+ is a complex scalar, H and z are real. Since we are dealing with a local gauge symmetry the would-be Goldstone modes ω^+ and z are eaten up by the gauge bosons as they become massive in unitary gauge. Here one sees that the gauge structure is not manifest anymore, i.e. the notation in equation (3.9) suggests that the $SU(2)_L \times U(1)_Y$ symmetry is broken. However, it is only the vacuum state which violates gauge symmetry; the SM Lagrangian still respects the imposed gauge symmetry which is the reason why we talk about a 'spontaneously broken symmetry'.

In unitary gauge, where ω^+ and z are absorbed by W_μ^+ and Z_μ the Higgs field can thus be written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}. \tag{3.12}$$

After SSB the square of the covariant derivatives of the Higgs field in (3.5) and (3.6) becomes

$$\begin{aligned}(D_\mu \phi^\dagger)(D^\mu \phi) \supset \frac{1}{8} v^2 g^2 (W_\mu^1 - i W_\mu^2)(W^{1\mu} + i W^{2\mu}) \\ + \frac{1}{8} v^2 (-g W^{3\mu} + g' B^\mu)(-g W_{3\mu} + g' B_\mu).\end{aligned} \tag{3.13}$$

A redefinition of the gauge field and a diagonalization into the mass basis allows us to read off the masses of the weak gauge bosons. We define

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \tag{3.14a}$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 - g' B_\mu) \tag{3.14b}$$

$$= W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W, \tag{3.14c}$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 + g' B_\mu) \tag{3.14d}$$

$$= W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W\,. \tag{3.14e}$$

From equation (3.13) we can read off

$$m_W = \frac{v}{2} g\,, \qquad m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}\,, \qquad m_A = 0. \tag{3.15}$$

The photon A_μ stays massless, which reflects the fact that one generator of $SU(2)_L \times U(1)_Y$ is still unbroken. As a consequence we obtain another gauge symmetry. SSB induces the breaking pattern $SU(2)_L \times U(1)_Y \to U(1)_Q$ with the generator $Q = T_L^3 + Y$. To get a feeling for the order of magnitude for the masses it should be added that the physical measured values are

$$m_W = 80.385 \pm 0.015\,\text{GeV}, \qquad \text{m}_\text{Z} = 91.1876 \pm 0.0021\,\text{GeV}, \qquad \text{v} \approx 246\,\text{GeV}, \tag{3.16}$$

which can be found in reference [21].

In (3.14e) and (3.14c) we introduced the Weinberg angle θ_W for which the following relations to the gauge couplings hold

$$\cos\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}\,, \qquad \sin\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}\,. \tag{3.17}$$

In particular, we find

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}\,. \tag{3.18}$$

All these relations are valid for tree-level considerations. Loop contributions break these relations which is closely related to the parameter $\Delta\rho \neq 0$ and the Peskin-Takeuchi parameters S, T, U which constrain the possibility of QFT-like new physics with a coupling to the electroweak sector. Furthermore, careful matching has to be performed between on-shell parameters and $\overline{\text{MS}}$-parameters to get the best precision in the running of the parameters. Matching here means the translation from physical on-shell parameters into $\overline{\text{MS}}$-parameters.

Next we want to see the impact of SSB on the physical field H concerning the interactions with the weak gauge bosons. Starting from (3.5) we obtain

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi) \tag{3.19}$$

$$= m_W^2 W^{\mu+} W_\mu^- \left(1 + \frac{H}{v}\right)^2 + \frac{1}{2} m_Z^2 Z^{0\mu} Z_\mu^0 \left(1 + \frac{H}{v}\right)^2 + \frac{1}{2}(\partial_\mu H)^2 - V(\phi) \tag{3.20}$$

and the Higgs potential becomes

$$V(\phi) \to -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4. \tag{3.21}$$

We see that the Higgs field has interactions with the massive weak gauge bosons as well as self-interactions. The self-interactions are a special feature about the Higgs field; no other massive particle in the SM has self-interactions. The second term in (3.21) denotes the tree-level mass term for the Higgs,

$$m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda} v\,. \tag{3.22}$$

This relation is also changed because of loop effects.

Let us now proceed to the Yukawa terms in (3.8) and see the consequences of SSB for them. These terms turn into

$$-\mathcal{L}_{\text{Yukawa}} = \bar{u}_{mL} Y^U_{mn} \frac{v}{\sqrt{2}} \left(1 + \frac{H}{v}\right) u_{nR} + (d, e) \text{ terms} + \text{h.c.} \tag{3.23}$$

$$= \bar{u}_L Y^U \frac{v}{\sqrt{2}} \left(1 + \frac{H}{v}\right) u_R + (d, e) \text{ terms} + \text{h.c.}, \tag{3.24}$$

where $u_L = (u_L, c_L, t_L)^T$. A similar definition holds for u_R. In (3.24) the generation of fermion masses through SSB is not yet obvious. To see that mass terms are generated one has to diagonalize the Yukawa matrices Y. We already emphasized that the Yukawa matrices are arbitrary 3×3 matrices. Diagonalization of the Yukawa matrices Y can be performed through a bi-unitary transformation of the chiral fermion fields using unitary matrices U_X and W_X. To put it in equations one arrives at

$$U_U Y^U W_U^\dagger = \text{diag}(y_u, y_c, y_t) = M_U \frac{\sqrt{2}}{v}, \tag{3.25a}$$

$$U_D Y^D W_D^\dagger = \text{diag}(y_d, y_s, y_b) = M_D \frac{\sqrt{2}}{v}, \tag{3.25b}$$

$$U_e Y^e W_e^\dagger = \text{diag}(y_e, y_\mu, y_\tau) = M_e \frac{\sqrt{2}}{v}. \tag{3.25c}$$

(3.25d)

We see that the tree-level masses of the fermions have been defined in (3.25a-3.25c). A redefinition of the quark fields should now be done in order to see physical consequences of the bi-unitary transformation. The change of variables for the right-handed quark fields is as follows:

$$u_R^n \to W_U^{nm} u_R^m, \qquad d_R^n \to W_D^{nm} d_R^m\,. \tag{3.26}$$

As a consequence the matrices W_U and W_D do not enter the Yukawa Lagrangian (3.8). Through this redefinition of fields the kinetic terms for the right-handed fields in (3.4) stay untouched since the unitary matrices commute with the covariant derivatives. So the matrices W_U and W_D disappear from the theory. However, the matrices U_U and U_D will appear in the theory as we see now. One defines

$$u_L^n \to U_U^{nm} u_L^m, \qquad d_L^n \to U_D^{nm} d_L^m. \tag{3.27}$$

As a consequence of this transformation the matrices U_U and U_D do enter the Lagrangian in (3.8). Let us now first go one step back to make the consequence of this redefinition manifest. We will write the kinetic term of the fermions (3.4) in terms of mass eigenstates of the weak bosons. Plugging in the definitions in (3.14a-3.14e) is straightforward and one ends up with

$$\mathcal{L}_f = \bar{L}_L(i\partial\!\!\!/)L_L + \bar{e}_R(i\partial\!\!\!/)e_R + \bar{Q}_L(i\partial\!\!\!/)Q_L + \bar{u}_R(i\partial\!\!\!/)u_R + \bar{d}_R(i\partial\!\!\!/)d_R \tag{3.28}$$

$$+ g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + eA_\mu J_{EM}^\mu. \tag{3.29}$$

Taking the definitions of the currents given in the appendix (A.14-A.19) we see that the neutral currents J_Z^μ and J_{EM}^μ are invariant under redefinition of the fields given in (3.27) and (3.26). However, the matrices U_U and U_D enter the charged currents. For example:

$$J^{\mu+} \subset \bar{u}_{nL}\gamma^\mu d_{nL} \to \bar{u}_{nL}\gamma^\mu (U_U^\dagger U_D)^{nm} d_{mL}. \tag{3.30}$$

We define

$$V = U_U^\dagger U_D \tag{3.31}$$

as the *Cabibbo–Kobayashi–Masakawa* (CKM) matrix. It is the only source of CP-violation in the SM.[2]

3.3 The Hierarchy Problem

The hierarchy problem is a fine-tuning problem which arises when the SM is embedded into another theory. Every particle should receive radiative corrections; however, the way these radiative corrections influence the parameters of the theory is very different for scalars, fermions or gauge bosons.

[2]CP-violation in QCD due to a term $F\tilde{F}$ has strong bounds and is usually omitted.

Fermion masses are generated in the SM through the Higgs mechanism. However, if we set the fermion masses to zero after SSB the symmetry of the Lagrangian is enhanced by a chiral symmetry for fermion masses. The consequence of this feature is that this symmetry protects fermion masses from arbitrary large radiative corrections. It means that the radiative corrections of fermion masses are proportional to the tree-level mass which means that in the limit of vanishing fermion mass no mass is generated through loop corrections.

Gauge bosons are also protected from acquiring mass but on a different footing. The Ward identity ensures that the gauge boson propagator stays transversal which results in massless gauge bosons. As an example one can take QED where exactly this happens. However, the role of the regulator used to evaluate the loop integrals should be emphasized here. Since the Ward identity arises from gauge invariance, it is crucial that one uses a regulator that respects gauge invariance. One should not be surprised to get a result which artificially violates the Ward identity if one uses a hard cut-off as a regulator which breaks both gauge and Lorentz invariance.

The role for scalar particles is different now. Within the SM we know no symmetry to protect scalar mass terms from radiative corrections. So, now we want to have a look at the one-loop corrections to the Higgs mass. They are given by the diagrams presented in figure 3.2 and 3.1.[3]

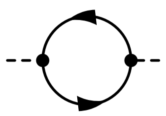


FIGURE 3.1: Radiative correction to a scalar mass m_H from a fermion with coupling $-\lambda_f H\bar{f}f$.

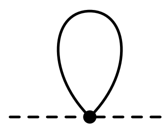


FIGURE 3.2: Radiative correction to a scalar mass m_H from an additional scalar particle S with a coupling $-\lambda_S|H|^2|S|^2$.

Figure 3.1 induces radiative corrections as (assuming a coupling $-\lambda_f H\bar{f}f$)

$$\Delta m_H^2 = -\frac{\lambda_f^2}{8\pi^2}\Lambda^2 + \ldots, \tag{3.32}$$

where the integral has been regulated with a cut-off and the dots denote terms which diverge at most logarithmically and are proportional to the mass of the fermion m_f. If the cut-off is large, say the Planck scale M_P the radiative corrections

[3]Gauge boson contributions are neglected here.

may be very large from a naive point of view. However, the reasoning that the cut-off should be interpreted as the scale of new physics is not correct. This is the exact situation within the SM and the fact of a quadratic divergence of the Higgs mass reflects the fact that the SM has to be renormalized. However, if the mass of the fermion m_f is big radiative corrections are big as well, i.e. $\Delta m_H^2 \sim m_f^2$ and a cancellation in the counterterms has to be performed. This is fine-tuning and reflects the one part of the hierarchy problem.

The situation for figure 3.2 is same where an additional scalar particle S with mass m_s is coupled. If we evaluate the diagram one obtains (assuming a coupling $-\lambda_S|H|^2|S|^2$)

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 - 2m_s^2 \ln\frac{\Lambda}{m_s} + \dots\right]. \tag{3.33}$$

The role of a regulator has already been pointed out in the context of the Ward identity. Here, we see again the consequences of a not well suited regulator: One could argue that the quadratic divergence should play the role of a new physics scale and that the radiative correction is quadratic divergent. However, if one regulates the diagram in figure 3.2 with dimensional regularization the quadratic divergence is not there which becomes obvious from a study of the divergence structure. The quadratic divergence is an artifact of a bad choice for the regulator. However, one observes that the logarithmic divergent piece stays and it is proportional to the mass of the additional scalar m_s. This means that if the additional scalar m_s is heavy, it is hard to understand why the mass of the scalar H is small. One has to adjust the counterterm exactly for that purpose which is fine-tuning.

In principle the same fine-tuning would if one introduced an additional vector-like fermion with mass m_f which does not couple to the Higgs directly but shares gauge interactions with the Higgs.[4] Through a two loop effect also a logarithmic divergence proportional to the fermion mass m_f appears. Due to the two loop effect the fine-tuning is milder but still large if m_f is heavy.

It should be emphasized again that the quadratic divergence introduced through the cut-off is an artifact of the regularization scheme and that there is no deeper physical meaning in the cut-off. One can understand this from the fact that the divergence structure in another regularization scheme, i.e. dimensional regularization does not yield a quadratic divergence. The hierarchy problem lies in the fact

[4]Note that a mass term of the form $m_f\bar{f}f$ is possible.

that heavy particles which couple to the scalar of the theory would pull the mass up to the heavy scale. If this should be avoided, the price to pay is a high amount of fine-tuning.

The most popular solutions for the hierarchy problem are Extra Dimensions and Supersymmetry. Recently also conformal symmetry became more and more popular and a promising way to evade the Hierarchy problem.

Supersymmetry: Every fermionic SM particle gets a bosonic super-partner and vice versa. Since bosons and fermions contribute with opposite signs to loops, the quadratic divergence vanishes naturally, i.e. the function $C(\lambda, g^2, ...) \to 0$ for exact supersymmetry. Unbroken super-symmetry actually even guarantees vanishing quadratic divergences to all orders in perturbation theory. Introducing super-symmetry is extending the symmetry of the Lagrangian by a symmetry between fermions and bosons. Since we know that in any renormalizable field theory fermion masses diverge at most logarithmically, the same holds for boson masses if super-symmetry is preserved. Unfortunately, super-symmetry cannot be an exact symmetry of nature, so it needs to be broken. This soft breaking reintroduces the fine-tuning problem depending on the mass-scale of the superpartners. The idea of an additional symmetry which is softly broken to explain a small parameter is exactly in the spirit of the notion of naturalness by t'Hooft.[5] A good introduction to super-symmetry is given in [22].

Extra-dimensions: The idea of extra-dimensions is that we actually live in more than 4 dimensions of space-time which are however compactified. This results in a shift of the Planck scale down to the electroweak scale, which shows that this theory does not suffer from a hierarchy problem [23].

Anthropic "Solution": The way to explain the big hierarchy between two scales and the high amount of fine-tuning which is needed to stabilize these different scales is that one assumes that such a high fine-tuning can only occur if there is intelligent life which is there to observe it. The universe in which we live has all properties which are necessary that intelligent life can exist and observe it. This "solution" has the big disadvantage that there is nothing to discover or explain anymore [24]. Everything what we observe and do not understand has to be the way it is because otherwise we wouldn't exist. In a certain way the anthropic principle is a motivated way of giving up.

[5]The radiative corrections to the Higgs mass are: $\Delta m_H^2 = m_{\text{soft}}^2 \frac{\lambda}{16\pi^2}\left[\ln\frac{\Lambda_{UV}}{m_{\text{soft}}} + \cdots\right]$.

Conformal symmetry: Also Conformal symmetry might play a role in solving the hierarchy problem. Bardeen argued that conformal symmetry might play a role in protecting the scalar mass term [25]. Whether this is true or not is not clear today and further research has to be done in this field.

The hierarchy problem will play an important role in the discussion of inflation. If a new scalar, the inflaton, is introduced and loop effects are taken into account the hierarchy problem arises.

3.4 The Need for Physics beyond the Standard Model

Until now in this thesis one might get the impression that the SM is a complete theory which gives an explanation for all phenomena in nature. The SM has withstood all experimental tests, however, it does not give satisfactory answers to everything. First, we didn't talk about gravity which is the fourth force of nature that we know apart from the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory we discussed before. We expect gravity to play an important role at latest when we approach the Planck scale, $M_P \sim 10^{19}$ GeV, since ordinary quantum field theory, which is based on a flat space-time, should not be applicable anymore. But also apart from gravity, which does not play an important role in daily life particle physics, there are evidences for BSM physics.

Neutrino masses: From experiments we know that neutrinos oscillate between different flavors [12], which is only possible if neutrinos are massive. The SM itself does not provide an explanation of the nature of neutrino masses. We also do not know whether neutrinos are Dirac or Majorana particles. Ongoing experiments are searching for neutrinoless double beta decay which is only allowed if neutrinos are their own antiparticles (and hence Marjorana particles) [26].

Dark matter: The Standard Model of Cosmology is a successful model fitting the Cosmic Microwave Background with a high accuracy. However, one needs a high amount ($\sim 26\%$) of dark matter to fit the data. Also direct observation of the rotation curves of galaxies and gravitational lensing indicate a high amount of matter which does not interact electromagnetically. However, the SM does not have a suitable candidate for a dark matter particle. From a theoretical perspective the idea of weakling interacting massive particles (WIMP) is an appealing concept which until today could not be identified in nature. Ongoing experiments concerning direct detection of dark matter such as XENON [27] or LUX [28] may clarify the nature of dark matter.[6] For a review on dark matter see for example reference [31].

Strong CP-problem: The strong CP-problem is a fine tuning problem which arises in QCD. The coupling of a possible term $\tilde{F}F$ in the Lagrangian is apparently extremely small (or even zero) which is unsatisfactory. A popular solution to the strong CP-problem was proposed by Pecci and Quinn [32] in the spirit of t'Hooft's notion of naturalness [33]. This solution predicts a new light pseudoscalar, the axion, which couples to gluons and photons. These axions could even make up dark matter but until today all searches for axions are negative and the simplest models of Pecci-Quinn symmetry are already ruled out but more sophisticated ideas might provide a hint [21].

Baryogenesis: There is an asymmetry of matter and anti-matter in our universe. To explain this asymmetry dynamically is the goal of baryogenesis. Successful scenarios of Baryogenesis need to fulfill the three Sakharov conditions, which are Baryon number violation, CP-violation and interactions out of thermal equilibrium [34]. The SM qualitatively meets all these conditions, but the CP-violation not sufficient to explain the observed asymmetry over baryons and anti-baryons quantitatively.[7] Baryogenesis requires the electroweak phase transition to be of first order, which is not possible with the observed Higgs mass. A nice overview of mechanisms of baryogenesis beyond the SM is given in [36].

Cosmology: The 2011 Nobel prize was to given to S. Perlmutter, B. P. Schmidt and A. G. Riess for the discovery of the accelerating expansion of the Universe through observations of distant supernovae [37]. The accelerated expansion is thought to be driven through dark energy. The problem of dark energy arises in the context of cosmology. The Standard Model of Cosmology augmented with a cosmological constant fits the data of the cosmic microwave background extremely well and as an outcome we need approximately 68% of Dark Energy to fit the data. However, we need an absolute scale, the cosmological constant, whose

[6]The DAMA collaboration claimed direct detection of dark matter but until now no other experiment is able to confirm their result but pushing down the limits on the interaction cross-section in tension with the DAMA claim or excluding it [29][30].

[7]Baryon number violation can occur in the SM via sphalerons in the early universe [35].

origin we cannot explain until today. We believe dark energy to be responsible for an accelerated expansion of the Universe but we do not understand why the cosmological constant is so small, yet not zero. Popular explanations of dark energy are quintessence scenarios, which basically add scalar fields to account for the negative pressure needed for an accelerated expansion of the Universe [38]. Another promising route is the idea of back-reaction models where one starts from Einsteins field equations of general relativity and takes the leading order terms of non-linearity into account which are neglected in the ΛCDM-model [39]. It may provide an explanation of dark energy in terms of geometry but the situation is unclear. For reviews on dark energy see for example reference [40, 41].

Inflation: Another window to new physics may be the theory of the early universe which provides an explanation for the initial conditions of the ΛCDM-model, inflation. Inflation explains in a nice way homogeneity and isotropy through an era of exponential expansion in the early Universe. Further details concerning inflation can be found in chapter 5.

All in all, we see that there are several good reasons for physics beyond the Standard Model. Whether these reasons imply a scale of new physics between the electro-weak scale and the Planck scale is another topic. The question whether fine-tuning and hierarchy arguments are actually a good motivation for BSM physics is completely unrelated to the need of BSM physics provided through experiments. Furthermore, we do not know whether this physics beyond the Standard Model is still within the scope of ordinary quantum field theory.

3.5 Right-handed Neutrinos and Type-I Seesaw

Until now there was no mechanism to give mass to neutrinos in this work. From oscillation experiments we know that neutrinos have a non-vanishing mass. In a certain way the "simplest" way of giving mass to neutrinos in complete analogy to the mass generation of other fermions in the SM is adding three right-handed neutrinos ν_{nR}. A consequence is that the Yukawa term is augmented by

$$\mathcal{L}_{M_\nu} = -Y^{\nu}_{mn} \bar{L}_{mL} (i\sigma\phi^\dagger) \nu_{nR}. \tag{3.34}$$

It is now possible to write down a neutrino mass term. However, it should be emphasized that a right-handed neutrino is by far not the only possibility. See for example reference [42] for a review. In the scenario with three additional right-handed neutrinos everything goes through as it went in the quark sector. An important consequence, however, is the unitary mixing matrix in complete analogy to the CKM matrix which is denoted by *Pontecorvo–Maki–Nakagawa–Sakata* (PMNS) matrix. In contrast to the quark sector the mixing angles in the lepton sector are big [43]. The PMNS matrix also provides another source of CP-violation apart from the know CP-violation of the CKM matrix which also might account on a quantitative level for the CP-violation needed to accommodate observations.

In the context of vacuum stability the presence of three right-handed neutrinos does not pose a problem at all. The right-handed neutrino have the same contribution as the top Yukawa coupling which is the reason the self-coupling is driven to smaller values for high energies. However, since neutrinos are orders of magnitude smaller than the top quark, one can completely ignore them in the stability discussion.

Note that there is in principle another term for the right-handed neutrinos which breaks lepton number. Now, we want to explore the consequences of such a term for the generation of neutrino masses. The neutrino part of the Lagrangian augmented with a right-handed neutrino and allowed Majorana term reads [44, 45]

$$-\mathcal{L}_{M_\nu} = Y^\nu_{mn}\bar{L}_{mL}(i\sigma\phi^\dagger)\nu_{nR} + \frac{1}{2}\bar{\nu}_R M_R \nu^c_R + \text{ h.c.}, \tag{3.35}$$

where the last term is the Majorana term which sets the second scale in the theory, apart from the μ^2-term in the Higgs potential. Note that M_R is a 3×3-matrix. Seeing as M_R is not connected to the electroweak scale, we may assume it to be much larger, $M_R \ll v$ This leads to the well known seesaw formula

$$m_\nu = v^2 Y^{\nu T} M_R^{-1} Y^\nu. \tag{3.36}$$

The big difference between this case and the case we showed before where no Majorana mass was apparent is that now the Yukawa couplings can be sizable which leads to crucial consequences in the RGE running of the parameters. However, neglecting for a moment the flavor structure we are able to estimate the neutrino mass $m_\nu = \mathcal{O}(0.1\,\text{eV})$ the Yukawa-coupling Y^ν is of order unity for a seesaw scale of $M_R = \mathcal{O}(10^{14}\,\text{GeV})$. Depending on the Majorana scale the Yukawa couplings can be chosen in order to fulfill experimental bounds on neutrino masses. In the picture of a seesaw mechanism the smallness of neutrino masses is explained through the suppression of a very heavy Majorana mass M_R.

The consequences of the type-I seesaw mechanism are in principle already clear from structure of the Lagrangian. Since the Yukawa coupling of the right-handed neutrino might be of order one it contributes as much as the top Yukawa coupling and, therefore, destabilizes the electroweak vacuum even further. This result is obtained on numerical footing in reference [46].

CHAPTER 4

STABILITY OF THE STANDARD MODEL

The goal of this chapter is to point out the interplay of physical parameters in the context of vacuum stability of the SM. The tools we use were presented in the previous chapters. We use the RGE running of the SM parameters to see the impact of them on the stability of the SM. This stability analysis is an important point for the discussion whether the SM might be valid all the way up to the Planck scale. There are two important points made by our interpretation of the LHC data: First, there is the discovery of the Higgs boson which completed the SM and in particular a measurement of the Higgs mass $m_H \approx 125\,\text{GeV}$ which also determines the self-coupling $\lambda \propto \frac{m}{v} \propto 0.13$. Second, they provide no evidence for BSM physics. Hence, it is quite tempting to use RGEs of the SM and let the parameters evolve to high scales and see their impact on vacuum stability. We follow the analysis of reference [47] with slight modifications.[1]

One has to be really careful about the statements and the plots shown in this section. The implicit assumption is that there is no new physics between the electroweak scale and the Planck scale, which is quite a strong statement. However, it is well motivated by the lack of any experimental sign of new physics because one observes a peculiar behavior for λ and β_λ near the Planck scale where both quantities almost vanish if RGE running is applied.

[1]The RGEs are augmented by three right-handed neutrino without a Majorana term to account for neutrino masses. - As it turns out in this framework the presence of right-handed neutrinos is irrelevant for the following discussion.

4.1 RGE Running of SM Parameters

We want to start with a general analysis of the SM gauge couplings. We use 2-loop RGEs obtained from reference [48] and let the couplings evolve from the electroweak scale to high scales. Figure 4.1 shows the running of the gauge couplings of the SM. We see that all gauge couplings almost meet at a scale of $\mathcal{O}(10^{16}\,\text{GeV})$ which gives rise to GUT motivated extensions of the SM. Note, however, that any new particles which might couple to the gauge bosons between the electroweak scale and the Planck scale can significantly change this running of parameters. It holds $\alpha_i = \frac{g_i^2}{4\pi}$, where α_1 is connected to the $SU(3)_C$ gauge coupling g. The same holds for α_2 and the $SU(2)_L$ gauge coupling and α_3 and the $U(1)_Y$ gauge coupling. We see that the situation for the size of the couplings is completely turned over

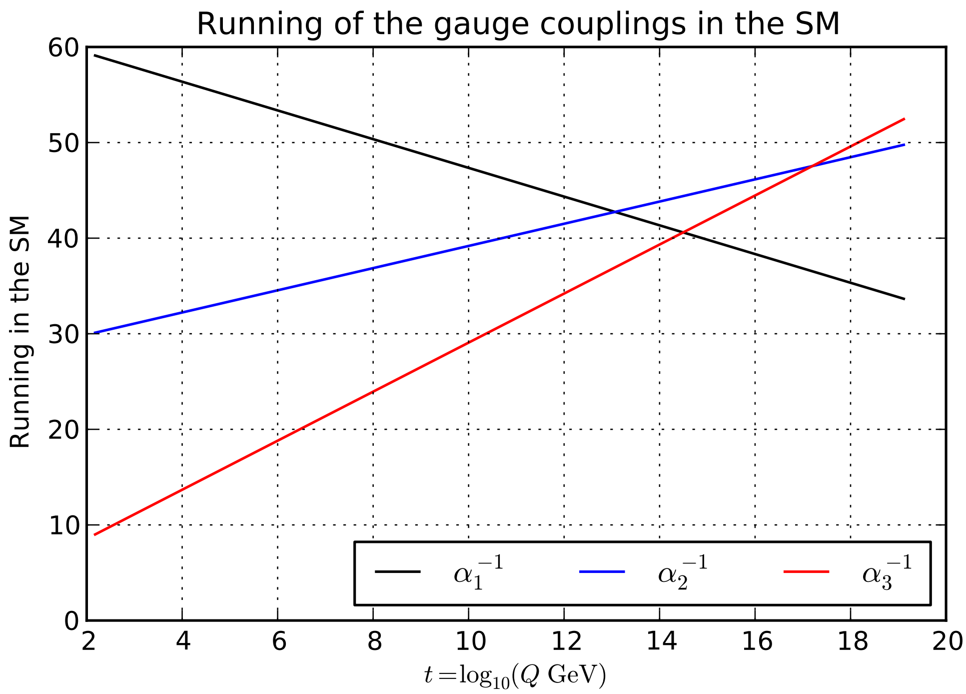


FIGURE 4.1: Running of the SM gauge couplings at 2-loop. One sees that the gauge couplings almost meet at a scale of $\mathcal{O}(10^{16}\,\text{GeV})$ which gives rise to supersymmetric extensions of the Standard Model. Note that the $U(1)_Y$ coupling is GUT-normalized which accounts for a factor of $\frac{5}{3}$. The RGEs are obtained from reference [48].

a high scales, i.e. the strong coupling of $SU(3)_C$ is not the strongest anymore. This is no surprise and reflects the fact of asymptotic freedom of the non-abelian

$SU(3)_C$-part of the SM.[2] One can also observe that the $U(1)$-coupling becomes bigger with rising energy. In this behavior one can already have a glimpse at the first fundamental conceptual failure of the SM, since the $U(1)_Y$ gauge coupling encounters a Landau pole at very high energies. However, one should not wonder too much about this conceptual problem since the $U(1)_Y$ Landau pole lies far beyond the Planck scale where we expect a UV-completion of the SM anyway.

4.2 Stability Analysis of the Standard Model

In order to analyze the SM potential one let the couplings run from the low scale to the high scale implicitly assuming that there is no new physics in between. The consequence is that the potential changes from its tree-level version given in equation (3.7).

An important fact one observes about the SM potential is that it is nearly conformal for high values, i.e. the SM potential can be well approximated by its RG-improved tree-level expression

$$V_{\text{eff}} \approx \frac{\lambda(\mu)}{4}\phi^4. \tag{4.1}$$

This means that an instability of the SM potential occurs at a scale μ where $\lambda(\mu)$ becomes negative. The SM vacuum is no longer bounded from below and one expects the SM vacuum to be unstable. This picture is only partly true, since the SM vacuum might also be very long-lived and as long as the lifetime of the SM vacuum exceeds the lifetime of our Universe the theory might still make sense. This means that long term existence of the electroweak vacuum is challenged. As a matter of fact, that is exactly the situation we find in the SM as we can see in figure 4.2. Note, however that here we used rather optimistic values for the top-mass [47]. The top mass and its error are the decisive parameter for the question whether the SM vacuum is stable or metastable. Furthermore, there are generic limits in the determination of m_t which we will encounter in section 4.6.

One observes in figure 4.2 that the Higgs potential becomes unbounded from below at a scale of $\mathcal{O}(10^{11}\,\text{GeV})$. One might be worried about this because this

[2] Asymptotic freedom is no general property of non-abelian gauge theories. QCD is asymptotically free because of the particle content of the SM which makes the β-function becomes negative.

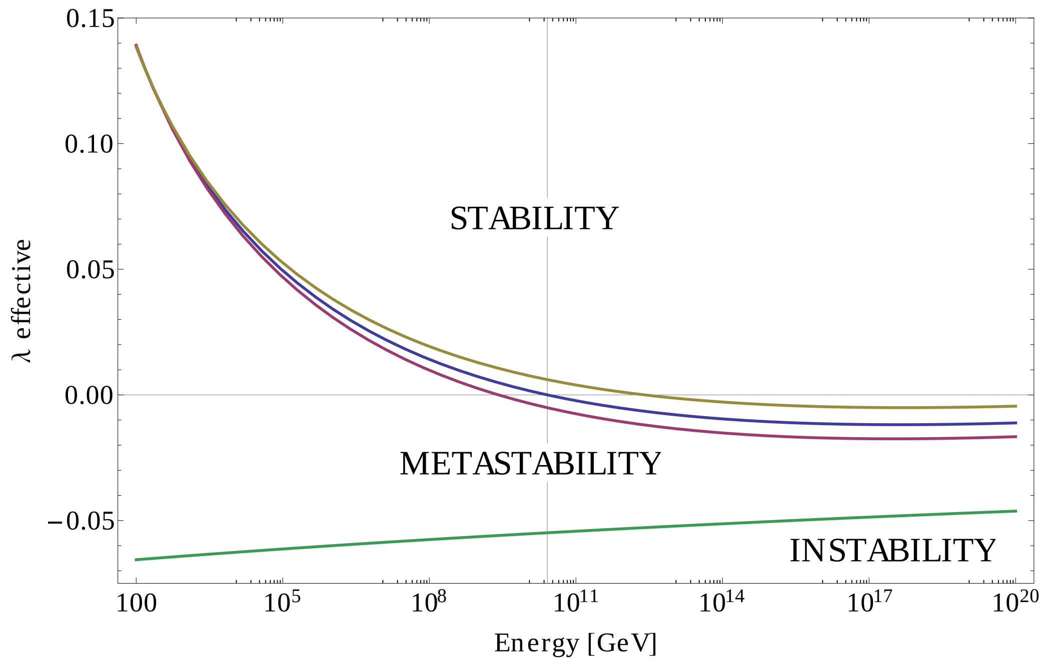


FIGURE 4.2: Running of the Higgs self coupling λ computed via 3-loop RGE running. The self-coupling becomes negative at a scale of $\mathcal{O}(10^{11}\,\text{GeV})$. This reflects the fact that the Higgs potential is unbounded from below in contrast with a desired stable vacuum. If the vacuum is not sufficiently long-lived this would be a clear sign of BSM physics, which should take care of this stability problem. The three lines correspond to different top masses namely 172.1 GeV (yellow),173, 1 GeV (blue), 174.1 GeV (red).

means that our vacuum is not absolutely stable. It provides a possibility for BSM physics if one demands that new physics should take care of this instability, i.e. new physics should step in at latest when $\lambda(\mu)$ turns negative. The job of any kind of new physics which might step in between the electroweak scale and the instability scale should be to overcome the top Yukawa coupling y_t which drives the self-coupling $\lambda(\mu)$ to negative values for high energies.

One solution of this would be to introduce a scalar singlet which couples to the Higgs and overcomes the top-Yukawa contribution in order to prevent the $\lambda(\mu)$ to turn negative. However, one has to be careful, since if one introduces another scalar in the theory; the running of $\lambda(\mu)$ might change in the desired way but one immediately runs into the hierarchy problem presented in section 3.3. For a recent analysis of RGE running with an additional scalar singlet as a dark matter candidate see reference [49].

The general question is whether the values for $\lambda(\mu)$ and $\beta_\lambda(\mu)$ at the Planck scale M_P are a coincident or if they have a special meaning in the sense of boundary conditions of a UV-completion of the SM. This question has been adressed in reference [50].

Another way out of this unfortunate situation is a tuning of the top-mass m_t such that absolute stability is still allowed. This tuning of a measured parameter does not come out of the blue: There are generic limitations in the determination of the top quark pole mass m_t at hadron colliders. One might argue about the error one is able to achieve today, but to be conservative an error of $\Delta_{m_t} \approx \mathcal{O}(\Lambda_{QCD})$ can be estimated. A detailed discussion of the role of the top-quark pole mass and the limitations of physical measurements in hadron colliders is given in section 4.6.

The running of $\lambda(\mu)$ in figure 4.2 is derived through 3-loop renormalization group equations for λ and suitable matching conditions for the other relevant parameters. The line separating the instability region from the metastability region is derived using

$$\lambda(\phi) > -\frac{8\pi^2/3}{4\log\left[\phi T_U \exp(\gamma)/2\right]}\,, \tag{4.2}$$

where T_U is the age of the Universe. Furthermore, one takes $\mu \approx \mathcal{O}(\phi)$. There are several standard simplifications which go into the expression displayed in (4.2) and further details can be found in reference [51].

In figure 4.3 we display the phase diagram of the SM. The only relevant parameters are m_H and m_t as one can see. The line separating the stability region and the metastability region is computed from the demand that $\lambda(M_P) = 0$. This means that in order to compute this line one fixes an arbitrary Higgs mass and then computes the value for the top-mass at which $\lambda(M_P) = 0$ is satisfied.[3] The error of this stability line comes mainly from the error in α_s. The take away-message from this plot is that vacuum stability is excluded by more than 2σ. But one should be careful with this statement because there is a strong dependence on the top quark pole mass and its error.

[3]This analysis can be done for different cut-offs below the Planck scale M_P changing this stability line.

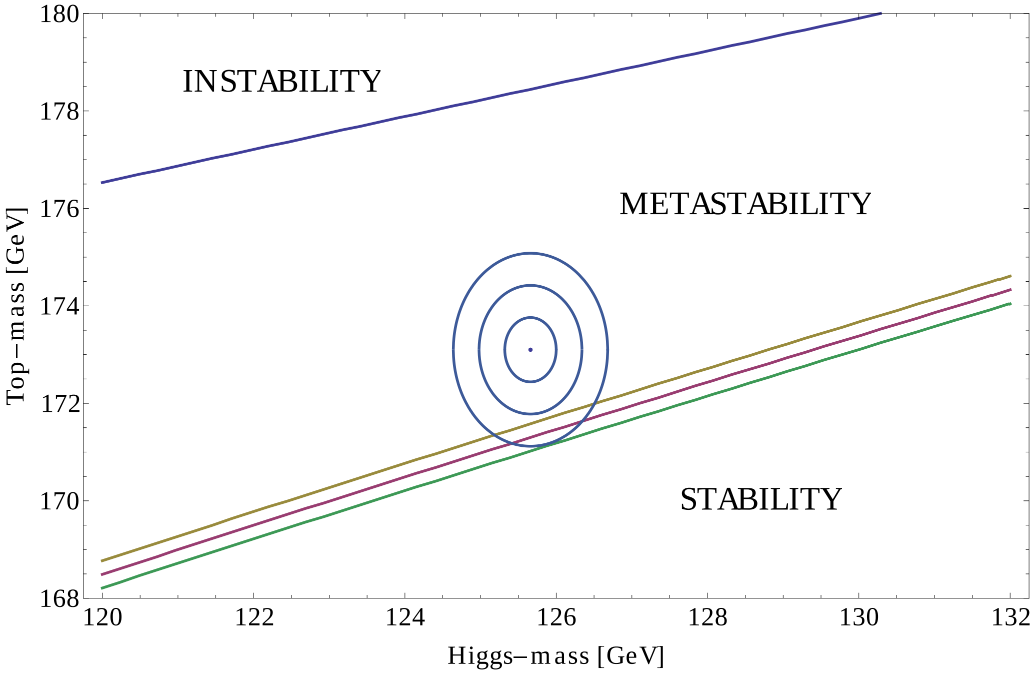


FIGURE 4.3: This figure shows the SM phase diagram in the Higgs-Top-plane. The line separating the metastability and the stability region is obtained by solving for which configuration of m_t and m_h the self-coupling λ turns negative at the Planck scale. The line separating the metastability and the stability region is obtained through the semi-classical estimation presented in [51]. Colored error-bars are obtained from error in α_s and the 1-, 2- and 3-σ level of the SM values for m_h and m_t. $m_t = (173.10 \pm 0.66)$ GeV and $m_h = (125.66 \pm 0.34)$ GeV in contrast to figure 4.9. The central value favors metastability of the electroweak vacuum. The small errors of m_t indicate that absolute stability is disfavored by more than 2σ. See section 4.6 for the role of the top quark.

4.3 Weyl Consistency Relation in the Context of the Standard Model

We have already pointed out that the SM possesses almost classical conformal symmetry at high energies, i.e. the only dimensional coupling in the Lagrangian, coupled to $H^\dagger H$, can be neglected. If one takes this feature serious, the question arises how to take care of the classical conformal behavior if one lets the couplings of the theory run. State of the art computations use a loop-number as high as possible to gain as much as precision as possible since the running of parameters has to be evolved over 16 orders of magnitude. It is true that one increases the precision for gauge, Yukawa and quartic couplings in the determination separately if one uses loop orders as high as possible. However, one violates the structure of classical conformal symmetry if one takes the same loop order for each gauge, Yukawa and quartic couplings. As a consequence, one has to count differently in the loops of respective couplings. Here we review briefly the work of [52]. As the authors of [52] point out one has to fulfill the Weyl consistency conditions which are a remnant of the almost classical conformal symmetry of the SM at high energies.

The idea is to keep track of the classical conformal symmetry after renormalization, which is a difficult task since the conformal symmetry is anomalous.[4] The idea is to first promote the couplings of the theory to functions of space-time, i.e. $h_i = h_i(x)$ and go on and work in an arbitrarily curved background. Now, a conformal transformation applied to the theory at hand implies a change of the space-time metric $g_{\mu\nu} \to e^{2\sigma(x)} g_{\mu\nu}$. This can be compensated by a change in the renormalized couplings as $h_i(\mu) \to h_i(e^{-\sigma(x)}\mu)$. Expressing this is possible if one performs a variation of the Schwinger functional W defined in 2.5. This can be parametrized as

$$\begin{aligned}\Delta_\sigma W &\equiv \int d^4x\sigma(x)\left(2g_{\mu\nu}\frac{\delta W}{\delta g_{\mu\nu}} - \beta_i\frac{\delta W}{\delta h_i}\right) \\ &= \sigma\left(aE(\gamma) + \chi^{ij}\partial_\mu h_i\partial_\nu h_j G^{\mu\nu}\right) + \partial_\mu\sigma w^i\partial_\nu h_i G^{\mu\nu} + \dots,\end{aligned} \tag{4.3}$$

where a, χ^{ij} and ω^i denote functions of the renormalized couplings and β_i is the corresponding β function to the coupling h_i. The important point is that the functions displayed on the right-hand side of (4.3) are not independent of each other. Furthermore, the Weyl anomaly should be abelian, which means

$$\Delta_\sigma\Delta_\tau W = \Delta_\tau\Delta_\sigma W. \tag{4.4}$$

This results in relations for the right-hand side of (4.3), namely

$$\frac{\partial\tilde{a}}{\partial g_i} = \left(-\chi^{ij} + \frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i}\right)\beta_j\,, \tag{4.5}$$

with $\tilde{a} \equiv a - w^i\beta_i$. If one works this out for the SM one observes that in order to respect the Weyl consistency conditions, one cannot apply an equal loop order to all relevant couplings. As it turns out, a 1-loop order in the self-coupling λ

[4] An anomalous symmetry is one that is broken by quantum corrections. One cannot find a measure $\mathcal{D}\phi$ in the path-integral which respects the symmetry.

requires a 2-loop order in the Yukawa-couplings and a 3-loop order in the gauge couplings in order to respect the Weyl consistency conditions. The naive thinking that the highest loop order possible helps the most in fact violates the relations derived in (4.5). The explicit equations for the case of the SM are given in the appendix B.

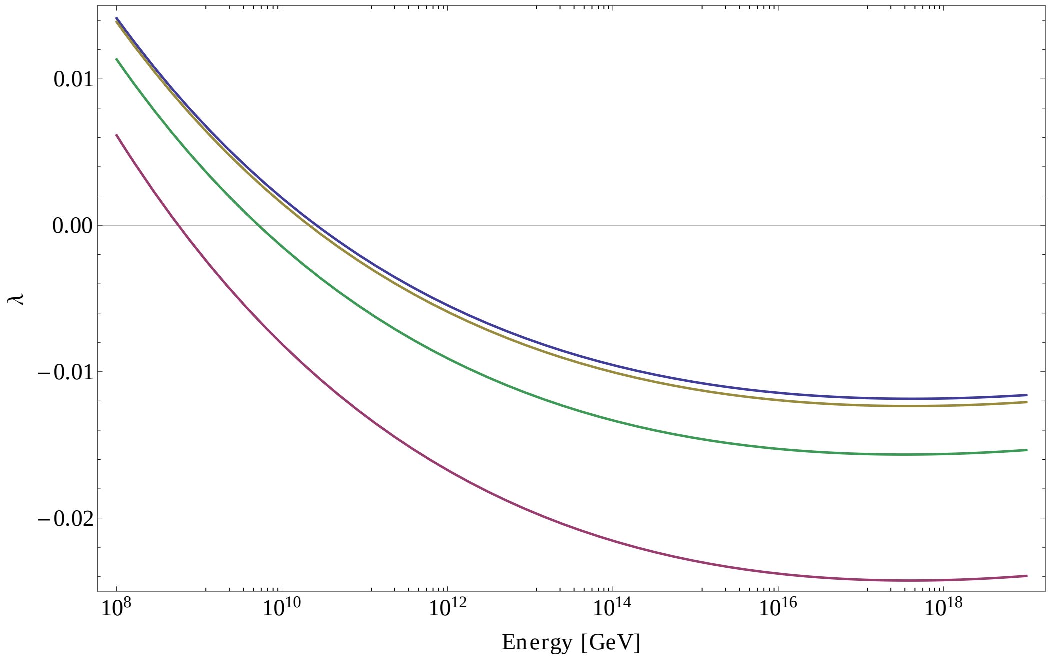


FIGURE 4.4: Running of λ according to different loop orders and matching with fixed $m_t = 173.1\,\text{GeV}$. Blue: 3-3-3 counting for gauge, Yukawa and self-coupling with state of the art matching conditions. Yellow: 3-2-1 counting with state of the art matching conditions. Green: 3-2-1 counting not exceeding the loop order imposed by Weyl consistency relation. Red: 3-2-1 counting with most conservative matching according to 2-loop in gauge, 1-loop in Yukawa and 0-loop in self-coupling.

Figure 4.4 shows the impact of the different counting schemes for the discussion of stability of the Standard Model. One observes that the scale at which the self-coupling $\lambda(\mu)$ becomes negative is less than 1 order of magnitude below $\Lambda \approx 10^{11}\,\text{GeV}$ (blue and yellow). So we see that including the Weyl consistency relations and therefore respecting the almost conformal nature of the Standard Model at high scales, has a rather small impact on the discussion of stability. The conclusions one draws are basically the same as if one takes that highest loop orders available. The only difference is the scale at which $\lambda(\mu)$ turns negative, but as we will discover in the next section, there are generic theoretical errors which are larger than 1 order of magnitude connected to the gauge dependence of the effective potential.

There is another important message to take away from figure 4.4: The biggest impact on the curves does not come from the different counting schemes but from the difference in the matching conditions which are imposed. The comparison between the blue and the yellow curve in figure 4.4 shows that the effect due to different counting schemes is really small. But we observe that if we change the loop order of the matching condition the impact is much bigger. The green curve represents NNLO matching conditions in all variables resulting in a change of more than 1 order of magnitude of the instability scale. The red curve, however, is matched due to 2-loop matching for the gauge couplings, 1-loop matching for the Yukawa coupling and 0-loop matching for the self-coupling. The difference to the blue line which corresponds to the state of the art computation with 3-3-3 counting and highest order in the matching conditions known is of 2 orders of magnitude. A table with the different values for the couplings with different matching conditions can be found in appendix B.2.

So, the different counting in the loop order for the different couplings is not important for the exact value where the instability occurs, since there are intrinsic errors which are bigger than the contribution from the Weyl consistency relations. However, if one wants to analyze classical conformal symmetries and their behavior on quantum level one should include loop orders of the respective couplings which respect the Weyl consistency relation, because otherwise we violate the classical conformal symmetry not only by the quantum nature of the theory but already by hand through an inconsistent choice of loop orders for the β functions of the theory.

4.4 Gauge Dependence of the Standard Model Vacuum Instability

An important question if one deals with the effective potential is which of the quantities one extracts from the effective potential are gauge independent and which are plagued with a gauge dependence and have to be considered to be unphysical to some extend. If we take all parameters of the SM as given except the Higgs mass m_h, the shape of the potential depends on several parameters.

These parameters are the Higgs mass m_h, the field ϕ and all parameters which fix a chosen gauge which we summarize as ξ. We can define a critical Higgs mass m_h^c which tells us when the RGE improved effective potential develops a new minimum at the same height as the electroweak minimum. This fact is illustrated in figure 4.5 [53, 54].

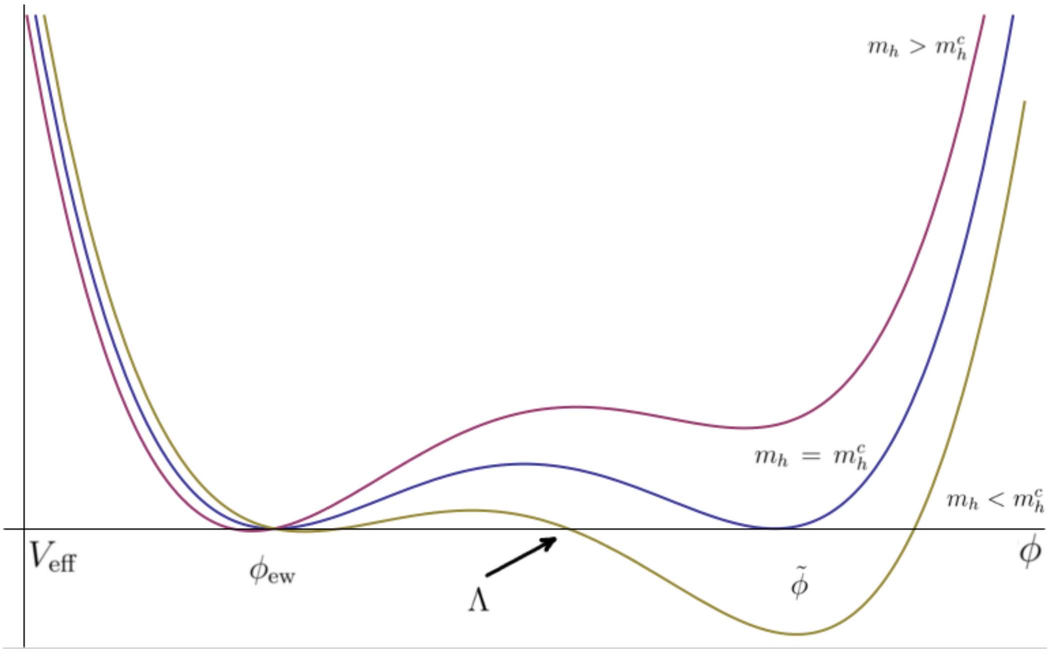


FIGURE 4.5: Sketch of the effective potential in a fixed gauge with different values for the Higgs mass. If $m_h < m_h^c$ the effective potential has an additional minimum which lies below the electroweak one (metastability/instability). For $m_h = m_h^c$ the two minima lie at the same level and for $m_h > m_h^c$ the second minimum lies above the electroweak minimum (absolute stability).

The condition for absolute stability of the effective potential can be put into mathematical formula through the equations

$$V_{\rm eff}(\phi_{\rm ew}, m_h^c; \xi) - V_{\rm eff}(\tilde{\phi}, m_h^c; \xi) = 0\,, \tag{4.6}$$

$$\left.\frac{\partial V_{\rm eff}}{\partial \phi}\right|_{\phi_{\rm ew}, m_h^c} = \left.\frac{\partial V_{\rm eff}}{\partial \phi}\right|_{\tilde{\phi}, m_h^c} = 0\,, \tag{4.7}$$

where $\phi_{\rm ew}$ denotes the electroweak minimum and $\tilde{\phi}$ the second minimum. If one takes into account that for large field values the effective potential can be approximated through (4.1), we obtain (see reference [55])

$$\lambda_{\text{eff}}(\tilde{\phi}, m_h^c; \xi) = 0\,, \tag{4.8}$$

$$\left.\frac{\partial \lambda_{\text{eff}}}{\partial \phi}\right|_{\tilde{\phi}, m_h^c} = 0\,.^5 \tag{4.9}$$

The most important tool for the analysis of physical quantities which can be extracted from the effective potential is the Nielsen identity [56]

$$\frac{\partial}{\partial \xi} V_{\text{eff}}(\phi, \xi) = -C(\phi, \xi) \frac{\partial}{\partial \phi} V_{\text{eff}}(\phi, \xi)\,, \tag{4.10}$$

where $C(\phi, \xi)$ is a correlator which involves the gauge-fixing functional and the ghost fields. The exact form of $C(\phi, \xi)$ is not important for the argument here. It is valid for the class of linear gauges and can be derived rigorously from BRST non-invariance of a composite operator involving the ghost fields and the gauge fixing functional.

The important point is the interpretation of equation (4.10). Since $C(\phi, \xi)$ is in general not zero we conclude that the extrema of the effective potential are gauge independent, i.e. spontaneous symmetry breaking is a gauge independent fact. If spontaneous symmetry breaking takes place in one gauge there is no other gauge where spontaneous symmetry breaking does not take place. However, we cannot conclude that scale Λ at which the instability occurs in a chosen gauge is a gauge-independent quantity. We will see that this is indeed not the case.

From an intuitive point of view one expects the critical Higgs mass to be gauge independent since by taking values below m_h^c in one gauge a new minimum develops and therefore it should happen in any gauge for the same m_h^c. This can be formally proven in the following way: One takes the condition for absolute stability given in (4.6) and combines it with the Nielsen identity (4.10). Performing the total differential of (4.6) with respect to ξ leads to

$$\underbrace{\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_{\text{ew}},m_h^c}}_{=0 \text{ (stat.cond.)}} \frac{\partial \phi_{\text{ew}}}{\partial \xi} + \left.\frac{\partial V_{\text{eff}}}{\partial m_h}\right|_{\phi_{\text{ew}},m_h^c} \frac{\partial m_h^c}{\partial \xi} + \underbrace{\left.\frac{\partial V_{\text{eff}}}{\partial \xi}\right|_{\phi_{\text{ew}},m_h^c}}_{=0 \text{ (4.10) and (4.7)}} =$$
$$\underbrace{\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\tilde{\phi},m_h^c}}_{=0 \text{ (stat.cond.)}} \frac{\partial \tilde{\phi}}{\partial \xi} + \left.\frac{\partial V_{\text{eff}}}{\partial m_h}\right|_{\tilde{\phi},m_h^c} \frac{\partial m_h^c}{\partial \xi} + \underbrace{\left.\frac{\partial V_{\text{eff}}}{\partial \xi}\right|_{\tilde{\phi},m_h^c}}_{=0 \text{ (4.10) and (4.7)}} . \tag{4.11}$$

Rewriting now (4.11) yields

$$\left(\left.\frac{\partial V_{\text{eff}}}{\partial m_h}\right|_{\phi_{\text{ew}},m_h^c} - \left.\frac{\partial V_{\text{eff}}}{\partial m_h}\right|_{\tilde{\phi},m_h^c} \right) \frac{\partial m_h^c}{\partial \xi} = 0 \,. \tag{4.12}$$

Since the bracket in (4.12) is usually not vanishing one concludes for the critical mass

$$\frac{\partial m_h^c}{\partial \xi} = 0 \,. \tag{4.13}$$

This statement holds to all orders in perturbation theory and proves that the critical mass m_h^c is a gauge independent quantity.

However, the instability scale which is often used in the standard analysis as a trigger that the potential becomes unbounded from below is a gauge dependent quantity which we will explore now following references [53, 54]. The instability scale Λ is the scale at which the running of λ becomes negative. So the instability scale is connected to the point where the effective potential has the same height as the electroweak minimum.[6] This can be expressed in equations through

$$V_{\text{eff}}(\Lambda; \xi) = V_{\text{eff}}(\phi_{\text{ew}}; \xi) \,. \tag{4.14}$$

We perform the total differential of equation (4.14) on both sides with respect to ξ. The right-hand side does not possess any gauge dependence since ϕ_{ew} is a minimum and vanishes after applying the Nielsen identity (4.10). This leads to

$$\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\Lambda} \frac{\partial \Lambda}{\partial \xi} + \left.\frac{\partial V_{\text{eff}}}{\partial \xi}\right|_{\Lambda} = 0 \,. \tag{4.15}$$

[6] This point do not need to be a minimum of the effective potential. See figure 4.5 for illustration.

Now, applying the Nielsen identity to the second term in (4.15) lets a factor $C(\Lambda, \xi)$ appear and yields

$$\left(\frac{\partial \Lambda}{\partial \xi} - C(\Lambda, \xi) \right) \frac{\partial V_{\text{eff}}}{\partial \phi} \bigg|_{\Lambda} = 0 \,. \tag{4.16}$$

If the instability scale Λ was a minimum, we would be back to the case where we deduced the gauge independence of m_h^c. So, since the instability scale Λ is not a minimum we obtain

$$\frac{\partial \Lambda}{\partial \xi} = C(\Lambda, \xi) \,. \tag{4.17}$$

Equation (4.17) shows that the instability scale Λ is a gauge dependent quantity. So the actual value of the instability scale is not a physical quantity and varies if one changes the gauge. A full analysis of the gauge dependence of the instability scale Λ for the 2-loop RGE improved effective potential has been carried out in reference [53]. They find an instrinsic dependence of the instability scale Λ which is of order of 1 magnitude. This also gives a hint for future treatment of the RGE-improved effective potential. Since there is a generic gauge dependence of the instability scale it probably does not make any sense in the context of vacuum stability to increase loop numbers in the analysis even further. Generic gauge dependence of the instability scale Λ on the one hand and errors in the top mass (see 4.6) limit the analysis the most not the precision due to computed loops.

4.5 Additional Non-renormalizable Scalar Operators

An important point in the discussion of the stability of the effective potential is that it develops a new minimum at $\phi \approx 10^{31}$GeV.[7] This point is usually ignored in the discussion and one expects that new physics interactions, which should step in at latest the Planck scale M_P, take care of this deficit. However, one argues that these new physics interactions should not affect the computation of the lifetime of the electroweak vacuum. So the situation we are dealing with is the following: One minimum of the effective potential V_{eff} is the electroweak minimum that we know and love, but it is not the true minimum of the theory. In this picture the true minimum lies outside the validity of quantum field theory; one expects that new physics should step in at latest at the Planck scale M_P. However, the nature

[7]The RGE improved effective potential develops this new minimum. The 1-loop effective potential is simply unbounded from below when crossing the instability scale.

of this UV-completion of the SM is unknown. As a first step towards the study of the impact of the UV-completion of the lifetime of the electroweak minimum, one can start by parameterizing this new physics by non-renormalizable operators.

Therefore, we augment the SM by non-renormalizable scalar operators and study their impact on the computation of the lifetime of the electroweak vacuum. An analysis similar to the one done here can be found in references [57, 58]. We restrict ourselves to the lowest possible higher dimensional operators, implicitly assuming all other ones suppressed. This means that the potential takes the form

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}. \tag{4.18}$$

The additional contribution from non-renormalizable operators to the Higgs self-coupling is [58]

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_P^2}. \tag{4.19}$$

Furthermore, the 1-loop β functions of the additional couplings are given as

$$16\pi^2\beta_{\lambda_6} = \frac{10}{7}\lambda_8\frac{m^2}{M_P^2} + 18\lambda_6 6\lambda - 6\lambda_6(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2) \tag{4.20}$$

$$16\pi^2\beta_{\lambda_8} = \frac{7}{5}28\lambda_6^2 + 30\lambda_8 6\lambda - 8\lambda_8(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2). \tag{4.21}$$

The idea is now that we let the SM evolve from the low scale physics that we know up to the Planck scale *without* the non-renormalizable operators since their impact in the low-energy regime is negligible. We solve the SM RGEs and note the value of all relevant couplings at the Planck scale. Then, we turn on the new physics interaction parametrized by scalar non-renormalizable operators suppressed by the Planck scale M_P and analyze their impact on the running for λ_{eff}.

Figure 4.6 shows the running of λ augmented by the non-renormalizable operators which might significantly change the running. Note that this figure might change if other values of λ_6 and λ_8 at the Planck scale are imposed. λ crosses the line of instability which renders the fact that the electroweak vacuum is too short-lived. Figure 4.6 shows, for a particular choice of the non-renormalizable couplings at the Planck scale M_P that the generic statement that the precise structure of the UV-completion of the Standard Model does not significantly change the computation of the lifetime of the electroweak vacuum is not true. A word of caution about this plot should be added since strictly speaking the effective

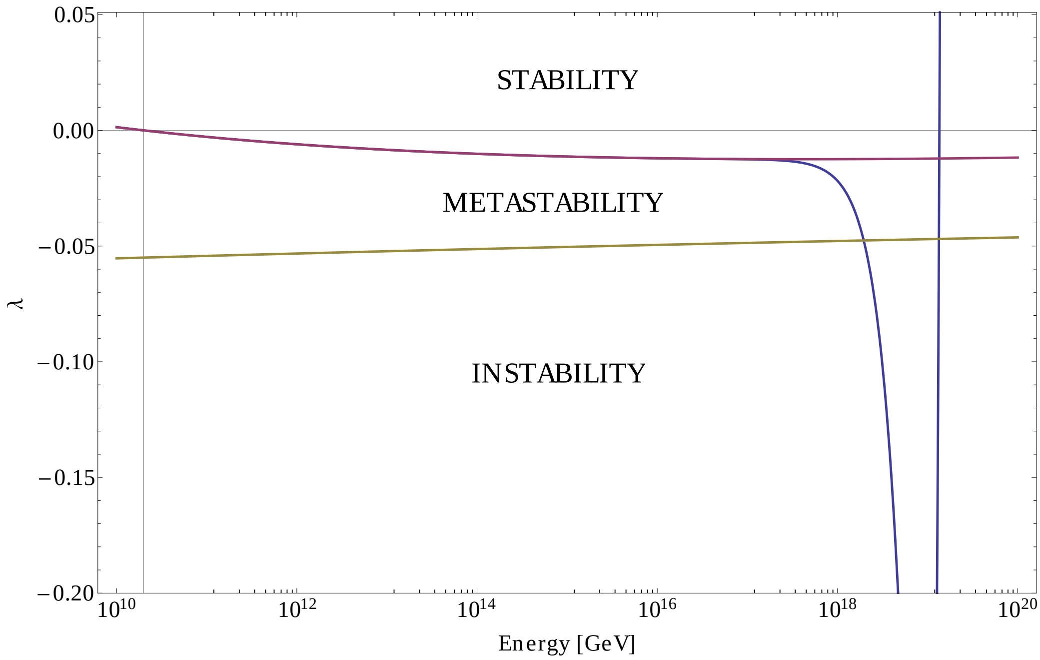


FIGURE 4.6: This plot shows in red the ordinary Higgs self-coupling $\lambda(\mu)$ in the SM which has already been shown in figure 4.2. In blue we show the impact of the non-renormalizable operators on the Higgs self-coupling. The function which is plotted in blue is $\lambda_{\text{eff}}(\mu) \approx \lambda(\mu) + \frac{\lambda_6(\mu)\phi^2}{6M_P^2} + \frac{\lambda_8(\mu)\phi^4}{8M_P^4}$ in the approximation of $\phi \approx \mathcal{O}(\mu)$ and running of the field itself is neglected. The values of the non-renormalizable operators at the Planck scale M_P are $\lambda_6(M_P) = -2$ and $\lambda_8(M_P) = 2.1$.

field theory approach breaks down in the vicinity of the Planck scale M_P and the non-renormalizable operators, which are irrelevant in the low energy regime, become very important and even dominant. In this sense it is important that the minimum still lies below the Planck scale M_P in order to ensure to that operators beyond order 8 are still suppressed to some extent. The closer one approaches the Planck scale the more important become the non-renormalizable operators and if the minimum lies beyond the Planck scale, the operators of higher order should have the most important impact. For different treatment see reference [58]. The vacuum in this scenario is too short-lived to be an acceptable scenario within the Standard Model. If the UV-completion, which is in this picture parametrized by the non-renormalizable operators has these values at the Planck scale M_P new physics between the electroweak scale and the Planck scale has to step in and take care of this deficit of the theory. Figure 4.7 and figure 4.8 show the running of the non-renormalizable couplings. Concerning perturbativity the big values of λ_8 should not worry us so much, since in the regime where the coupling is strong, the

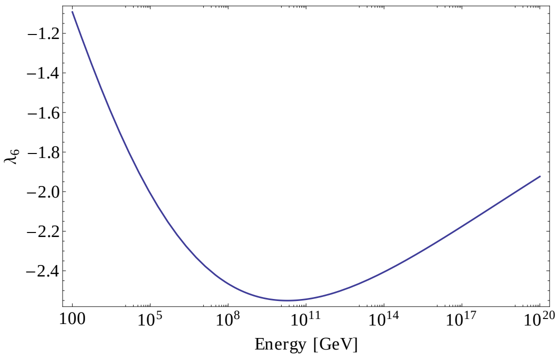


FIGURE 4.7: Running of the non-renormalizable coupling λ_6 for $\lambda_6(M_P) = -2$

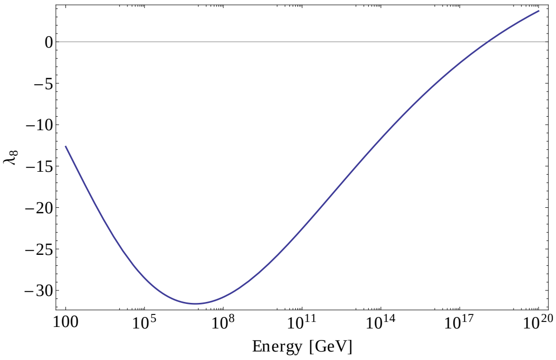


FIGURE 4.8: Running of the non-renormalizable coupling λ_8 for $\lambda_8(M_P) = 2.1$

suppression of the non-renormalizable operators is big.[8] Figure 4.7 and figure 4.8 show the running of the non-renormalizable operators. The big coupling of λ_8 concerning perturbativity should not worry us so much since in the regime where the couping is strong the suppression of the non-renormalizable operators is big. However, clearly this analysis is not complete yet and lacks severals features such as derivatives which should be included for a full analysis.

In order to compute the lifetime τ of the electroweak vacuum one has to calculate [59, 60]

$$\frac{1}{\tau} = T_U^3 \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' [-\partial^2 + V''(\phi_b)]}{\det [-\partial^2 + V''(v)]} \right|^{-1/2} e^{-S[\phi_b]} , \tag{4.22}$$

where $\phi_b(r)$ is the $O(4)$ symmetric solution to the so-called *bounce* which corresponds to the euclidean equation of motion. $S[\phi_b]$ denotes the action for the bounce and $[\partial^2 + V''(\phi_b)]$ is the fluctuation operator. The computation of the determinant is rather involved and to make physical meaning of the computation one has to remove zero modes of the determinant, which is denoted through the prime in the determinant of the numerator in equation (4.22). The computation performed in [57] shows that the potential for small field values can be well approximated through the ordinary SM effective potential. So they find that up to a scale of $\eta \approx 0.780 M_P$ the potential can be approximated by $V_{\text{eff}}^{new}(\phi) = \frac{\lambda_{\text{eff}}}{4}\phi^4$ where $\lambda_{\text{eff}} = \lambda + \frac{\lambda_6(\mu)\phi^2}{6M_P^2} + \frac{\lambda_8(\mu)\phi^4}{8M_P^4}$. For the region $\phi > \eta$ bends down steeply for this choice of λ_6 and λ_8 at the Planck scale which we also observe in figure 4.6. They argue that a new minimum close to the Planck scale develops at $\phi_{min}^{(2)} = 0.979 M_P$ and that because of this fact a linearization of the potential can be performed

[8]However, for a more realistic treatment one should still expect higher order terms to be relevant.

which yields

$$V(\phi) = \left[\frac{\lambda_{\text{eff}}}{4}\eta^4 - \frac{\lambda_{\text{eff}}\eta^3}{\gamma}(\phi-\eta)\right], \tag{4.23}$$

where γ is given as

$$\gamma = -\,\lambda_{\text{eff}}\ \eta^3 \left(\lambda\eta^3 + \lambda_6\frac{\eta^5}{M_P^2} + \lambda_8\frac{\eta^7}{M_P^4}\right)^{-1}. \tag{4.24}$$

They claim bounce solutions for (4.23) as

$$\phi_b(r) = \begin{cases} 2\eta - \eta^2\sqrt{\frac{|\lambda_{\text{eff}}|}{8}}\frac{r^2+\overline{R}^2}{\overline{R}} & 0 < r < \overline{r}, \\ \sqrt{\frac{8}{|\lambda_{\text{eff}}|}}\frac{\overline{R}}{r^2+\overline{R}^2} & r > \overline{r}, \end{cases} \tag{4.25}$$

where

$$\overline{r}^2 = \frac{8\gamma}{\lambda_{\text{eff}}\eta^2}(1+\gamma), \qquad \overline{R}^2 = \frac{8}{|\lambda_{\text{eff}}|}\frac{\gamma^2}{\eta^2}. \tag{4.26}$$

This means that solutions for equation (4.25) exist only in a certain range for γ, namely $-1 < \gamma < 0$. In this way $\overline{R}$ is the size of the bounce and its action is given at ϕ_b as

$$S[\phi_b] = \left(1-(\gamma+1)^4\right)\frac{8\pi^2}{3|\lambda_{\text{eff}}|}. \tag{4.27}$$

Other bounce solutions are given as

$$\phi_b^{(2)}(r) = \sqrt{\frac{2}{|\lambda_{\text{eff}}|}}\frac{2\,R}{r^2+R^2}, \tag{4.28}$$

where the allowed values for the size of the bounce can now lie in the range $\sqrt{\frac{8}{|\lambda_{\text{eff}}|}}\frac{1}{\eta} < R < \infty$. However, if we take $|\phi| \ll M_P$ and replace λ_{eff} by λ in equation (4.28) the bounce action is degenerate in R and is given as

$$S = \frac{8\pi^2}{3|\lambda|}. \tag{4.29}$$

For a precise treatment the contribution of (4.28) should be taken into account for the computation of the lifetime τ of the electroweak vacuum. However, as it turns out, the contribution is exponentially suppressed if we only take into account the tree level contributions coming from equations (4.22), (4.27) and (4.29) under the assumption that a solution which requires $-1 < \gamma < 0$ exists. We see that we meet these requirements for the case of interest, namely $\lambda_6(M_P) = -2$ and

$\lambda_8(M_P) = 2.1$ since $\gamma \approx -0.963$.

The authors of reference [57] now follow the treatment presented in [61] to compute the fluctuation determinant. They proceed as follows:

$$\log\left(\frac{\det'(-\partial^2 + \mathrm{V}''(\phi_\mathrm{b}))}{\det(-\partial^2)}\right)^{1/2} = \frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2 \ln \rho_l, \tag{4.30}$$

where $\rho_l = \lim_{r\to\infty}\rho_l(r)$. In equation (4.30) $\rho_l(r)$ is the solution of

$$\rho_l''(r) + \frac{(2l+d-1)}{r}\rho_l'(r) - V''(\phi_b(r))\rho_l(r) = 0, \tag{4.31}$$

where suitable boundary conditions have to be imposed like $\rho_l(0) = 1$ and $\rho_l'(0) = 0$. Derivatives in equation (4.31) are understood to be taken with respect to r. Special care has to be taken during the computation of the sum in equation (4.30) since the eigenvalue for $l = 0$ is related to a negative mode and the eigenvalues for $l = 1$ are related to translational modes, which are zero. These modes should be treated separately and one has to compute the divergent sum without $l = 0$ and $l = 1$. The procedure which takes care of the divergence is renormalization. The $\overline{\text{MS}}$ renormalized sum of equation (4.30) is given as (see [57, 62]):

$$\begin{aligned}\left[\frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2 \ln \rho_l\right]_r &= \frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2 \ln \rho_l \\ -\frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2&\left[\frac{\int_0^\infty drrV''}{2(l+1)} - \frac{\int_0^\infty drr^3(V'')^2}{8(l+1)^3}\right] \\ &-\frac{1}{8}\int_0^\infty drr^3(V'')^2\left[\ln\left(\frac{\mu r}{2}\right) + \gamma_E + 1\right].\end{aligned} \tag{4.32}$$

Through a truncation of the sum for suitable angular momentum $L = 5$ (standard renormalization procedure) they obtain ($L = \overline{R}M_P$):

$$\left[\frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2 \ln \rho_l\right]_r = -2.49 - 5.27\,\ln\left(\frac{1.48\mu}{M_P}\right). \tag{4.33}$$

Replacing the ρ_0 ($l = 0$) with its absolute value (see [62]) they find the contribution to the sum (4.30) as $\frac{1}{2}\ln|\rho_0| = -0.806$. The treatment of the zero modes which correspond to ($l = 1$) is the following: ρ_1 has to be replaced through ρ_1' as:

$$\rho_1' = \lim_{k\to 0}\frac{\rho_1^k}{k^2}. \tag{4.34}$$

and ρ_1^k corresponds to the solution of equation (4.31) with $V''(\phi_b) + k^2$ instead of $V''(\phi_b)$. Observing that ρ_1' possesses the dimension of length square yields that is given in terms of $\overline{R}$. All in all the contribution of the zero modes can be summed up in $\frac{1}{2} \cdot 4 \, \ln \rho_1' = 2 \, \ln (0.0896 \overline{R}^2)$.

In order to easier compare their results with the standard results from the effective potential they chose the same renormalization scale $\mu = 1.32 \cdot 10^{17}$ GeV and find

$$\tau = 5.45 \cdot 10^{-212} \, T_U, \tag{4.35}$$

contradicting the standard result which we already displayed in figure 4.3. The lifetime of the electroweak vacuum is way too short if one augments the Higgs potential with ϕ^6 and ϕ^8 non-renormalizable operators and choses the values given above at the Planck scale M_P. Clearly this analysis is lacking features such as loop corrections to the tree level contributions and a full treatment of dimension 8 operators. However, the work of [57] gives a hint that the common belief that new physics at the Planck scale should not significantly influence the computation of the lifetime of the electroweak vacuum may be wrong. On the contrary, they find a significant change resulting in a really short-lived, unstable vacuum.

4.6 Top-mass Measurements

The top quark pole mass is usually deduced through the reconstruction of decay products of the top at colliders. However, the technique used involves Monte Carlo generators which model the process in consideration. The extracted mass parameter is then nothing more and nothing less than the Monte-Carlo mass of the event generator used, which introduces a model dependence in the measurement. The translation between the mass parameter which has been extracted and the pole mass is a very delicate process which is not yet fully understood. The process of translating the extracted Monte-Carlo mass to the mass of a known renormalization scheme introduces an uncertainty of $\mathcal{O}(1\,\text{GeV})$ since the Monte Carlo generators require modeling of jets, missing energy, initial state radiation contributions, as well as of the hadronization part. This problem gets worse because many event generators do not go beyond leading order and leading logarithm [63].

The precise definition of a top quark pole mass is difficult from a theoretical point of view. The definition of a mass is usually due to the first pole in the propagator of a particle, which can be done rigorously for non-colored particles, e.g. electrons. After renormalization of QED, the first pole of the propagator of the electron is the physical "on-shell" mass of the electron. The implicit assumption of this picture is somehow a free propagation of the particle since we compare asymptotic states of the S-matrix. This is the first point where the definition of a top quark pole mass has a shortcoming from a theoretical perspective, because (top) quarks are colored and therefore do not meet the requirement of free propagation for a suitable amount of time. QCD confinement does not allow us to define a pole mass in a self-consistent way. Even worse, one is plagued with infrared renormalons which yield an ambiguity of $\mathcal{O}(\Lambda_{QCD})$ due to non-perturbative effects. Any measurement of a top quark pole mass which is performed at hadron colliders (i.e. Tevatron, LHC) is limited by Λ_{QCD}. To the extend of our knowledge today, it is impossible to perform measurements with a higher precision through kinematical reconstruction of decay products of top quarks, i.e. final state leptons and jets. This view is also shared in references [64, 65].

This deficit in the definition of a pole mass can be cured if one uses the $\overline{\text{MS}}$ scheme in the process of renormalization. The mass then becomes a parameter dependent on the energy scale in consideration: $m_t^{\overline{\text{MS}}}(\mu)$. Determination of the $\overline{\text{MS}}-$mass is then possible in any process which is precisely measured on the one hand and determined beyond leading order in QCD perturbation theory on the other hand. In this way one can extract the $m_t^{\overline{\text{MS}}}(\mu)$ and then proceed in the calculation and determine the on-shell mass which is given through a relation between bare and renormalized parameters, namely

$$\theta_0 = \theta_{\text{OS}} - \delta\theta_{\text{OS}} = \theta(\bar{\mu}) - \delta\theta_{\overline{\text{MS}}}. \tag{4.36}$$

Equation (4.36) can be used to express $\overline{\text{MS}}$ quantities in terms of physical, on-shell parameters. This reflects the fact that suitable matching conditions have to be imposed in order to relate an $\overline{\text{MS}}$ mass to a pole mass. A process which meets the requirements mentioned above is the total production cross section for top pair production $\sigma(t\bar{t} + X)$ [66]. Probably only at a future linear collider (ILC) is the correct determination of the top quark pole mass possible within an accuracy of a few hundred MeV [64].[9]

Furthermore, there is in principle the possibility that new physics might influence the measurements of the top quark pole mass. Usually experimental measurements are compared with SM predictions which leaves the possibility open that there might be a Bias in the extraction of physical masses. However, it is unlikely that these contributions cause large corrections.

The readers interested in the technical aspects of the extraction of the top quark pole mass and the techniques involved are referred to references [67, 68].

As a summary of current top mass measurements one might give the following relation between data extracted from experiment and physical pole mass as

$$\boxed{m_t^{\text{pole}} = m_t^{\text{MonteCarlo}} \pm \Delta.} \tag{4.37}$$

The important question is now which value for Δ. Some authors claim that one may put $\Delta \sim \mathcal{O}(\Lambda_{QCD}) \sim 250 - 500\,\text{MeV}$ [69] which motivates the plot shown in figure 4.2 and figure 4.3 with rather optimistic values for the error of the m_t. But others are more conservative and estimate the error to be $\Delta \sim 1\,\text{GeV}$ [63].

This sheds a different light on figure 4.3 since the value used there is rather optimistic with an error of only $\Delta m_t = 0.66\,\text{GeV}$, which lies below the (conservative) estimate of the intrinsic error in the determination of the top quark pole mass which is introduced through the uncertainty in relation (4.37).

All this discussion concerning the scheme used during renormalization may appear awkward since we know that calculated in any scheme at sufficiently high order the scheme dependence should disappear. However, in actual calculations it might be important to keep in mind that scheme dependence is introduced of which we should take care of by imposing suitable matching conditions to relate the parameters in different schemes of the theory.

The question whether the SM vacuum is stable or not is a difficult subject and poses problems on theoretical and experimental grounds. First of all we do not know if there is new physics between the electroweak scale and the Planck scale. If there is new physics the discussion of vacuum stability of the SM might be obsolete. On theoretical grounds the errors due perturbation theory seem to

[9]Note that current measurements of m_t through $\sigma(t\bar{t}+X)$ at the LHC still allow for absolute vacuum stability [64].

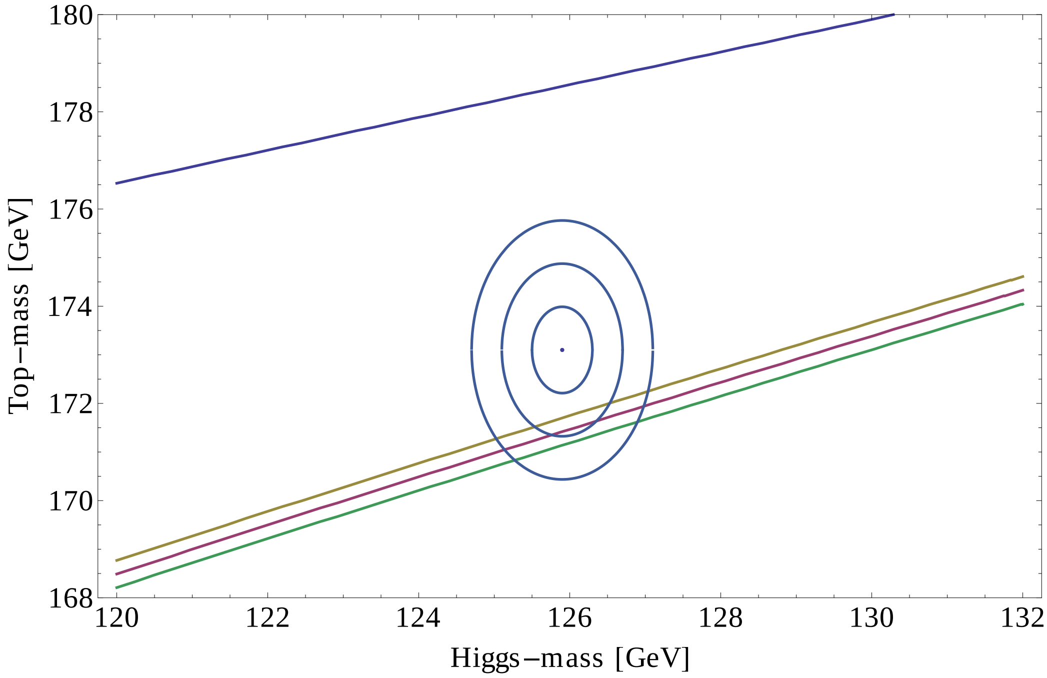


FIGURE 4.9: The plot shows the same as in figure 4.3 but with the more conservative values $m_t = (173.1 \pm 0.9)$ GeV and $m_h = (125.9 \pm 0.4)$ GeV. Metastability is favored through the central value of m_h and m_t. However, absolute stability is still allowed within less than 2-σ.

be small if one compares the gain in precession from e.g. two-loop order to three-loop order. Also new physics which is connected to the Planck scale which can be parametrized through non-renormalizable operators might significantly change the results of standard vacuum stability analysis. However, on experimental grounds the intrinsic error in the measurement of the top quark pole mass has to be reduced in order to see whether the vacuum is stable or not. Even if the values measured right now seem to indicate that meta-stability is preferred it is too early for a final statement. Measurements of the top quark pole mass/top Yukawa coupling at a future ILC provides the possibility to definitely answer the question of vacuum stability of the SM. Also run 2 of the LHC might help to solve the question of vacuum stability, not through a better measurement of the top mass but through the discovery of new physics between the electroweak scale and the Planck scale. One potential source of new physics is inflation which might be connected to an energy scale of $\mathcal{O}(10^{16}\,\text{GeV})$ which was suggested through the result of BICEP2 [14]. We will go on and review the standard results of inflation to see whether Higgs inflation is still a viable scenario in the context of the BICEP2 claim.

CHAPTER 5

STABILITY AND INFLATION IN THE LIGHT OF BICEP2

The aim of this chapter is to introduce the concept of inflation and study the impact of possible detection of primordial gravitational waves in the context of vacuum stability. The idea of inflation can be first found in references [70–72] and is today the leading working hypothesis to explain the boundary conditions of ΛCDM-model. Recently, there was the claim of detection of B-modes by the BICEP2 collaboration [14] which would be evidence for primordial gravitational waves generated by inflation. However, there is an ongoing debate whether the measurement implies a signal or not [73, 74]. We will first start to motivate inflation and then see the possible consequences of the BICEP claim in the context of vacuum stability in a Higgs inflation scenario. In order to have a solid ground we introduce the basic concepts of cosmology and then develop the tools to study inflationary scenarios.

The first building block of cosmology is the cosmological principle, which states that the Universe is homogeneous and isotropic on sufficiently large scales. Taking the field equations from general relativity with this assumption leads to the Friedmann equations which are the starting point for standard cosmology; the model which seems to be most compatible with observations is the ΛCDM-model. It works extremely nice as a fitting model; however, the initial conditions of the Universe have to be extremely fine-tuned in order to comply with observations. These problems go by the name of flatness problem and horizon problem. The flatness problem raises the question why our Universe is so flat nowadays.[1] The horizon problem states that patches of space-time which are causally disconnected also seem to be extremely homogeneous. In other words: Why do points in space-time which cannot talk to each other have (so many of) the same properties? This fine-tuning is a priori a clear shortcoming of the ΛCDM model and makes extensions desirable.

Inflation provides a dynamical explanation for the fine-tuning problems which arise in the context of standard cosmology. In this section we will present the flatness problem and the horizon problem in more detail and develop the idea of inflation as a solution. To explain the basic concepts of inflation we follow the lecture presented in [75] to give a short overview. Later we will explore the observables which have to be accommodated if one takes the idea of inflation as a serious possibility in the early Universe.

Throughout this chapter we will only consider single-field inflation, since multi-field goes beyond the scope of this work. However, one has to keep in mind that significant deviations of the relations presented in this section may be achieved if one starts with the hypothesis of a multi-field inflation scenario.

5.1 Standard Cosmology and Inflation

It is convenient to derive the Friedmann equations to have a solid footing on which we can start to talk about cosmology and finally about inflation.

5.1.1 Friedmann Equations

First, we take the Einstein equations of general relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}\,, \tag{5.1}$$

where the Einstein tensor $G_{\mu\nu}$ is defined as $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$. $R_{\mu\nu}$ and R are the Ricci tensor and the Ricci scalar, respectively and $T_{\mu\nu}$ is the energy-momentum tensor of the Universe. Λ is here the cosmological constant and G

[1] This corresponds to a fine-tuning problem since $\Omega = 1$ is an unstable fixpoint. See equation (5.8) below.

is the cosmological constant $G = 1/M_P^2$. Λ in equation (5.1) must not be confused with the instability scale of chapter 4. The gravitational constant Λ is very small $\Lambda \propto 10^{-122} M_P^4$ in Planck units [76]. It is not understood why it is so small, yet not zero. Comments on the value of the cosmological constant can be found in references [77, 78]. We will set $8\pi G = 1$ many times throughout this chapter. Now, we can derive the Friedmann equations under important assumptions. The assumptions are homogeneity, isotropy and a special form of a energy-momentum tensor

$$T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p), \tag{5.2}$$

which corresponds to a perfect fluid in a comoving frame. In equation (5.2) ρ and p are the matter energy density and the isotropic pressure, respectively. Renaming $\rho - \Lambda \to \rho$ and $p + \Lambda \to p$ together with these assumptions the Einstein equations (5.1) reduce to

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2}\,, \tag{5.3}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)\,. \tag{5.4}$$

Details about the employed Friedmann–Robertson–Walker (FRW) metric and cosmology in general can be found in appendix C. Putting equation (5.3) and (5.4) together we also obtain the continuity equation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0\,. \tag{5.5}$$

With this at hand we can proceed to the flatness and horizon problem.

5.1.2 Fine-tuning Problems in Cosmology

The flatness problem and the horizon problem are in principle no problems of ΛCDM, but it remains unclear why the initial conditions have to be fine-tuned like this in order to explain homogeneity and flatness. From a physical point of view it is not satisfactory that the theory itself cannot predict the homogeneity and isotropy in a natural way but these two conditions rather have to be put in by hand. This is what we will explore now.

5.1.2.1 Flatness Problem

If we take the Friedmann equation (5.3) we can rewrite it to make the flatness problem more apparent. We write it as

$$1 - \Omega(a) = \frac{-k}{(aH)^2}\,, \tag{5.6}$$

where

$$\Omega(a) \equiv \frac{\rho(a)}{\rho_{\text{crit}}(a)}\,, \qquad \rho_{\text{crit}}(a) \equiv 3H(a)^2\,. \tag{5.7}$$

Differentiation of equation (5.6) together with the continuity equation (5.5) yields

$$\frac{d\Omega}{d\ln a} = (1+3w)\Omega(\Omega-1)\,, \tag{5.8}$$

where w denotes the equation of state parameter, i.e. $w = p/\rho$. It is apparent from equation (5.8) that $\Omega = 1$ is an unstable fixpoint if $(1+3w) > 0$. The parameter Ω has to be fine-tuned very much in order to keep the system at this unstable fixpoint. From the dynamics it is not natural for the system to stay at this configuration: Already a little deviation from $\Omega = 1$ would change the evolution dramatically which is not compatible with observations.

5.1.2.2 Horizon Problem

The comoving horizon τ is defined to be the maximum distance a light ray can travel between the time $t = 0$ and the time $t = t'$:

$$\tau \equiv \int_0^{t'} \frac{dt}{a(t)} = \int_0^{a} \frac{da}{Ha^2} = \int_0^{a} d\ln a \left(\frac{1}{aH}\right)\,. \tag{5.9}$$

In equation (5.9) the comoving horizon was expressed in terms of the comoving Hubble radius, $(aH)^{-1}$. The comoving horizon τ corresponds to the causal horizon, which means that particles which are separated over a distance greater than τ could have *never* communicated with each other. Note the difference to the comoving Hubble radius: Particles which are separated over a distance greater than $(aH)^{-1}$ cannot communicate at *present*.

For a Universe which is dominated by a fluid the comoving Hubble radius takes the following form:

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}\,. \tag{5.10}$$

The important observation about equation (5.10) is the dependence on the sign of $(1+3w)$. Depending on positive or negative sign, the comoving Hubble radius grows or shrinks. This means that in a Big Bang scenario where the scale factor $a(t)$ grows monotonically, the comoving horizon τ scales like

$$\tau \propto a^{\frac{1}{2}(1+3w)} . \tag{5.11}$$

This means that if we look at the extreme situations of a matter dominated Universe (MD) ($p = 0$) or a radiation dominated Universe (RD) ($p = \frac{\rho}{3}$) we find for the comoving horizon τ

$$\tau = \int_0^a \frac{da}{Ha^2} \propto \begin{cases} a & \text{RD} \\ a^{1/2} & \text{MD} \end{cases} . \tag{5.12}$$

So we see that the comoving horizon grows monotonically in time. This means that light entering the horizon comes from causally disconnected patches. But the observation from CMB data tells us that the Universe was extremely homogeneous. How can this be?

The crucial point in the discussion above was the monotonically increasing comoving Hubble radius. It poses the serious problem to understand why causally disconnected patches of the Universe seem to be equally homogeneous. This is exactly where inflation steps in to change the evolution of the comoving Hubble radius.

5.1.3 Conditions for Inflation

Inflation provides a solution to the flatness and horizon problems presented above through an era of decreasing comoving Hubble radius. This shrinking comoving Hubble radius also implies an accelerated expansion of the Universe. Furthermore, it can be related to the pressure of the Universe in the following way

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \quad \Rightarrow \quad \frac{d^2a}{dt^2} > 0 \quad \Rightarrow \quad \rho + 3p < 0 . \tag{5.13}$$

Depending on which quantity one looks at one encounters the three equivalent criteria for successful inflationary models. Shrinking comoving Hubble radius, accelerated expansion of the Universe or negative pressure are these three criteria as we can see from equation (5.13). A closer look at the accelerated expansion reveals a relation between the second derivative of the scale parameter and the Hubble rate as

$$\frac{\ddot{a}}{a} = H^2(1-\varepsilon)\,, \qquad \text{where} \quad \varepsilon \equiv -\frac{\dot{H}}{H^2}\,. \tag{5.14}$$

This means that we can quantify the acceleration as

$$\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1\,, \tag{5.15}$$

where dN measures the number of e-folds which has to be around 60 in order to match with observations from the CMB because otherwise inflation would be too short.

In the next section we will give the general treatment of inflation in terms of a new scalar field called inflaton which has a potential with a certain shape, in order to meet observations.

5.2 Simple Models of Inflation

The idea of inflation has already been presented, but the open question is how to achieve the conditions for successful inflationary scenarios. In order to do this one usually introduces a new scalar field, the inflaton. The action is given through a minimal coupling of the inflaton to gravity as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] = S_{\rm EH} + S_\phi\,. \tag{5.16}$$

Assuming FRW metric (see appendix C) the scalar energy momentum tensor takes the form of a perfect fluid with pressure p and density ρ as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{5.17}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{5.18}$$

This leads to the equation of state

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}\,, \tag{5.19}$$

which opens the possibility for negative pressure for certain configurations of the potential $V(\phi)$. The potential has to dominate over the kinetic term $\frac{1}{2}\dot{\phi}^2$ to ensure a negative pressure. The dynamics of the scalar field and the FRW metric are governed by the equations

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \qquad \text{and} \qquad H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{5.20}$$

where the $3H\dot{\phi}$-term is a friction term. Since the potential $V(\phi)$ dominates over the kinetic term $\frac{1}{2}\dot{\phi}^2$, the conditions for the potential are formulated as slow-roll conditions where we already encountered the first slow-roll parameter in equation (5.15). In order to have a sufficiently long accelerated expansion of the Universe the friction term $3H\dot{\phi}$ in equation (5.20) as well as the derivative of the potential $V_{,\phi}$ have to dominate over the second derivative of ϕ, i.e. $|\ddot{\phi}| \ll |3H\dot{\phi}|, |V_{,\phi}|$. This can be ensured through a second slow-roll parameter which is

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN}. \tag{5.21}$$

Relating the two slow-parameters to the actual form of the potential yields

$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad \eta_{\rm v}(\phi) \equiv M_{\rm pl}^2\frac{V_{,\phi\phi}}{V}. \tag{5.22}$$

In order to have successful inflationary scenarios one has to match certain observations from the CMB. Important quantities which have to be met are the number of e-folds as $N \approx 60$ and the scalar spectral index $\Delta_s^2 \sim 10^{-9}$. Successful inflation also needs to fulfill the slow-roll parameters as $\epsilon_V, \eta_V < 1$. The claim of the BICEP2 collaboration is a tensor-to-scalar ratio of $r = 0.20^{+0.07}_{-0.05}$. All the requirements are summarized in table 5.1. See reference [13] for further details.

5.2.1 Cosmological Perturbations

Until now the physics of inflation is entirely described as a classical process. It may explain the boundary conditions of the ΛCDM-model and solve the horizon problem and the flatness problem but it does not give an explanation so far how structure in the Universe may be formed. Combining inflation with quantum mechanics provides a possibility to generate the initial seeds of all structure in the Universe. The formal derivation is cumbersome and involved and can be found for example in reference [75]. The important result which we quote here is the tensor-to-scalar ratio r which is given as

$$r \equiv \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\,\varepsilon_\star\,, \tag{5.23}$$

where $\Delta_{\rm t}^2$ is the amplitude of tensor perturbations in the CMB and $\Delta_{\rm s}^2$ the corresponding quantity for scalar perturbations. Note, the relation to the ϵ-parameter where the star denotes that it has to be evaluated at the horizon exit. This can be related to the energy density in the early Universe through

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\text{GeV}\,. \tag{5.24}$$

Equation (5.24) is an important relation to discuss the consequences of a possible detection of primordial graviational waves in the context of vacuum stability.

Quantity	Calculation	Value
ϵ_V	$\frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$	< 1
η_V	$M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V}$	< 1
Δ_s^2	$\approx \frac{1}{24\pi^2}\frac{V}{M_P^4}\frac{1}{\epsilon_{\rm v}}\Big\vert_{k=aH}$	10^{-9}
N	$\approx \int_{\phi_{\rm end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon_V}}$	60
r	$16\epsilon_V^\star = \frac{\Delta_t^2}{\Delta_s^2}$	$0.20^{+0.07}_{-0.05}$ (BICEP2 claim)

Table 5.1: Quantities in single-field slow-roll inflation which have to be met according to observations and theoretical predictions. The values given are the values during inflation. See reference [13] for further details.

5.3 BICEP2 Measurement and Possible Issues

In march 2014 the BICEP2 collaboration announced the detection of a large tensor-to-scalar ratio of $0.20^{+0.07}_{-0.05}$ [14], which would imply a very high energy density in the early Universe if one assumes a single-field inflation scenario.[2]

[2]From an economic point of view single-field inflation scenarios are desired since the theory loses its predictive power dramatically if one introduces multiple inflaton fields.

The status about the BICEP2 claim is not entirely clear: There are several criticisms that the dust foreground is not entirely understood by the collaboration [73, 74] and what the collaboration interprets as a signal might be compatible with the foreground. However, ongoing experiments like Keck Array and the expected presentation of data from PLANCK might clarify the situation. Furthermore, BICEP and PLANCK are working together now, so we will know about the fate of the claim soon.

However, until now the BICEP2 claim is only a claim that merits investigation in terms of possible consequences of a correct measurement and possible issues with the observation.

5.4 Higgs Inflation

The idea of Higgs inflation (HI) is rather simple and seems quite natural: Since we now know one elementary scalar particle, the Higgs, we take the Higgs as the inflaton. This is an economic solution since inflationary scenarios introducing a new particle suffer from a hierarchy problem. The idea of HI is not new and was presented even before the Higgs discovery [79]. At first sight the Higgs potential does not seem to be a good candidate since we need a flat potential for slow roll. Obviously, the Higgs potential in the SM is not flat but a big non-minimal coupling to gravity might help to obtain a scalar potential compatible with observations regarding inflation.

The first key which maybe provides the possibility to take the Higgs as the inflaton is a modification of the gravitational interaction. Scalar particles may be coupled non-minimally to gravity in the following way which modifies the action as

$$\delta S_{\mathrm{NM}} = \int d^4x \sqrt{-g}\Big[-\xi \Phi^\dagger \Phi R \Big], \tag{5.25}$$

where R is the Ricci scalar and Φ is a scalar particle. As a consequence the action of the SM Higgs changes; it is given in the Jordan frame as

$$S_J = \int d^4x \sqrt{-g}\left[-\frac{M^2}{2}R - \xi\frac{h^2}{2}R + \frac{\partial_\mu h \partial^\mu h}{2} - V(h)\right], \tag{5.26}$$

with the standard Higgs potential given by

$$V(h) = \frac{\lambda}{4}(h^2 - v^2)^2\,. \tag{5.27}$$

In equation (5.26) and (5.27) h denotes the Higgs field and v the vacuum expectation value as introduced in chapter 3, where both are written down in the Jordan frame. However, the non-minimal coupling to gravity poses problems, since the equations of motion are coupled and computations become hard. To avoid this unfortunate situation one performs a conformal transformation which gives the theory in the Einstein frame. The consequence is that the non-minimal coupling ξ is removed and the relations deduced earlier are applicable and allow an analysis at tree-level. The conformal transformation is defined to be

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = \frac{M^2 + \xi h^2}{M_P^2}, \tag{5.28}$$

resulting in an effective Planck mass $M_{P,\text{eff}} \sim M^2 + \xi h^2$. A difference between M and M_P is apparent if the VEV $\langle h \rangle = v$ is non-zero.[3] The drawback of the transformation (5.28) is that the kinetic term for the Higgs has a non-minimal mixing. Therefore one replaces the scalar h by a canonically normalized scalar χ such that

$$\frac{\partial\chi}{\partial h} = \sqrt{\frac{\Omega^2 + \frac{3}{2}M_P^2(\Omega^2)'^2}{\Omega^4}} = \sqrt{\frac{1 + (\xi + 6\xi^2)h^2/M_P^2}{(1 + \xi h^2/M_P^2)^2}} \tag{5.29}$$

holds. The transformation and the redefinition lead to the theory in the Einstein frame as

$$S_E = \int d^4x\sqrt{-\hat{g}}\left[-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\right]. \tag{5.30}$$

The price to pay (which is actually desired) is a change of the form of the potential as well as changes of the metric and the Ricci scalar.[4] The consequence of the non-minimal coupling of the Higgs to gravity is that the potential becomes flat in the region of interest. This is shown in figure 5.1.

The potential $U(\chi)$ for large field values as $h, \chi \gg M_P/\xi$ is given as

$$U(\chi) \simeq \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2. \tag{5.31}$$

[3] The difference between M_P and $M_{P,\text{eff}}$ is negligible in most of the cases, because $\xi v \ll M_P$.

[4] Beyond tree-level the situation is much more complicated, since it is not entirely clear how to treat the theory due to non-renormalizablity. The equivalence of the theory in Einstein frame and Jordan frame beyond tree-level has not yet been studied very intensively. Some comments can be found in reference [80].

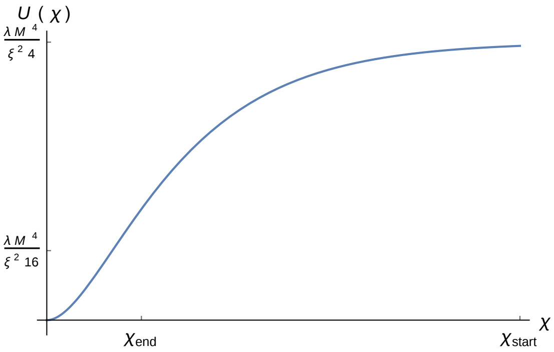


FIGURE 5.1: Sketch of the potential in HI in the Einstein frame viable for large field region $h, \chi \gg M_P/\xi$ with fine-tuned top mass which accounts for absolute stability and constant λ.

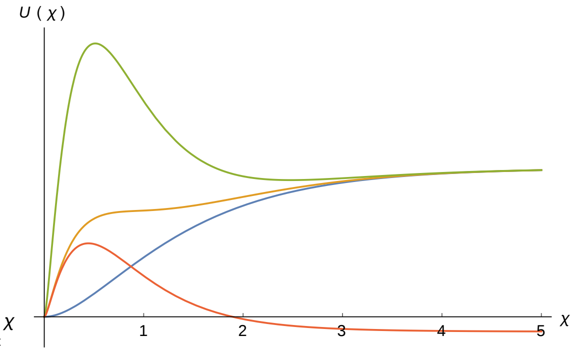


FIGURE 5.2: The schematic change of the effective potential in the Einstein frame depending on the value of λ_0 relative to $b/16$. Green: $\lambda_0 \gg \frac{b}{16}$, yellow: $\lambda_0 = \frac{b}{16}$, blue: $\lambda_0 \ll \frac{b}{16}$, dark orange: unstable potential for $\lambda_0 < 0$.

However, for small field values $h, \chi \ll M_P/\xi$ the potential turns into the usual SM quadratic potential. In order to fulfill the constraints from observations the non-minimal coupling to gravity is not arbitrary and is $\xi \simeq 47000$ if only tree-level contributions are taken into account [81]. This is a really big coupling and one should be rather skeptical whether this theory makes sense for perturbative evaluation.

Generic tree-level predictions of HI are the correct amount of e-folds and a successful slow-roll. Furthermore, on tree-level the prediction is a small tensor-to-scalar ratio r of $\mathcal{O}(0.003)$. However, the situation changes for critical values of the Higgs mass and top mass.[5] The BICEP2 measurement, if taken seriously, provides the possibility to determine the top quark pole mass on the grounds of inflation and not on the grounds of the question of vacuum stability. However, in the end also this boils down to the question whether the self-coupling λ is positive up to the energy scale of inflation: otherwise the quantum fluctuations in the early Universe would drive the system into the vacuum at high field values and not into the electroweak vacuum. This will be discussed in more detail in section 5.5.

Furthermore, it should be pointed out that absolute stability of the electroweak vacuum is indispensable for any consideration of HI. The top quark pole mass has to be tuned in order to guarantee that there is no instability at a high scale. As we will see this tuning gets even worse if one tries to accommodate a high value for r in the theory.

[5] A high amount of fine-tuning for ξ and m_t is necessary to achieve this.

The BICEP2 claim initiated the study of HI beyond tree-level and some interesting results were presented in reference [82–84] which we want to present briefly here.

Starting point is the parametrization of the renormalization-group-improved effective potential in the Einstein frame as in (5.31). The running of the self-coupling λ can be parametrized as

$$\lambda(z) = \lambda_0 + b\,(\log z)^2\,, \tag{5.32}$$

where $z = \frac{\mu}{qM_P}$. Now the important point is that the self-coupling λ as well as its β-function are close to zero near the Planck scale. Evaluating the self-coupling at $z' = \frac{1}{\kappa}(1 - e^{\frac{2}{\sqrt{6}}\frac{\chi}{M_P}})$ and varying the values for λ_0 and b yields the effective potential in the Einstein frame. In the end the unphysical parameters λ_0 and b have to be translated into physical quantities such as m_h and m_t. A sketch of the influence of the parameters on the potential in the Einstein frame is given in figure 5.2.

A more profound analysis of the loop contribution during HI can be found in reference [82] where an analysis of HI at the critical point is done. This corresponds to the point where $\lambda_0 = \frac{b}{16}$ in figure 5.2. The authors find that the non-minimal coupling can be reduced to $\xi \simeq \mathcal{O}(10)$ whereas the tensor-to-scalar ratio r is also compatible with the BICEP2 result. The tensor-to-scalar ratio r and the scalar spectral index n_s depending on the parameters ξ and κ can be found in the figures 5.3 and 5.4. These figures show that HI can account for a tensor-to-scalar ratio of $\mathcal{O}(0.1)$ since the shape of the potential develops a high sensitivity on the parameters of the theory near the critical point. The correct amount of e-folds and the slow roll conditions also have been checked.

The interesting question now is if the tuning of the inflationary theory is compatible with observations from particle physics. Since HI is closely connected to the form of the SM potential one expects a dependence of the potential during HI on the top mass m_t in the analogy to the dependence of the SM potential on the top mass m_t.

However, since one performs a conformal transformation, the relation between the measured values of the top mass m_t and the relevant value for the top mass during inflation is not obvious. One cannot directly use the measured values in the Einstein frame since the field χ does not correspond to the Higgs field. The main

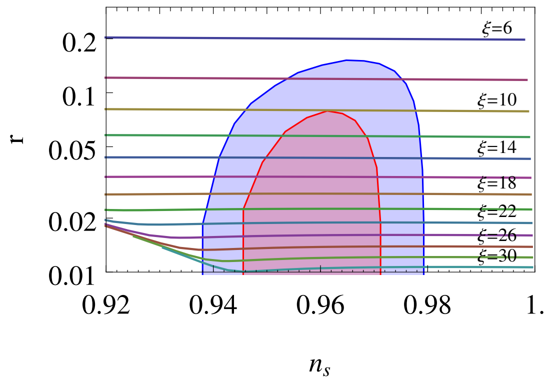


FIGURE 5.3: Inflationary indexes r and n_s which depend on ξ and κ. λ_0 is fixed from COBE normalization. Along the lines ξ is fixed and κ varies between $\{0.9, 1.1\}$. Also the PLANCK result is displayed with 1 and 2 σ contours [13]. This plot is taken from reference [82].

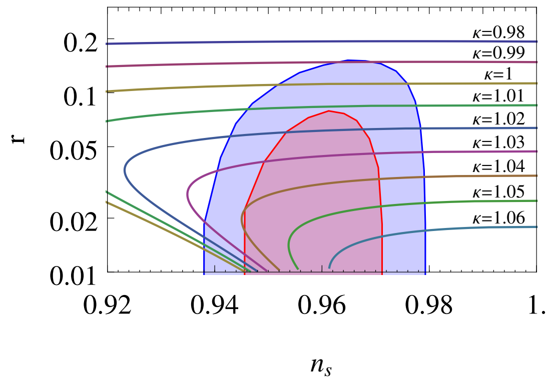


FIGURE 5.4: Same plot as in figure 5.3 with constant κ. ξ varies between $\{5, 30\}$. This plot is taken from reference [82].

problem is that the SM in the Einstein frame is non-polynomial and thus non-renormalizable. This reflects the fact that further assumptions have to be made in order to connect the SM during inflation and the low energy theory of the SM. In this case the assumption is that there is no new physics between the electroweak scale and the Planck scale in order to be able to perform the RG running of the SM.

To start to make the connection between the theory in the Einstein frame for inflation and the SM at low energies, one sees that the interaction between the Higgs field and the top quark is given as [82]

$$L = \frac{y_t}{\sqrt{2}} \bar{t} t F(\chi), \tag{5.33}$$

where $F(\chi) = \frac{h}{\Omega}$. Note that the field h is not canonically normalized and has to be connected to the field χ with the help of equation (5.29). The aim is the removal of divergencies in the arbitrary background fields which can be achieved by adding counterterms in the action. This results in a modification of the $\bar{t}th$ vertex through a change of the top Yukawa coupling y_t and the self-coupling λ

$$y_t \to y_t + \frac{y_t^3}{16\pi^2}\left(\frac{3}{\epsilon} + C_t\right) F'^2 \tag{5.34}$$

$$\lambda \to \lambda - \frac{y_t^4}{16\pi^2}\left(\frac{6}{\epsilon} - C_\lambda\right) F'^4, \tag{5.35}$$

where the modification of λ comes from the top Yukawa contribution in the evolution of λ. ϵ is the parameter of dimensional regularization, C_t and C_λ are the constant parts of the counterterms and $F' = dF/d\chi$. Note that these constant parts cannot be fixed from theoretical calculation but rely on observations or on a UV complete theory which hosts the SM at low energies which is not known to us today. It holds for small field values $h \ll M_P/\xi$ that $F' \approx 1$. As a consequence the parameter C_t and C_λ are absorbed in the low energy definition of y_t and λ which means that they cannot be observed. However, in the inflationary region where $h > M_P/\xi$ it holds $F' \to 0$. This means that only the left-hand side of equations (5.34) and (5.35) contribute in during inflation. To make the transition between these two regimes suitable approximations can be made according to reference [82]. The consequence are modified masses during inflation which are given as

$$m_t^* = m_t \left(1 - \frac{y_t^2 C_t}{16\pi^2}\right), \quad m_h^* = m_h \left(1 - \frac{y_t^4 C_\lambda}{16\pi^2} \frac{v^2}{m_h^2}\right). \tag{5.36}$$

The presence of the constants C_λ and C_t reflects the uncertainty which arises from the non-minimal coupling to gravity which renders the theory to be not renormalizable. Combining now particle physics with cosmology they are able to fix the unknown parameters C_t and C_λ. They conclude as an example to achieve $r = 0.12$ with $m_h^* \simeq 122.6$ GeV and $m_t^* \simeq 169.8$ GeV, $\xi \simeq 8$. In order to match with observables as Higgs mass and top mass one can fix $C_\lambda \simeq 1$ and $C_t \simeq 1.5$ to get $m_h = 125.6\,\text{GeV}$ and a top mass $m_t \simeq 171.5\,\text{GeV}$ which is consistent within 2 σ of the measured value.

5.5 Stability and Higgs Inflation

A high tensor-to-scalar ratio implies big quantum fluctuations of the Higgs in the early Universe which makes our Universe highly unlikely as pointed out in [85]. They point out that the survival probability of the SM vacuum is given as

$$P_\Lambda \propto e^{-\frac{H_\star^2 N}{32\Lambda^2}}, \tag{5.37}$$

where $H_\star$ is the Hubble rate during inflation which is assumed to be constant.

The price to pay is a high amount of fine-tuning in the top mass to make HI still a viable scenario. However, this fine-tuning is within reasonable errors of the top-quark pole mass. The whole point is to make the electroweak potential absolutely stable on the one hand and tune the value of the top mass in order to get the right answers for inflation on the other hand.

The situation is the following: The energy density in the early Universe was very high if the BICEP2 claim is right which one can see from equation (5.24): $V \simeq 10^{16}\,\text{GeV}$. This would imply that we would rather end up in the minimum at the high scale and not in the electroweak one. An easy "way out" is the fine-tuning of the top mass in order to obtain absolute stability for the SM potential. Other authors also deal with the topic of HI in the context of vacuum stability see for example reference [86].

From an effective field theory point of view the situation in HI is not satisfactory. In order to accommodate the correct value of the tensor-to-scalar ratio r large field inflation scenarios are preferred also in HI. Large field inflation scenarios involve trans-Planckian field values which cannot be understood from standard quantum field theoretic considerations. If one takes QFT seriously any kind of non-renormalizable operators which appear due to an effective field theory approach should appear in the potential. These non-renormalizable operators would be the dominant contribution for large field inflation and should have dramatic impact on the behavior of the potential in the Einstein frame. Only with a high amount of fine-tuning these operators could be kept under control in the region of interest. However, in order to avoid a reasoning like this one can introduce a shift-symmetry at the Planck scale which would forbid such terms.

This brings a connection to section 4.5 where the impact of non-renormalizable operators on the SM potential was discussed. It is clear that even a small value for the coupling of the non-renormalizable operators has a severe impact on the potential and therefore on the discussion whether HI is still a viable scenario. This fact is also observed in reference [87] where the authors carry out an analysis of the effective potential augmented by non-renormalizable operators in the context of HI. However, they neglect a non-minimal gravitational coupling of the Higgs to the Ricci scalar which seems to be indispensable in order to accommodate the theory with observation.

A similar study of HI for a Higgs mass $m_h \simeq 126\,\text{GeV}$ has been carried out in [88]. Going beyond the usual assumptions of HI by adding conformal symmetry a study if HI can be a viable scenario is presented in reference [89]. They conclude that it is hard to accommodate the theory with the BICEP2 claim in this framework.

5.6 Testability of the Great Desert Scenario in the Context of Inflation

Since the top quark pole mass plays the decisive role in the considerations of whether the SM can be a viable theory up to high scales and whether HI might be the correct scenario to explain the evolution of the early Universe an exact determination of the top quark pole mass is necessary. This might be done for example at a future linear collider like the international linear collider (ILC). The determination of a top quark pole mass at the ILC provides the possibility to rule out HI (provided the BICEP2 claim turns out to be a discovery) but it also gives the possibility to probe the self-consistency of the SM in terms of vacuum stability.

The fine-tuning in order to make HI still a viable scenario in the context of the top quark pole mass is rather high. Only within a range of a few hundred MeV HI is still a viable scenario provided that the high tensor-to-scalar ratio claimed by BICEP2 turns out to be correct. This is then a sharp and precise "prediction" which provides a possibility to rule out HI. However, if the tensor-to-scalar ratio is not as high as claimed by BICEP2 this sharp constraint from a theoretical perspective loses its power. The task to rule out HI becomes a harder again but is still possible with a good measurement of the top mass.

Clearly any sign of BSM which is connected to a new physics scale would rule out the hypothesis that the SM is valid all the way up to the Planck scale. If this new physics is connected to a big amount of fine-tuning we cannot argue anymore that the hierarchy problem is actually a problem and the main necessity/attractiveness for HI is gone.

Table 5.2 summarizes the possibilities depending on the future of the BICEP2 claim and the actual value of the top quark mass. If no new physics is found in LHC run 2 naturalness arguments become weaker and weaker and fine-tuning seems to be unavoidable if new QFT-like physics should be accommodated. At the moment the most promising route to go seems to be the determination of the

	SM	SM + HI
$m_t > 173\,\mathrm{GeV} + r$ big	$(\surd)$ not stable	↯ ⇒New physics needed
$m_t \sim 171\,\mathrm{GeV} + r$ big	$\surd$ stable	$(\surd)$ ⇒Tuning of m_t of $\mathcal{O}(100\,\mathrm{MeV})$
$m_t > 173\,\mathrm{GeV} + r$ small	$(\surd)$ not stable	↯ ⇒New physics needed
$m_t \sim 171\,\mathrm{GeV} + r$ small	$\surd$ stable	$\surd$ ⇒Tuning of m_t

Table 5.2: Overview of the different possibilities of possible measurements of the top quark mass and the tensor-to-scalar ratio r. Note that the fine-tuning in order to make HI a viable scenario in the top mass is very big if r is big. The table assumes no other new physics is found. A tuning of the non-minimal coupling to gravity ξ is always implied. Note further that more assumptions about the underlying UV-completion of the SM have to be made in order to connect the measured Higgs and top mass to inflation.

top mass m_t and/or the top Yukawa coupling y_t. Deviations in the top Yukawa coupling from SM calculations may also give a hint for new physics.

Right now the SM seems to be perfectly fine if one accepts the discussion of meta-stability which is maybe a little bit too optimistic at the moment assuming optimistic errors of the top mass.

CHAPTER 6

CONCLUSION AND OUTLOOK

The Standard Model, with slight extensions not related to high scale physics, might be able to explain all observations in nature. Augmenting the SM with three right-handed neutrinos has no severe consequences in terms of vacuum stability.[1] We reviewed the standard results concerning vacuum stability and saw that a metastable vacuum is a viable scenario if there is no new scale between the Planck scale and the electroweak scale. However, the question whether the vacuum is actually stable or metastable cannot be answered today since the error in the top quark pole mass is too big. The issues with the definition and the measurement of a top quark pole mass have been reviewed. Furthermore, the impact of additional non-renormalizable operators has been discussed, as well as the impact of Weyl consistency relations.

In the context of vacuum stability understanding arising scheme-dependences and the correct use of the effective potential might still be improved. Some authors even claim the vacuum to be unstable. They come to this result through a mass-dependent renormalization scheme but they do not manage yet to treat everything in a consistent way since they are forced to use $\overline{\text{MS}}$-running for the RGEs beyond 1-loop [90, 91]. One should be skeptical about the result and wait for further improvement of these techniques. Other authors also see hints that the SM vacuum is actually stable [92–94] using the RG flow technique. However, also this treatment is not complete yet since the actual computation for the SM is rather involved and not done yet. So from a theoretical perspective the situation is

[1] A Majorana term which gives rise to seesaw type-I is suppressed.

not so clear and one should wait for improvements in the techniques and actually apply them to the SM rather than to toy models. Perturbation theory actually seems to be a good developed technique on which one can rely.

The actual problem in determining the fate of the SM vacuum seems to be actually more an experimental problem in determining the top quark pole mass. Future experiments, possibly carried out at an ILC, might help to improve the understanding of a top quark pole mass as well as reduce the error bars, which are crucial to judge the stability. But also any other deviation from SM physics might provide a hint on how to solve this uncomfortable situation of vacuum stability.

The situation that there is no new physics between the Planck scale and the electroweak scale might not be true and a possible detection of a high tensor-to-scalar ratio from the BICEP2 experiment would imply a very high energy density of $\mathcal{O}(10^{16}\,\mathrm{GeV})$ in the early universe near the GUT scale. The general predictions of inflation have been reviewed and the possible consequences of the BICEP2 claim have been stated. From a naive tree-level analysis Higgs inflation seems to be ruled out. However, analysis beyond tree-level might still give the possibility to accommodate theoretical predictions with observations. It is crucial to understand that going beyond tree-level is a hard task due to the non-renormalizability of the theory which is introduced because of a non-minimal coupling to gravity of the Higgs and the ambiguities which come from the transformation between Jordan frame to Einstein frame. Without further assumptions one cannot say anything about quantum corrections in Higgs inflation. This already shows how delicate business is, since we almost know nothing about a possible UV-completion of the SM.

For the future a better understanding of inflation is very important. Most analysis is only carried out at tree-level and the proper treatment of quantum corrections is hard. Even simple inflationary scenarios which seem to be good at tree-level have to be fine tuned very much because they suffer from a hierarchy problem once quantum corrections are taken into account, which one should do for more realistic scenarios.

The nature of reheating, i.e. the process where the inflaton decays into the SM particles, is not well understood and an improvement in terms of quantum field theoretic treatment is highly desirable and might also help to exclude models of inflation which seem to be reliable today.

If the BICEP2 claim of a large tensor-to-scalar ratio of $r \sim \mathcal{O}(0.1)$ is confirmed from other experiments and the top mass is above the critical value excluding absolute stability this poses a new serious problem for the SM. Our existence in the universe would then not be understood because of the quantum fluctuations in the early universe should have driven us into the vacuum at the high scale rather than in the electroweak vacuum. But also if the BICEP2 claim turns out to be wrong, the question of vacuum stability is still important. Is a theory with a long-lived metastable vacuum really okay? And is that actually the case for the SM? If one wants to build theories which provide absolute stability new physics has to step in below the instability scale. This might be motivated also in terms of the known problems which are not solved in the pure SM.

Even if there is no sign of new physics right now, we cannot be sure that the simplest explanations for the problems of the SM are correct and further investigation on the experimental side as well as in theory has to be done. Theory should focus equally on better understanding on formal grounds as well as phenomenologially motivated extensions of the SM which should be preferably testable in future experiments.

LHC Run 2 next year provides the possibility to probe the SM further and eventually BSM signs show up which may give the possibility to rule out the great desert scenario between the electroweak scale and the Planck scale. But also low-energy experiments where the new physics may show up in loops provide the possibility to find BSM contributions which would also be good news.

APPENDIX A

DEFINITIONS, CONVENTIONS AND DETAILS

A.1 Metric and Unit Convention

The metric used in this work is

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} , \tag{A.1}$$

which can be used to raise and lower Lorentz indices. The unit convention used here implies $\hbar = c = 1$, which goes by the name of natural units. If one multiplies the quantity under consideration with suitable powers of $\hbar$ and c and replaces their values by the SI convention one obtains the quantity under consideration in SI units. For example, a mass of 125 GeV corresponds to $125\,\frac{\mathrm{GeV}}{\mathrm{c}^2} = 2.2 \times 10^{-25}\,\mathrm{kg}$.

A.2 Calculational Details in Dimensional Regularization

Calculation of the area of a d-dimensional unit sphere:

$$\left(\sqrt{\pi}\right)^d = \left(\int dx\; e^{-x^2}\right)^d = \int d^d x \exp\left(-\sum_{i=1}^{d} x_i^2\right) \tag{A.2}$$

$$= \int d\Omega_d \int_0^\infty dx\; x^{d-1} e^{-x^2} = \left(\int d\Omega_d\right) \cdot \frac{1}{2} \int_0^\infty d(x^2)(x^2)^{\frac{d}{2}-1} e^{-(x^2)} \tag{A.3}$$

$$= \left(\int d\Omega_d\right) \cdot \frac{1}{2} \Gamma(d/2). \tag{A.4}$$

As a result we get:

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}. \tag{A.5}$$

Further details to the calculation in 2.3.2 are given here:

$$\int_0^\infty dl \frac{l^{d-1}}{(l^2+\Delta)^2} = \frac{1}{2} \int_0^\infty d(l^2) \frac{(l^2)^{\frac{d}{2}-1}}{(l^2+\Delta)^2} \tag{A.6}$$

$$= \frac{1}{2} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \int_0^1 dx\; x^{1-\frac{d}{2}} (1-x)^{\frac{d}{2}-1}. \tag{A.7}$$

A.3 Clifford Algebra and Dirac Matrices

The Dirac matrices satisfy the Clifford algebra. They are given as

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{A.8}$$

with the anti-commutator $\{\cdot,\cdot\}$. Another identity the γ-matrices have to satisfy is

$$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 = \gamma_\mu. \tag{A.9}$$

The *chiral* representation of the Dirac matrices is given by

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}. \tag{A.10}$$

A shorthand notation for the chiral representation of the γ-matrices is

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{A.11}$$

where $\sigma^\mu \equiv (\mathbb{1}, \vec{\sigma})$ and $\bar{\sigma}^\mu \equiv (\mathbb{1}, -\vec{\sigma}) = \sigma_\mu$. The chiral projection operators are defined as

$$P_L = \frac{1}{2}(\mathbb{1} - \gamma^5), \qquad P_R = \frac{1}{2}(\mathbb{1} + \gamma^5). \tag{A.12}$$

A.4 Covariant Derivatives of the Standard Model Fields

We give here only the gauge transformation under the electroweak gauge group, since the QCD part is not important for this work. The covariant derivatives of the SM fields (and right-handed neutrino ν_R) are defined as follows

$$D_\mu Q_L = \left(\partial_\mu + \frac{ig}{2}\vec{\sigma}\cdot\vec{W}_\mu + \frac{ig'}{6}B_\mu\right) Q_L, \tag{A.13a}$$

$$D_\mu L_L = \left(\partial_\mu + \frac{ig}{2}\vec{\sigma}\cdot\vec{W}_\mu - \frac{ig'}{2}B_\mu\right) L_L, \tag{A.13b}$$

$$D_\mu u_R = \left(\partial_\mu + \frac{2ig'}{3}B_\mu\right) u_R, \tag{A.13c}$$

$$D_\mu d_R = \left(\partial_\mu - \frac{ig'}{3}B_\mu\right) d_R, \tag{A.13d}$$

$$D_\mu e_R^- = \left(\partial_\mu - ig'B_\mu\right) e_R^-, \tag{A.13e}$$

$$D_\mu \nu_R = \partial_\mu \nu_R. \tag{A.13f}$$

A.5 Currents of the Standard Model

The charged and neutral currents of the SM are given as:

$$J_W^{\mu+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L), \tag{A.14}$$

$$J_W^{\mu-} = \frac{1}{\sqrt{2}}(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L), \tag{A.15}$$

$$J_Z^\mu = \frac{1}{\cos\theta_W}\left[\bar{\nu}_L\gamma^\mu\left(\frac{1}{2}\right)\nu_L + \bar{e}_L\gamma^\mu\left(-\frac{1}{2}+\sin^2\theta_W\right)e_L + \bar{e}_R\gamma^\mu(\sin^2\theta_W)e_R\right. \tag{A.16}$$

$$+\bar{u}_L\gamma^\mu\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)u_L + \bar{u}_R\gamma^\mu\left(-\frac{2}{3}\sin^2\theta_W\right)u_R \tag{A.17}$$

$$\left.\bar{d}_L\gamma^\mu\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)d_L + \bar{d}_R\gamma^\mu(\frac{1}{3}\sin^2\theta_W)d_R\right] \tag{A.18}$$

$$J_{EM}^\mu = \bar{e}\gamma^\mu e + \bar{u}\gamma^\mu\left(+\frac{2}{3}\right)u + \bar{d}\gamma^\mu\left(-\frac{1}{3}\right)d \tag{A.19}$$

Where implicit summation over the three generations are understood.

APPENDIX B

STANDARD MODEL RELATIONS

B.1 Weyl Consistency Relations for the SM

$$2\frac{\partial}{\partial\alpha_t}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_t}{\alpha_t}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.1}$$

$$4\frac{\partial}{\partial\alpha_1}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_1}{\alpha_1^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.2}$$

$$\frac{4}{3}\frac{\partial}{\partial\alpha_2}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.3}$$

$$2\frac{\partial}{\partial\alpha_1}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_1}{\alpha_1^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.4}$$

$$\frac{2}{3}\frac{\partial}{\partial\alpha_2}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.5}$$

$$\frac{1}{4}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.6}$$

$$\frac{1}{3}\frac{\partial}{\partial\alpha_2}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial\alpha_1}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.7}$$

$$\frac{1}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial\alpha_1}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.8}$$

$$\frac{3}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_2}{\alpha_2^2}\right) = \frac{\partial}{\partial\alpha_2}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right) \tag{B.9}$$

$$\beta_1 = 2\alpha_1^2\Bigg\{\frac{1}{12}+\frac{10n_G}{9}+\left(\frac{1}{4}+\frac{95n_G}{54}\right)\alpha_1+\underbrace{\left(\frac{3}{4}+\frac{n_G}{2}\right)\alpha_2}_{Eq.\ (B.7)}+\underbrace{\frac{22n_G}{9}\alpha_3}_{Eq.\ (B.8)}$$
$$+\left(\frac{163}{1152}-\frac{145n_G}{81}-\frac{5225n_G^2}{1458}\right)\alpha_1^2+\left(\frac{87}{64}-\frac{7n_G}{72}\right)\alpha_1\alpha_2-\frac{137n_G}{162}\alpha_1\alpha_3$$
$$+\left(\frac{3401}{384}+\frac{83n_G}{36}-\frac{11n_G^2}{18}\right)\alpha_2^2+\left(\frac{1375n_G}{54}-\frac{242n_G^2}{81}\right)\alpha_3^2-\frac{n_G}{6}\alpha_2\alpha_3$$
$$+\alpha_t\Bigg[\underbrace{-\frac{17}{12}}_{Eq.\ (B.4)}-\frac{2827}{576}\alpha_1-\frac{785}{64}\alpha_2-\frac{29}{6}\alpha_3+\left(\frac{113}{32}+\frac{101n_t}{16}\right)\alpha_t\Bigg]$$
$$+\underbrace{\alpha_\lambda\left(\frac{3}{4}\alpha_1+\frac{3}{4}\alpha_2-\frac{3}{2}\alpha_\lambda\right)}_{Eq.\ (B.2)}\Bigg\}, \tag{B.10}$$

$$\beta_2 = 2\alpha_2^2\Bigg\{-\frac{43}{12}+\frac{2n_G}{3}+\underbrace{\left(\frac{1}{4}+\frac{n_G}{6}\right)\alpha_1}_{Eq.\ (B.7)}+\left(-\frac{259}{12}+\frac{49n_G}{6}\right)\alpha_2+\underbrace{2n_G\alpha_3}_{Eq.\ (B.9)}$$
$$+\left(\frac{163}{1152}-\frac{35n_G}{54}-\frac{55n_G^2}{162}\right)\alpha_1^2+\left(\frac{187}{64}+\frac{13n_G}{24}\right)\alpha_1\alpha_2-\frac{n_G}{18}\alpha_1\alpha_3$$
$$+\left(-\frac{667111}{3456}+\frac{3206n_G}{27}-\frac{415n_G^2}{54}\right)\alpha_2^2+\frac{13n_G}{2}\alpha_2\alpha_3+\left(\frac{125n_G}{6}-\frac{22n_G^2}{9}\right)\alpha_3^2$$
$$+\alpha_t\Bigg[\underbrace{-\frac{3}{4}}_{Eq.\ (B.5)}-\frac{593}{192}\alpha_1-\frac{729}{64}\alpha_2-\frac{7}{2}\alpha_3+\left(\frac{57}{32}+\frac{45n_t}{16}\right)\alpha_t\Bigg]$$
$$+\underbrace{\alpha_\lambda\left(\frac{1}{4}\alpha_1+\frac{3}{4}\alpha_2-\frac{3}{2}\alpha_\lambda\right)}_{Eq.\ (B.3)}\Bigg\}, \tag{B.11}$$

$$\beta_3 = 2\alpha_3^2\Bigg\{-\frac{11}{2}+\frac{2n_G}{3}+\underbrace{\frac{11n_G}{36}\alpha_1}_{Eq.\ (B.8)}+\underbrace{\frac{3n_G}{4}\alpha_2}_{Eq.\ (B.9)}+\left(-51+\frac{38n_G}{3}\right)\alpha_3$$
$$+\left(-\frac{65n_G}{432}-\frac{605n_G^2}{972}\right)\alpha_1^2-\frac{n_G}{48}\alpha_1\alpha_2+\frac{77n_G}{54}\alpha_1\alpha_3+\left(\frac{241n_G}{48}-\frac{11n_G^2}{12}\right)\alpha_2^2$$
$$+\frac{7n_G}{2}\alpha_2\alpha_3+\left(-\frac{2857}{4}+\frac{5033n_G}{18}-\frac{325n_G^2}{27}\right)\alpha_3^2$$
$$+\alpha_t\Bigg[\underbrace{-1}_{Eq.\ (B.6)}-\frac{101}{48}\alpha_1-\frac{93}{16}\alpha_2-20\alpha_3+\left(\frac{9}{4}+\frac{21n_t}{4}\right)\alpha_t\Bigg]\Bigg\}, \tag{B.12}$$

$$\beta_t = 2\alpha_t \Bigg\{ \frac{9}{4}\alpha_t - \underbrace{4\alpha_3}_{Eq.\ (B.6)} - \underbrace{\frac{17}{24}\alpha_1}_{Eq.\ (B.4)} - \underbrace{\frac{9}{8}\alpha_2}_{Eq.\ (B.5)} + \underbrace{3\alpha_\lambda^2 - 6\alpha_t\alpha_\lambda}_{Eq.\ (B.1)} - 6\alpha_t^2 + 18\alpha_3\alpha_t$$
$$+ \alpha_3^2\Big(-\frac{202}{3} + \frac{40n_G}{9}\Big) + \alpha_t\Big(\frac{131}{32}\alpha_1 + \frac{225}{32}\alpha_2\Big) + \frac{1187}{432}\alpha_1^2 - \frac{3}{8}\alpha_1\alpha_2$$
$$+ \frac{19}{18}\alpha_1\alpha_3 - \frac{23}{8}\alpha_2^2 + \frac{9}{2}\alpha_3\alpha_2 \Bigg\}, \tag{B.13}$$

$$\beta_\lambda = \underbrace{\frac{9}{16}\alpha_2^2 - \frac{9}{2}\alpha_\lambda\alpha_2}_{Eq.\ (B.3)} + \underbrace{\frac{3}{16}\alpha_1^2 - \frac{3}{2}\alpha_\lambda\alpha_1}_{Eq.\ (B.2)} + \underbrace{\frac{3}{8}\alpha_1\alpha_2}_{Eqs.\ (B.2-B.3)} + 12\alpha_\lambda^2$$
$$+ \underbrace{6\alpha_\lambda\alpha_t - 3\alpha_t^2}_{Eq.\ (B.1)} . \tag{B.14}$$

n_G stands for the number of generations, which we set to three and n_t represents the number of top-like quarks, which is taken to be one. The coloring should make it easier to see the different contributions coming from equation (B.1-B.9). Below each term the relevant equation is noted. Note that only the β-functions for 3 loops in gauge, 2 loops in top Yukawa and 1 loop the self-coupling are displayed [52].

B.2 Matching Conditions in Different Loop Orders

$\overline{\mu} = m_t$	λ	y_t	g_2	g_Y
LO	0.13023	0.99425	0.65294	0.34972
NLO	0.12879	0.94953	0.64755	0.35937
NNLO	0.12710	0.93849	0.64822	0.35760

TABLE B.1: Values of the relevant couplings computed in different loop orders for the renormalization scale $\overline{\mu} = m_t$, corresponding to $m_h = 125.66$ GeV and $m_t = 173.1$ GeV [47]

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APPENDIX C

FURTHER DETAILS OF COSMOLOGY

C.1 Friedmann–Robertson–Walker Metric

Under the assumption of homogeneity and isotropy of the Universe the line element ds^2 is given as

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{C.1}$$

where t is the time and r, θ and ϕ are spherical coordinates. $a(t)$ is the scale factor which characterizes the relative size of spacelike hypersurfaces at different times and k denotes the curvature which can be either $+1, 0, -1$, corresponding to positive curvature, flat space or negative curvature, respectively. One should keep in mind that equation (C.1) uses comoving coordinates, which means that if we assume $a(t)$ to be increasing, i.e. an expanding Universe, the coordinates of galaxies r, θ and ϕ stay untouched without forces acting on them.

Depending on the curvature of space k, equation (C.1) can be parametrized as

$$ds^2 = -dt^2 + a^2(t)\left[d\chi^2 + \Phi_k(\chi^2)(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{C.2}$$

where the parameter χ may take the following form

$$r^2 = \Phi_k(\chi^2) \equiv \begin{cases} \sinh^2\chi & k=-1 \\ \chi^2 & k=0 \\ \sin^2\chi & k=+1 \end{cases} . \tag{C.3}$$

If the Universe expands or shrinks, the physical distance between two point without peculiar motion cannot stay unchanged. However, in a comoving frame the coordinates r, θ and ϕ stay the same. To obtain the physical distance between two points one has to compute $R = a(t)r$. This means that the property of the Universe, whether it expands or shrinks, is encoded in the evolution of the scale factor $a(t)$.

Another important quantity which should be mentioned is the Hubble rate H. It defines the evolution of the Universe, i.e. whether it expands or shrinks and therefore the definition of H involves the scale factor $a(t)$. The precise definition is

$$H = \frac{\dot{a}}{a} . \tag{C.4}$$

C.2 Definitions of Einstein's Gravity

Here we give the missing definitions which were not important in chapter 5, but play an important role to understand the equations which appeared there.

The Einstein tensor is given as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R , \tag{C.5}$$

in terms of the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R,

$$R_{\mu\nu} = {\Gamma^\alpha}_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + {\Gamma^\alpha}_{\beta\alpha}\Gamma^\beta_{\mu\nu} - \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha} , \qquad R \equiv g^{\mu\nu} R_{\mu\nu} , \tag{C.6}$$

where

$${\Gamma^\mu}_{\alpha\beta} \equiv \frac{g^{\mu\nu}}{2} \left[g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu} \right] . \tag{C.7}$$

Commas denote partial derivatives. Note that the Christoffel symbols ${\Gamma^\mu}_{\alpha\beta}$ are not proper tensors. One can to find a coordinate frame in which the Christoffel symbols vanish but if they were tensors they would vanish in every coordinate frame. From a physical point of view this means that one can always introduce for every point in space which are locally flat.

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 04.08.2014

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Tim Wolf