Cavity output field suppression due to interference effects

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We show how interference effects are responsible for suppression of the output electromagnetic field of an optical micro-resonator in the good-cavity limit. The system of interest consists in a moderately strongly pumped two-level emitter embedded in the optical cavity. When an additional weaker laser of the same frequency is pumping the combined system through one of the resonator's mirror then the output cavity electromagnetic field can be almost completely suppressed. This is due to the interference among the scattered light by the strongly pumped atom into the cavity mode and the incident weaker laser field. The result applies to photonic crystal environments as well.

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I. INTRODUCTION

Light interference is a widely investigated topic and its importance for various applications is enormous [1–5]. Due to quantum interference effects for instance elimination of spectral lines or complete cancellation of the spontaneous decay can occur. Spatial interference shows interesting features as well [2, 4, 5]. Furthermore, suppression of the resonance fluorescence in a lossless cavity was demonstrated in [6] whereas cavity-field-assisted atomic relaxation and suppression of resonance fluorescence at high intensities was shown in [7], and suppression of fluorescence in a squeezed vacuum was demonstrated in [8], respectively. Suppression of Bragg scattering by collective interference of spatially ordered atoms within a high-Q cavity mode was demonstrated in [9]. On the other side, cavity-enhanced single-atom spontaneous emission was observed in Ref. [10], while suppression of spontaneous decay at optical frequencies in [11]. A direct measurement of the field of a radiating dipole in a trap is provided in [12], while the pumped one-atom cavity spectra is given in [13] and photon statistics in Ref. [14], respectively. Actually, the bichromatic driving of single atoms was intensively investigated recently emphasising interesting interference phenomena. In particular, the resonance fluorescence of a two-level atom in a strong bichromatic field was analysed in [15] and the response of a two-level system to two strong fields was experimentally studied in [16], correspondingly. The decay of bichromatically driven atom in a cavity was investigated in [17]. Broadband high-resolution x-ray frequeny combs were obtained via bichromatically pumping of three-level $\Lambda-$type atoms [18]. Moreover, bicromatic driving of a solid-state cavity quantum electrodynamics system was investigated in Ref. [19]. Finally, photonic crystal's influence on quantum dynamics of pumped few-level qubits was investigated in detail as well [20–22].

Here, we investigate the feasibility of controlling the cavity output electromagnetic field in a system consisting of a strongly pumped two-level emitter. If a second coherent driving is applied through one of the mirrors and perpendicular to the first laser-beam then the output cavity field can be almost completely inhibited in the good-cavity limit. Notice that the lasers are in resonance with the cavity mode frequency. We have found that the interference between the second weaker light beam and the light scattered by the two-level radiator into the cavity mode due to stronger pumping is responsible for the suppression effect. Furthermore, the inhibition requires the laser frequency to be out of atomic frequency resonance while for photonic crystals surroundings it can be even on resonance.

The article is organized as follows. In Sec. II we describe the analytical approach and the system of interest, while in Sec. III we analyze the obtained results. The summary is given in Sec. IV.

II. QUANTUM DYNAMICS OF A PUMPED TWO-LEVEL ATOM INSIDE A DRIVEN MICROCAVITY

The Hamiltonian describing a two-level atomic system having the transition frequency ω_0 and interacting with a strong coherent source of frequency ω_1 while embedded in a pumped micro-cavity of frequency ω_c, in a frame rotating at $\omega = \omega_1 = \omega_2$ (See Figure 1), is:

$$H = \hbar\Delta S_z + \hbar\delta a^\dagger a + \hbar g(a^\dagger S^- + aS^+) + \hbar\Omega(S^+ + S^-) + \hbar\epsilon(a^\dagger + a), \quad (1)$$

where $\Delta = \omega_0 - \omega$, and $\delta = \omega_c - \omega$. In the Hamiltonian (1) the components, in order of appearance, describe the atomic and the cavity free energies, the interaction of the two-level emitter with the micro-cavity mode and the atom's interaction with the first laser field with Ω being the corresponding Rabi frequency, and respectively, the interaction of the second driving field with the cavity mode with ϵ being proportional to the input laser

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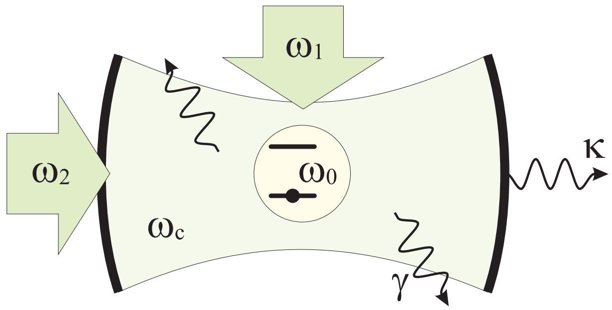

FIG. 1: (color online) The schematic of the model: A two-level radiator possessing the transition frequency ω_0 embedded in a single-mode (ω_c) micro-cavity is pumped with an intense laser field of frequency ω_1. A second coherent source of frequency ω_2 is driving the entire system through one of the mirrors. γ is the single-atom spontaneous decay rate, while κ describes the cavity photon leaking rate, respectively.

field strength amplitude. The atomic bare-state operators $S^+ = |e\rangle\langle g|$ and $S^- = [S^+]^\dagger$ obey the commutation relations for su(2) algebra, i.e., $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$. Here, $S_z = (|e\rangle\langle e| - |g\rangle\langle g|)/2$ is the bare-state inversion operator. $|e\rangle$ and $|g\rangle$ are, respectively, the excited and ground state of the atom while $a^\dagger$ and a are the creation and the annihilation operator of the electromagnetic field (EMF) in the resonator, and satisfy the standard bosonic commutation relations, namely, $[a, a^\dagger] = 1$, and $[a, a] = [a^\dagger, a^\dagger] = 0$ [23, 24].

We shall describe our system using the laser-qubit semiclassical dressed-state formalism defined as [4]:

$$\begin{aligned} |+\rangle &= \sin\theta|g\rangle + \cos\theta|e\rangle, \\ |-\rangle &= \cos\theta|g\rangle - \sin\theta|e\rangle, \end{aligned} \tag{2}$$

with $\tan 2\theta = 2\Omega/\Delta$. Applying this transformation to (1) one arrives then at the following dressed-state Hamiltonian

$$\begin{aligned} H = H_0 &+ \hbar g(\cos^2\theta R^- - \sin^2\theta R^+)a^\dagger \\ &+ \hbar g(\cos^2\theta R^+ - \sin^2\theta R^-)a, \end{aligned} \tag{3}$$

with

$$H_0 = \hbar\bar{\Omega}R_z + \hbar\delta a^\dagger a + \hbar(\epsilon + g_0 R_z)(a^\dagger + a). \tag{4}$$

Here, $\bar{\Omega} = \sqrt{\Omega^2 + (\Delta/2)^2}$ while $g_0 = (g/2)\sin 2\theta$. The new quasi-spin operators, i.e., $R^+ = |+\rangle\langle -|$, $R^- = [R^+]^\dagger$ and $R_z = |+\rangle\langle +| - |-\rangle\langle -|$ are operating in the dressed-state picture. They obey the following commutation relations: $[R^+, R^-] = R_z$ and $[R_z, R^\pm] = \pm 2R^\pm$.

Considering that $\delta \ll \bar{\Omega}$ the last two terms in Eq. (3) can be ignored under the secular approximation. Therefore, the master equation describing the laser-dressed two-level atom inside a leaking pumped resonator and damped via the vacuum modes of the surrounding EMF reservoir is:

$$\begin{aligned} \frac{d}{dt}\rho(t) &+ \frac{i}{\hbar}[H_0, \rho] = -\kappa[a^\dagger, a\rho] - \Gamma_0[R_z, R_z\rho] \\ &- \Gamma_+[R^+, R^-\rho] - \Gamma_-[R^-, R^+\rho] + H.c.. \end{aligned} \tag{5}$$

Here

$$\begin{aligned} \Gamma_0 &= (\gamma_0 \sin^2 2\theta + \gamma_d \cos^2 2\theta)/4, \\ \Gamma_+ &= \gamma_+ \cos^4\theta + (\gamma_d/4)\sin^2 2\theta, \\ \Gamma_- &= \gamma_- \sin^4\theta + (\gamma_d/4)\sin^2 2\theta. \end{aligned}$$

Respectively, $\gamma_{0,\pm}$ are the single-atom spontaneous decay rates at the dressed-state frequencies $\{\omega_L, \omega_L \pm 2\bar{\Omega}\}$, while γ_d signifies the pure dephasing rate. Note that the master equation (5) was obtained under the intense-field approximation, i.e., it is valid when $\bar{\Omega} \gg \{\delta, g, \epsilon, \Gamma_0, \Gamma_\pm\}$.

The equations of motion for the variables of interest can be easily obtained from the Master Equation (5). Therefore, the steady-state quantum dynamics is described by the following system of linear algebraic equations:

$$\begin{aligned} 0 &= -ig_0\langle R_z a\rangle_s - i\epsilon\langle a\rangle_s + ig_0\langle R_z a^\dagger\rangle_s + i\epsilon\langle a^\dagger\rangle_s \\ &+ 2\kappa\langle a^\dagger a\rangle_s, \\ 0 &= (\kappa + i\delta + 2\Gamma_+ + 2\Gamma_-)\langle R_z a\rangle_s \\ &+ 2(\Gamma_+ - \Gamma_-)\langle a\rangle_s + i\epsilon\langle R_z\rangle_s + ig_0, \\ 0 &= (\kappa - i\delta + 2\Gamma_+ + 2\Gamma_-)\langle R_z a^\dagger\rangle_s \\ &+ 2(\Gamma_+ - \Gamma_-)\langle a^\dagger\rangle_s - i\epsilon\langle R_z\rangle_s - ig_0, \\ 0 &= (\kappa + i\delta)\langle a\rangle_s + ig_0\langle R_z\rangle_s + i\epsilon, \\ 0 &= (\kappa - i\delta)\langle a^\dagger\rangle_s - ig_0\langle R_z\rangle_s - i\epsilon. \end{aligned} \tag{6}$$

In the system of equations (6), we have used the fact that the steady-state dressed-state inversion is

$$\langle R_z\rangle_s = -(\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-),$$

together with the trivial condition $R_z^2 = 1$ which is the case for a single-qubit system.

In the following Section, we shall discuss our results, i.e., the possibility of inhibiting the cavity output field via interference effects.

III. OUTPUT CAVITY FIELD SUPPRESSION

One of the solutions of system (6) represents the steady-state mean-photon number in the micro-cavity mode, namely:

$$\langle a^\dagger a\rangle_s = A\epsilon^2 + B\epsilon + C. \tag{7}$$

For $\delta = 0$ and $\gamma_0 = \gamma_\pm \equiv \gamma$, the coefficients A, B and C are given by the following expressions:

$$\begin{aligned} A &= \frac{1}{\kappa^2}, \\ B &= -\frac{2g\gamma\Delta\Omega}{\kappa^2(\gamma\Delta^2 + 2(\gamma + \gamma_d)\Omega^2)}, \\ C &= \frac{g^2\Omega^2}{\kappa^2(\gamma\Delta^2 + 2(\gamma + \gamma_d)\Omega^2)} \\ &\times \frac{\gamma(\kappa + 2\gamma)\Delta^2 + 2\kappa(\gamma + \gamma_d)\Omega^2}{(\kappa + 2\gamma)\Delta^2 + 4(\kappa + \gamma + \gamma_d)\Omega^2}. \end{aligned} \tag{8}$$

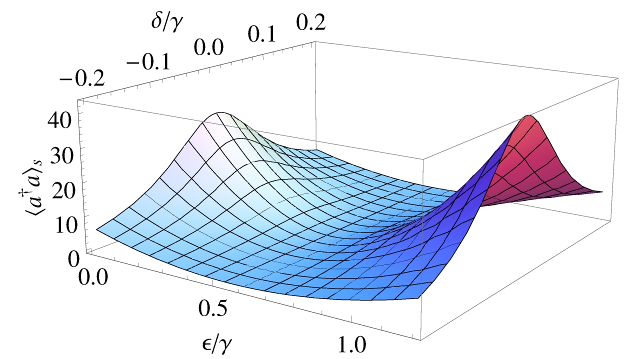

FIG. 2: (color online) The steady-state dependence of the micro-cavity mean photon number $\langle a^\dagger a\rangle_s$ versus the variables ϵ/γ and δ/γ. Other parameters are: $\gamma_d/\gamma = 0.01$, $\kappa/\gamma = 0.1$, $g/\gamma = 2$, $\Delta/\Omega = 3$.

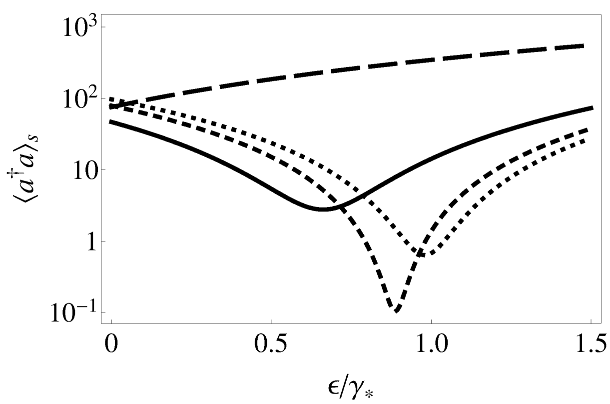

FIG. 3: The steady-state dependences of the mean-photon number $\langle a^\dagger a\rangle_s$ as a function of ϵ/γ_*. The solid-line is for $\gamma_* \equiv \gamma_+ = \gamma_-$, while the long-dashed curve stands for $\gamma_* = \gamma_-$ and $\gamma_+ \to 0$. Further, the short-dashed line is for $\gamma_* = \gamma_+$ and $\gamma_- \to 0$, whereas the dotted curve corresponds to $\Delta = \delta = 0$. Other parameters are the same as in Fig. (2) with $\Delta/\Omega = 1$ and $\delta/\gamma_* = 0$.

Because of the quadratic dependence on ϵ, the minimum value of the mean-photon number is:

$$\langle a^\dagger a\rangle_s^{\min} = C - \frac{B^2}{4A}. \tag{9}$$

The above value is achieved at:

$$\epsilon^{\min} = -B/(2A).$$

Based on Eqs. (8) and (9) it follows that $\epsilon^{\min}$ is independent on $\{\kappa, \delta\}$ and its value does not exceed $\frac{g\sqrt{2}}{4}(1+\gamma_d/\gamma)^{-1/2}$. Particularly, in Fig. (2) the minimum value of the steady-state mean-photon number is $\langle a^\dagger a\rangle_s^{\min} \approx 0.06$ and is achieved when $(\epsilon/\gamma)^{\min} \approx 0.54$. An explanation of the steady-state behaviours shown in Fig. (2) can be found if one represents the mean-photon number given by (7) as follows:

$$\langle a^\dagger a\rangle_s = \frac{\epsilon}{\kappa^2}\{\epsilon + g_0\langle R_z\rangle_s\} + \frac{g_0}{\kappa(\kappa + 2\Gamma_+ + 2\Gamma_-)} \times \{g_0 + \epsilon\langle R_z\rangle_s - 2(\Gamma_+ - \Gamma_-)(g_0\langle R_z\rangle_s + \epsilon)/\kappa\}. \tag{10}$$

From the above expression (10), one can see that for $\delta = 0$ the mean-photon number due to weaker external pumping of the cavity mode is proportional to ϵ^2 while that due to stronger driving of the two-level qubit to g_0^2, respectively. There is also a cross-contribution proportional to ϵg_0. All these terms demonstrate interference effects among the contributions due to two pumping lasers and, hence, the minima's nature in Fig. (2). Notice that on resonance, i.e., $\Delta = 0$, the inhibition effects are absent when $\gamma_+ = \gamma_-$ since $\langle R_z\rangle_s = 0$ (see Eq. 10). Also, one can obtain small values for $\langle a^\dagger a\rangle_s$ if $\kappa > \gamma$. However, in this case we are in the bad-cavity limit and, therefore, lower values for mean-photon number or even zero are expected [14, 23, 25]. Thus, in contrary, the cavity output field suppression reported here occurs in the good-cavity limit, i.e., when $\gamma > \kappa$ and $g > \{\kappa, \gamma\}$.

Figure (3) shows the mean-photon numbers when the two-level radiator is surrounded by a photonic crystal environment. In this case, the output cavity field can be suppressed even on atom-laser frequency resonance, i.e., when $\Delta = 0$ (see the dotted curve). This is due to the fact that in photonic crystal environments γ_+ can be different than γ_- [20–22] and, thus, the population will be distributed unequally among the dressed states, that is,

$$\langle R_z\rangle_s = \frac{\gamma_- - \gamma_+}{\gamma_- + \gamma_+ + 2\gamma_d} \neq 0, \quad \text{if} \quad \Delta = 0.$$

Negative values for the dressed-state inversion lead to cavity output field suppression (see Fig. 3). For the sake of comparison, the solid curve stands for ordinary vacuum-cavity environments. Thus, finalizing, we have shown here how the output cavity field can be minimized due to interference effects.

IV. SUMMARY

Summarizing, we have demonstrated cavity output field suppression due to interference effects. The system of interest is formed from a strongly pumped two-level atom placed in an optical micro-resonator. A second weak laser being in resonance with the cavity mode frequency is probing the whole system through one of the cavity's mirror. Interference effects occur among the light scattered in the cavity mode by the strongly pumped atom and the incident weaker laser field leading to output cavity field inhibition. The idea works for photonic crystal environments as well.

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