Residual zonal flow level in stellarators for arbitrary wavelengths

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Introduction

Understanding and controlling the mechanisms that generate plasma microturbulence is crucial for the design and operation of future magnetic confinement fusion reactors. Zonal flows, *i.e.* flows associated to electrostatic potential perturbations that are constant on magnetic surfaces, are considered of great importance in the formation of internal transport barriers that improve confinement by reducing the turbulent transport. Although the dynamics of zonal flows is non-linear and influenced by collisions, the study of their linear collisionless evolution still provides useful information.

In the linear collisionless limit, a zonal potential perturbation in a toroidal plasma relaxes, in general, to a non-zero residual value [1]. An analytical expression for the residual value in general toroidal geometry (tokamak or stellarator) can be derived. This expression involves certain averages over the particle trajectories. We have extended the code CAS3D-K [2, 3] to work out those averages. We have validated the results from CAS3D-K against the results from two gyrokinetic codes, EUTERPE [4, 5] and GENE [6], by calculating the same quantities in an independent way.

Linear collisionless evolution of zonal flows in toroidal devices

We use the linear collisionless gyrokinetic theory to study the damping of a zonal flow like perturbation. Following the seminal work by Rosenbluth and Hinton [1], we solve an initial value problem in toroidal devices for any radial wavelength. The evolution of the electrostatic potential in the Laplace space is given by

$$\widehat{\phi}_k(p) = \sum_s Z_s \left\{ \frac{e^{-\mathrm{i}k_\psi \delta_s} J_{0s}\, \overline{e^{\mathrm{i}k_\psi \delta_s} f_s(0)/F_{s0}}}{p + \mathrm{i}k_\psi \overline{\omega_s}} \right\}_s \left[\sum_s \frac{Z_s^2 e}{T_s} \left\{ 1 - p\, \frac{e^{-\mathrm{i}k_\psi \delta_s} J_{0s}\, \overline{e^{\mathrm{i}k_\psi \delta_s} J_{0s}}}{p + \mathrm{i}k_\psi \overline{\omega_s}} \right\}_s \right]^{-1}, \quad (1)$$

where $\phi(t) = \phi_k(t) e^{\mathrm{i}k_\psi \psi}$. Here, k_ψ is the radial wavenumber of the perturbation, p is the Laplace space variable, $J_{0s} := J_0(k_\perp \rho_s)$ is the Bessel function of the first kind of species s with $k_\perp := k_\psi |\nabla\psi|$ and ρ_s the Larmor radius. δ_s and ω_s are the mean radial displacement and the radial magnetic drift frequency of species s respectively and both are related through the magnetic differential equation $\omega_s = \overline{\omega_s} + v_\parallel \widehat{\mathbf{b}} \cdot \nabla \delta_s$, where $v_\parallel$ is the component of the velocity parallel to the magnetic field line $\widehat{\mathbf{b}}$. For any function Q, we have used the definition $\{Q\}_s := \left\langle \int \mathrm{d}^3 v\, F_{0s}\, Q \right\rangle_\psi$, where $F_{0s} = F_{0s}(\psi, v)$ is a Maxwellian distribution function and the flux surface average is defined by $\langle Q \rangle_\psi := V'^{-1} \int \mathrm{d}\theta \int \mathrm{d}\varphi \sqrt{g}\, Q$ with $V' = \int \mathrm{d}\theta \int \mathrm{d}\varphi \sqrt{g}$, here θ and φ represent poloidal and toroidal angles respectively and $\sqrt{g}$ is the Jacobian. We have also used averaging operations involving spatial coordinates:

$$\overline{Q} = \begin{cases} \left\langle QB/|v_\parallel| \right\rangle_\psi \left\langle B/|v_\parallel| \right\rangle_\psi^{-1} & \text{for passing particles,} \\ \left(\oint Q\, \mathrm{d}l/v_\parallel \right) \left(\oint \mathrm{d}l/v_\parallel \right)^{-1} & \text{for trapped particles.} \end{cases}$$

Equation (1) is valid for any toroidal geometry and can be particularized for tokamaks by taking $\overline{\omega_s} = 0$ and for stellarators by taking $\overline{\omega_s} = 0$ only for passing particles. An initial perturbed distribution function $f_s(0)$ has to be specified which must fulfill the quasineutrality condition. The residual level, defined by $\phi_k(\infty)/\phi_k(0)$, is found by taking the limit $\lim_{p\to 0} p\, \widehat{\phi}_k(p)/\phi_k(0)$.

CAS3D-K, EUTERPE and GENE codes

CAS3D-K [2, 3] classifies particles by their trajectories. It is this classification what makes the code very useful to calculate certain average operations. The code is parallelized using MPI making the computation separable by flux surfaces and can work in any toroidal magnetic configuration from VMEC. We have extended CAS3D-K to perform the phase-space integrations and averages required in Eq. (1). We have also implemented the solution of various magnetic differential equations.

EUTERPE [4, 5] is a global gyrokinetic code in 3D geometry of the Particle In Cell type with a Lagrangian scheme, uses a δf discretization with markers and can be used in linear and non-linear modes.

GENE [6] is an Eulerian gyrokinetic δf code which can be run in radially global, full flux surface or flux tube simulation domains. Here, we restrict to flux tubes only.

CAS3D-K benchmark in Large Aspect Ratio Tokamaks with a source term

We calculate, with CAS3D-K, the residual level with a constant source term given in Refs. [7, 8] which can be found by adding to the quasineutrality equation a constant source and taking $f_s(0) = 0$ in Eq. (1). The expression for the residual level with adiabatic electrons is given by $\phi_k(\infty)/\phi_k(0) = \{1 - J_{0i}^2\}_i / \{1 - J_{0i} e^{-ik_\psi \delta_i} \overline{J_{0i} e^{ik_\psi \delta_i}}\}_i$ and with kinetic electrons by $\phi_k(\infty)/\phi_k(0) = \sum_s \{1 - J_{0s}^2\}_s / \sum_s \{1 - J_{0s} e^{-ik_\psi \delta_s} \overline{J_{0s} e^{ik_\psi \delta_s}}\}_s$. We calculate these expressions with CAS3D-K and compare them against the same quantities obtained by two independent calculations: we compare the case with adiabatic electrons against the analytical result in Ref. [7] and the case with kinetic electrons against a gyrokinetic simulation with GENE.

In Fig. 1 (left) we show the analytical result, taken from Ref. [7] with adiabatic electrons,

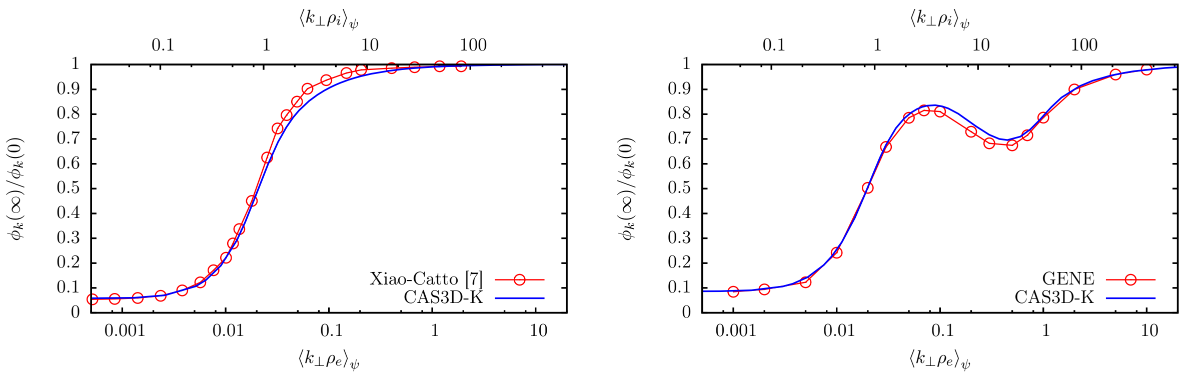

Figure 1: *CAS3D-K benchmark against an analytical solution (left), and a local gyrokinetic simulation (right), in a large aspect ratio tokamak with a source term.*

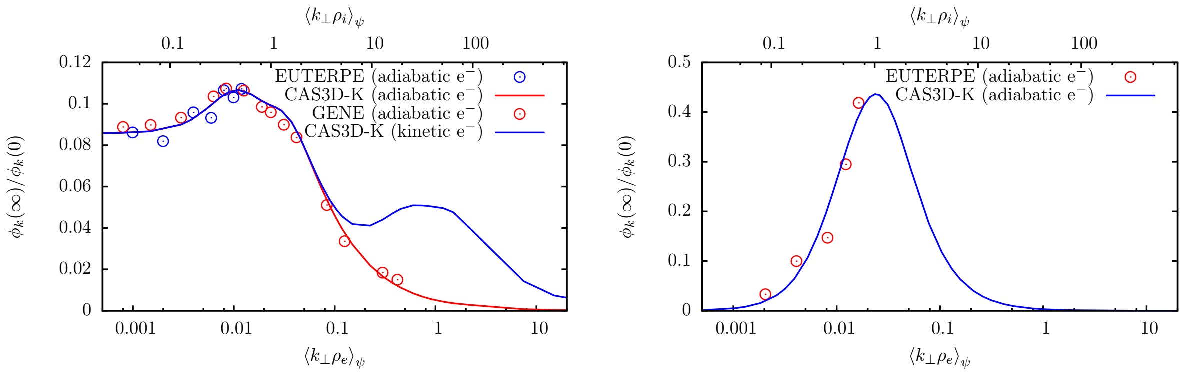

Figure 2: *Residual level obtained by a semi-analytical calculation with CAS3D-K and numerical simulations with the gyrokinetic codes GENE and EUTERPE in a large aspect ratio tokamak with adiabatic and kinetic electrons (left) and in the Wendelstein 7-X stellarator with adiabatic electrons (right).*

obtained in an analytical circular large aspect ratio tokamak, and we compare it against the computed with CAS3D-K in a large aspect ratio tokamak with the same parameters $q = 2$ and $\varepsilon = 0.2$. The results show good agreement.

In Fig. 1 (right) we compare the result obtained with CAS3D-K, with kinetic electrons, and a gyrokinetic simulation with GENE. Both results show also good agreement for all radial wavelengths.

Residual zonal flow level for the initial value problem

We compare the residual level obtained from Eq. (1) by different methods, a calculation with CAS3D-K and gyrokinetic simulations with EUTERPE and GENE, in two toroidal devices: a large aspect ratio tokamak, with major radius $R = 1.7$ m and minor radius $a = 0.4$ m, and the Wendelstein 7-X stellarator standard configuration.

In Fig. 2 (left) we show the residual level obtained for the large aspect ratio tokamak, at a radial position $\psi = 0.25$, calculated from CAS3D-K, EUTERPE and GENE with adiabatic electrons and the residual from CAS3D-K with kinetic electrons. The agreement between the semi-analytical calculation and the gyrokinetic simulations with adiabatic electrons is really good for all radial wavelengths.

In Fig. 2 (right) we show the residual level obtained for the Wendelstein 7-X stellarator, at a radial position $\psi = 0.25$, obtained from CAS3D-K and EUTERPE, with adiabatic electrons.

Conclusions

The derived Eq. (1) is valid for any wavelength and any magnetic toroidal geometry. In particular we have obtained the residual zonal flow level in a large aspect ratio tokamak and the Wendelstein 7-X stellarator.

CAS3D-K has been extended to perform the phase-space integrations required in Eq. (1) for any radial wavelength and to solve the required magnetic differential equations.

The main advantage of using CAS3D-K is that we can calculate the residual level with much less computational cost than gyrokinetic simulations. This advantage is even more clear when using kinetic rather than adiabatic electrons.

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