A gyrokinetic model for studies of magnetic reconnection in periodic and bounded plasmas

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Abstract

A gyrokinetic Particle-in-Cell (PIC) code to investigate magnetic reconnection in the low-β limit is presented. With a magnetic field geometry similar to the VINETA II experiment [1], the model can be applied to study the kinetic effects of local plasma parameters and plasma boundary conditions on magnetic reconnection.

Introduction

Laboratory plasmas are confined by vessel walls. In addition to infinite model systems, boundary conditions such as the plasma sheath are important for the particle kinetics. Also for natural occurrences in space, e.g. many magnetic field lines in the solar atmosphere have base points on the conducting surface of the sun [2], boundary conditions do play a role.

The spatial dimensions vary from small-scale dynamics in the current sheet to large-scale global boundary conditions which both influence each other [3]. To model such phenomena the huge time and spatial scales involved in magnetic reconnection pose a problem. Gyrokinetic simulations are a basic tool to study low frequency instabilities, turbulence and their anomalous transport in magnetized plasmas on larger scales than normal kinetic codes. In order to discriminate the influence of bounded currents on magnetic reconnection a gyrokinetic Particle-in-Cell (PIC) code [4] is adapted

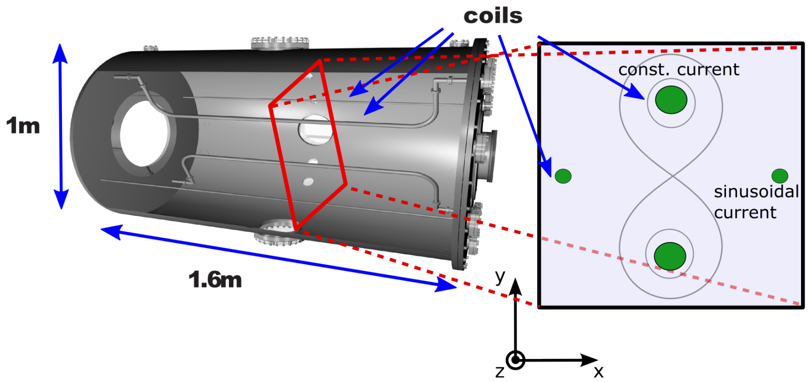

Figure 1: *A schematic of the VINETA II experiment is shown (left). The experiment is mimicked in the slab model which is indicated on the right hand side. The model consists of two sets of coils - one pair to generate the in-plane x-point topology and one to drive the flux surfaces. Image: courtesy of Hannes Bohlin.*

to study driven magnetic reconnection in the strong guide field limit. To conceive magnetic reconnection the currents in the system need to be understood. Instead of calculating the full particle dynamics a reduced set of equations is used. The fast gyromotion is neglected and only finite Larmor radius (FLR) effects are taken into account. This decreases the temporal and spatial demands for many problems so that they become feasible to calculate.

The code closely mimics the magnetic field configuration of the VINETA II experiment [5]. The setup of the linear, cylindrical device is seen in Fig. 1. The simulation intrinsically contains a strong guide field along the axial direction of the experiment. It contains two axial coils with steady current in order to produce an x-point configuration. Two additional coils have a sinusoidal current to perturb the magnetic x-point topology and to drive the reconnection process.

The gyrokinetic model

The PIC code solves the Vlasov-Poisson-Ampère system of equations for a low-β plasma in vector potential A and canonical momentum p notation [4]. It follows the characteristics of the nonlinear electrostatic gyrokinetic Vlasov equation.

Particle position and velocity of the ith particle of species s are denoted by $\underline{R}_i^s$ and $\underline{v}_i^s$, respectively. The index z indicates the z-component. Using the canonical momentum $\underline{p}_{i,z}^s = \underline{v}_{i,z}^s + \frac{q_s}{m_s}\underline{A}_z(R_i^s,t)$ and a generalized potential $\Psi(R_i^s,t) = \bar{\Phi}(R_i^s,t) - \underline{v}_{i,z}^s\underline{A}_z(R_i^s,t)$ the equations of motion reduce to

$$\frac{\mathrm{d}\underline{R}_i^s}{\mathrm{d}t} = \underline{v}_{z,i}^s\hat{\underline{b}} - \frac{\nabla\Psi \times \underline{b}}{B_0} \tag{1}$$

$$\frac{\mathrm{d}\underline{p}_{i,z}^s}{\mathrm{d}t} = -\frac{q_s}{m_s}\underline{b}\nabla\Psi\,. \tag{2}$$

The gyroradius is denoted by ρ and the magnetic field B is described in terms of the vector potential $\underline{B} = \nabla \times \underline{A}$. m_s, q_s and $\hat{\underline{b}} = \underline{b} + (\underline{A}_z \times \underline{b})/B_0$ are the mass, the charge and the unit vector along the magnetic field, respectively. The notations $\bar{\Phi}(R) = \langle \int \Phi(\underline{r})\delta(\underline{r} - \underline{R} - \rho)dr\rangle_\varphi$ and $\langle\rangle_\varphi = \oint d\varphi/2\pi$ are used.

The electrostatic potential is calculated from the gyrokinetic Poisson equation:

$$\nabla^2\bar{\Phi} + \frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{ci}}^2}\nabla_\perp^2\bar{\Phi} = \frac{q_e}{\varepsilon_0}\sum_j^{N_e}\delta(\underline{R} - \underline{R}_j^e) + \frac{q_i}{\varepsilon_0}\sum_j^{N_i}\delta(\underline{R} - \underline{R}_j^i), \tag{3}$$

where ω_{pi} is the ion plasma frequency, $\nabla_\perp$ is the gradient perpendicular to the axial magnetic field and δ is the delta distribution. The second term on the left hand side represents the ion polarization shielding effects. Ampère's law becomes in the canonical momentum formulation

$$-\delta_e^2 \nabla_\perp^2 A_z = \frac{1}{N_0}\left[\left(\sum_{N_e} p_e - \sum_{N_i} p_i\right) - \left(\left\{\sum_{N_e} A_z \delta(\underline{R} - \underline{R}_j^e)\right\} + \frac{m_e}{m_i}\left\{\sum_{N_i} A_z \delta(\underline{R} - \underline{R}_j^i)\right\}\right)\right].$$

The electron skin depth is denoted by $\delta_e = c/\omega_{\rm pe}$.

The code uses three-dimensional cartesian coordinates. The 2D model applies a slab geometry (x and y direction) with periodic boundaries in the axial z-dimension. The in-plane fields are bounded by conducting walls and particles are reflected. Along the guide field (z-direction) the boundaries are periodic for the fields and the particles leaving the system are entering the simulated domain from the other side. Calculations with various particle density profiles are possible to study the magnetic field penetration [6]. A full 3D model with reflecting or absorbing conducting walls and logical sheath boundaries is currently being implemented. Various boundary conditions are needed in order to calculate the nonlinear effects on the particle kinetics similar to the metal wall end plates in the linear VINETA II experiment. A necessity is a strong axial guide field to have electrons and ions fully magnetized, whereas the in-plane magnetic field is generated by currents in external coils parallel to the guide field. This allows to study reconnection rates and the dynamics of the generated currents as functions of plasma density profiles and different ratios of in-plane to guide magnetic field ratios.

The logical sheath model

In order to calculate the influence of the plasma sheath effects on magnetic reconnection a logical sheath model is used. On earth most laboratory plasmas are confined within a vacuum vessel. Such bounded plasmas are in contact with the surface of a wall resulting in a plasma sheath at the plasma edge. The spatial extent of the sheath is about a few Debye lengths. A standard electrostatic PIC code must have a grid size below the Debye length in order to resolve the sheath potential drop. In gyrokinetics the cell size is larger. Thus, the sheath can not be calculated self-consistently. The idea of the logical sheath is not to resolve those spatial scales, but to mimic the equilibrium conditions at the plasma edge [7].

If ions and electrons hit a conducting surface they recombine and are lost to the plasma. Such surface acts as a perfect sink for the particles. Normally, electrons in a plasma move faster than ions, so that initially the electron flux to the wall exceeds the ion flux by far. The flux imbalance will charge up the wall negatively and generates a potential drop which cannot be resolved in a gyrokinetic model. This barrier repels electrons and attracts ions until both fluxes are equal. Thereby, the potential drop adjusts and steady-state is achieved. This flux balance is applied to

all particles exceeding the extents of the simulation. Ions are streaming freely out of the system and are absorbed if they exceed particle boundaries. If N_i is the number of absorbed ions per pusher step, then only the fastest, up to N_i, electrons are absorbed and the remaining slow particles are reflected as if repelled by the sheath potential drop.
In the case where the number of absorbed ions is greater than the number of electrons hitting the particle boundary all electrons are absorbed and the excessive "charge" is stored for the next particle push. The potential drop can be estimated from the speed of the fastest repelled electron $\Delta\Phi = \frac{m}{2e} v_{\max}^2$.

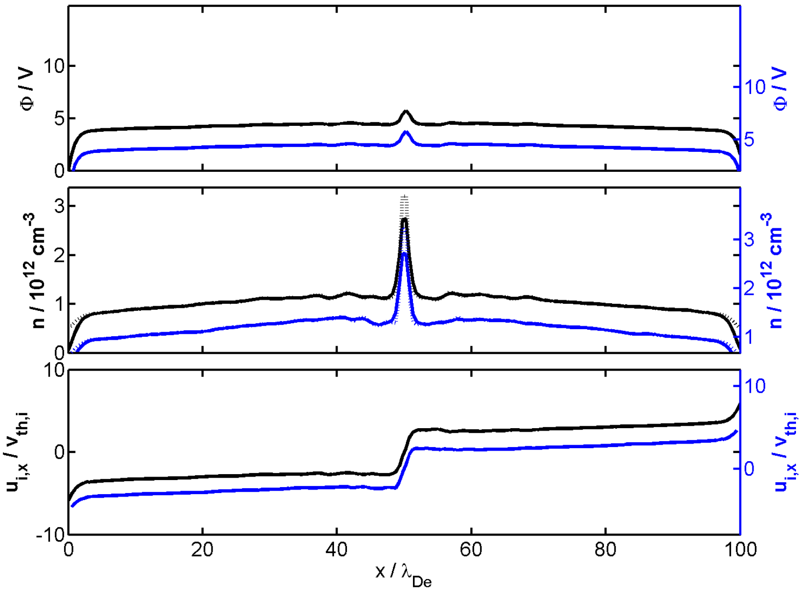

Figure 2: *Comparison of the logical sheath solution (blue, right axis) with the self-consistent calculation (black, left axis) of a 1D electrostatic PIC code. Shown are the electrostatic potential* Φ*, the particle density* n *and the mean velocity of the ions* u_i*. To discriminate both solutions the axis for the logical sheath is shifted vertically.*

The logical sheath has been tested separately in a one-dimensional electrostatic PIC code [8], see Fig. 2, and is now being implemented into the gyrokinetic model.

References

[1] H. Bohlin, A. von Stechow, K. Rahbarnia, *et al.*, Rev. Sci. Instrum. **85**(2), 023501 (2014)

[2] M. Yamada, Rev. Mod. Phys. **82**(1), 607 (2010)

[3] A. Kuritsyn, H. Ji, *et al.*, Geophys. Res. Lett., **34**, L16106, (2007)

[4] R.D. Sydora, Phys. Plasmas **8**(5), 1929 (2001)

[5] A. von Stechow, H. Bohlin, K. Rahbarnia, *et al.*, Europhys. Conf. Abs. **38**, P4.111 (2014)

[6] J. Clementson, L. Lewerentz, O. Grulke, *et al.*, Europhys. Conf. Abs. **38**, P4.118 (2014)

[7] S.E. Parker, R.J. Procassini, *et al.*, J. Comput. Phys., **104**(2), 41 (1993)

[8] K. Matyash, R. Schneider, Contrib. Plasma Phys., **44**, (2004)