Force by a cross-field current between electrodes on a plasma confined by a cusp magnetic field

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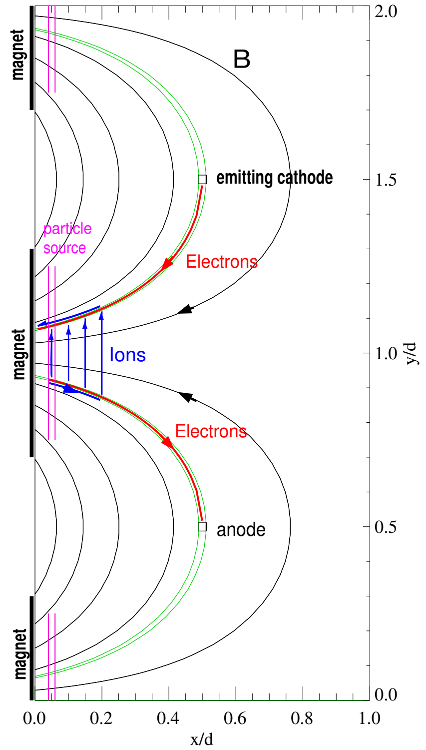
In a recent experiment a plasma confined by a cusp magnetic field was set into rotation by the $j \times B$ force due to a current between electrodes inserted into the edge of the plasma [1]. The aim of a planned experiment is to the create a dynamo in the magnetic field free bulk plasma. In order to find out how much momentum can be transferred to the plasma in such an experiment, we studied the process of plasma acceleration with particle-in-cell simulations resolving the full gyro-motion. The strong localised electric field can increase the ion gyro radius up to $E_\perp/B\omega_c$ such that the usual expressions for particle drifts do not apply. The force on the plasma and the resulting plasma velocity in the direction of $j \times B$ are calculated.

A slab model is used for modelling a plasma confined by a periodic array of magnets (period $2d$) with alternating polarity at $x = 0$. A corresponding array of electrodes with alternating potential is located at $x = x_s$. The magnetic field is taken as $B_x = \hat{B}\exp(-\pi x/d)\cos(\pi y/d)$, $B_y = \hat{B}\exp(-\pi x/d)\sin(\pi y/d)$. The simulation domain is a rectangle with dimensions $L_x = 2d$ and $L_y = 2d$, where d is the distance between the electrodes. The value of L_x is chosen such that the magnetic field and the electric field are so small at L_x that the forces are negligible (the decay length is d/π for B and for the electric vacuum field of the electrodes). The geometry of the model is shown in Fig. 1, where half of the computational domain is depicted with magnetic field lines and the position of the electrodes and magnets.

The simulations are performed with a 5D particle-in-cell (PIC) code (2 spatial dimensions, 3 velocity components) [2]. The full particle orbits are calculated by alternately solving the equations of motion for the particles and the Poisson equation for the electric potential in each time step. Initially the simulation region is filled with a uniform charge neutral plasma of density n_0 and the particles have Maxwellian velocity distributions with a temperature ratio $T_e/T_i = 10$ like in the experiment with ECR heating. All particles hitting the wall with the magnets are lost, but their charge contributes to the local surface charge. This corresponds to an insulating wall. At $x = L_x$ particles are reflected by changing the sign of v_x and periodic boundary conditions in y are used. In a source layer near $x = 0$ electrons and ions are created in equal numbers with Maxwellian velocity distributions with the same temperatures as the initial plasma. In most of the calculations the cathode is strongly emitting electrons with $T_{\text{em}}/T_e \approx 0.02$. The plasma and the fields in our model are uniform in z direction. The electric potential is split into two parts,



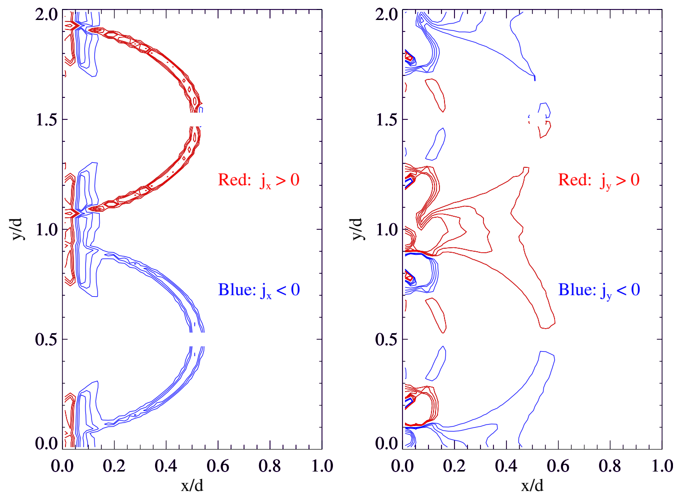
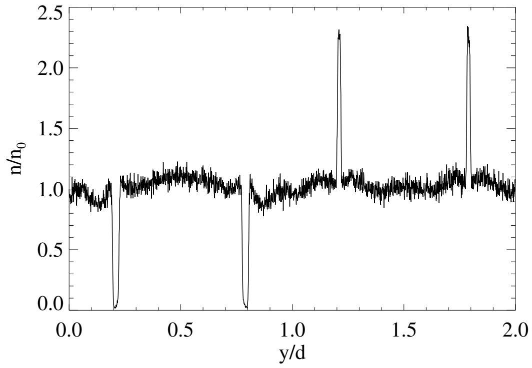


Fig. 1 (left): Left half of the simulation domain with field lines, current flow and position of magnets and sources indicated. Figs. 2,3: Contour lines of parallel el. current (left) and cross-field ion current (right).

$\Phi = \Phi_1 + \Phi_2$. The first part, Φ_1, is obtained by solving the Poisson equation $\Delta\Phi_1 = \rho_{el}/\varepsilon_0$, where ρ_{el} is the charge density of the plasma which includes also the charge of those particles that have hit the electrodes and were removed from the plasma. Their charges are assigned to the volume of the electrodes. This corresponds to electrodes with a fixed potential difference, but together floating with respect to the plasma potential. The boundary conditions are: at $x = 0$ the electric field is given by the local surface charge density, corresponding to an insulating wall, while at L_x the potential is set to zero. The second part of the electric potential, Φ_2, is proportional to the vacuum potential around two electrodes at opposite unit voltage fulfilling $E_x = 0$ at $x = 0$. The amplitude factor of Φ_2 is chosen so that the required potential difference $\Delta\Phi$ between the electrodes is obtained: $\Delta\Phi_2 = \Delta\Phi - \Delta\Phi_1$, where $\Delta\Phi_1$ is the potential difference due to the plasma charge. A computer with many compute cores ($N_p = 160$ or 320) is used. The rectangular domain is divided into subdomains $L_x \times (L_y/N_p)$, each of which (with additional guard cells) is assigned to a compute core together with all the particles located in that subdomain. The Poisson equation is solved on a grid with spacing $0.5\lambda_D$ with periodic boundary conditions in y by employing the Fast Fourier Transform (FFT) in y and solving a matrix equation in x. For the FFTs the grid is repartitioned into subgrids $(L_x/N_p) \times L_y$ [2].

The parameters used are close to that in the experiment: distance d is varied between $300\,\lambda_D$ and $800\,\lambda_D$, which for $n = 10^{11}\mathrm{cm}^{-3}$ and $T_e = 10\,\mathrm{eV}$ equals about 6 cm. $(\omega_{ce}/\omega_{pe})_{\mathrm{wall}}$ is varied between 1 and 4. A mass ratio of $m_i/m_e = 7000$ is used corresponding to He ions and a potential



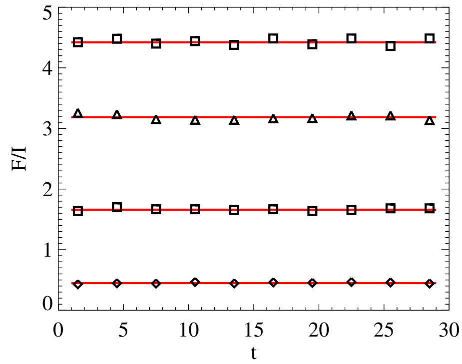


Fig. 4 (left): Density between wall and electrodes versus y/d in case without source.

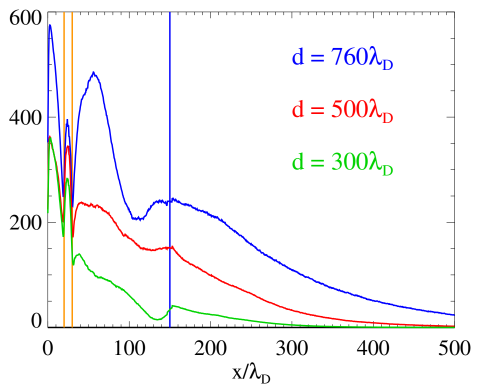
Fig. 5 (right): F/I (force on plasma divided by the current) versus time. Top to bottom: $x_s/d = 0.2, 0.3, 0.5, 0.9$

difference of 100 V is applied to the electrodes. The particle source is located between $40\,\lambda_D$ and $60\,\lambda_D$. The results are: Without electron emission at the cathode the current is limited to the ion saturation current as expected. With electron emission at the cathode a much higher current is flowing. Near the electrodes the current is carried by electrons parallel to the field lines, while the cross-field current is carried by ions (Fig. 2,3). Without a strong particle source the current decays rapidly, since the flux tube connected with the anode is drained of particles as shown in Fig. 4. The electrons are collected by the anode, while the ions are pushed across field lines by the electric field. Therefore, a particle source as described above was included in the simulations. The electrons emitted by the cathode cannot reach the anode, they are flowing to the wall as indicated in Fig. 1. The density in the flux tube connected with the cathode increases (see Fig. 4). The force on the plasma, $F = \int (j_x B_y - j_y B_x)\, dV$, is proportional to the electric current between the electrodes (see Fig. 5). Expressing the magnetic field by derivatives of the flux per unit length in z, $B_x = \partial\psi/\partial y$, $B_y = -\partial\psi/\partial x$, we get with $\nabla \cdot j = 0$

$$F = -\int (j \cdot \nabla\psi)\, dV = -\int \nabla \cdot (\psi j)\, dV = -\int \psi j \cdot dS = I(\psi_2 - \psi_1), \tag{1}$$

where I is the current and $\psi_2 - \psi_1 = \Delta\psi$ is the magnetic flux passing between the electrodes. The last equality holds if the current is flowing in the plasma ($j \cdot dS \neq 0$ only at the electrodes) and if the electrodes are sufficiently small for $\psi_{1,2} = const.$ The symbols in Fig. 5 refer to different ratios x_s/d and the lines show the values of $F/I = \Delta\psi = 2(d/\pi)\hat{B}\exp(-\pi x_s/d)$.

In Fig. 6 the force density in the plasma at the time $t = 10^4 \omega_{pe}^{-1}$ ($2.9\omega_{ci}^{-1}$ at the magnet, $1.1\omega_{ci}^{-1}$ at the electrodes) is shown for three different values of d with fixed electrode position $x_s = 150\lambda_D$. The force is large near the magnets, but some cross field current further inside the plasma also contributes to the force. The resulting plasma momentum at the same time is



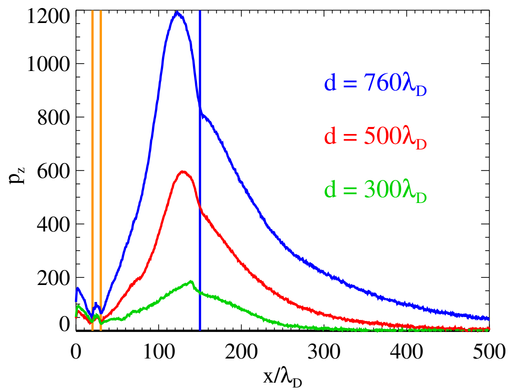
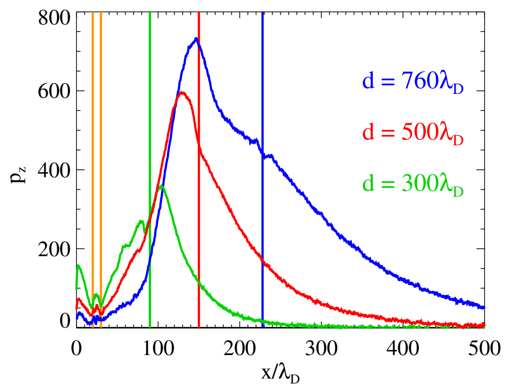


Fig. 6 (left): Force density $\int (j_x B_y - j_y B_x) dy / e n_0 c_s B_0 \Delta y$ versus x/λ_D for three values of d and fixed electrode position. Fig. 7 (right): Plasma momentum $\int m_i n_i u_z dy / m_i n_0 c_s \Delta y$ for the same cases. Positions of source (yellow) and electrodes (blue) are indicated, $t = 10^4 \omega_{pe}^{-1}$.



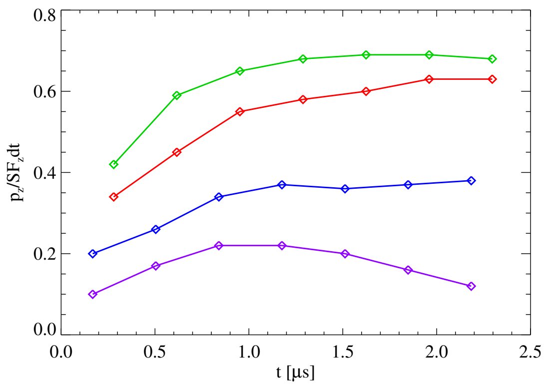


Fig. 8 (left): Plasma momentum $\int m_i n_i u_z dy / m_i n_0 c_s \Delta y$ for three values of d and $x_s/d = 0.3$. Fig. 9 (right): Ratio between momentum in the bulk plasma ($x > x_s$) and time integrated force (transferred momentum) for $x_s/d = 0.2$(top)$, 0.3, 0.5, 0.9$; $t \leq 4 \cdot 10^4 \omega_{pe}^{-1} \sim 6\omega_{ci}^{-1}$.

depicted in Fig. 7 for the same cases and in Fig. 8 for three different values of d with fixed x_s/d. The strong dependence of the momentum on d in Fig. 7 is due to the strong increase of the force like $d \exp(-\pi x_s/d)$. At fixed ratio x_s/d (Fig. 8) the force increases only linearly with d. The momentum density is small near the wall due to losses to the wall, since ions have to flow towards the wall together with the electrons from the cathode. Only between 20 and 70 percent of the transferred momentum reach the bulk plasma to the right of the electrodes (Fig. 9); when the distance x_s between the electrodes and the wall is larger, a larger fraction of the momentum remains in the volume between the wall and the electrodes. Thus, with electrodes closer to the wall the force (hence the transferred momentum) is larger and a higher fraction of the transferred momentum reaches the bulk plasma.

[1] C. Collins *et al.*, PRL 108, 115001 (2012)

[2] A. Bergmann, Phys. of Plasmas 1, 3598 (1994)