Numerical studies on sawtooth crashes

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1. Introduction

As a typical phenomenon in tokamak discharges, sawtooth crash has attracted much research interest [e.g. 1-6]. In this paper sawtooth crashes are studied numerically based on both single and two fluid equations. Using the large aspect-ratio tokamak approximation, the magnetic field is defined as $\boldsymbol{B}=B_{0t}(\boldsymbol{e}_t-\boldsymbol{e}_\theta k_t/k_\theta)+\nabla\psi\times\boldsymbol{e}_t$, where ψ is the helical flux function, k_θ=m/r and k_t=n/R, m and n are the poloidal and toroidal mode numbers, r and R the minor and major radius, and the subscript 0 denotes an equilibrium quantity. The ion velocity $v=v_{||}\boldsymbol{e}_{||}+\boldsymbol{v}_\perp$, where $\boldsymbol{v}_\perp=\nabla\phi\times\boldsymbol{e}_t$, ϕ is the stream function, and the subscripts || and ⊥ denote the parallel and perpendicular components. The electron continuity equation, generalized Ohm's law, the equation of motion in the parallel and the perpendicular direction (after taking the operator $\mathbf{e}_t\cdot\nabla\times$), and the electron energy transport equation are solved. Normalizing the length to the plasma minor radius a, the time t to the resistive time $\tau_R=a^2/\eta$, ψ to aB_{0t}, and the electron density n_e and temperature T_e to their values at the magnetic axis, one has [7]

$$\frac{dn_e}{dt} = d_1\nabla_{||}j - \nabla_{||}(n_e v_{||}) + \nabla\cdot(D_\perp\nabla n_e) + S_n \quad (1)$$

$$\frac{d\psi}{dt} = E_0 - \eta j - \frac{\eta}{\nu_{ei}}\frac{dj}{dt} - \eta\frac{\mu_e}{\nu_{ei}}\nabla_\perp^2 j + \Omega(\nabla_{||}n_e + \nabla_{||}T_e) \quad (2)$$

$$\frac{dv_{||}}{dt} = -C_s^2\nabla_{||}p / n_e + \mu\nabla_\perp^2 v_{||} \quad (3)$$

$$\frac{dU}{dt} = S^2\nabla_{||}j + \mu\nabla_\perp^2 U \quad (4)$$

$$\frac{3}{2}n_e\frac{dT_e}{dt} = d_1 T_e\nabla_{||}j - T_e n_e\nabla_{||}v_{||} + n_e\nabla\cdot(\chi_{||}\nabla_{||}T_e) + n_e\nabla\cdot(\chi_\perp\nabla_\perp T_e) + S_p \quad (5)$$

where $d/dt=\partial/\partial t+\boldsymbol{v}_\perp\cdot\nabla$, $j=-\nabla_\perp^2\psi-2nB_{0t}/mR$ is the plasma current density along the $\mathbf{e}_t$ direction, $U=-\nabla_\perp^2\phi$ is the plasma vorticity, μ the ion viscosity, χ the heat conductivity, and D the particle diffusivity. S_n and S_p are the particle and heat source, E_0 is the equilibrium electric field, $S=\tau_R/\tau_A$, $p=p_e=n_eT_e$, $d_1=\omega_{ce}/\nu_{ei}$, $\Omega=\beta_e d_1$, $C_s=[T_e/m_i]^{1/2}/(a/\tau_R)$, $\beta_e=4\pi n_eT_e/B_{0t}^2$, ω_{ce} is the electron cyclotron frequency, μ_e the perpendicular electron viscosity, υ_{ei} the electron-ion collisional frequency, $\tau_A=a/V_A$, and V_A is defined using B_{0t}. Above equations are utilized in Ref. [7] except for the electron inertia and perpendicular viscosity in Ohm's law.

2. Numerical results

The single fluid results are obtained by neglecting the electron inertia and pressure gradient in the generalized Ohm's law. For low Lundquist number S, $S<10^7$, only a growing m/n=1/1 island is found, which finally occupies the whole region inside the q<1 surface accompanying the shrinking of the original core, similar to earlier results [3].

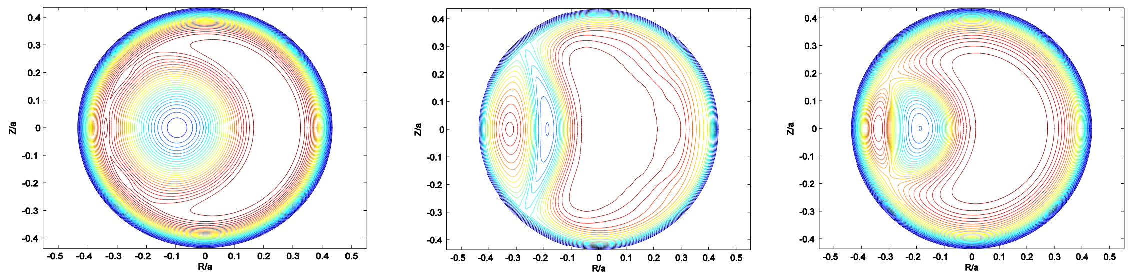

Figure 1 Constant-ψ contours on the R-Z plane for $S=2.65\times10^8$, where R=0 corresponds to the major radius of the original magnetic axis, and Z is along the vertical direction. (a/left) Small secondary islands grow on the left hand side of the core region. (b/middle) Original core region is squeezed into a thin layer by the 1/1 and the secondary island. (c/ right) In late phase when both the m/n=1/1 and secondary island change slowly in time.

For higher S values, however, the secondary islands (plasmoids) are found to grow. As an example, constant-ψ contours are shown in figure 1 for $S=2.65\times10^8$ at different times. A monotonic original q-profile is assumed with the q=1 surface located at $r_s=0.3a$, and $q_{(r=0)}=0.9$ at the original magnetic axis. The viscosities are taken to be $\mu=18.8(a^2/\tau_R)$ and $\mu_e/\upsilon_{ei}=10^{-4}a^2$. During the early nonlinear phase, there is only a growing m/n=1/1 island similar to the low S cases. At later times, however, the thin current sheet created by the 1/1 island breaks up, leading to the formation of small secondary islands as shown in Figure 1a. Afterwards, one secondary island grows into a large size in a short time scale in parallel to the 1/1 island, and the original plasma core is squeezed into a thin layer in between the two islands (figure 1b). This leads to additional small secondary islands during the mode growth [4]. The ultimate result of the reconnection process is a quasi-steady helical state with two coexisting islands (figure 1c) which persists on a long time scale, characteristic for the current re-arrangement within the q=1 surface. It is seen from figures 1b and 1c that, the 1/1 island has the maximum width during its fast growing phase before it enters into the quasi-steady state. For $S\geq10^9$, the island saturates at a width being significantly smaller than r_s.

The reconnection time is shown in figure 2 as a function of S. The black curve is the growth time of the m/n=1/1 island from the width W=0.1a up to the maximum ($S<10^9$) or saturated ($S\geq10^9$) island width. The blue line is the reconnection time calculated from

Kadomtsev's model [2]. The red dashed curve is the scaled reconnection time, obtained from the black curve by multiplying a factor $r_s/W_{max/sat}$, where $W_{max/sat}$ is the maximum ($S<10^9$) or saturated ($S\geq10^9$) island width. The reconnection time agrees with Kadomtsev's model for $S<10^7$ but is much shorter for larger S values. The formation of secondary islands allows faster reconnection for high S values.

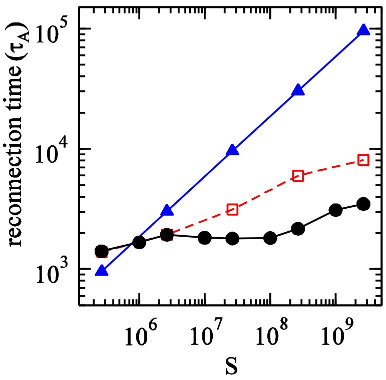

Figure 2 Reconnection time versus S. The black curve is the m/n=1/1 island growth time from the width w=0.1a up to the maximum or saturated island width. The red curve is scaled time. The blue line is calculated from Kadomtsev's model.

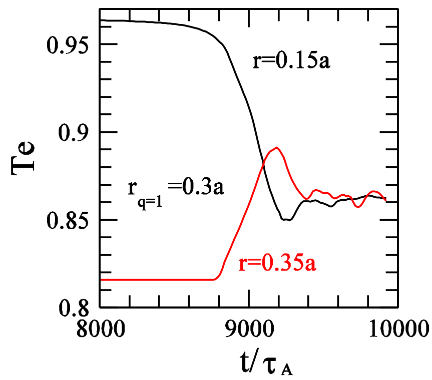

Figure 3 Results obtained from two-fluid equations: The time evolution of the electron temperature inside (r=0.15a) and outside (r=0.35a) the q=1 surface. The electron temperature decreases in a time period about 50μs.

The simulations based on two-fluid equations use typical ASDEX Upgrade plasma parameters as input: B_{0t}=2T, a=0.5m, R=1.7m, T_e=2keV and $n_e=3\times10^{19}m^{-3}$ at q=1 surface, which lead to $S=2.6\times10^8$, $\Omega=9.4\times10^4$, $C_s=2.0\times10^7(a/\tau_R)$, $d_I=3.1\times10^7$, and $\nu_{ei}=2.2\times10^4$/s. It is assumed that $\mu_e/\upsilon_{ei}=10^{-4}a^2$, $\chi_{\|}/\chi_\perp=8.0\times10^8$, $\chi_\perp=\mu_\perp=0.2/m^2/s=19(a^2/\tau_R)$ and $D_\perp=\mu/5$. The original equilibrium electron temperature profile is $T_e=T_{e0}\,[1-(r/a)^2]^2$.

The time evolution of the m/n=0/0 component of the electron temperature at different normalized minor radius, r/a, is shown in figure 3. The electron temperature decreases in a time period about 50μs, in agreement with ASDEX Upgrade experimental results [6].

The secondary island also exists in two-fluid calculations for a sufficiently high value of S (or low Ω). As an example, the constant-ψ contour during the mode growth is shown in figure 4 for $\Omega=3\times10^4$. The secondary island usually survives only for a short period of time during the mode growth, possibly caused by the diamagnetic drift. The ratio between the amplitude of $\psi_{2/2}$ and $\psi_{1/1}$ is usually in the range 0.3-0.5, no matter whether there are secondary islands or not.

Shear plasma flow is found to be generated by the internal kink mode. The radial profiles of the m/n=0/0 component of poloidal rotation velocity are shown in figure 5 at different time. In the original equilibrium there is no plasma rotation, and the plasma current is in the negative direction. The driven plasma rotation is in the counter (co-) current direction inside (outside) the q=1 surface in the linear phase and then propagates towards the magnetic axis during the mode growth. After sawtooth crash, the driven plasma rotation is in the co- (counter-) current direction inside (outside) the q=1 surface, in agreement with TCV experimental observations [8]. The shear flow results from the m/n=0/0 component of electromagnetic torque, caused by the difference between the mode frequency and the local electron fluid frequency.

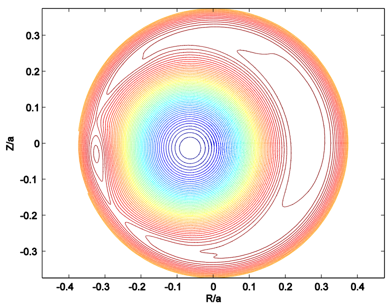

Figure 4 Constant-ψ contour for $\Omega=3\times10^4$, obtained from two fluid equations. The secondary island is formed during the mode growth but only survives for a short period of time.

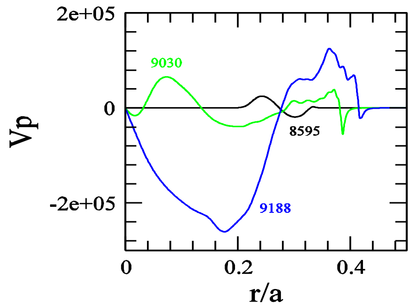

Figure 5 Radial profiles of m/n=0/0 component poloidal rotation velocity V_p in the early ($t=8595\tau_A$) and later ($9030\tau_A$) nonlinear phase and after sawtooth crash ($91188\tau_A$),

3. Summary

(a) From single fluid equations secondary islands are found to grow for a sufficiently high Lundquist number, leading to fast reconnection and ultimately a quasi-steady state with two coexisting islands. Kadmotsev's model is only applicable for low S values.

(b) Based on two-fluid equations, fast sawtooth crash, ~50μs, is obtained for typical ASDEX Upgrade parameters. Shear plasma flows is found to be driven by the internal kink mode.

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