Changes in the radial structure of EPMs during the chirping phase taking the uncertainties of the time-frequency transforms into account

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The understanding of energetic particle modes (EPMs) in tokamaks plays a key role in preparation of future burning plasma experiments. The aim of the present paper is to investigate the properties of chirping beta-induced Alfven eigenmodes (BAEs) [1, 2]. Our goal is to find out if there is any change in the radial mode structure during the non-linear chirping phase.

Mode amplitude and uncertainties

In order to reach our goal the soft X-ray (SXR) diagnostics of ASDEX Upgrade was used. We analyzed the signals from six line of sights (LOS) of camera J measuring the plasma core, covering the region ($\rho_{\rm pol} < 0.25$) where the BAEs were observed. The different LOSs represent different parts of the plasma. An estimation of the mode structure of the BAEs is given by estimating the mode amplitude on each LOS.

Continuous linear time-frequency analysis is ideal for studying of transient wave-like phenomena, thus we have chosen short-time Fourier transform (STFT) [3], which is time- and frequency shift invariant, and it is very suitable to study fluctuations in a limited frequency range. The squared absolute value of the STFT (called spectrogram) is defined as an energy density on the time-frequency plane. A spectrogram calculated from one LOS of the SXR camera is presented on figure 1. Downwards chirping $n = 1$ BAE modes are visible in the range of 65 - 90 kHz. Using a maximum-searching algorithm, the time evolution of the maxima of the mode frequency can be traced, which is called the ridge of the STFT transform. These frequency ridges are indicated with dashed black lines on figure 1. Following the ridge, the time evolution of the mode amplitude can be estimated. In a given time instant the amplitude of the mode can be calculated by taking the value of STFT on the ridge [4]:

$$a_0(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}} \left| \mathrm{STFT}(t, \omega = \omega_{\rm ridge}) \right| , \qquad (1)$$

when the amplitude and frequency of the mode is constant on the time scale of σ, the width of the Gaussian window used to evaluate the STFT.

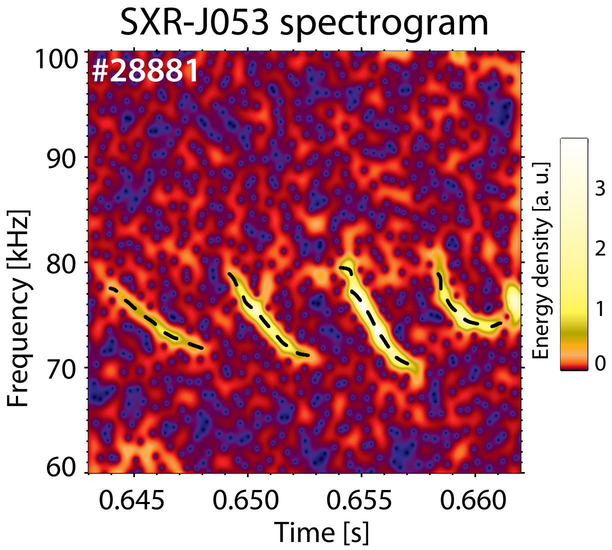

Figure 1: *Chirping $n = 1$ BAE modes with decreasing frequency in the range of 65 - 90 kHz are visible on the SXR spectrogram. Time evolution of the frequency ridge is indicated with black dashed line.*

However, in the case of chirping modes the fast changes in the frequency and amplitude require a well established criterion to determine the application limits of this approximation. We have modelled this mode as a frequency and amplitude modulated chirp: $f(t) = a(t)\cos[\phi(t)]$, where the derivative of the phase gives the instantaneous frequency $(\phi(t)' \equiv \omega(t))$. In order to observe the effect of modulation on the values of the STFT on the ridge the 1st order Taylor expansion of the amplitude $\left(a(t) \approx a(u) + a'(u)t\right)$ and the 2nd order Taylor expansion of the phase $\left(\phi(t) \approx \phi(u) + \phi'(u)t + \phi''(u)t^2/2\right)$ was calculated. Substituting these formulas into the definition of the STFT, we get the relation between the instantaneous amplitude of the chirp and the value of the STFT on the ridge:

$$a_1(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}} \sqrt[4]{1 + 4\phi''(u)^2\sigma^4} \left|\mathrm{STFT}(t, \omega = \omega_{\mathrm{ridge}})\right| . \qquad (2)$$

The difference between the zeroth order (1) and the first order (2) approximation is a factor of $\sqrt[4]{1 + 4\phi''(u)^2\sigma^4}$, where $\phi''(u)$ is the slope of the frequency. The application of this first order approximation can give a better characterization of the time evolution of the mode frequency, nevertheless the localization of the frequency ridge is biased by the background noise which can lead to artifacts in the correction. Furthermore, the calculation of the correction requires evaluation of a numerical derivative, which can also complicate the interpretation of the result. The calculation of the numerical derivative must be preceded by an interpolation which is done by a reproducing kernel [3]. This method can reconstruct the values of the continuous transform in any arbitrary time-frequency point, but it is very CPU intensive to perform. On the other hand this correction is also proportional to the σ width of the Gaussian, which means that with a narrower window the contribution of the correction term is reduced. A narrower window in time means a worse resolution in frequency, which increases the uncertainty of the ridge determination, leading to a necessary trade-off. The optimum resolution is found by choosing a window size where the contribution of the correction factor is smaller than the uncertainty emerging from background noise, while still providing a sufficient frequency resolution. The estimation of the amplitude is also biased by the background noise, which has to be taken into account. The background noise is assumed to be a broadband, statistically independent, additive white noise. Since the STFT is a linear transformation the effect of the noise on the amplitudes can be taken into account by estimating the mean value of the background noise on the spectrogram. In each case we selected an area on the time-frequency plane where the background noise is the dominant signal component and no other coherent modes appear. The mean of the STFT in this area is chosen to represent the uncertainty of the measurement.

Time evolution of the radial eigenfunction

We present the analysis of 3 BAE chirps observed in the ramp-up phase of discharge #28881 on the ASDEX Upgrade tokamak. Due to the ramp-up phase and the time distance between the three chirps, it is possible that each three of these chirps exhibit a different mode structure and time evolution. The soft X-ray spectrograms are shown in figure 2a-c, where the frequency ridges are indicated with a thin black line. The instantaneous amplitude of the mode on each LOS is calculated by using the zeroth order approximation (1). The uncertainty of the measurement is calculated from the mean background noise as it was described before. Since different channels observe different parts of the plasma the radial distribution of the mode amplitude can be calculated. Each LOS is labeled with the normalized poloidal flux of the magnetic flux surface to which the LOS is tangential. This way the shape of the radial eigenfunction can be evaluated in any time instant. The spatially dependent, line integrated mode amplitudes (normalized to their weighted average) in figure 2g-i are not showing radial shift within uncertainties.

If the shape of the radial eigenfunction does not change during the chirping phase, the *ratio* of the amplitudes measured on two different LOSs is constant. The ratios are calculated between each channel pairs and normalized by their weighted average. The normalized ratios are shown on figure 2d-f. The uncertainty of the normalized ratios are calculated using standard error propagation and the value of variance is indicated with dashed lines. If the radial eigenfunction does not change in time, the expectation value of the normalized ratio is 1. The normalized ratios cross 1 within the errors bars most of the time which suggests that if there is change in the radial eigenfunction it is smaller than the uncertainty of our measurement. Significant changes of the eigenfunctions cannot be shown with the current level of the plasma background noise.

Conclusions

We presented a method to evaluate the instantaneous frequency and amplitude of chirping modes. We investigated the effect of fast frequency- and amplitude modulation of the instantaneous amplitude estimates. We introduced a correction factor that compensates for the systematic errors in the amplitude determination. However, the proper calculation of the correction

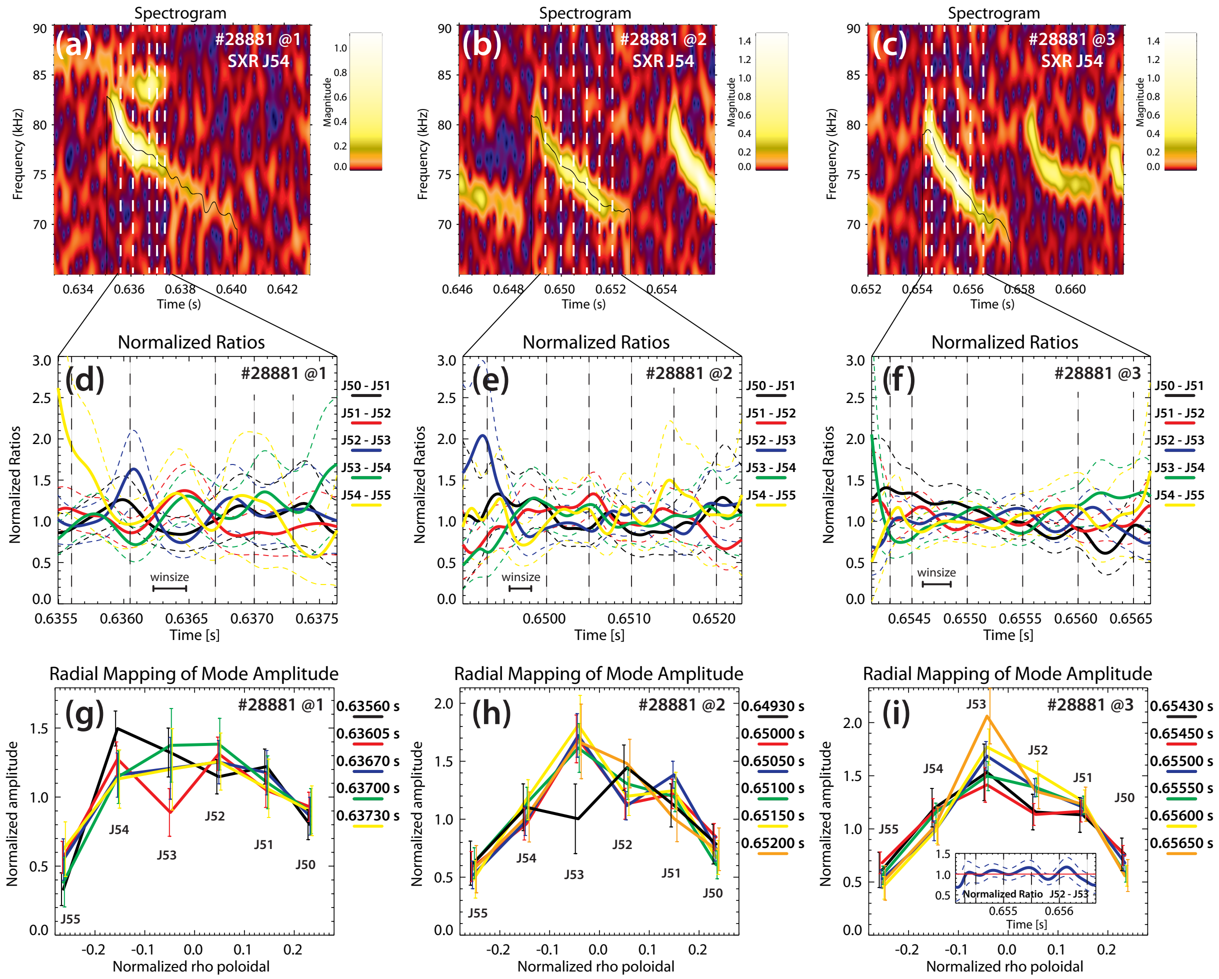

Figure 2: **(a-c)** *Soft X-ray spectrograms of the three investigated chirps. The frequency ridges are indicated with a thin black line.* **(d-f)** *The normalized ratios between neighbouring channel pairs. If the radial eigenfunction does not change in time, the expectation value of the normalized ratios is 1.* **(g-i)** *Normalized radial eigenfunctions in time instants indicated with white lines on spectrograms.*

factors requires the application of an interpolation based on STFT convolution, which is very CPU-intensive. In the physical cases studied in this paper a proper choice of the time-frequency window eliminates the necessity for such correction, as the systematic errors are smaller than the uncertainty from the background noise, leading to significantly reduced signal processing times. Our investigation shows that the changes in radial eigenfunction of the observed chirping modes is smaller than the uncertainty of our measurement. The analysis will be extended in the near future to a higher number of chirping modes under various plasma conditions. We also plan on using other diagnostic tools (such as ECE imaging).

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