Application of the SIESTA code to the calculation of MHD equilibria for the Wendelstein 7-X Stellarator

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Introduction

Bootstrap currents may cause deviations from the optimum configuration of W7-X if left unchecked, having an impact on the location of the $5/5$ island chain on which divertor configurations are based, that might be drawn into the plasma. Sensitivity studies of this kind have been previously studied at W7-X with the VMEC/Extender tool, that permits the calculation of the magnetic field created outside of the last closed surface from a previously computed VMEC equilibrium solution, with the proper current profile adjusted to include the expected bootstrap currents, within the plasma. The possibility of actively healing these deleterious effects by means of external current drive was also investigated using VMEC/Extender previously. It was shown [4] that this technique may be adequate to keep the $5/5$ resonance location outside of the plasma and under control. However, neither VMEC nor VMEC/Extender tells us anything about the effect of bootstrap current inside of the LCFS since VMEC assumes from the start the existence of good magnetic surfaces and VMEC/Extender does not compute the fields inside. Recently, a new MHD code (SIESTA [1]) has been developed that can overcome this constraint, while keeping many of the good convergence and accuracy properties that VMEC has. In this contribution, we describe the preliminary results of applying SIESTA to this problem.

The SIESTA Code

Magnetohydrodynamics (MHD) is a very important tool when designing and operating magnetic confinement devices for fusion plasmas. *SIESTA* [1] is a recently developed code that allows to calculate MHD equilbria for three dimensional configurations without precluding the existence of magnetic islands or stochastic regions. It does so by implementing an iterative technique that minimizes the total energy under proper constraints ensuring the conservation of magnetic flux and plasma density. The ideal MHD equations are used to obtain finite constrained independent variations of the magnetic field $\mathbf{B}$ and plasma pressure, p, combined with Ohm's law ($\mathbf{E}+\mathbf{v}\times\mathbf{B}=0$) and the adiabaticity condition $p=n^{\Gamma}$:

$$\delta\mathbf{B}(\xi) = \nabla\times(\mathbf{v}\times\mathbf{B}) \tag{1}$$

$$\delta p(\xi) = (\Gamma-1)\xi\cdot\nabla p - \Gamma\nabla\cdot(p\xi) \tag{2}$$

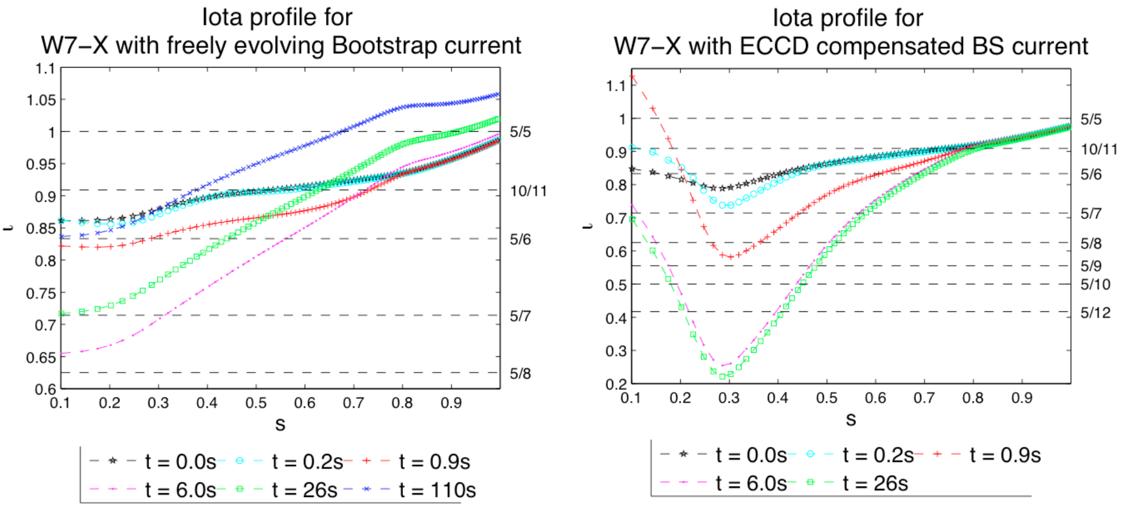

Figure 1: *Rotational transform of configurations with (right) and without (left) ECCD active correction.*

The plasma displacement vector $\xi = \mathbf{v}\Delta t$ is treated as the independent variable upon which the variation of the energy is carried out, and that carries the weight of the iterative scheme that ends up in a (local) minimum energy state in which the MHD force vanishes.

SIESTA is quite different from more traditional iterative MHD equilibrium solvers, since it does not require any *along the line* calculation, solving instead a set of coupled PDEs for the components of the displacement vector. This makes the calculation noticeably faster and more accurate. SIESTA is usually run after a VMEC solution is available for the same problem. It uses the 3D inverse coordinates found by VMEC as its background (fixed) coordinate system, and the VMEC solution as the starting point for the iterative procedure. Convergence is further facilitated by the use of a preconditioned Newton scheme, combined with other techniques that diminish the bad impact of small eigenvalues and large condition numbers.

W7-X configurations under analysis

In this work, we consider the so-called W7-X standard configuration. Although the transport optimization for W7-X attempts to make the bootstrap current vanish, it can nonetheless become non-zero if the magnetic configuration departs from the high mirror one. Typically, unbalanced currents of a few tens of *kA* are to be expected. It is difficult to predict exactly what the magnitude, the radial distribution and time evolution of the bootstrap current profile will be, although some idea is available from transport simulations [5, 4].

The results of one of these simulations are shown in the left frame of Fig. 1, that shows the temporal evolution of the rotational transform, together with the radial location of the lower-order rational surfaces inside of the plasma. The application of VMEC/Extender to the MHD equilibria obtained by VMEC for these profiles showed that the 5/5 island chain (that forms the

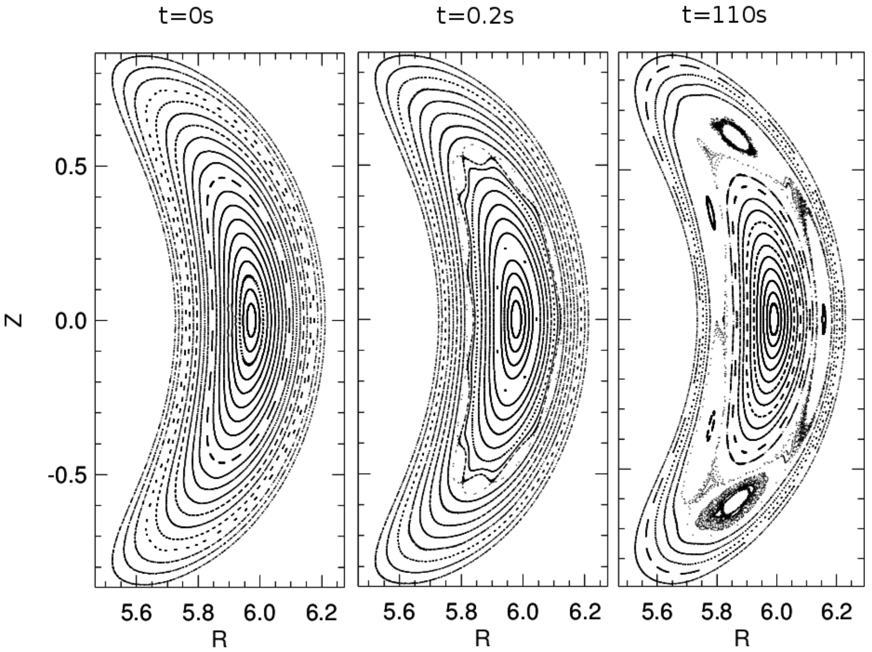

Figure 2: *Poincaré plots at* $\phi = 0$ *of the SIESTA equilibria for cases with free-evolving bootstrap current.*

W7-X divertor) ends up moving inside the LCFS as time advances, as can be seen for $t = 26s$ in the figure. In that work, it was also sown that ECCD could be used to compensate the bootstrap current, which resulted in modified rotational transform profiles that managed to keep the $5/5$ outside (shown in the right frame of Fig. 1).

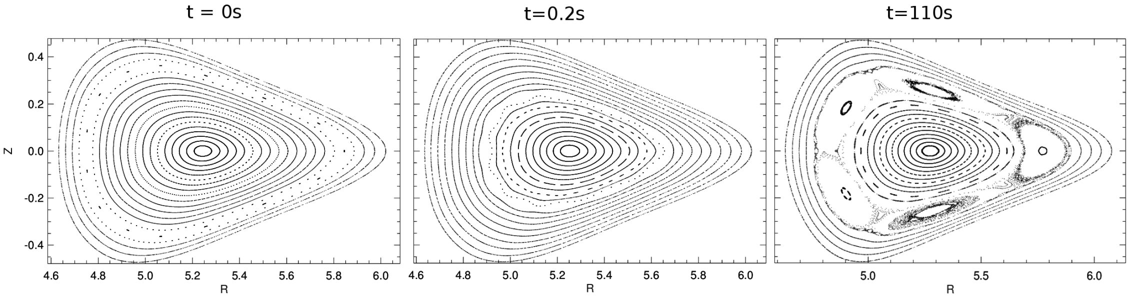

Figure 3: *Poincaré plots at* $\phi = \pi$ *of the SIESTA equilibria for cases with free-evolving bootstrap current.*

VMEC/Extender cannot tell us much of what happens within the LCFS, though. The objective of this work is to use SIESTA to do it. The first results obtained are discussed next. We have started by applying it to the set of equilibria corresponding to the freely-evolving bootstrap current cases. The first results are shown, in the form of a Poincare plot taken at toroidal angles $\phi = 0$ and $\phi = \pi$, in Figs. 2 and 3. The first frame on the left of each figure shows the equilibrium in the absence of bootstrap current. Clearly, very well defined magnetic surfaces exist, as should be expected since there are no low-order resonances within the plasma, that coincides quite well with those provided by VMEC for the case. The last frame on the right of each figure shows the equilibrium at time $110s$, when the rotational transform in Fig. 1 already shows the $5/5$ rational surface well within the plasma. The SIESTA solution clearly shows the $5/5$ island chain inside, around $s = 0.7$ (s here is the square root of the toroidal flux, as it is used within SIESTA). This is consistent with the predictions of VMEC/Extender in Ref. [4], that could trace the inward motion of the resonance toward the plasma as time advanced. Caution must be exercised however on the actual size of SIESTA's islands, since no current constraint is imposed within them in the calculation, and no flattening of profiles is carried out. The island size obtained is consistent with the constraints imposed on the SIESTA calculation (conservation of magnetic flux and mass), as well as with the energy allocated to the perturbation of the equilibria and how it is distributed among all possible instabilities.

Conclusions

Preliminary analysis with SIESTA on the standard configuration of W7-X are consistent with previous work with VMEC/Extender, confirming that unbalanced bootstrap currents can cause the introduction of the $5/5$ island chain into the plasma, effectively reducing the plasma volume. Future work will be devoted to understanding better the effects (of the bootstrap-current modified rotational transform profile) inside of the plasma, since other lower-order rational surfaces, such as the $5/6$ or $5/7$ are also introduced from the axis. In addition, the consequences within the plasma of the current drive schemes proposed to actively control the $5/5$ island divertor will also be explored.

References

[1] S.P. Hirshman, R. Sanchez and C.R. Cook, Phys. Plasmas **18**, 6 (2011)

[2] Yasuhiro Suzuki *et al.*, Nucl. Fusion **46**, 11 (2006)

[3] A. Reiman and H. Greenside, Comp. Phys. Comm. **43**, 1 (1986)

[4] J Geiger *et al.*, Contrib. Plasma Phys. **50**, 8 (2010)

[5] Yu. Turkin *et al.*, Fusion Sci. Technol. **50**, 387 (2006)

[6] M. Drevlak, D. Monticello and A. Reiman, Nucl. Fusion **45**, 7 (2005)