Evaluation of non-ambipolar particle fluxes driven by external non-resonant magnetic perturbations in a tokamak *

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Introduction

The toroidal torque driven by external non-resonant magnetic perturbations (neoclassical toroidal viscosity) is an important momentum source affecting the toroidal plasma rotation in tokamaks. The well-known force-flux relation directly links this torque to the non-ambipolar neoclassical particle fluxes arising due to the violation of the toroidal symmetry of the magnetic field. A universal approach for this problem is usually provided by δf Monte Carlo methods [1, 2] where the linearized drift-kinetic equation (LDKE) is solved directly (without bounce-averaging). Here, a more efficient universal approach for the numerical computation of these fluxes (without model simplifications of the LDKE) is described where the problem dimension is reduced by one using the quasilinear approximation. The only limiting condition is that the non-axisymmetric perturbation field is small enough such that the effect of the perturbation field on particle motion within the flux surface is negligible. Therefore, in addition to most of the transport regimes described by the banana (bounce averaged) kinetic equation [3, 4] also such regimes as, e.g., ripple-plateau and resonant diffusion regimes are naturally included in this approach. The approach has been realized in the code NEO-2 [5] for general tokamak geometry using the full linearized collision operator. The quasilinear approach and the results of benchmarking the code NEO-2 with various existing models are the topic of this report.

Definitions

In Boozer coordinates (r, ϑ, φ) with re-defined flux surface label, $\langle|\nabla r|\rangle = 1$, the poloidal and toroidal components of the ion fluid velocity in the covariant notation are

$$V^\vartheta = \frac{ckB_\varphi}{e_i\sqrt{g}\langle B^2\rangle}\frac{\mathrm{d}T_i}{\mathrm{d}r}, \qquad V^\varphi = \frac{c}{\sqrt{g}B^\vartheta}\left(E_r - \frac{1}{e_i n_i}\frac{\mathrm{d}(n_i T_i)}{\mathrm{d}r}\right) + qV^\vartheta, \tag{1}$$

where c, e_i, n_i, T_i, E_r, $\sqrt{g}$, q and $\langle\dots\rangle$ are speed of light, ion charge, ion density, ion temperature, radial electric field, metric determinant, safety factor and the "flux surface" average, respectively. Here the coefficient $k = 5/2 - D_{32}/D_{31}$ is determined by the neoclassical transport coefficients D_{jk}, which link the thermodynamic forces A_k and fluxes I_j by the relations

$$I_j = -n_\alpha \sum_{k=1}^{3} D_{jk}A_k. \tag{2}$$

The thermodynamic fluxes are expressed through flux surface averaged particle ($I_1 = \Gamma_\alpha$) and heat flux density ($I_2 = Q_\alpha/T_\alpha$) and through the parallel flow density ($I_3 = n_\alpha\langle V_{\|\alpha}B\rangle$), whereas

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the thermodynamic forces are specified by

$$A_1 = \frac{1}{n_\alpha}\frac{\mathrm{d}n_\alpha}{\mathrm{d}r} - \frac{e_\alpha E_r}{T_\alpha} - \frac{3}{2T_\alpha}\frac{\mathrm{d}T_\alpha}{\mathrm{d}r}, \qquad A_2 = \frac{1}{T_\alpha}\frac{\mathrm{d}T_\alpha}{\mathrm{d}r}, \qquad A_3 = \frac{e_\alpha}{T_\alpha}\frac{\langle E_\| B\rangle}{\langle B^2\rangle}, \tag{3}$$

where α denotes the species and $E_\|$ is the inductive electric field. The evolution of the radial electric field is described by a simplified toroidal rotation equation neglecting contributions for neutral beam injection or other external sources

$$\frac{\partial}{\partial t}\sum_\alpha m_\alpha \left\langle g_{\varphi\varphi} n_\alpha V_\alpha^\varphi \right\rangle + \frac{1}{S}\frac{\partial}{\partial r} S \Pi_\varphi^r = T_\varphi^{NA}, \tag{4}$$

where m_α is α species mass, Π_φ^r is the momentum flux density due to neoclassical and anomalous transport and T_φ^{NA} is the density of the toroidal torque driven by external non-axisymmetric magnetic field perturbations. This toroidal torque density is linked with non-ambipolar particle fluxes Γ_α^{NA} via the flux-force relation,

$$T_\varphi^{NA} = -\frac{1}{c}\sqrt{g}B^\vartheta \sum_\alpha e_\alpha \Gamma_\alpha^{NA} \;=\; -\nu_{\mathrm{t}} m_i n_i \left\langle g_{\varphi\varphi}\left(V^\varphi - V_{\mathrm{in}}^\varphi\right)\right\rangle, \tag{5}$$

whereby the rotation relaxation rate ν_{t} (toroidal viscosity frequency) and the "intrinsic" ("offset") rotation velocity V_{in}^φ take a particular simple form if the electron particle flux is negligible,

$$\nu_{\mathrm{t}} = \frac{e_i^2 g \left(B^\vartheta\right)^2 D_{11}^{NA}}{c^2 m_i T_i \left\langle g_{\varphi\varphi}\right\rangle}, \qquad V_{\mathrm{in}}^\varphi = \frac{c\, k_{NA}}{e_i\sqrt{g}B^\vartheta}\frac{\mathrm{d}T_i}{\mathrm{d}r} \quad \text{with} \quad k_{NA} = \frac{D_{12}^{NA}}{D_{11}^{NA}} - \frac{5}{2} + \frac{B_\varphi^2\, k}{\langle B^2\rangle \left\langle g_{\varphi\varphi}\right\rangle}. \tag{6}$$

Quasilinear ansatz

For the evaluation of non-ambipolar particle fluxes within the standard neoclassical approach the solution of the LDKE (which is 4D) is required. Within quasilinear approach, which reduces the problem dimension by one, the perturbation field is split into an axisymmetric and small non-axisymmetric part, $B = B_0 + \delta B$. It is sufficient to find the solution to LDKE up to linear order in the perturbation field amplitude, $f_1 = f_{10} + f_{11}$. Expanding the non-axisymmetric parts of the magnetic field and of the solution in a Fourier series over a field aligned toroidal angle, $\varphi_0 = \varphi - q\vartheta$,

$$\delta B(\vartheta,\varphi_0) = \mathrm{Re}\sum_{n=1}^{\infty} B_n(\vartheta)\mathrm{e}^{in\varphi_0}, \qquad f_{11}(\vartheta,\varphi_0) = \mathrm{Re}\sum_{n=1}^{\infty} f_n(\vartheta)\mathrm{e}^{in\varphi_0}, \tag{7}$$

the set of equations for the axisymmetric and the non-axisymmetric parts is

$$\hat{L}_{(0)} f_{10} \;=\; \frac{m_\alpha c B_\varphi v^2 (2-\eta B_0)}{2 e_\alpha \sqrt{g_0} B_0^3}\frac{\partial B_0}{\partial \vartheta}\frac{\partial f_M}{\partial r}, \tag{8}$$

$$\begin{aligned} \hat{L}_{(n)} f_n \;&=\; \frac{m_\alpha c B_\varphi v^2 (2-\eta B_0)}{2 e_\alpha \sqrt{g_0} B_0^3}\left(\frac{\partial B_n}{\partial \vartheta} - \frac{in B_0^2 B_n}{B_0^\vartheta B_\varphi} - \frac{2B_n}{B_0}\frac{\partial B_0}{\partial \vartheta}\right)\frac{\partial f_M}{\partial r} \\ &+\; v_\| \frac{B_0^\vartheta}{B_0^2}\left(\frac{B_n}{B_0}\frac{\partial B_0}{\partial \vartheta} - \frac{\partial B_n}{\partial \vartheta}\right)\eta\frac{\partial f_{10}}{\partial \eta} - \frac{B_n}{B_0}\hat{L}_{cL} f_{10}. \end{aligned} \tag{9}$$

Here, the velocity space variables are (v,η) where $\eta = v_\perp^2/(v^2 B_0)$, f_M is a local Maxwellian,

$$\hat{L}_{(n)} \equiv v_\| \frac{B_0^\vartheta}{B_0}\frac{\partial}{\partial \vartheta} - \hat{L}_{cL} + in(\Omega_{tE} + \Omega_{tB}), \tag{10}$$

$\hat{L}_{cL}$ is the full linearized collision operator and Ω_{tE} and Ω_{tB} are toroidal rotation frequencies due to the electric and magnetic drift, respectively. The non-ambipolar particle flux density, which determines the coefficients D_{11}^{NA} and D_{12}^{NA}, is expressed through Fourier amplitudes as

$$\Gamma_\alpha^{NA} = \sum_{n=1}^{\infty} \frac{\pi m_\alpha c}{4e_\alpha \sqrt{g_0} B_0^\vartheta} \left(\int_0^{2\pi} \frac{d\vartheta}{B_0^2} \right)^{-1} \int_0^{2\pi} \frac{d\vartheta}{B_0^2} \int_0^\infty dv v^4 \int_0^{1/B_0} d\eta \frac{2-\eta B_0}{\sqrt{1-\eta B_0}} \sum_{\sigma=\pm 1} n \, \mathrm{Im} f_n B_n^*. \qquad (11)$$

In NEO-2, the dependence of f_{10} and f_n on the velocity module v is discretized by a series expansion over associated Laguerre polynomials of the order 3/2 and the resulting set of coupled 2D equations is solved using a finite-difference scheme on an adaptive (ϑ, η) grid.

Benchmarking results

For benchmarking, a tokamak configuration with circular cross-section and aspect ratio $A = 3.8$ is used. The results for the full linearized collision model correspond here to the ion component. The perturbation field is taken in the form of a single harmonic, $\delta B = \varepsilon_M B_0(r, \vartheta) \cos(m\vartheta + n\varphi)$. Since transport coefficients have a simple, quadratic dependence on ε_M, this quantity has been set to 1 in all plots below. For the non-linear models, such as the DKES code [6], results are obtained for $\varepsilon_M = 10^{-3}$ and are then rescaled. Results of benchmarking with the DKES code are presented in Fig. 1. There scans over the collisionality parameter $\nu^* = 2\nu q R v_T^{-1}$ of the diffusion coefficient D_{11} normalized to the mono-energetic plateau diffusion coefficient $D_p = \pi q v_T \rho_L^2 / 16R$ are shown for various perturbation modes and for various radial electric fields given in terms of toroidal Mach numbers (normalized toroidal rotation velocity values) $M_t = \Omega_{tE} R v_T^{-1}$. Besides bounce-averaged regimes the ripple plateau and resonant diffusion regimes are clearly reproduced by both codes.

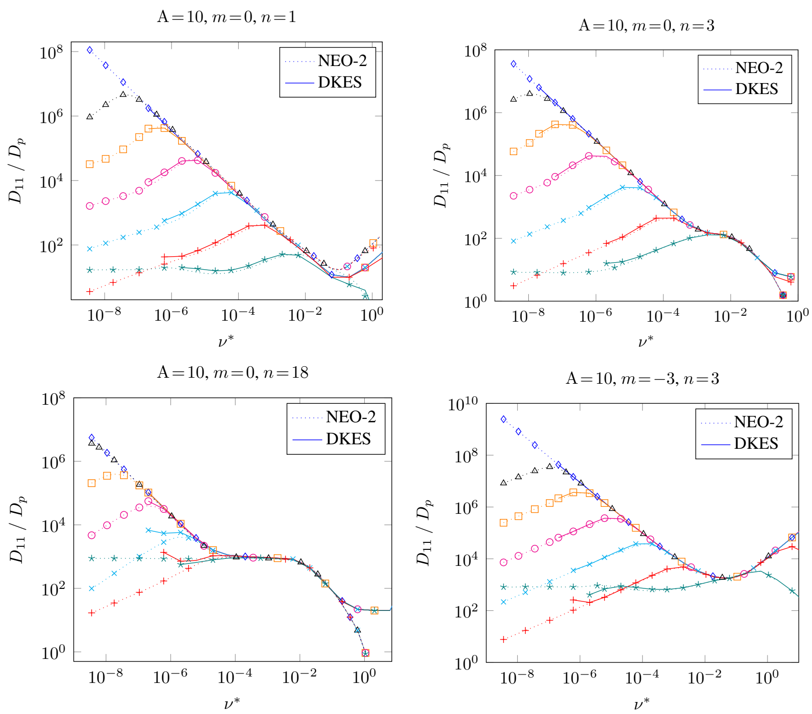

Fig. 1. Normalized coefficient $D_{11} D_p^{-1}$ from NEO-2 (Lorentz collision model) and DKES [6] as a function of collisionality ν^* for various perturbation modes and toroidal Mach numbers $M_t = 0$ ($\Diamond$), $2.8 \cdot 10^{-7}$ ($\triangle$), $2.8 \cdot 10^{-6}$ ($\Box$), $2.8 \cdot 10^{-5}$ ($\circ$), $2.8 \cdot 10^{-4}$ ($\times$), $2.8 \cdot 10^{-3}$ ($+$) and $2.8 \cdot 10^{-2}$ ($\star$). The toroidal rotation frequency due to magnetic drift is set to zero for the four cases shown above. Aspect ratio and mode numbers are indicated in the titles.

Results of the computation with the full linearized collision operator and the comparison to the universal formula of Shaing et al [3] are shown in Fig. 2. A good agreement is seen in the domain of validity of the bounce-averaged approach.

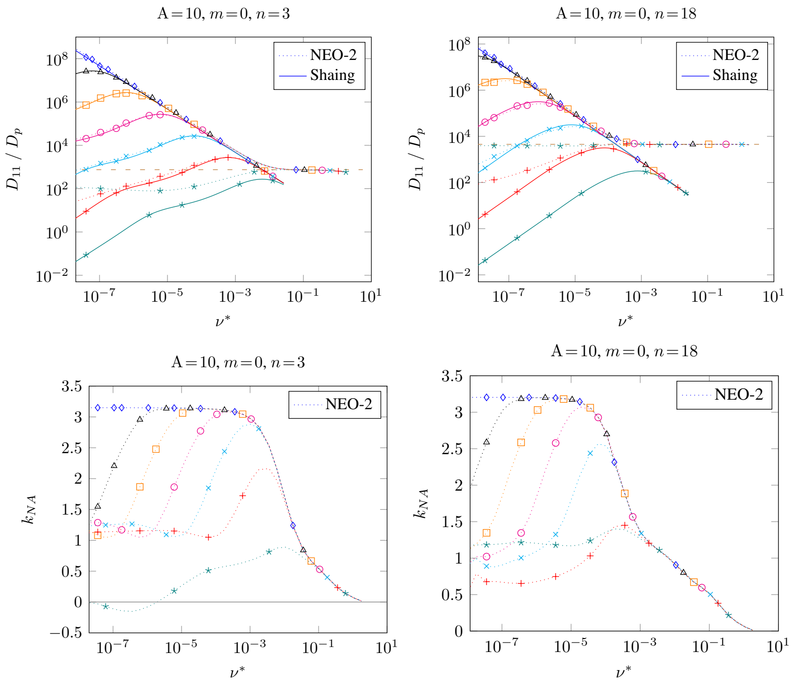

Fig. 2. Normalized coefficient $D_{11}D_p^{-1}$ from NEO-2 (full collision model) and bounce-averaged model of Shaing [3] (upper panel) and "offset" rotation coefficient k_{NA} from NEO-2 (lower panel) as functions of collisionality ν^* for various perturbation modes and toroidal Mach numbers $M_t = 0$ ($\Diamond$), $2.8 \cdot 10^{-7}$ ($\triangle$), $2.8 \cdot 10^{-6}$ ($\Box$), $2.8 \cdot 10^{-5}$ ($\circ$), $2.8 \cdot 10^{-4}$ ($\times$), $2.8 \cdot 10^{-3}$ ($+$) and $2.8 \cdot 10^{-2}$ ($\star$). The ripple-plateau diffusion coefficient [7] is shown with a dashed line at the upper panel.

The comparison of NEO-2 results with the full collision operator in the superbanana plateau regime to the respective asymptotic formula of Shaing et al [3] is shown in Fig. 3. The toroidal magnetic drift is fixed by setting $cT_\alpha \left(e_\alpha \psi_a\right)^{-1} = |\Omega_{tE}|$ where ψ_a is the toroidal flux at the edge.

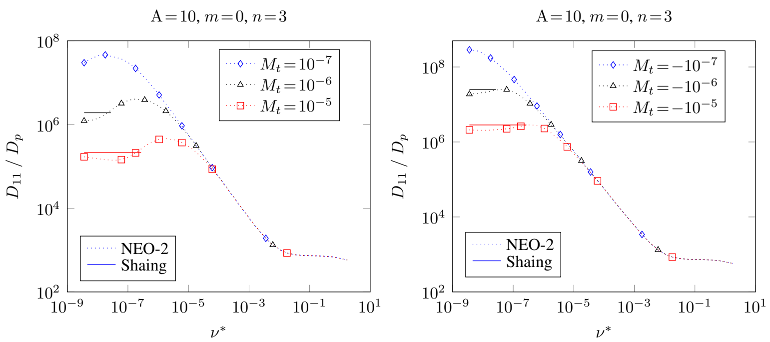

Fig. 3. Normalized coefficient $D_{11}D_p^{-1}$ from NEO-2 (full collision model) and asymptotical formula of Shaing [3] as a function of collisionality ν^* for various toroidal Mach numbers. Aspect ratio and mode number are indicated in the title.

Conclusions

The developed quasilinear version of the drift kinetic equation solver NEO-2 allows for an efficient evaluation of the non-ambipolar particle flux, which determines the torque, in the whole collisionality range. The only limiting assumption of sufficient smallness of the perturbation field is valid in most cases of practical interest. The code has been benchmarked with a few analytical and semi-analytical models as well as with the DKES code and stays in good agreement in the validity domain of those models. As shown in the benchmarking, bounce averaged models can give significant errors already in case of mild toroidal Mach numbers ($M_t \sim 0.03$ in the examples here) not only for short scale perturbations typical for the toroidal field ripple but also for medium scale perturbations such as perturbations produced by ELM mitigation coils.

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