Low Scale Unification of Gauge Interactions

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We investigate the possibility to realize the unification of the Standard Model gauge interactions at the low scale in four dimensions. We find that the fields needed to define a minimal theory where baryon and lepton numbers are local symmetries, allow for unification at the low scale in agreement with experiments. In these scenarios the proton is stable and we briefly discuss the implications for cosmology.

I. INTRODUCTION

Grand Unified Theories (GUTs) are considered as one of the most appealing extensions of the Standard Model where one understands the origin of the gauge interactions. In the context of GUTs [1] the Standard Model interactions are just different manifestations at low-energy of the same fundamental interaction. Unfortunately, these theories are realized at very high energy scales, $M_{GUT} \sim 10^{15-16}$ GeV [2], and one cannot hope to test these ideas directly in experiments. Supersymmetric unification [3] is very appealing and works very well in the context of the Minimal Supersymmetric Standard Model. However, the unification is also only possible at energy scales around 10^{16} GeV or even at the String scale, see for example [4]. The most generic prediction coming from grand unification is proton decay [5] and in most of the GUT models one needs an extra suppression mechanism to satisfy the experimental limits on the proton lifetime. This is the main reason to assume the large energy gap between the electroweak and GUT scale which is known as the great desert.

It is well-known that in the Standard Model baryon and lepton numbers are conserved symmetries at the classical level. In our modern view, the Standard Model is just an effective theory which describes physics at the electroweak scale. In general, one should think about the impact of all possible higher-dimensional operators which modify the Standard Model predictions. For example, in the Standard Model one can have the following higher order effective operators

$$\mathcal{L} \supset \frac{c_L}{\Lambda_L} \ell_L \ell_L H^2 + \frac{c_B}{\Lambda_B^2} q_L q_L q_L \ell_L$$
$$+ \frac{c_F}{\Lambda_F^2} (\bar{q}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu q_L) + \dots, \tag{1}$$

where the first operator violates lepton number, the second violates baryon number and the third breaks the flavour symmetry of the Standard Model gauge sector. The experimental bounds demand $\Lambda_L < 10^{14}$ GeV, $\Lambda_B > 10^{15}$ GeV [5], and $\Lambda_F > 10^{3-4}$ TeV [6], when c_L, c_B and c_F are of order one. Since in any generic GUT one generates the second operator due to new gauge interactions, it is impossible to achieve unification at the low scale without predicting an unstable proton and therefore one needs the great desert. If one could imagine a theoretical framework where the second operator is absent, the proton is stable and the unification scale is only con-
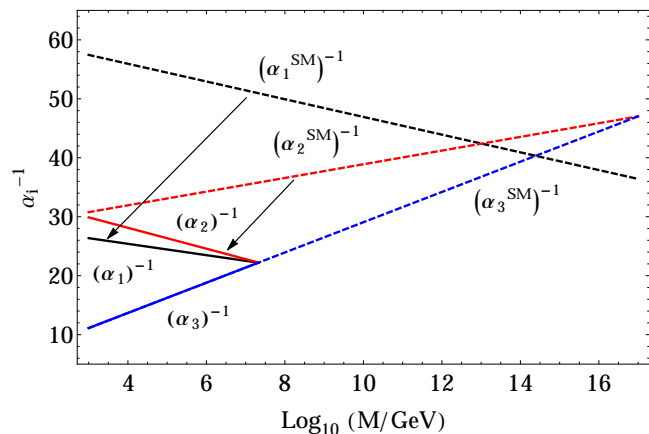

FIG. 1: Evolution of the gauge couplings in the Standard Model represented by dashed lines and in the new model where the unification scale is realized at 10^4 TeV. See the text for details.

strained by flavor physics and has to be above 10^4 TeV. This is the main goal of this article.

In this article, we investigate the possibility to achieve unification at the low scale in the context of theories where baryon and lepton numbers are local symmetries spontaneously broken at the low scale. In these theories, the proton is stable and the new fields needed to define an anomaly free theory allow for unification at the low scale. One can imagine different scenarios where the unification scale could be between 10^4 TeV and the Planck scale. If the unification scale is smaller than 10^4 TeV, one needs an extra mechanism to suppress the flavor violating effective operators. Following the recent results presented in Refs. [7–10] we find that the lepto-baryons needed to cancel the baryonic and leptonic anomalies allow us to have unification at the low scale. We discuss in detail the possible scenarios where the unification scale is around 10^4 TeV and point out the main features of the model. See Fig. 1 for the unification of Standard Model gauge interactions at 10^4 TeV.

II. UNIFICATION OF GAUGE FORCES

In order to investigate the unification of gauge interactions at the low scale it is important to make sure that the proton

is stable or the baryon number violating operators are highly suppressed. Recently, the minimal theory [10] where the baryon and lepton numbers can be defined as local symmetries has been pointed out. This theory is based on the gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L,$$

and the new fermions called "lepto-baryons" needed for anomaly cancellation are

$$\Psi_L \sim (1, 2, 1/2, B, L), \Psi_R \sim (1, 2, 1/2, -B, -L),$$
$$\Sigma_L \sim (1, 3, 0, -B, -L), \text{ and } \chi_L \sim (1, 1, 0, -B, -L),$$

where $B = L = 3/2n_F$, and n_F is the number of copies. Notice that these fields change only the evolution of the $SU(2)_L$ and $U(1)_Y$ gauge couplings.

The Higgs sector of this theory is composed of the fields $S_B \sim (1, 1, 0, 2B, 2L)$ and $S_L \sim (1, 1, 0, 0, 2)$. Therefore, when S_L acquires a vacuum expectation value we will have $\Delta L = \pm 2$ processes. When $U(1)_B$ is spontaneously broken and $n_F \neq 3n$ (n is an integer number) the proton is stable because one only has $\Delta B = \pm 3/n_F$ interactions. The evolution of the Standard Model gauge couplings at one-loop level is described by the equation

$$k_i \alpha_i^{-1}(M) = \alpha_i^{-1}(M_Z) - \frac{B_i}{2\pi} \text{Log}\left(\frac{M}{M_Z}\right), \quad (2)$$

where $k_i = (k_1, k_2, k_3)$ are the normalization factors. In the simplest Grand Unified Theory based on $SU(5)$ one has $k_i = (5/3, 1, 1)$. However, in general the k_1 value depends of the embedding in a given theory. The coefficients

$$B_i = b_i^{\text{SM}} + \theta(M - M_F) n_F b_i^{\text{new}} \times \frac{\text{Log}(M/M_F)}{\text{Log}(M/M_Z)}, \quad (3)$$

contain all possible contributions from the low scale, M_Z, to the unification scale, M_U. Here, M_F is the mass scale of the new fermions, $\theta(x)$ is the step function, and $b_i^{\text{SM}} = (41/6, -19/6, -7)$ are the coefficients in the Standard Model.

Using the above equations one can solve for the unification scale as a function of the values of the gauge couplings at the M_Z scale and the mass of the new fields

$$\text{Log}(M_U) = \frac{C(M_Z) + n_F(b_2^{\text{new}} - b_3^{\text{new}})\text{Log}(M_F)}{b_2^{\text{SM}} - b_3^{\text{SM}} + n_F(b_2^{\text{new}} - b_3^{\text{new}})}, \quad (4)$$

where

$$C(M_Z) = 2\pi(\alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z)) + (b_2^{\text{SM}} - b_3^{\text{SM}})\text{Log}(M_Z). \quad (5)$$

In the same way, we can find the expression for the value of k_1 for given values of M_F

$$k_1 = \frac{2\pi\alpha_1^{-1}(M_Z) - b_1^{\text{SM}}\text{Log}\left(\frac{M_U}{M_Z}\right) - n_F b_1^{\text{new}}\text{Log}\left(\frac{M_U}{M_F}\right)}{2\pi\alpha_3^{-1}(M_Z) - b_3^{\text{SM}}\text{Log}\left(\frac{M_U}{M_Z}\right) - n_F b_3^{\text{new}}\text{Log}\left(\frac{M_U}{M_F}\right)}. \quad (6)$$

n_F	$M_U(\text{TeV})$	k_1
1	$1.24 \cdot 10^9$	2.05
2	$4.96 \cdot 10^6$	2.67
4	$2.14 \cdot 10^4$	3.62
5	$4.58 \cdot 10^3$	3.99

TABLE I: Solutions for the unification at the low scale in models with gauged baryon and lepton numbers.

In the model discussed above, where baryon and lepton numbers are local symmetries, the new coefficients are $b_i^{\text{new}} = (2/3, 2, 0)$. Now, we are ready to understand the numerical results assuming the unification of gauge interactions. In Table I we show different scenarios where n_F takes the values from one to five, excluding $n_F = 3$. Assuming for simplicity that the new fermions are at the same scale, $M_F = 500$ GeV, we compute the unification scale and the allowed values for the k_1 parameter. As one can see the unification scale can be as low as 10^4 TeV in agreement with the constraints from flavor violation. In all these scenarios the proton is stable.

In Fig. 1 we show the evolution of the gauge couplings in the Standard Model and in the model with lepto-baryons. The lepto-baryons at the low scale change the evolution of α_2 and α_1 dramatically without affecting α_3. We assume four copies of lepto-baryons with baryon and lepton numbers equal to 3/8. Since they have different baryon and lepton numbers than the Standard Model fields we never induce new sources of flavor violation and the unification scale M_U is larger than 10^4 TeV in order to suppress the flavor violating effective operators discussed in the introduction.

The lightest neutral field in the new sector is always stable after symmetry breaking as discussed in Ref. [10] and one has a candidate to describe the cold dark matter in the Universe. The collider bounds on the new fermions are weak because they can only be produced through electroweak interactions at the Large Hadron Collider (LHC). Therefore, this type of model is in agreement with all constraints coming from collider experiments and cosmology.

A. Baryonic and Leptonic Couplings

In this minimal theory for local baryon and lepton numbers the evolution of the new couplings is defined by the equation

$$\alpha_X^{-1}(M_Z) = k_X \alpha_X^{-1}(M_U) + \frac{B_X}{2\pi} \text{Log}\left(\frac{M_U}{M_Z}\right), \quad (7)$$

where $X = B, L$. The B_X coefficients are given by

$$B_B = b_B^{\text{SM}} + \theta(M_U - M_F) b_B^{\text{new}} \times \frac{\text{Log}(M_U/M_F)}{\text{Log}(M_U/M_Z)}, \quad (8)$$

and

$$B_L = b_L^{\text{SM}} + \theta(M_U - M_F) b_L^{\text{new}} \times \frac{\text{Log}(M_U/M_F)}{\text{Log}(M_U/M_Z)}. \quad (9)$$

Notice that the values of these gauge couplings at different scales and the k_X are barely constrained. However, one can envision that it is possible to define a theory where all gauge couplings are unified, the Standard Model couplings, α_1, α_2, α_3, and the new couplings α_B and α_L. Therefore, assuming unification at the M_U scale, we can find the values of α_B and α_L at the M_Z scale for given values of k_B and k_L. The contribution of the Standard Model fields to the running of the baryonic and leptonic couplings is given by the coefficients $b_B^{\rm SM} = 8/3$ and $b_L^{\rm SM} = 6$. The new fields contribute as follows

$$b_B^{\rm new} = \left(\frac{16}{3} n_F + \frac{4}{3} \right) B^2 = 3 \frac{(4n_F + 1)}{n_F^2}, \qquad (10)$$

$$b_L^{\rm new} = b_B^{\rm new} + \frac{10}{3}. \qquad (11)$$

Notice that the right-handed neutrinos and S_L contribute to the running of the leptonic coupling, defining the difference between the running of α_B and α_L. Hence, we can calculate
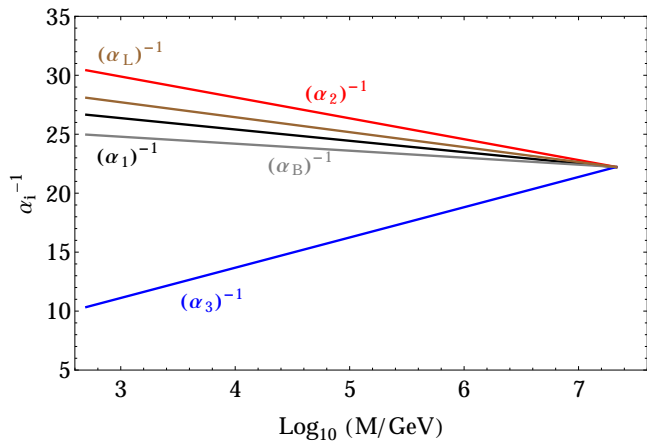

FIG. 2: Evolution of the gauge couplings with lepto-baryons where the unification scale is $M_U \sim 2 \times 10^4$ TeV. Here we assume for simplicity $k_B = k_L = k_1$ for the running of the α_B and α_L. At the M_Z scale we find $\alpha_B^{-1}(M_Z) = 25.17$ and $\alpha_L^{-1}(M_Z) = 28.69$.

the values of these couplings assuming unification at the low scale. For simplicity, we have assumed the same k_i factor for all Abelian symmetries in the theory.

In Fig. 2 we show the numerical results for the running of all gauge couplings. As one can appreciate the couplings α_B and α_L are small at the electroweak scale. This result is welcome because in this way one can satisfy the bounds on the leptophobic Z_B and quarkphobic Z_L gauge couplings coming from collider experiments. See Refs. [11, 12] for the bounds on the leptophobic gauge bosons.

Now, let us explicitly write down the relevant interactions for the model in order to understand how the different fields can obtain mass after symmetry breaking. The relevant interactions are given by

$$\begin{aligned} - \mathcal{L} \supset \; & h_1 \bar{\Psi}_R H \chi_L + h_2 H^\dagger \Psi_L \chi_L + h_3 H^\dagger \Sigma_L \Psi_L \\ & + h_4 \bar{\Psi}_R \Sigma_L H + \lambda_\Psi \bar{\Psi}_R \Psi_L S_B^* + \lambda_\chi \chi_L \chi_L S_B \\ & + \lambda_\Sigma \operatorname{Tr} \Sigma_L^2 S_B + Y_\nu \ell_L H \nu^c + \lambda_R \nu^c \nu^c S_L + \text{h.c.}, \end{aligned}$$
$$(12)$$

where $\nu^c = (\nu_R)^c$ are the right-handed neutrinos and $H \sim (1, 2, 1/2, 0, 0)$ is the Standard Model Higgs boson. In general for n_F copies of the fields the couplings h_1, h_2, h_3, h_4, λ_Ψ, λ_χ, and λ_Σ are $n_F \times n_F$ matrices. Notice that when S_B breaks $U(1)_B$ all the new fields can have large masses even before the electroweak symmetry is broken. The leptonic $U(1)_L$ symmetry is broken by the vacuum expectation value of S_L, and in the same way we generate large right-handed neutrino masses.

In Ref. [10] we have investigated the possible implications for baryogenesis if the local baryon number is broken at the low scale for $n_F = 1$. In this scenario the sphalerons must satisfy the extra condition that total baryon number is conserved. As one can see in Ref. [10] it is possible to have a consistent relation between the final baryon asymmetry and the $B - L$ asymmetry generated by some mechanisms in the early Universe such as leptogenesis. These results can be easily generalized for several copies of the new fermions. In Ref. [10] we also discussed the existence of a dark matter candidate which is the lightest Majorana fermion in the new sector. If we have several copies of these fields, the same candidate can be used to describe the cold dark matter in the Universe.

Before we summarize our main results, it is important to discuss the impact on the running of the gauge couplings in the context of other models where the cancellation of the baryonic and leptonic anomalies is possible. In Ref. [7] the anomaly cancellation was realized using an extra chiral family. Unfortunately, this solution is ruled out by the LHC due to the existence of a Standard Model-like Higgs. In Ref. [8] the possibility to use vector-like families for anomaly cancellation was pointed out. These vector-like families change the running of the gauge couplings, but the unification is always realized at high scales because the new quarks change the evolution of the strong coupling as well.

The possibility to cancel the B and L anomalies using lepto-baryons was also discussed in Ref. [9]. In this scenario, the new fields do not change the evolution of the strong coupling and they are singlets or live in the fundamental representation of $SU(2)$. Since the fundamental representation of $SU(2)$ contributes less than the adjoint representation one will need many copies of these fields to achieve the low scale unification. Therefore, we focus on the scenarios studied in this article since they are the minimalistic setting where we can make sure that the proton is stable after symmetry breaking and the unification can be realized at the low scale as shown in Fig. 1 and Fig. 2. We have presented our results at one-loop level in order to illustrate the main idea. It is also important to understand the predictions at two-loop level and in a given unified theory the threshold effects could be very important.

III. CONCLUDING REMARKS

In this article, we have shown that the unification of the Standard Model gauge interactions can be achieved at the low scale. In order to investigate this issue we focused on theories where baryon and lepton numbers are defined as local gauge symmetries spontaneously broken at the low scale. In this class of theories, the proton is stable and the unification of the gauge couplings can be realized at the low scale in a consistent way. We have illustrated the numerical results in Fig. 1 and Fig. 2 for the case $n_F = 4$, where the unification scale is $M_U \sim 2 \times 10^4$ TeV, in agreement with the constraints from flavor violating decays. In our opinion, one can have an even lower unification scale adding more copies of the new fermions, but it is only reasonable if there is a mechanism to suppress flavor violation. We listed one example in Table I where the unification is below 10^4 TeV in order to complete our studies.

The low unification scale which can be achieved with lepto-baryons motivates the possibility to test the idea of grand unification at future colliders. One should say that these results are just one step towards the possibility to have a complete unified theory at the low scale. We have identified which type of theories allow us to realize low scale unification. In this context, using different copies of lepto-baryons needed for anomaly cancellation we find the allowed values for the k_i coefficients for exact unification. These results should motivate the model builders in the GUT and String communities since now one can search for alternative ways to have unification at the low scale. In our opinion, these results tell us that the dream of testing the unification of gauge interactions is maybe not too far. The main priority in this field is to find the simplest unified theory which could allow us to have the embedding of this low energy theory with gauged baryon and lepton numbers.

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