Modeling of magnetic island modifications by ECCD using XTOR-2F

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Introduction

By reducing the maximum achievable β, Tearing Modes limit the performances of fusion devices. They can however be controlled and suppressed using Electron Cyclotron Current Drive (ECCD) as demonstrated in many tokamaks. In this work, simulations of tearing modes under the influence of an external current source have been carried out, and the results are then compared with analytical and empirical models.

Rutherford equation

The dynamic of the island size, here denoted W, can be described by the so-called extended Rutherford equation. The effect of an external current source on the island can be modeled by adding a term $\Delta'_{RF} = -\frac{D_{RF}}{W^2}\eta_{RF}$ in the Rutherford equation, with $D_{RF} = \frac{16\mu_0 R}{\pi s \psi'_s r_s} I_{RF}$, where I_{RF} is the total current injected by the RF-source and η_{RF} a measure of how efficiently the current is deposed inside the O-point. The latter depends on the source shape and position, as well as on its width compared to the island size. A definition of η_{RF} can be found in equation (16) of [3].

Physical model

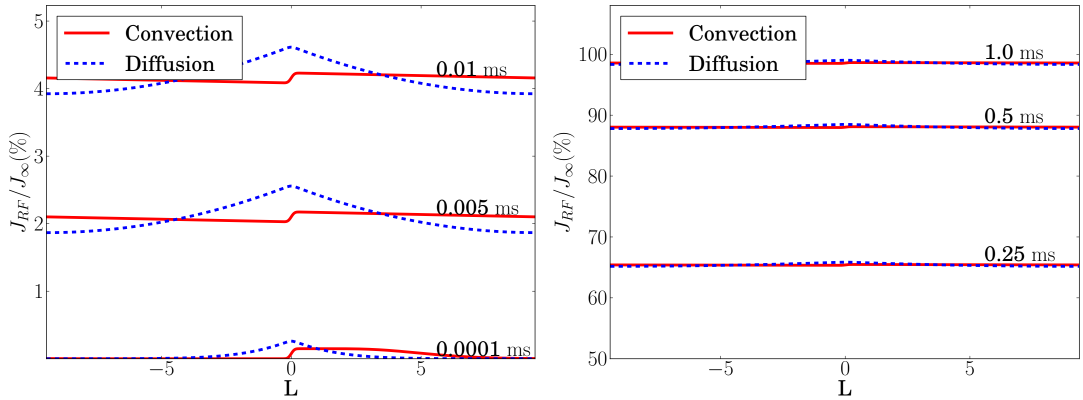
The nonlinear MHD code XTOR-2F [2] solves the two fluid 3D MHD equations in a torus. However, in the framework of this study, we restrict ourselves to a single-fluid case. The current induced by the RF source is implemented as a parallel term $\mathbf{J_{RF}} = J_{RF}\frac{\mathbf{B}}{|\mathbf{B}|}$ in the Ohm's law, $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta\,(\mathbf{J} - \mathbf{J}_{RF})$. To properly describe the propagation and the equilibration of this current along the magnetic field lines, a time-evolution equation for J_{RF} must be added. The physics of current drive generation by RF waves suggests to use a convective propagation of the current density along field lines [6] , as shown on equation (1), where ν_f is the collisionality of the fast electrons, expressed as $\nu_f = \nu_{ei}\left(\frac{\mathrm{v}_{th}}{\mathrm{v}_{res}}\right)^3$ [6], with $\mathrm{v}_{th} = \sqrt{2T_e/m_e}$ and v_{res} the velocities of the thermal and fast electrons, respectively. In the following, we have set $\frac{\mathrm{v}_{res}}{\mathrm{v}_{th}} = 2$. $\chi_\perp$ is taken equal to the perpendicular diffusion coefficient of the thermal particles, such that $\frac{\chi_\perp}{\eta} = 150$.

$$\frac{\partial J_{RF}}{\partial t} = \nu_f\,(J_s - J_{RF}) + \chi_\perp \nabla^2 J_{RF} + \mathrm{v}_{res}\nabla_{||} J_{RF} \tag{1}$$

This model leads to a rapid homogenization of the current density on the flux surfaces on a time scale of 5×10^{-4} ms for the plasma parameters considered here. The current rises to its final value with a characteristic timescale $\tau_{RF} = \nu_f^{-1}$ which is of the order of a quarter of a millisecond for the values considered in the following simulations. In the following, we show that due to the short time scale of the homogenization compared to the current rise, the convective propagation can be replaced by a diffusive one, as done in [4] for instance, without changing the physics of RF impact on magnetic island. This model is shown on equation (2).

$$\frac{\partial J_{RF}}{\partial t} = \nu_f \left(J_s - J_{RF}\right) + \chi_\perp \nabla^2 J_{RF} + \chi_{||} \nabla_{||}^2 J_{RF} \tag{2}$$

The figures 1a and 1b, computed with a simple 1D model corresponding to the projections of equations (1) and (2) on a closed field line, show the current evolution dynamic, for a Gaussian source centered in the middle of the field line, for $\nu_{ei} \approx 3.4 \times 10^3$ s^{-1}, v$_{th} \approx 2.7 \times 10^7$ m.s^{-1}, and $\chi_{||} \approx 2.7 \times 10^7$ m^2.s^{-1}. On a very short timescale, one can see the difference of propagation between the two models (figure 1a). Nevertheless, the current density is almost homogeneous on a field line, and the discrepancy between the two models is small (figure 1b). Note that in equation (2), we consider $\chi_{||}$ as a free parameter, that we choose so that the homogenization is fast compared to the island dynamic and the current rise, with a resulting current density homogeneous on a flux surface. For numerical reasons, we use the diffusive model (eq. (2)) to simulate the establishment of the current in the presented XTOR simulations.



(a) Current density along a magnetic field line, for homogeneization timescale

(b) Current density along a magnetic field line, for current-rise timescale

Figure 1: Evolution of the current density along a magnetic field line for the convective and diffusive model. The values are scaled to J_∞, the mean value of the source along a field line

Numerical simulations

We use an AUG-Like equilibrium. For simplicity, the shape of the separatrix is modified to be up-down symmetric. Pressure and density profiles are fitted from AUG pulse #29682, but the pressure is artificially reduced so as to deal with linearly unstable tearing modes. The central density is $n_i(0) = 8.2 \times 10^{19}$ m^{-3}, the magnetic field on the axis is $B_0 = 2.65$ T. In the following, the radial coordinates and widths are expressed in square root of the normalized toroidal flux unit, $\sqrt{\phi}$. At saturation, we observe a large 2/1-mode, whose size is $W_{sat} \approx 0.10$. When the RF source term is added, one has $\left.\frac{\mathrm{d}W}{\mathrm{d}t}\right|_{t=0} \propto -\frac{D_{RF}}{W_{sat}^2}\eta_{RF}(W_{sat})$. The surface $q = 2/1$ is located on $r_s = \sqrt{\phi_s} = 0.51$, while the O-points of the island are located on $r_O = 0.49$. The current source term is defined as $J_s(r,\theta,\phi) = J_s^0 \exp\left(\frac{-(r-r_{RF}^0)^2}{2\sigma_{r_{RF}}^2} \frac{-(\theta-\theta_{RF}^0)^2}{2\sigma_{\theta_{RF}}^2} \frac{-(\phi-\phi_{RF}^0)^2}{2\sigma_{\phi_{RF}}^2}\right)$, with $\sigma_r = 0.01$, $\sigma_\theta = 0.01$ rad, and $\sigma_\phi = 0.1$ rad. The total injected current is 2.5% of the total plasma current, corresponding to $I_{RF} \approx 18.75$ kA, and this quantity is kept constant while changing the size of the source.

Effect of total current injected and source width on island growth rate.

The definition of D_{RF} shows that $\left.\frac{\mathrm{d}W}{\mathrm{d}t}\right|_{t=0}$ depends linearly on I_{RF}. As shown on figure 2, we retrieve this behavior within our simulations. The figure 3 shows the dependence of η/η_{RF}^0 with the source width $\delta_I = 2.355\sigma_r$, where η_{RF}^0 is the efficiency obtained for a narrow ($\sigma_r = 10^{-3}$) and precisely O-point localized source and δ_I is the radial full width at half maximum of the source. The behavior observed in XTOR is in excellent agreement with Hegna's model.

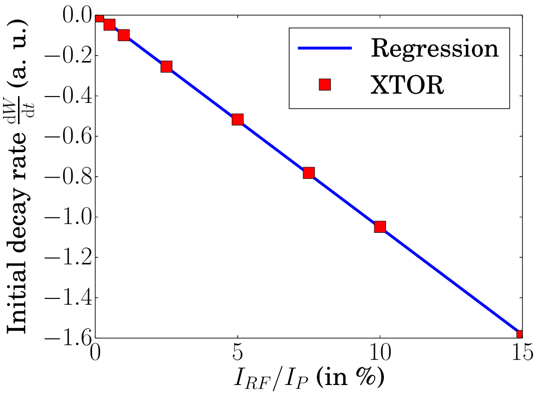


Figure 2: Total injected current Scan.

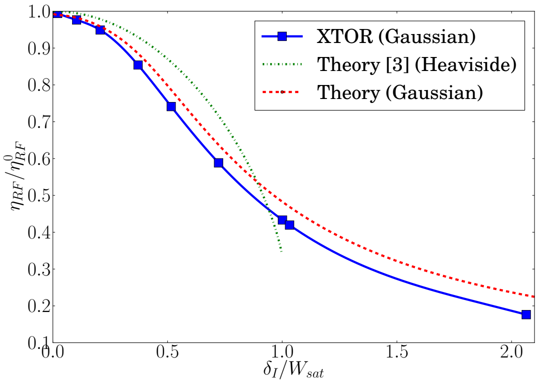


Figure 3: Scan along radial width of the source

Effect of misalignment

We vary the position of the source around the surface r_s, its width being held constant. The figure 4 shows the radial extension of the scan, as well as the source size compared to the island. The figure 5 shows the comparison of the results obtained with the XTOR simulations and the computation using Hegna's model. The efficiency is maximum for a radial position inside the resonance, on the surface r_O, due to the fact the O-point of the island do not lie exactly on the resonant surface. Since in Hegna's model, it is assumed that $r_s = r_O$, on figure 5, we have shifted the curve corresponding to the Hegna's model so that its origin is on r_O and not on r_s. For radial positions inside the island, we retrieve the stabilization suggested by the computation of η_{RF}.

We however observe an asymmetry, which is not present in Hegna's model. This asymmetry is likely due to the asymmetry of the shape of the island. The shape of the curve obtained is also consistent with the results obtained by Perkins in [5]. In [7], La Haye suggests that η could be fitted as $\eta \approx \exp\left(-\frac{5}{3}\frac{\Delta R}{\delta_I}\right)$, where ΔR is the radial misalignment. This fit is plotted on figure 5, and appears to be too narrow. In [8], La Haye comes to the same conclusion and suggests to broaden the ECCD deposition size such that $\delta_I \approx W_{sat}$, to fit the experimental data. This is also plotted on figure 5 (La Haye (broad)). This leads to a better fit, yet too broad. It appears that in our case, a good fit can be obtained using La Haye's model assuming that $\delta_I \approx \frac{1}{2}W_{sat}$.

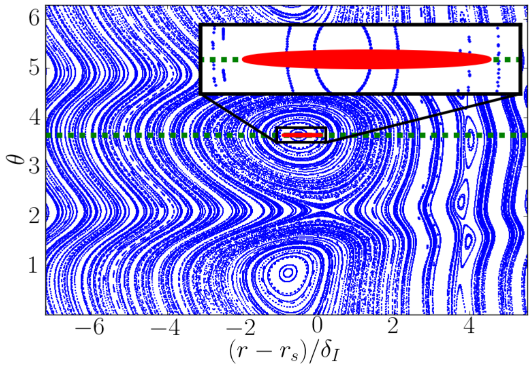


Figure 4: Poincaré plot of the island. The dotted-line shows the radial extension of the scan. The red ellipse shows the mid-height size of the RF current source.

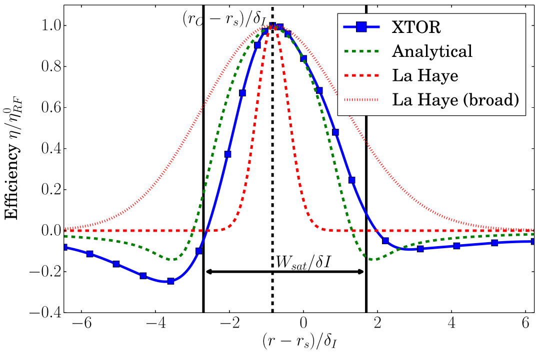


Figure 5: Comparison between the efficiencies defined using Hegna's model and the XTOR simulations.

Conclusion

In this work, simulations of tearing modes modification under the influence of an external current source have been carried out using the toroidal nonlinear full MHD code XTOR, in which the proper terms have been implemented. The results have been compared with analytical and empirical models, and show a good agreement. This work paves the way to simulations of NTMs behavior under ECCD, as well as their control.

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