

# Gamma-ray Line from Nambu-Goldstone Dark Matter in a Scale Invariant Extension of the Standard Model

Jisuke Kubo,<sup>1,\*</sup> Kher Sham Lim,<sup>2,†</sup> and Manfred Lindner<sup>2,‡</sup>

<sup>1</sup>*Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

<sup>2</sup>*Max-Planck-Institut für Kernphysik, 69117 Heidelberg, Germany*

## Abstract

A recently proposed scale invariant extension of the standard model is modified such that it includes a Dark Matter candidate which can annihilate into gamma-rays. For that a non-zero  $U(1)_Y$  hypercharge  $Q$  is assigned to the fermions in a QCD-like hidden sector. The Nambu-Goldstone bosons, that arise due to dynamical chiral symmetry breaking in the hidden sector, are cold Dark Matter candidates, and the extension allows them to annihilate into two photons, producing a  $\gamma$ -ray line spectrum. We find that the  $\gamma$ -ray line energy must be between 0.7 TeV and 0.9 TeV with the velocity-averaged annihilation cross section  $10^{-30} \sim 10^{-26} \text{ cm}^3/\text{s}$  for  $Q = 1/3$ . With a non-zero hypercharge  $Q$ , the hidden sector is no longer completely dark and can be directly probed by collider experiments.

---

\* [jik@hep.s.kanazawa-u.ac.jp](mailto:jik@hep.s.kanazawa-u.ac.jp)

† [kher sham.lim@mpi-hd.mpg.de](mailto:kher sham.lim@mpi-hd.mpg.de)

‡ [lindner@mpi-hd.mpg.de](mailto:lindner@mpi-hd.mpg.de)

## I. INTRODUCTION

With the discovery of the Higgs particle [1, 2] the standard model (SM) is now complete. However, the SM must be extended since it does not contain a Dark Matter (DM) candidate and since finite neutrino masses must also be included. From a pure theoretical point of view there exist also severe conceptual problems and one of them is that the SM cannot explain the origin of its energy scale. Theoretically, we can imagine a world without any energy scale, but in the real world of elementary particles scale invariance is broken. In the SM the mass term of the Higgs field is the only term in the Lagrangian that violates (at tree level) scale invariance. Although the SM does not explain the origin of its energy scale, the measured mass  $m_h$  of the Higgs particle seems to suggest how to go beyond the SM, because this mass value together with the top quark mass implies that the SM remains perturbative at least up to the Planck scale [3–5]; an ultraviolet (UV) completion of the SM is not needed. Any extension which modifies the high energy behavior of the SM should therefore be well motivated (see e.g. [6]), since it may require a UV completion at lower scales.

Introducing an explicit Higgs mass term in the SM does not only break classical scale invariance, but it also leads to another severe issue known as the gauge hierarchy problem, namely the quadratic sensitivity of quantum corrections to high scales. It is therefore tempting to start from classically scale invariant theories where the SM scale emerges from dimensional transmutation. Various attempts to introduce an energy scale in this way exist in the literature [7]–[34]. Scale invariance is broken at the quantum level even in perturbation theory [35], but it has been argued that the protective features of conformal symmetry are not completely destroyed [36]. Specifically logarithmic sensitivities would exist, while quadratic divergencies would be absent.

We follow the idea that the energy scale in a classically scale invariant theory is generated by  $D\chi$ SB in a QCD-like hidden sector, which is transmitted via a SM singlet messenger field to the SM sector [37–40]<sup>1</sup>. So we assume that the fermions in the hidden sector are SM singlet and allow the presence of fundamental scalar fields. In fact, the messenger is assumed to be the simplest possibility, a real SM singlet scalar  $S$ . Note that this avoids the well known phenomenological problems of technicolor models and the model looks very much like the SM, since the hidden sector couples only via the Higgs portal.

The possibility that DM annihilates into  $\gamma$ -ray lines has recently received much attention and we want to discuss this possibility therefore in this paper. Specifically we consider a simple extension of the above mentioned model, where we assign a  $U(1)_Y$  hypercharge  $Q$  to the hidden sector fermions such that they are electrically charged with a charge  $Q$ . Since the coupling is vector-like, no breaking of  $U(1)_Y$  is caused by  $D\chi$ SB in the hidden sector. As we will see, this non-zero charge makes it possible that DM particles, which are the pseudo Nambu-Goldstone particles in this model, can be annihilated into two photons, producing a  $\gamma$ -ray line spectrum. Monochromatic  $\gamma$ -ray lines from DM annihilation exist in other DM models, too [42, 43], and in fact experimental searches for  $\gamma$ -ray lines have been undertaken with Fermi LAT [45, 46] and HESS [47] for a wide range of high energies. We find that the energy of the  $\gamma$ -ray line in our model lies between 0.7 TeV and 0.9 TeV. (We are not

---

<sup>1</sup> See also [41].

aiming to explain the recent observations of the galactic keV X-ray [48, 49] here.) The upper limits on the velocity-averaged annihilation cross section  $\langle v\sigma \rangle$  given by Fermi LAT and HESS constrain the electric charge  $Q$  of the hidden fermions. We find that the  $\langle v\sigma \rangle$  is  $10^{-30} \sim 10^{-26} \text{ cm}^3/\text{s}$  for  $Q = 1/3$ , which can well satisfy the experimental constraints of Fermi LAT and HESS. Since  $\langle v\sigma \rangle$  is proportional to  $Q^4$ , our calculations can be simply extended to the case of an arbitrary  $Q$ .

In this model not only DM particles but also hidden baryons are stable. For an arbitrary  $Q$  the electric charge of the hidden baryons are fractionally charged. Note that the consistent range of the DM mass between 0.7 TeV and 0.9 TeV is independent of  $Q$  and hence the scale  $\Lambda$  of the hidden sector is roughly fixed (regardless of  $Q$ ), which means that the mass of the stable hidden baryons is  $\sim 3 \text{ TeV}$ . Using the fact that the hidden sector is basically described by a scaled-up QCD, we have found that the relic abundance of the hidden baryons  $\Omega_{hB} h^2$  in the Universe is at most  $10^{-4}$ , which is independent of  $Q$ . This is sufficiently below the upper bound given in [50] and the constraint in the  $Q$ -DM mass plane given in [51] is also satisfied. Consequently there is practically no constraint (except for those from FermiLat and HESS) on the fractionally charged hidden baryons.

Since the hidden sector (strictly speaking it is no longer a hidden sector, because the fermions are electrically charged) can now communicate through gauge boson exchange (photon and  $Z$  boson) with the SM sector, the hidden sector could be produced at the ILC. We postpone these interesting processes for future studies, as our main priority in this paper is to find a prescription to obtain gauge invariant amplitudes. This is because we approximate the strongly coupled QCD-like sector by the Nambu-Jona-Lasinio model (NJL) [52, 53] (see [54, 55] for reviews), which is defined with a finite cutoff  $\Lambda$  that violates gauge invariance. To overcome this problem, we propose least subtraction procedure. In the NJL model the cutoff  $\Lambda$  is a physical parameter and a finite  $\Lambda$  is essential to describe effectively  $D\chi\text{SB}$ . We therefore stress that we keep the subtraction terms to the minimum necessary.

## II. THE MODEL

We consider an extension of the model studied in [37–40] which consists of a hidden QCD-like sector coupled via a real singlet scalar  $S$  to the SM. The fermion  $\psi$  in the hidden sector belongs to the fundamental representation of the hidden gauge group  $SU(3)_H$ . With this setting  $D\chi\text{SB}$  in the hidden sector does not break the SM gauge symmetries, thereby avoiding the FCNC problem. This is one of the main differences to technicolor model. If we further assume that the Yukawa coupling  $\bar{\psi}\psi S$  respects  $SU(N_f)_V$  flavor symmetry, there is only one coupling constant  $y$  for the Yukawa coupling, so that in the hidden sector there are only two independent parameters; the gauge coupling constant  $g_H$  and the Yukawa coupling  $y$ .

In extending the model we impose that neither the SM gauge symmetry nor the  $SU(N_f)_V$  flavor symmetry is broken in the hidden sector. If we further impose that the matter content remains unchanged, then there is a unique possibility for the extension that the hidden (Dirac) fermion carries a common  $U(1)_Y$  charge  $Q$ <sup>2</sup>. This implies that the hidden sector

<sup>2</sup> The new gauge coupling contributes only to  $\Pi_{YY}$  of the gauge boson self-energy diagrams so that the

Lagrangian of the extended model is written as

$$\mathcal{L}_H = -\frac{1}{2}\text{Tr} F^2 + \text{Tr} \bar{\psi}(i\gamma^\mu\partial_\mu + g\gamma^\mu G_\mu + g'Q\gamma^\mu B_\mu - yS)\psi , \quad (1)$$

where  $G_\mu$  is the gauge field for the hidden QCD, and  $B$  is the  $U(1)_Y$  gauge field. The trace is taken over the flavor as well as the color indices. The  $\mathcal{L}_{\text{SM}+S}$  part of the total Lagrangian  $\mathcal{L}_T = \mathcal{L}_H + \mathcal{L}_{\text{SM}+S}$ , which contains the SM gauge and Yukawa interactions along with the scalar potential

$$V_{\text{SM}+S} = \lambda_H(H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{HS}S^2(H^\dagger H) , \quad (2)$$

is unchanged<sup>3</sup>.  $H^T = (H^+ , (h + iG)\sqrt{2})$  is the SM Higgs doublet field, with  $H^+$  and  $G$  as the would-be Nambu-Goldstone fields.

Here we follow [40] in which the NJL model is used to describe D $\chi$ SB in the hidden sector, restricting ourselves to  $N_c = N_f = 3$ , because in this case the NJL model parameters, up-to an overall scale, can be fixed from hadron physics [55, 56]. So at low energy we replace the Lagrangian  $\mathcal{L}_H$  by

$$\mathcal{L}_{\text{NJL}} = \text{Tr} \bar{\psi}(i\gamma^\mu\partial_\mu + g'Q\gamma^\mu B_\mu - yS)\psi + 2G \text{Tr} \Phi^\dagger\Phi + G_D (\det \Phi + h.c.) , \quad (3)$$

where

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu , \quad g' = e/\cos\theta_W , \quad (4)$$

$$\Phi_{ij} = \bar{\psi}_i(1 - \gamma_5)\psi_j = \frac{1}{2}\lambda_{ji}^a \text{Tr} \bar{\psi}\lambda^a(1 - \gamma_5)\psi , \quad (5)$$

and  $\lambda^a$  are the Gell-Mann matrices with  $\lambda^0 = \sqrt{2/3} \mathbf{1}$ . The last term in (3), which exhibits a six fermi interaction, is present due to chiral anomaly of the axial  $U(1)_A$ . The chiral symmetry  $U(3)_L \times U(3)_R$  is explicitly broken down to its diagonal subgroup  $U(3)_V = SU(3)_F \times U(1)_V$  by the Yukawa coupling with the singlet  $S$ . To deal with the non-renormalizable Lagrangian (3) we have used in [40] a self-consistent mean-field approximation which has been intensely studied by Hatsuda and Kunihiro [55, 56] for hadron physics. The effective Lagrangian  $\mathcal{L}_{\text{NJL}}$  has three dimensional parameters  $G, G_D$  and the cutoff  $\Lambda$ , which have canonical dimensions of  $-2, -5$  and  $1$ , respectively. Since the original Lagrangian  $\mathcal{L}_H$  has only one independent scale, the parameters  $G, G_D$  and  $\Lambda$  are not independent. We obtain the NJL parameters for the hidden QCD from the upscaling of actual values of  $G, G_D$  and the cutoff  $\Lambda$  from QCD hadron physics. That is, we assume that the dimensionless combinations

$$G^{1/2}\Lambda = 2.0 , \quad (-G_D)^{1/5}\Lambda = 2.1 , \quad (6)$$

which are satisfied for hadrons, remain unchanged for a higher scale of  $\Lambda$  [40].

---

<sup>3</sup>  $S, T, U$  parameters remain unchanged.

<sup>3</sup> This classically scale invariant model is perturbatively renormalizable, and the Green's functions are infrared finite [57, 58].

In what follows we briefly outline the approximation method [55, 56]. One assumes that the dynamics of the theory creates a chiral symmetry breaking condensate

$$\langle 0 | \bar{\psi}_i \psi_j | 0 \rangle = -\frac{1}{4G} \text{diag}(\sigma, \sigma, \sigma) , \quad (7)$$

which is treated as a classical field  $\sigma$ . The vacuum  $|0\rangle$  is defined by the annihilation operator of the constituent fermion  $\psi$  in the background of the mean fields. We restrict our discussion (in a more complete treatment, one may add terms involving  $\eta$  or  $\rho$  mesons) to the mean fields collected in

$$\varphi \equiv \langle 0 | \bar{\psi} (1 - \gamma_5) \lambda^a \psi | 0 \rangle = -\frac{1}{4G} (\text{diag}(\sigma, \sigma, \sigma) + i(\lambda^a)^T \phi_a) , \quad (8)$$

where we denote the pseudo Nambu-Goldstone boson after spontaneous chiral symmetry breaking as  $\phi_a$ . These dark pions are stable due to flavor symmetry and they serve as good DM candidates. In the self-consistent mean field approximation one splits up the NJL Lagrangian (3) into the sum

$$\mathcal{L}_{\text{NJL}} = \mathcal{L}_0 + \mathcal{L}_I ,$$

where  $\mathcal{L}_I$  is normal ordered (i.e.  $\langle 0 | \mathcal{L}_I | 0 \rangle = 0$ ), and  $\mathcal{L}_0$  contains at most fermion bilinears which are not normal ordered. After some manipulations, one finds the following form for  $\mathcal{L}_0$  [40]:

$$\begin{aligned} \mathcal{L}_0 = & \text{Tr} \bar{\psi} \gamma^\mu (i\partial_\mu + g' Q B_\mu) \psi - \left( \sigma + yS - \frac{G_D}{8G^2} \sigma^2 \right) \text{Tr} \bar{\psi} \psi - i \text{Tr} \bar{\psi} \gamma_5 \phi \psi - \frac{1}{4G} \sum_{a=1}^8 \phi_a \phi_a \\ & - \frac{3\sigma^2}{8G} + \frac{G_D}{8G^2} \left( -\text{Tr} \bar{\psi} \phi^2 \psi + \sum_{a=1}^8 \phi_a \phi_a \text{Tr} \bar{\psi} \psi + i\sigma \text{Tr} \bar{\psi} \gamma_5 \phi \psi + \frac{\sigma^3}{2G} + \frac{\sigma}{2G} \sum_{a=1}^8 (\phi_a)^2 \right) . \quad (9) \end{aligned}$$

Note that this Lagrangian no longer contains the four and six fermi interactions. At the non-trivial lowest order only  $\mathcal{L}_0$  is relevant for the calculation of the effective potential, the DM mass  $m_{\text{DM}}$  and the DM interactions. The mass spectrum for all the CP-even particles, namely  $h, S$  and  $\sigma$  can be obtained from the minimum of the effective potential, once the free parameters of the model i.e.  $y, \lambda_H, \lambda_{HS}, \lambda_S$  are given. See Ref. [40] for more details in the calculation of the effective potential. The dimensionless couplings  $y, \lambda_H, \lambda_{HS}, \lambda_S$  are required to satisfy perturbativity and vacuum stability up the Planck scale. Once the global minimum of the effective potential is obtained, the effective couplings between the bosons and the DM properties are determined. The  $U(1)_Y$  coupling does not contribute to the effective potential and the mass matrix for  $h, S, \sigma$  in the lowest order.

As we have mentioned that the CP-odd pseudo Nambu-Goldstone bosons  $\phi_a$  are the DM candidates for our model, let us investigate their properties in more details. Like the CP-even bosonized  $\sigma$  field, the DM particles have no tree level kinetic term and their masses are defined as the zero of the inverse propagator

$$\begin{aligned} \Gamma_\phi(p^2) = & -\frac{1}{2G} + \frac{G_D \langle \sigma \rangle}{8G^3} + \frac{G_D N_c}{G^2} \int \frac{d^4 k}{i(2\pi)^4} \frac{M}{(k^2 - M^2)} \\ & + 2N_c \left( 1 - \frac{G_D \langle \sigma \rangle}{8G^2} \right)^2 \int \frac{d^4 k}{i(2\pi)^4} \frac{\text{Tr}(\not{k} - \not{p} + M) \gamma_5 (\not{k} + M) \gamma_5}{((k-p)^2 - M^2)(k^2 - M^2)} , \quad (10) \end{aligned}$$

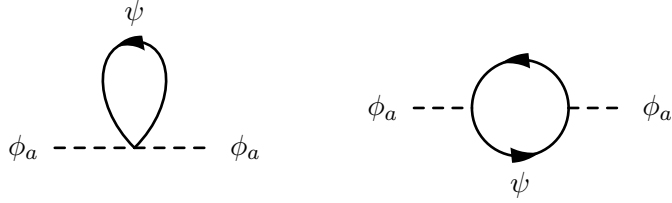


Figure 1. One-loop contributions of the heavy dark fermions to the DM mass.

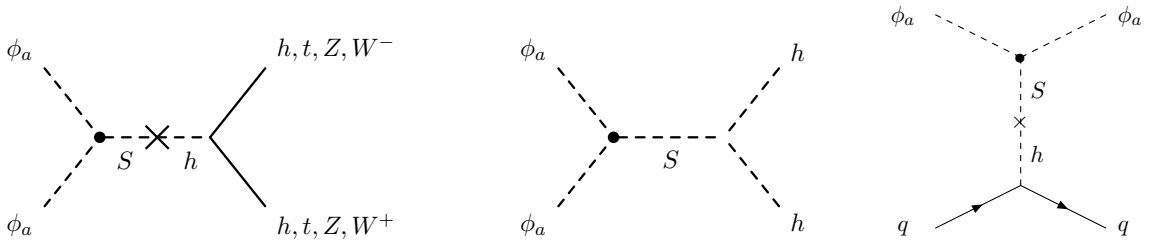


Figure 2. The left and middle diagrams are the s-channel DM annihilation diagrams. The right diagram contributes to the DM scattering off the nucleon. The coupling marked with a dot is a one-loop three-point vertex given in [40].

where  $M = \sigma + yS - G_D\sigma^2/8G^2$  is the constituent hidden sector fermion mass when all the CP-even scalar fields obtained their vacuum expectation values (VEV). The first two terms in Eq. (10) stem from the tree level effective Lagrangian (9) while the heavy dark fermions contribute to the one-loop radiative correction for the DM inverse propagator. The relevant one-loop diagrams are given in Fig. 1. From Eq. (10) the DM mass and its wave function renormalization constant can be calculated

$$\Gamma_\phi(m_{\text{DM}}^2) = 0, \quad Z_\phi^{-1} = \left. \frac{d\Gamma_\phi(p^2)}{dp^2} \right|_{p^2=m_{\text{DM}}^2}. \quad (11)$$

As  $y \rightarrow 0$ , the chiral symmetry of the fermions should be restored and  $m_{\text{DM}} \rightarrow 0$ <sup>4</sup>, hence the size of the DM mass is controlled by the Yukawa coupling  $y$ . The additional  $U(1)_Y$  coupling however does not contribute to the DM mass.

### III. RELIC ABUNDANCE OF DM AND ITS DIRECT DETECTION

Before we start to compute the relic abundance  $\Omega h^2$ , let us discuss the parameter space. In our previous paper [40] the dimensionless coupling constants,  $y, \lambda_{S,HS,H}$ , are constrained by the vacuum stability and by the absence of the Landau pole. It turns out that with a non-zero  $Q$  (at least for  $Q \lesssim 1/3$ ) the allowed parameter space does not practically change.

<sup>4</sup> In this case  $\phi$  is a true Nambu-Goldstone boson.

Note that the annihilation processes of DM occur at the one-loop level through the one-loop  $\phi\phi\text{-}S$  amplitude and the one-loop  $\phi\phi\text{-}S\text{-}S$  amplitude (if  $m_S < m_{\text{DM}}$ ). The  $\phi\phi\text{-}S$  amplitude can be calculated from the one-loop diagram

$$\begin{aligned} \Gamma_{\phi\phi S} = & 4N_c y \left(1 - \frac{G_D \langle \sigma \rangle}{8G^2}\right)^2 \int \frac{d^4 k}{i(2\pi)^4} \frac{\text{Tr}(\not{k} + M)\gamma_5(\not{k} - \not{p} + M)(\not{k} + \not{p}' + M)\gamma_5}{((k-p)^2 - M^2)(k^2 - M^2)((k+p')^2 - M^2)} \\ & + N_c y \frac{G_D}{4G^2} \int \frac{d^4 k}{i(2\pi)^4} \frac{\text{Tr}(\not{k} - \not{p}' + M)(\not{k} + \not{p} + M)}{((k-p')^2 - M^2)((k+p)^2 - M^2)}, \end{aligned} \quad (12)$$

which is crucial for determining the relic abundance and the direct detection cross section of the DM. The momenta  $p, p'$  represent the incoming momenta of the dark pions. The one-loop effective couplings are represented as  $\bullet$  in Fig. 2. These amplitudes are small for small  $y$  as the amplitudes scale like  $\mathcal{A}(\phi\phi \rightarrow S) \sim y$  and  $\mathcal{A}(\phi\phi \rightarrow SS) \sim y^2$  respectively. As mentioned above, the size of the DM mass is controlled by  $y$ , i.e. a small  $y$  implies a small DM mass and for a larger  $y \gtrsim 0.2$  the DM mass  $m_{\text{DM}}$  can become larger than the fermion constituent mass  $M$ , which will develop imaginary parts in these one-loop amplitudes. In [40] we forbid the occurrence of the imaginary parts, yielding an upper bound on  $y$  for a given set of  $\lambda_{S,HS,S}$ . As the parameter  $y$  is bounded from above, the  $\phi\phi\text{-}S\text{-}S$  amplitude contributes negligibly to the relic abundance calculation and only the  $\phi\phi\text{-}S$  amplitudes are important. However in this parameter space we have found that the only way to enhance the annihilation rate of DM is via a resonance effect in the s-channel annihilation processes shown in Fig. 2 (left and middle). That is,  $2m_{\text{DM}} \simeq m_S$  has to be satisfied. The direct detection rate of DM is however strongly suppressed ( $\lesssim 10^{-48} \text{ cm}^2$ ) because it is a t-channel process shown in Fig. 2 (right), constraining the parameter space into phenomenologically unattractive corner.

In this paper we allow the occurrence of the imaginary parts in the one-loop diagrams, as they are related to the real parts due to the dispersion relation, which has proven to be successful in describing the QCD hadron physics (see [55] for instance). We set the upper bound at  $m_{\text{DM}} < 2M$ , which should be compared with  $m_{\eta'} = 0.958 \text{ GeV}$  and  $M_s = 0.5 \text{ GeV}$  in the usual QCD physics, where  $M_s$  is the constituent mass of the strange quark. In fact, in the optimistic range  $y \gtrsim 0.4$  with  $m_S < m_{\text{DM}}$ , the  $\phi\phi\text{-}S\text{-}S$  amplitude (which is generated at one-loop as shown in Fig. 3 ~ 5) is no longer small and can become large enough to give a correct relic abundance of DM. Therefore, we choose below the parameter space  $y \gtrsim 0.4$  and open the channel  $\phi\phi \rightarrow SS$ . In this parameter region, the s-channel processes contributed by  $\phi\phi\text{-}S$  amplitudes are negligibly suppressed and can be ignored. The annihilation diagrams in Fig. 3 ~ 5 for  $\phi\phi \rightarrow SS$  yield

$$\mathcal{A}(\phi\phi \rightarrow SS) = 2N_c y^2 \left[ \frac{G_D}{4G^2} I_a + \left(1 - \frac{G_D \langle \sigma \rangle}{8G^2}\right)^2 (4I_b + 2I_c) \right], \quad (13)$$

where  $I_i$  represents the integral for the respective  $i$ th loop diagram in Fig. 3 ~ 5. We obtain the DM annihilation cross section

$$\langle v\sigma(\phi\phi \rightarrow SS) \rangle = \frac{Z_\phi^2}{32\pi m_{\text{DM}}^3} |\mathcal{A}(\phi\phi \rightarrow SS)|^2 \left(1 - \frac{m_S^2}{m_{\text{DM}}^2}\right)^{1/2}, \quad (14)$$

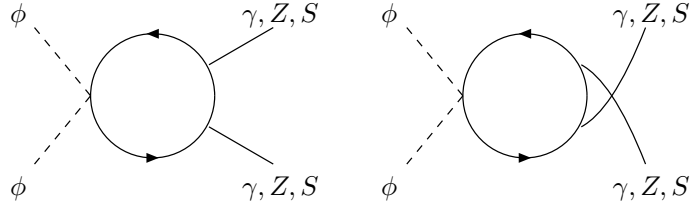


Figure 3. The DM annihilation with  $\phi\phi\bar{\psi}\psi$  coupling (a)

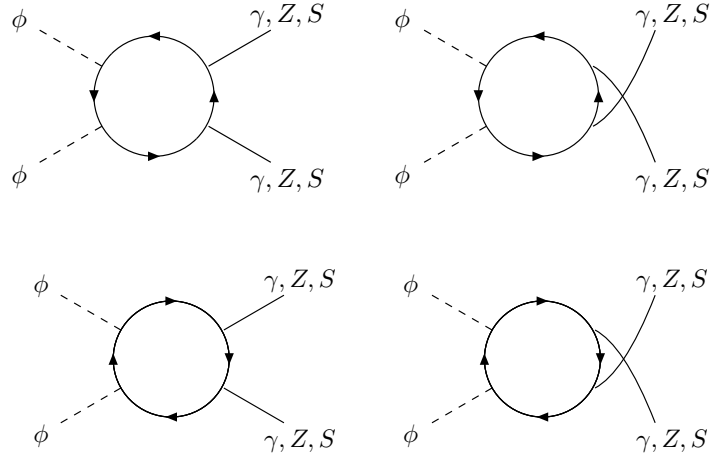


Figure 4. The DM annihilation (b)

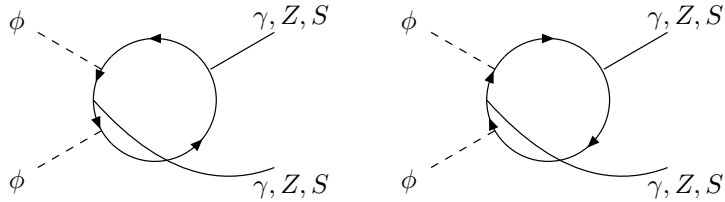


Figure 5. The DM annihilation (c).

where  $Z_\phi$  is given in Eq. (11). We do not include the annihilation modes into  $\gamma\gamma, \gamma Z, ZZ$  as the annihilation cross section of these modes is proportional to  $\alpha^2 Q^4$  (see Eq. (19)) while the DM annihilation cross section to  $S$  particles is dominated by  $y^4$ . Unless the electric charge  $Q \gtrsim 1$ , the annihilation modes into  $\gamma\gamma, \gamma Z, ZZ$  can be ignored in the relic abundance calculation (see also the comment in the footnote on page 12). The annihilations into these modes are calculated in the next section. We find, imposing the constraint on the relic abundance  $\Omega h^2 = 0.1187 \pm 0.005(3\sigma)$  [59], that the spin independent annihilation cross section is just below the XENON100 [60] and LUX [61] constraints and above the XENON1T sensitivity [62]. This is shown in Fig. 6.



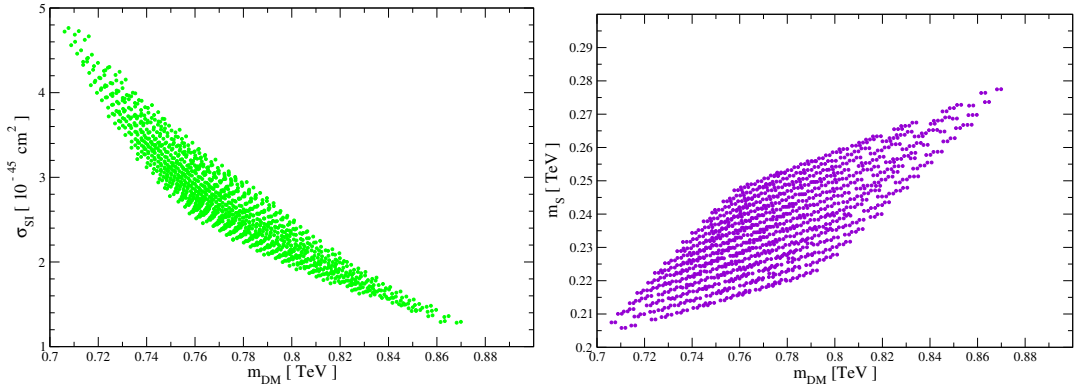


Figure 6. Left: The spin-independent cross section off nucleon against  $m_{\text{DM}}$ , where  $\Omega h^2 = 0.1187 \pm 0.005(3\sigma)$  [59] is imposed. The XENON100 [60] and LUX [62] limits are  $\sim 10^{-44} \text{ cm}^2$  for  $m_{\text{DM}} = 0.7 \text{ TeV}$ , while the XENON1T sensitivity is two orders of magnitudes higher than that of XENON100 [62]. Right: The mass of the singlet  $S$  against  $m_{\text{DM}}$ . If  $m_{\text{DM}} < m_S$ , then  $\sigma_{\text{SI}} \lesssim 10^{-48} \text{ cm}^2$  [40].

The DM mass  $m_{\text{DM}}$  is constrained in the present model. The lower limit  $m_{\text{DM}} \gtrsim 0.7 \text{ TeV}$  comes from the fact that  $y$  has to be large enough so that the size of the annihilation process  $\phi\phi \rightarrow SS$  yields a correct relic abundance of DM. If on the other hand  $y$  is too large, the annihilation cross section into two singlet scalars becomes too large so that the relic abundance falls below the observed value. The Yukawa coupling  $y$  is also constrained from above to avoid the triviality bound, which gives the upper limit  $m_{\text{DM}} \lesssim 0.9 \text{ TeV}$ .

Note that because of the  $SU(3)_V$  flavor symmetry and an accidental  $U(1)_{\text{hB}}$  (hidden baryon number), not only the DM candidates, but also the lightest hidden baryons are stable. In the case that  $N_c = N_f = 3$  in the hidden sector and  $Q = 1/3$  for the hidden fermions, there is no stable hidden hadron with a fractional electric charge. The hidden mesons for our model are neutral, while the charge of the hidden baryons formed by three hidden fermions is one if  $Q = 1/3$ . There might be a tiny amount of relic stable hidden baryons and anti-baryons in the universe, which if a large number of them are not annihilated, could spoil the large scale structure formation. Let us roughly estimate fraction of this hidden baryon. As the hidden sector is described by a scaled-up QCD, so that because the coupling  $G_{\phi B\bar{B}}$  is dimensionless, the hidden meson-baryon coupling  $G_{\phi B\bar{B}}$  is approximately the same as in QCD, i.e.  $G_{\phi B\bar{B}} \sim 13$ . Using this fact, we have estimated the relic abundance  $\Omega_{\text{hB}} h^2$  of the hidden baryons to be  $\sim 10^{-4}$  for the hidden baryon mass of 3 TeV. Note that this result is independent of  $Q$ . Therefore, we may fairly ignore the stable charged hidden baryons in discussing the relic abundance of DM.

#### IV. RESTORING GAUGE INVARIANCE

The cutoff  $\Lambda$  breaks gauge invariance explicitly and to restore gauge invariance we have to subtract non-gauge invariant terms from the original amplitude. In renormalizable theories there is no problem to define a finite renormalized gauge invariant amplitude, i.e. In the limit of  $\Lambda \rightarrow \infty$  the gauge non-invariant terms are a finite number of local terms, which can be

cancelled by the corresponding local counter terms so that the subtracted amplitude is, up to its normalization, independent of the regularization scheme (see for instance [63]). To achieve such a uniqueness in cutoff theories, one needs an additional prescription. For instance, we can define the real part of an amplitude using dispersion relation, as it was done in the original paper by Nambu and Jona-Lasinio [53] (see also [55]). This procedure yields a gauge invariant real part of the amplitude in one-loop, because the imaginary part of the amplitude is gauge invariant in one-loop order. In this method, however, the one-loop tadpole diagram cannot be reproduced from its imaginary part as the tadpole diagram does not contain any imaginary part. Another way<sup>5</sup> is to utilize a gauge invariant regularization such as the Pauli-Villars regularization [54, 65] which preserves gauge invariance by construction but breaks chiral symmetry explicitly. The drawback is however, for a finite regulator mass, it is not clear whether the breaking of chiral symmetry results from the regulator or from non-perturbative effect. Moreover, the regulator fields are “ghost” fields, which are not completely decoupled at a finite cutoff  $\Lambda$ .

We will propose another method, which we call “*least subtraction procedure*”. In the NJL model as a cutoff theory the cutoff  $\Lambda$  is a physical parameter, and a finite  $\Lambda$  is essential to describe effectively D $\chi$ SB. If we subtract too much from the amplitude to restore gauge invariance, we may lose information on non-perturbative effects. Therefore, we stress that we keep the subtraction terms to the minimum as necessary. The details of least subtraction procedure is given in Appendix A, where we consider the photon self-energy, the  $S$ - $\gamma$ - $\gamma$  as well as the  $\phi$ - $\phi$ - $\gamma$ - $\gamma$  vertex functions. The results are applied to the next section for the calculation of the DM annihilation cross section into two  $\gamma$ 's.

## V. MONOCHROMATIC $\gamma$ -RAY LINE FROM DM ANNIHILATION

The charge  $Q$  of the hidden fermion is a free parameter. It can be constrained from the indirect detection of DM, e.g. the upper bound on  $\sigma(\phi\phi \rightarrow \gamma\gamma)$  for  $\gamma$ -ray lines given in [45–47]. The four-point  $\phi$ - $\phi$ - $\gamma$ - $\gamma$  coupling<sup>6</sup> is generated at one-loop as is shown in Fig. 3 ~ 5, which predicts the DM annihilation into two monochromatic photons of energy  $m_{\text{DM}}$ . Similar processes have been calculated in a universal extra dimension model [43], for instance. In Appendix A it is shown how to restore gauge invariance of the four-point amplitude, with the result given in (A32). If we neglect the mass of  $Z$  against  $m_{\text{DM}}$ , the four-point functions  $\mathcal{A}_{\mu\nu}^R(\gamma Z)$  and  $\mathcal{A}_{\mu\nu}^R(ZZ)$  can be approximated by (A32) as well, with the replacement of  $e^2$  by  $-e^2 t_W$  and  $e^2 t_W^2$ , respectively.

For  $p = p' = (m_{\text{DM}}, \mathbf{0})$  the photon momenta take the form  $k = (m_{\text{DM}}, \mathbf{k})$  and  $k' = (m_{\text{DM}}, -\mathbf{k})$ , with their polarization tensors  $\epsilon(k) = (0, \boldsymbol{\epsilon}(k))$  and  $\epsilon(k') = (0, \boldsymbol{\epsilon}(k'))$  satisfying

$$0 = \epsilon(k) \cdot k = \epsilon(k) \cdot k' = \epsilon(k) \cdot p = \epsilon(k) \cdot p' \quad (15)$$

$$0 = \epsilon(k') \cdot k = \epsilon(k') \cdot k' = \epsilon(k') \cdot p' = \epsilon(k') \cdot p, \quad (16)$$

<sup>5</sup>  $\zeta$ -function regularization was also used in [64] to obtain a gauge invariant effective potential in the presence of the electromagnetic field as an external field.

<sup>6</sup> The  $U(1)_Y$  gauge invariance and the  $SU(3)_F$  flavor symmetry together with the reality of  $\phi$  forbid the existence of the  $\phi\phi B_\mu$  coupling.

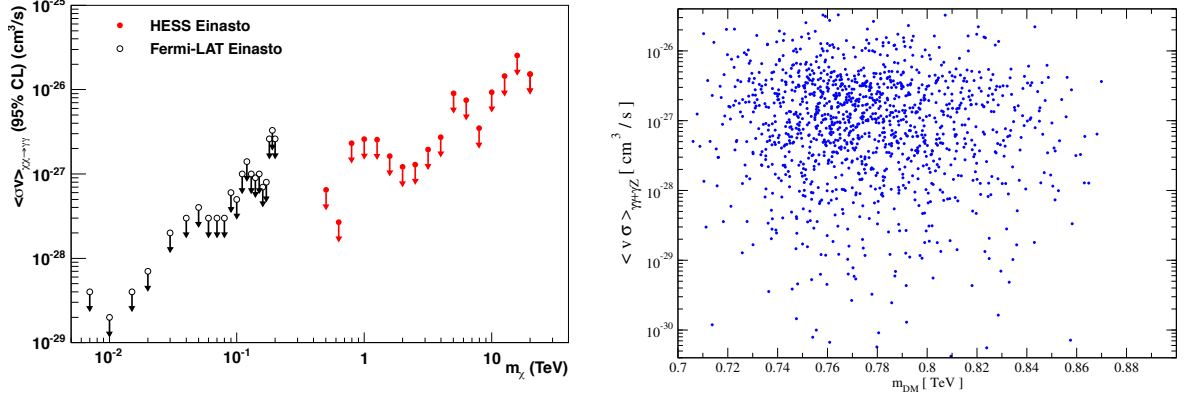


Figure 7. Left: The Fermi Lat [46] (black) and HESS[45] (red) upper bounds on the velocity-averaged DM annihilation cross section for monochromatic  $\gamma$ -ray lines, where this graph is taken from [45]. Right: the velocity-averaged DM annihilation cross section  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z}$  as a function of  $m_{\text{DM}}$  with  $Q = 1/3$ , where  $\Omega h^2 = 0.1187 \pm 0.005(3\sigma)$  [59] is imposed. Since  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z}$  is proportional to  $Q^4$ , our calculations can be simply extended to the case of an arbitrary  $Q$ .

respectively. Therefore, only  $g_{\mu\nu}$  terms of the subtracted gauge invariant four-point function  $\Gamma_{\mu\nu}(\gamma\gamma)$  contributes:

$$\Gamma_{\mu\nu}(\gamma\gamma) = g_{\mu\nu} (\mathcal{A}^{R(a)} + \mathcal{A}_g^R) = i\frac{\alpha}{\pi} Q^2 g_{\mu\nu} \mathcal{A}(\gamma\gamma), \quad (17)$$

where  $\mathcal{A}^{R(a)}$  (defined in (A12)) is the contribution from Fig. 3, while  $\mathcal{A}_g^R$  (defined in (A28)) is the contribution from Fig. 4 and 5. The other ones can be approximated as

$$\Gamma_{\mu\nu}(a b) \simeq i\frac{\alpha}{\pi} Q^2 g_{\mu\nu} \mathcal{A}(\gamma\gamma) \times \begin{cases} a b \\ -t_W \gamma Z \\ t_W^2 Z Z \end{cases}. \quad (18)$$

Then (the s-wave part of) the corresponding velocity-averaged annihilation cross sections are given by

$$\langle v\sigma(\phi\phi \rightarrow a b) \rangle = \frac{\alpha^2 Q^4 Z_\phi^2}{16\pi^3 m_{\text{DM}}^2} \mathcal{A}^2(\gamma\gamma) \times \begin{cases} a b \\ (1/2) \gamma \gamma \\ t_W^2 (1 - m_Z^2/4m_{\text{DM}}^2) \gamma Z \\ (3/4)t_W^4 (1 - m_Z^2/m_{\text{DM}}^2)^{1/2} Z Z \end{cases}, \quad (19)$$

where  $Z_\phi$  is the wave function renormalization constant which is given in [40]. The energy  $E_\gamma$  of  $\gamma$ -ray line produced in the annihilation into  $\gamma Z$  is  $m_{\text{DM}}(1 - m_Z^2/4m_{\text{DM}}^2)$ . In practice, however, due to finite detector energy resolution this line cannot be distinguished from the  $E_\gamma = m_{\text{DM}}$  line. Therefore, we simply add both cross sections. So we compute  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z} = \langle v\sigma(\phi\phi \rightarrow \gamma\gamma) \rangle + \langle v\sigma(\phi\phi \rightarrow \gamma Z) \rangle$  with  $Q = 1/3$  as a function of  $m_{\text{DM}}$  for different values of  $\lambda_H$ ,  $\lambda_S$  and  $\lambda_{HS}$ . As noticed in the previous section, we have not included the annihilation modes into  $\gamma\gamma, \gamma Z, ZZ$  in calculating the relic abundance. In this way we can obtain a

separate information on the size of the annihilation cross section producing the line  $\gamma$ -ray spectrum of DM in this model <sup>7</sup>.

As we see from Fig. 7 (left) strong constraints are given for  $m_{\text{DM}} \simeq 0.6$  (0.5) TeV:  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z} \lesssim 3$  (7)  $\times 10^{-28}$  cm<sup>3</sup>/s. Since our DM is heavier than 0.7 TeV (see Fig. 6), these strong constraints do not apply. Above 0.7 TeV, the upper bound is about one order of magnitude larger than that for  $m_{\text{DM}} = 0.6$  TeV, so that the constraints can well be satisfied even for  $Q > 1/3$ , as we can see from Fig. 7 (right). An interesting feature of the present model is that the  $\gamma$ -ray line energy is constrained between  $\sim 0.7$  TeV and  $\sim 0.9$  TeV, because the DM mass  $m_{\text{DM}}$  is constrained as it is explained in the previous section. Another feature of the model related to  $\gamma$ -ray lines is that the production cross section of  $\gamma$ -ray lines is in the same order in  $1/N$  expansion (i.e. in one-loop order) as the total annihilation cross section of DM. That is,  $\langle v\sigma \rangle_{HH, f\bar{f}, WW, \dots} \sim \langle v\sigma \rangle_{\gamma\gamma+\gamma Z}$  in the present model. This is similar to one of three exceptions, forbidden channels, considered in [66]. In the case of the forbidden channels the tree-level processes are kinematically forbidden, which should be contrasted to the present case in which the Nambu-Goldstone DM has no contact with the messenger field  $S$  at the tree-level.

The differential  $\gamma$ -ray flux is given by

$$\frac{d\Phi}{dE_\gamma} \propto \langle v\sigma \rangle_{\gamma\gamma} \frac{dN^{\gamma\gamma}}{dE_\gamma} + \langle v\sigma \rangle_{\gamma Z} \frac{dN^{\gamma Z}}{dE_\gamma} \simeq \langle v\sigma \rangle_{\gamma\gamma+\gamma Z} \delta(E_\gamma - m_{\text{DM}}). \quad (20)$$

Prospects observing such line spectrum is discussed in detail in [43, 44]. Obviously, with an increasing energy resolution the chance for the observation increases. Observations of  $\gamma$ -ray lines of energies between  $\sim 0.7$  TeV and  $\sim 0.9$  TeV not only fix the charge of the hidden sector fermion, but also yields a first experimental hint on the hidden sector.

## VI. CONCLUSION

The Nambu-Goldstone theorem predicts in the presented model for the hidden sector, where chiral symmetry is dynamically broken and hence a scale is created, the existence of a DM candidate. This generated scale is transmitted to the SM sector via a real SM singlet scalar  $S$  to trigger spontaneous breaking of electroweak gauge symmetry. With a non-zero  $U(1)_Y$  hypercharge  $Q$  of the hidden sector fermion the hidden sector is no longer dark, and new possibilities to test experimentally the hidden sector are open. We studied in this paper the possibility of DM annihilation and found that this model allows DM to annihilate into two photons, producing a  $\gamma$ -ray line spectrum. We found that the  $\gamma$ -ray line energy must be between 0.7 TeV and 0.9 TeV with the velocity-averaged annihilation cross section  $10^{-30} \sim 10^{-26}$  cm<sup>3</sup>/s for  $Q = 1/3$ , which satisfies easily the recent limits given by Fermi LAT [45, 46] and HESS [47].

<sup>7</sup> The contribution can become important for  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z} \gtrsim \langle v\sigma(\phi\phi \rightarrow SS) \rangle \simeq 8 \times 10^{-27}$  cm<sup>3</sup>/s. But this approximate inequality can not be satisfied for  $m_{\text{DM}}$  between 0.7 TeV and 0.9 TeV, if the HESS constraint for  $m_{\text{DM}} = 0.8$  TeV, i.e.  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z} \lesssim 2 \times 10^{-27}$  cm<sup>3</sup>/s, is satisfied for this range of  $m_{\text{DM}}$ . If the HESS constraint for  $m_{\text{DM}} = 0.8$  TeV does not apply and there is no cosmological constraint for this range of  $m_{\text{DM}}$ , we should control the size of  $\langle v\sigma \rangle_{\gamma\gamma+\gamma Z}$  by varying  $Q$  when the approximate inequality above is satisfied.

With a non-zero  $Q$  the hidden sector is doubly connected with the SM sector. The connection via photon and  $Z$  opens possibilities to probe the hidden sector at collider experiments such as  $e^+e^-$  collision [67]. In the parameter range, where the annihilation of DM into two singlets  $SS$  is dominant and a correct relic abundance of DM is obtained, the constituent mass  $M$  of the fermion is comparable with  $m_{\text{DM}}$ , i.e.  $0.7 \text{ TeV} \lesssim M \lesssim 0.9 \text{ TeV}$ . This is the energy region of hidden hadron physics and the scale of the hidden sector itself is  $\sim 0.7 \text{ TeV}$ , compared to  $\Lambda_{\text{QCD}} \approx 1 \text{ GeV}$ . The hidden strong interaction becomes therefore perturbative at about one order of magnitude above this energy region,  $\gtrsim 10 \text{ TeV}$ , and the hidden fermion becomes massless and could be produced directly to yield hidden sector jets at collider experiments.

**Acknowledgements:** We would like to thank Teiji Kunihiro for useful discussions. J. K. would like to thank the theory group of the Max-Planck-Institut für Kernphysik in Heidelberg for their hospitality. J. K. is partially supported by the Grant-in-Aid for Scientific Research (C) from the Japan Society for Promotion of Science (Grant No.22540271). K. S. L. acknowledges support by the International Max Planck Research School for Precision Tests of Fundamental Symmetries.

## Appendix A: Least Subtraction Procedure

Here we elucidate least subtraction procedure which can be applied to any cutoff theory in principle to obtain gauge invariant amplitudes. The basic idea is to keep the subtraction terms to the minimum necessary. This works as follows. Consider an unsubtracted amplitude

$$\mathcal{A}_{\mu_1 \dots \mu_{n_g}}(\Lambda; k_1 \dots k_{n_g}, p_1 \dots p_{n_s}), \quad (\text{A1})$$

with  $n_g$  photons and  $n_s$  scalars (scalars and axial scalars)<sup>8</sup>. Expand the amplitude in the external momenta  $k$ 's and  $p$ 's:

$$\mathcal{A}_{\mu_1 \dots \mu_{n_g}} = \sum_{m=0} \mathcal{A}_{\mu_1 \dots \mu_{n_g}}^{(m)}, \quad (\text{A2})$$

where  $\mathcal{A}_{\mu_1 \dots \mu_{n_g}}^{(m)}$  consists of  $m$ -th order monomials of the external momenta. In general,  $\mathcal{A}_{\mu_1 \dots \mu_{n_g}}^{(0)} = \mathcal{A}_{\mu_1 \dots \mu_{n_g}}(\Lambda; 0, \dots, 0)$  is non-vanishing and we can subtract it because it is not gauge invariant. We keep the tensor structure of  $\mathcal{A}_{\mu_1 \dots \mu_{n_g}}^{(0)}$  as the tensor structure of the counter terms for  $\mathcal{A}_{\mu_1 \dots \mu_{n_g}}^{(m)}$  ( $m > 0$ ) until a new tensor structure for the counter terms is required. We continue this until no more new tensor structure is needed. At each step we stress the minimal number of the new tensor structures for the counter terms.

### •Photon self-energy

As an example we consider the one-loop photon self-energy. Using the usual technique,

---

<sup>8</sup> We impose that the on-shell conditions (except for the self-energy) and the momentum conservation for the external momenta are satisfied.

introducing a Feynman parameter  $x$  for the denominator of the propagators, going to the Euclidean momentum space, shifting the internal momentum appropriately, we obtain the unsubtracted self-energy tensor

$$\begin{aligned} \Pi_{\mu\nu}(\Lambda; k) = & i \frac{e^2 Q^2 N_c N_f}{8\pi^2} \int_0^1 dx \left[ \frac{4\Lambda^2(1-x)x(g_{\mu\nu}k^2 - k_\mu k_\nu) - \Lambda^4 g_{\mu\nu}}{\Lambda^2 + B^2} \right. \\ & \left. - 4(g_{\mu\nu} k^2 - k_\mu k_\nu)(1-x)x \ln(1 + \Lambda^2/B^2) \right] \end{aligned} \quad (\text{A3})$$

with  $B^2 = M^2 - (1-x)x k^2$ . According to least subtraction procedure, we expand  $\Pi_{\mu\nu}(\Lambda; k)$  in  $k$  and find

$$\begin{aligned} \Pi_{\mu\nu}(\Lambda; k) = & i \frac{e^2 Q^2 N_c N_f}{8\pi^2} [g_{\mu\nu} \mathcal{A}_g(\Lambda; k^2) + k_\mu k_\nu \mathcal{A}_{kk}(\Lambda; k^2)] , \\ \mathcal{G}(\Lambda; k^2) = & \mathcal{A}_g(\Lambda; k^2) + k^2 \mathcal{A}_{kk}(\Lambda; k^2) = -\frac{\Lambda^4}{\Lambda^2 + M^2} - \frac{k^2 \Lambda^4}{6(\Lambda^2 + M^2)^2} - \frac{k^4 \Lambda^4}{30(\Lambda^2 + M^2)^3} \\ & - \frac{k^6 \Lambda^4}{140(\Lambda^2 + M^2)^4} - \frac{k^8 \Lambda^4}{630(\Lambda^2 + M^2)^5} + \dots , \end{aligned} \quad (\text{A4})$$

which would vanish if the amplitude were gauge invariant. Further,

$$\Pi_{\mu\nu}^{(0)} = \Pi_{\mu\nu}(\Lambda; 0) = i \frac{e^2 Q^2 N_c N_f}{8\pi^2} g_{\mu\nu} \mathcal{A}_g(\Lambda; 0) = -i \frac{e^2 Q^2 N_c N_f}{8\pi^2} g_{\mu\nu} \Lambda^4 / (\Lambda^2 + M^2) . \quad (\text{A5})$$

This defines the tensor structure for the counter terms, because this term is not gauge invariant and has to be subtracted. Therefore, the subtracted amplitude is  $\mathcal{A}_g^R(\Lambda; k^2) = \mathcal{A}_g(\Lambda; k^2) - \mathcal{G}(\Lambda; k^2)$ . Obviously, in this example, all the non-gauge invariant terms can be canceled by the counter terms of this tensor structure. That is, no more new tensor structure is needed for the counter terms.

Since in this example we know the closed expression for the amplitude, it is not necessary to implement least subtraction procedure. As we see from (A3), the  $\Lambda^4 g_{\mu\nu}$  term is not gauge invariant. This non-gauge invariant term, which is a photon mass function  $\Pi_{\mu\nu}^\Lambda(\Lambda; k)$ , can not be made gauge invariant by adding  $k_\mu k_\nu$  terms without introducing a singularity in  $k^2$ . Therefore, we have to subtract  $\Pi_{\mu\nu}^\Lambda(\Lambda; k)$  from the self-energy  $\Pi_{\mu\nu}(\Lambda; k)$ , in accord with least subtraction procedure as described above.

The gauge invariant term proportional  $\propto \Lambda^2$  in (A3) gives a wrong normalization so that further counter terms are needed. Finally, we have the normalized subtracted gauge invariant self-energy of the photon:

$$\begin{aligned} \Pi_{\mu\nu}^R(\Lambda; k) = & i \frac{e^2 Q^2 N_c N_f}{8\pi^2} (g_{\mu\nu} k^2 - k_\mu k_\nu) \int_0^1 dx \left[ \left( \frac{4\Lambda^2(1-x)x}{\Lambda^2 + B^2} - \frac{2\Lambda^2/3}{\Lambda^2 + M^2} \right) \right. \\ & \left. + 4(1-x)x \left\{ \ln \left( 1 - \frac{(1-x)xk^2}{M^2} \right) - \ln \left( 1 - \frac{(1-x)xk^2}{\Lambda^2 + M^2} \right) \right\} \right] . \end{aligned} \quad (\text{A6})$$

#### • S- $\gamma$ - $\gamma$ amplitude

The next example is the  $S(p)$ - $\gamma(k)$ - $\gamma(k')$  three-point function. There are two diagrams at

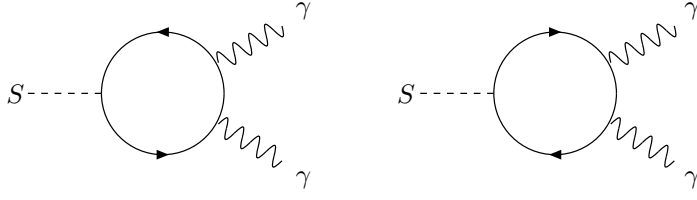


Figure 8.  $S$ - $\gamma$ - $\gamma$  coupling

the one-loop level as shown in Fig. 8. Again using the usual technique, we obtain the unsubtracted amplitude

$$\mathcal{A}_{\mu\nu}(\Lambda; k, k') = i \frac{e^2 Q^2 y N_c N_f M}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{2\Lambda^4}{(\Lambda^2 + D^2)^2} \times \left[ g_{\mu\nu} - \frac{(1-4xy)(g_{\mu\nu} k \cdot k' - k_\mu k'_\nu)}{D^2} \right], \quad (\text{A7})$$

where

$$D^2 = M^2 - 2xy k \cdot k'. \quad (\text{A8})$$

The last term in the square bracket has an imaginary part and gauge invariant. The first  $g_{\mu\nu}$  term in the square bracket is not gauge invariant. It is obvious that least subtraction procedure implies a complete subtraction of this term. The subtracted amplitude is

$$\mathcal{A}_{\mu\nu}^R(\Lambda; k, k') = -i \frac{e^2 Q^2 y N_c N_f M}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{2\Lambda^4}{(\Lambda^2 + D^2)^2} \frac{(1-4xy)(g_{\mu\nu} k \cdot k' - k_\mu k'_\nu)}{D^2}. \quad (\text{A9})$$

#### • $\phi$ - $\phi$ - $\gamma$ - $\gamma$ amplitude

The next example is the  $\phi(p)$ - $\phi(p')$ - $\gamma(k)$ - $\gamma(k')$  four-point function. The diagrams at the one-loop level are shown in Fig. 3~5 with the sum of the unsubtracted amplitudes:

$$\mathcal{A}_{\mu\nu}(\Lambda; k, k', p, p') = \mathcal{A}_{\mu\nu}^{(a)}(\Lambda; k, k', p, p') + \mathcal{A}_{\mu\nu}^{(b)}(\Lambda; k, k', p, p') + \mathcal{A}_{\mu\nu}^{(c)}(\Lambda; k, k', p, p'). \quad (\text{A10})$$

The diagrams of Fig. 3, which give  $\mathcal{A}_{\mu\nu}^{(a)}$  in (A10), are simpler to compute, because the structure is the same as Fig. 8. Again using the usual technique, we obtain the unsubtracted amplitude (with  $N_f = 3$ )

$$\mathcal{A}_{\mu\nu}^{(a)}(\Lambda; k, k') = -i \left[ \frac{e^2 Q^2 N_c}{2\pi^2} \right] \left[ \frac{G_D M}{8G^2} \right] \int_0^1 dx \int_0^{1-x} dy \times \frac{2\Lambda^4}{(\Lambda^2 + D^2)^2} \left[ g_{\mu\nu} - \frac{(1-4xy)(g_{\mu\nu} k \cdot k' - k_\mu k'_\nu)}{D^2} \right], \quad (\text{A11})$$

where  $D^2$  is given in (A8). The non-gauge invariant terms have the same structure as (A7). Therefore, we subtract the non-gauge invariant  $g_{\mu\nu}$  term:

$$\mathcal{A}_{\mu\nu}^{R(a)} = (g_{\mu\nu} k \cdot k' - k_\mu k'_\nu) \mathcal{A}^{R(a)} = \mathcal{A}_{\mu\nu}^{(a)} \text{ with the first } g_{\mu\nu} \text{ term omitted.} \quad (\text{A12})$$



The tensor structure of the other diagrams is more complicated. The amplitude has to satisfy the Bose symmetry:

$$\mathcal{A}_{\mu\nu}(\Lambda; k, k', p, p') = \mathcal{A}_{\nu\mu}(\Lambda; k', k, p, p') \text{ and } \mathcal{A}_{\mu\nu}(\Lambda; k, k', p, p') = \mathcal{A}_{\mu\nu}(\Lambda; k, k', p', p) \text{ (A13)}$$

To proceed we introduce the Mandelstam variables:

$$S = (p + p')^2 = 2m_{\text{DM}}^2 + 2p \cdot p' = (k + k')^2 = 2k \cdot k' , \quad (\text{A14})$$

$$T = (p - k)^2 = m_{\text{DM}}^2 - 2p \cdot k , \quad U = (p - k')^2 = m_{\text{DM}}^2 - 2p \cdot k' . \quad (\text{A15})$$

All the dot products of the momenta and  $m_{\text{DM}}^2$  can be expressed as a function of  $S, T$  and  $U$ . The most general tensor structure, which is consistent with the Bose symmetry (A13) is<sup>9</sup>

$$\begin{aligned} \mathcal{A}_{\mu\nu}^{(b+c)}(\Lambda; k, k', p, p') &= \mathcal{A}_{\mu\nu}^{(b)}(\Lambda; k, k', p, p') + \mathcal{A}_{\mu\nu}^{(c)}(\Lambda; k, k', p, p') \\ &= i \left[ \frac{e^2 Q^2 N_c}{2\pi^2} \right] \left( 1 - \frac{G_D \langle \sigma \rangle}{8G^2} \right)^2 \{ g_{\mu\nu} \mathcal{A}_g(\Lambda; S, T, U) \\ &\quad + k_\mu k'_\nu \mathcal{A}_{kk}(\Lambda; S, T, U) + (p_\mu p_\nu + p'_\mu p'_\nu) \mathcal{A}_p(\Lambda; S, T, U) \\ &\quad + p'_\mu p_\nu \mathcal{A}_{p'}(\Lambda; S, T, U) + p_\mu p'_\nu \mathcal{A}_{p'}(\Lambda; S, U, T) \\ &\quad + (k_\mu p'_\nu + p_\mu k'_\nu) \mathcal{A}_k(\Lambda; S, T, U) + (k_\mu p_\nu + p'_\mu k'_\nu) \mathcal{A}_k(\Lambda; S, U, T) \} , \end{aligned} \quad (\text{A16})$$

where the amplitudes  $\mathcal{A}_i$ 's have to satisfy

$$\mathcal{A}_{g,kk,p}(\Lambda; S, T, U) = \mathcal{A}_{g,kk,p}(\Lambda; S, U, T) . \quad (\text{A17})$$

Gauge invariance means that the following quantities vanish:

$$k^\nu \mathcal{A}_{\mu\nu}(\Lambda; k, k', p, p') = k_\mu \mathcal{G}_1(\Lambda; S, T, U) + p_\mu \mathcal{G}_2(\Lambda; S, T, U) + p'_\mu \mathcal{G}_3(\Lambda; S, T, U), \quad (\text{A18})$$

$$k'^\mu \mathcal{A}_{\mu\nu}(\Lambda; k, k', p, p') = k'_\nu \mathcal{G}'_1(\Lambda; S, T, U) + p_\nu \mathcal{G}'_2(\Lambda; S, T, U) + p'_\nu \mathcal{G}'_3(\Lambda; S, T, U), \quad (\text{A19})$$

where we impose the on-shell condition  $k^2 = k'^2 = 0, p^2 = p'^2 = m_{\text{DM}}^2$  along with the four momentum conservation  $p + p' = k + k'$ , and  $\mathcal{G}_i$ 's are defined as

$$\begin{aligned} \mathcal{G}_1(\Lambda; S, T, U) &= \mathcal{G}'_1(\Lambda; S, T, U) \\ &= \mathcal{A}_g(\Lambda; S, T, U) + \frac{S}{4} [2\mathcal{A}_{kk}(\Lambda; S, T, U) + \mathcal{A}_k(\Lambda; S, T, U) + \mathcal{A}_k(\Lambda; S, U, T)] \\ &\quad + \frac{1}{4}(T - U) [\mathcal{A}_k(\Lambda; S, T, U) - \mathcal{A}_k(\Lambda; S, U, T)] , \end{aligned} \quad (\text{A20})$$

$$\begin{aligned} \mathcal{G}_2(\Lambda; S, T, U) &= \mathcal{G}'_3(\Lambda; S, T, U) = \mathcal{G}_3(\Lambda; S, U, T) \\ &= \frac{1}{4}(S - T + U) \mathcal{A}_p(\Lambda; S, T, U) + \frac{S}{2} \mathcal{A}_k(\Lambda; S, T, U) + \frac{1}{4}(S + T - U) \mathcal{A}_{p'}(\Lambda; S, U, T), \end{aligned} \quad (\text{A21})$$

<sup>9</sup> Because of the on-shell gauge invariance, we have suppressed terms proportional to  $k_\nu$  and  $k'_\mu$  in (A16).



$$\begin{aligned} \mathcal{G}_3(\Lambda; S, T, U) &= \mathcal{G}'_2(\Lambda; S, T, U) = \mathcal{G}_2(\Lambda; S, U, T) \\ &= \frac{1}{4}(S + T - U)\mathcal{A}_p(\Lambda; S, U, T) + \frac{S}{2}\mathcal{A}_k(\Lambda; S, U, T) + \frac{1}{4}(S - T + U)\mathcal{A}_{p'}(\Lambda; S, T, U). \end{aligned} \quad (\text{A22})$$

Gauge invariance requires that all  $\mathcal{G}_i$ 's should vanish identically. We have calculated them for  $\mathcal{A}_{\mu\nu}^{(b+c)} = \mathcal{A}_{\mu\nu}^{(b)} + \mathcal{A}_{\mu\nu}^{(c)}$  explicitly for a small external momenta  $\sim \mathcal{O}(\epsilon)$  and find

$$\epsilon = 0 : \mathcal{G}_1 = \frac{\Lambda^4}{(\Lambda^2 + M^2)^2}, \quad \mathcal{G}_2 = 0, \quad (\text{A23})$$

$$\epsilon^2 : \mathcal{G}_1 = \frac{11\Lambda^4}{40(\Lambda^2 + M^2)^3} [S + 2(T + U)], \quad \mathcal{G}_2 = \frac{3\Lambda^4}{40(\Lambda^2 + M^2)^3} (T - U), \quad (\text{A24})$$

$$\begin{aligned} \epsilon^4 : \mathcal{G}_1 &= \frac{\Lambda^4}{280(\Lambda^2 + M^2)^4} [82S^2 + 275S(T + U) + 64 \{4(T^2 + U^2) + 7TU\}], \\ \mathcal{G}_2 &= \frac{3\Lambda^4}{280(\Lambda^2 + M^2)^4} (T - U) [16S + 13(T + U)], \end{aligned} \quad (\text{A25})$$

$$\begin{aligned} \epsilon^6 : \mathcal{G}_1 &= \frac{\Lambda^4}{1008(\Lambda^2 + M^2)^5} [517S^3 + 2168S^2(T + U) + 10S \{337(T^2 + U^2) + 586TU\} \\ &\quad + 3 \{761(T^3 + U^3) + 1591(T^2U + TU^2)\}], \end{aligned} \quad (\text{A26})$$

$$\mathcal{G}_2 = \frac{\Lambda^4}{336(\Lambda^2 + M^2)^5} (T - U) [116S^2 + 239S(T + U) + 17 \{7(T^2 + U^2) + 10TU\}],$$

$$\begin{aligned} \epsilon^8 : \mathcal{G}_1 &= \frac{\Lambda^4}{1584(\Lambda^2 + M^2)^6} [2142S^4 + 10058S^3(T + U) + 2S^2(\{9701(T^2 + U^2) + 17284TU\} \\ &\quad + 15S \{1399(T^3 + U^3) + 3137(T^2U + TU^2)\} + 3 \{3993(T^4 + U^4) \\ &\quad + 9178(T^3U + TU^3) + 11098T^2U^2\}], \\ \mathcal{G}_2 &= \frac{\Lambda^4}{1584(\Lambda^2 + M^2)^6} (T - U) [1512S^3 + 4007S^2(T + U) \\ &\quad + 15S \{319(T^2 + U^2) + 512TU\} + 630 \{3(T^3 + U^3) + 5(T^2U + TU^2)\}]. \end{aligned} \quad (\text{A27})$$

As we see from (A23)  $\sim$  (A27) that the non-gauge invariant function  $\mathcal{G}_1$  has the same structure as  $\mathcal{G}$  in (A4) for the photon self-energy. Therefore, we subtract this term from the amplitude so that the function  $\mathcal{A}_g$  is replaced by

$$\mathcal{A}_g^R = \mathcal{A}_g - \mathcal{G}_1, \quad (\text{A28})$$

where  $\mathcal{A}_g$  and  $\mathcal{G}_1$  are defined in (A16) and (A20), respectively. At  $\mathcal{O}(\epsilon^2)$   $\mathcal{G}_2$  becomes non-zero. As we see from (A18) and (A19), this non-gauge invariant term requires an introduction of a new tensor structure for counter terms. We see from (A23)  $\sim$  (A27) that  $\mathcal{G}_2$  is proportional to  $(T - U)$ , so that we can rewrite it as

$$\mathcal{G}_2(\Lambda; S, T, U) = (T - U)\hat{\mathcal{G}}_2(\Lambda; S, T, U), \quad (\text{A29})$$

which will be justified below to all orders in the expansion of  $k$ 's and  $p$ 's. Therefore, we can cancel this non-gauge invariant term by adding counter terms such that  $\mathcal{A}_p$  and  $\mathcal{A}_{p'}$  change according to

$$\mathcal{A}_p \rightarrow \mathcal{A}_p^R = \mathcal{A}_p + 2\hat{\mathcal{G}}_2, \quad \mathcal{A}_{p'} \rightarrow \mathcal{A}_{p'}^R = \mathcal{A}_{p'} - 2\hat{\mathcal{G}}_2, \quad (\text{A30})$$

where  $\mathcal{A}_p$  and  $\mathcal{A}_{p'}$  are defined in (A16). Since  $\mathcal{A}_p$  has to satisfy the Bose symmetry (A17),  $\hat{\mathcal{G}}_2$ , too, has to satisfy the same symmetry. We have numerically checked that

$$\mathcal{G}_2(\Lambda; S, T, T) = 0, \quad \mathcal{G}_2(\Lambda; S, T, U) = -\mathcal{G}_2(\Lambda; S, U, T) \quad (\text{A31})$$

is satisfied within an accuracy that we can get, which implies that  $\hat{\mathcal{G}}_2(\Lambda; S, T, U) = \hat{\mathcal{G}}_2(\Lambda; S, U, T)$ .

Finally, we have the gauge invariant  $\phi$ - $\phi$ - $\gamma$ - $\gamma$  four-point function:

$$\begin{aligned} \mathcal{A}_{\mu\nu}^R(\Lambda; k, k', p, p') = & \mathcal{A}_{\mu\nu}^{R(a)}(\Lambda; k, k', p, p') + i \left[ \frac{e^2 Q^2 N_c}{2\pi^2} \right] \left( 1 - \frac{G_D \langle \sigma \rangle}{8G^2} \right)^2 \{ g_{\mu\nu} \mathcal{A}_g^R(\Lambda; S, T, U) \\ & + k_\mu k'_\nu \mathcal{A}_{kk}(\Lambda; S, T, U) + (p_\mu p_\nu + p'_\mu p'_\nu) \mathcal{A}_p^R(\Lambda; S, T, U) \\ & + p'_\mu p_\nu \mathcal{A}_{p'}^R(\Lambda; S, T, U) + p_\mu p'_\nu \mathcal{A}_{p'}^R(\Lambda; S, U, T) \\ & + (k_\mu p'_\nu + p_\mu k'_\nu) \mathcal{A}_k(\Lambda; S, T, U) + (k_\mu p_\nu + p'_\mu k'_\nu) \mathcal{A}_k(\Lambda; S, U, T) \} , \end{aligned} \quad (\text{A32})$$

where  $\mathcal{A}^{R(a)}$  is given in (A12). Note that for the case  $T = U$  the function  $\mathcal{G}_2$  vanishes. In this case, therefore, all the counter terms are proportional to the metric tensor. In the center of mass system,  $T = U$  implies  $p = p' = (m_{\text{DM}}, \mathbf{0})$ , and only  $k$  and  $k'$  are independent Lorentz vectors ( $p_\mu = (k_\mu + k'_\mu)/2$ ). In this system, the non-gauge invariant terms of  $\mathcal{A}_{\mu\nu}$  are proportional to  $g_{\mu\nu}$ . No Lorentz transformation can produce non-gauge invariant terms proportional to  $k_\mu p_\nu$  or  $p_\mu p_\nu$  from the  $g_{\mu\nu}$  term. This is the reason why  $\mathcal{G}_2$  vanishes when  $T = U$ , and therefore the assumption that this function takes the form  $\mathcal{G}_2 = (T - U)\hat{\mathcal{G}}_2$  is justified to all orders in the expansion of  $k$ 's and  $p$ 's. The amplitude with  $p = p' = (m_{\text{DM}}, \mathbf{0})$  will be used for the annihilation of two DM's into two photons in sect. V.

The tensor structure (A16) is the most general one. However, using the on-shell conditions  $k + k' = p + p'$  we can eliminate  $k$  and  $k'$ , because  $k_\nu$  and  $k'_\mu$  terms do not contribute neither to the physical amplitude nor to the on-shell gauge invariance conditions (A18) and (A19) and hence can be suppressed as we have done in (A16). In this basis,  $\mathcal{A}_{\mu\nu}^{(b+c)}(\Lambda; k, k', p, p')$  of (A16) becomes

$$\begin{aligned} \mathcal{A}_{\mu\nu}^{(b+c)}(\Lambda; k, k', p, p') = & i \left[ \frac{e^2 Q^2 N_c}{2\pi^2} \right] \left( 1 - \frac{G_D \langle \sigma \rangle}{8G^2} \right)^2 \\ & \times \left\{ g_{\mu\nu} \mathcal{A}_g(\Lambda; S, T, U) + (p_\mu p_\nu + p'_\mu p'_\nu) \tilde{\mathcal{A}}_p(\Lambda; S, T, U) \right. \\ & \left. + p'_\mu p_\nu \tilde{\mathcal{A}}_{p'}(\Lambda; S, T, U) + p_\mu p'_\nu \tilde{\mathcal{A}}_{p'}(\Lambda; S, U, T) \right\} , \end{aligned} \quad (\text{A33})$$

where

$$\begin{aligned} \tilde{\mathcal{A}}_p(\Lambda; S, T, U) = & \mathcal{A}_p(\Lambda; S, T, U) + \mathcal{A}_{kk}(\Lambda; S, T, U) + \mathcal{A}_k(\Lambda; S, T, U) \\ & + \mathcal{A}_k(\Lambda; S, U, T) , \end{aligned} \quad (\text{A34})$$

$$\tilde{\mathcal{A}}_{p'}(\Lambda; S, T, U) = \mathcal{A}_{p'}(\Lambda; S, T, U) + \mathcal{A}_{kk}(\Lambda; S, T, U) + 2\mathcal{A}_k(\Lambda; S, U, T) . \quad (\text{A35})$$

Then the gauge invariance conditions (A18) and (A19) mean that

$$\begin{aligned}\tilde{\mathcal{G}}_2(\Lambda; S, T, U) &= \tilde{\mathcal{G}}_3(\Lambda; S, U, T) \\ &= \frac{1}{4}(S - T + U)\tilde{\mathcal{A}}_p(\Lambda; S, T, U) + \frac{1}{4}(S + T - U)\tilde{\mathcal{A}}_{p'}(\Lambda; S, U, T) \quad (\text{A36})\end{aligned}$$

$$= \mathcal{G}_1(\Lambda; S, T, U) + (T - U)\hat{\mathcal{G}}_2(\Lambda; S, T, U) \quad (\text{A37})$$

has to vanish, where  $\mathcal{G}_1$  and  $\hat{\mathcal{G}}_2$  are defined in (A20) and (A29), respectively. From (A36) and (A37) it is now obvious how to restore gauge invariance: From (A37) we see that  $\tilde{\mathcal{G}}_2(\Lambda; S, T, U)$  can be uniquely divided into the even and odd part under the interchange  $T \leftrightarrow U$ , because  $\mathcal{G}_1$  and  $\hat{\mathcal{G}}_2$  are even functions. The odd part can be canceled by

$$\tilde{\mathcal{A}}_p \rightarrow \tilde{\mathcal{A}}_p^R = \tilde{\mathcal{A}}_p + 2\hat{\mathcal{G}}_2, \quad \tilde{\mathcal{A}}_{p'} \rightarrow \tilde{\mathcal{A}}_{p'}^R = \tilde{\mathcal{A}}_{p'} - 2\hat{\mathcal{G}}_2, \quad (\text{A38})$$

as in the case of  $\mathcal{A}_p$  and  $\mathcal{A}_{p'}$  (see (A30)). The even part can be canceled by the redefinition of  $A_g$  which is defined in (A28). The resulting gauge invariant subtracted amplitude is identical with (A32) up to on-shell conditions.

- 
- [1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716** (2012) 1 [arXiv:1207.7214 [hep-ex]].
  - [2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716** (2012) 30 [arXiv:1207.7235 [hep-ex]].
  - [3] M. Holthausen, K. S. Lim and M. Lindner, JHEP **1202** (2012) 037 [arXiv:1112.2415 [hep-ph]].
  - [4] G. Degrandi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, JHEP **1208** (2012) 098 [arXiv:1205.6497 [hep-ph]]; D. Buttazzo, G. Degrandi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, JHEP **1312** (2013) 089 [arXiv:1307.3536].
  - [5] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl and M. Shaposhnikov, JHEP **1210** (2012) 140 [arXiv:1205.2893 [hep-ph]].
  - [6] J. Kubo, K. S. Lim and M. Lindner, arXiv:1403.4262 [hep-ph].
  - [7] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7** (1973) 1888.
  - [8] J. P. Fatelo, J. M. Gerard, T. Hambye and J. Weyers, Phys. Rev. Lett. **74** (1995) 492.
  - [9] R. Hempfling, Phys. Lett. B **379** (1996) 153 [hep-ph/9604278].
  - [10] T. Hambye, Phys. Lett. B **371** (1996) 87 [hep-ph/9510266].
  - [11] K. A. Meissner and H. Nicolai, Phys. Lett. B **648** (2007) 312 [hep-th/0612165]; K. A. Meissner and H. Nicolai, Phys. Lett. B **660** (2008) 260 [arXiv:0710.2840 [hep-th]]; K. A. Meissner and H. Nicolai, Phys. Rev. D **80** (2009) 086005 [arXiv:0907.3298 [hep-th]].
  - [12] R. Foot, A. Kobakhidze and R. R. Volkas, Phys. Lett. B **655** (2007) 156 [arXiv:0704.1165 [hep-ph]]; Phys. Rev. D **84** (2011) 075010 [arXiv:1012.4848 [hep-ph]].
  - [13] R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D **76** (2007) 075014 [arXiv:0706.1829 [hep-ph]]; Phys. Rev. D **77** (2008) 035006 [arXiv:0709.2750 [hep-ph]]; arXiv:1310.0223 [hep-ph].
  - [14] W.-F. Chang, J. N. Ng and J. M. S. Wu, Phys. Rev. D **75** (2007) 115016 [hep-ph/0701254 [HEP-PH]].

- [15] T. Hambye and M. H. G. Tytgat, Phys. Lett. B **659** (2008) 651 [arXiv:0707.0633 [hep-ph]].
- [16] S. Iso, N. Okada and Y. Orikasa, Phys. Lett. B **676** (2009) 81 [arXiv:0902.4050 [hep-ph]]; Phys. Rev. D **80** (2009) 115007 [arXiv:0909.0128 [hep-ph]]; PTEP **2013** (2013) 023B08 [arXiv:1210.2848 [hep-ph]].
- [17] M. Holthausen, M. Lindner and M. A. Schmidt, Phys. Rev. D **82** (2010) 055002 [arXiv:0911.0710 [hep-ph]].
- [18] K. Ishiwata, Phys. Lett. B **710**, 134 (2012) [arXiv:1112.2696 [hep-ph]].
- [19] V. V. Khoze, JHEP **1311** (2013) 215 [arXiv:1308.6338 [hep-ph]].
- [20] Y. Kawamura, PTEP **2013** (2013) 11, 113B04 [arXiv:1308.5069 [hep-ph]].
- [21] F. Gretschnig and A. Monin, arXiv:1308.3863 [hep-th].
- [22] C. D. Carone and R. Ramos, Phys. Rev. D **88** (2013) 055020 [arXiv:1307.8428 [hep-ph]].
- [23] V. V. Khoze and G. Ro, JHEP **1310** (2013) 075 [arXiv:1307.3764].
- [24] E. Gabrielli, M. Heikinheimo, K. Kannike, A. Racioppi, M. Raidal and C. Spethmann, Phys. Rev. D **89**, 015017 (2014) [arXiv:1309.6632 [hep-ph]].
- [25] C. Englert, J. Jaeckel, V. V. Khoze and M. Spannowsky, JHEP **1304** (2013) 060 [arXiv:1301.4224 [hep-ph]].
- [26] A. Farzinnia, H. -J. He and J. Ren, Phys. Lett. B **727** (2013) 141 [arXiv:1308.0295 [hep-ph]].
- [27] S. Abel and A. Mariotti, arXiv:1312.5335 [hep-ph].
- [28] M. Ibe, S. Matsumoto and T. T. Yanagida, arXiv:1312.7108 [hep-ph].
- [29] C. T. Hill, arXiv:1401.4185 [hep-ph].
- [30] J. Guo and Z. Kang, arXiv:1401.5609 [hep-ph].
- [31] B. Radovic and S. Benic, arXiv:1401.8183 [hep-ph].
- [32] V. V. Khoze, C. McCabe and G. Ro, arXiv:1403.4953 [hep-ph].
- [33] H. Davoudiasl and I. M. Lewis, arXiv:1404.6260 [hep-ph].
- [34] P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, arXiv:1404.0548 [hep-ph].
- [35] C. G. Callan, Jr., Phys. Rev. D **2** (1970) 1541; K. Symanzik, Commun. Math. Phys. **18** (1970) 227.
- [36] W. A. Bardeen, FERMILAB-CONF-95-391-T.
- [37] T. Hur, D. -W. Jung, P. Ko and J. Y. Lee, Phys. Lett. B **696** (2011) 262 [arXiv:0709.1218 [hep-ph]].
- [38] T. Hur and P. Ko, Phys. Rev. Lett. **106** (2011) 141802 [arXiv:1103.2571 [hep-ph]].
- [39] M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann and K. Tuominen, arXiv:1304.7006 [hep-ph].
- [40] M. Holthausen, J. Kubo, K. S. Lim and M. Lindner, arXiv:1310.4423 [hep-ph].
- [41] M. J. Strassler and K. M. Zurek, Phys. Lett. B **651** (2007) 374 [hep-ph/0604261]; T. Han, Z. Si, K. M. Zurek and M. J. Strassler, JHEP **0807** (2008) 008 [arXiv:0712.2041 [hep-ph]].
- [42] T. Bringmann, L. Bergstrom and J. Edsjo, JHEP **0801** (2008) 049 [arXiv:0710.3169 [hep-ph]].
- [43] G. Bertone, C. B. Jackson, G. Shaughnessy, T. M. P. Tait and A. Vallinotto, Phys. Rev. D **80** (2009) 023512 [arXiv:0904.1442 [astro-ph.HE]].
- [44] R. Laha, K. C. Y. Ng, B. Dasgupta and S. Horiuchi, Phys. Rev. D **87**, no. 4, 043516 (2013) [arXiv:1208.5488 [astro-ph.CO]].
- [45] M. Ackermann *et al.* [LAT Collaboration], Phys. Rev. D **86** (2012) 022002 [arXiv:1205.2739

- [astro-ph.HE]].
- [46] M. Gustafsson [ for the Fermi-LAT Collaboration], arXiv:1310.2953 [astro-ph.HE].
  - [47] A. Abramowski *et al.* [H.E.S.S. Collaboration], Phys. Rev. Lett. **110** (2013) 041301 [arXiv:1301.1173 [astro-ph.HE]].
  - [48] E. Bulbul, M. Markevitch, A. Foster, R. K. Smith, M. Loewenstein and S. W. Randall, arXiv:1402.2301 [astro-ph.CO].
  - [49] A. Boyarsky, O. Ruchayskiy, D. Iakubovskiy and J. Franse, arXiv:1402.4119 [astro-ph.CO].
  - [50] A. D. Dolgov, S. L. Dubovsky, G. I. Rubtsov and I. I. Tkachev, Phys. Rev. D **88**, no. 11, 117701 (2013) [arXiv:1310.2376 [hep-ph]].
  - [51] P. Langacker and G. Steigman, Phys. Rev. D **84**, 065040 (2011) [arXiv:1107.3131 [hep-ph]].
  - [52] Y. Nambu, Phys. Rev. Lett. **4** (1960) 380.
  - [53] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345; Phys. Rev. **124** (1961) 246.
  - [54] S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.
  - [55] T. Hatsuda and T. Kunihiro, Phys. Rept. **247** (1994) 221 [hep-ph/9401310].
  - [56] T. Kunihiro and T. Hatsuda, Prog. Theor. Phys. **71** (1984) 1332; Phys. Rev. Lett. **55**, 158 (1985); Phys. Lett. B **206** (1988) 385 [Erratum-ibid. **210** (1988) 278].
  - [57] J. H. Lowenstein and W. Zimmermann, Commun. Math. Phys. **46** (1976) 105; Commun. Math. Phys. **44** (1975) 73 [Lect. Notes Phys. **558** (2000) 310].
  - [58] E. C. Poggio and H. R. Quinn, Phys. Rev. D **14** (1976) 578.
  - [59] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5062 [astro-ph.CO].
  - [60] E. Aprile *et al.* [XENON100 Collaboration], Phys. Rev. Lett. **109** (2012) 181301 [arXiv:1207.5988 [astro-ph.CO]].
  - [61] D. S. Akerib *et al.* [LUX Collaboration], arXiv:1310.8214 [astro-ph.CO].
  - [62] E. Aprile [XENON1T Collaboration], arXiv:1206.6288 [astro-ph.IM].
  - [63] J. M. Jauch and F. Rohrlich, “The Theory of Photons and Electrons”, Addison-Wesley Pub. Company, Inc. 1959
  - [64] H. Suganuma and T. Tatsumi, Annals Phys. **208** (1991) 470.
  - [65] S. P. Klevansky, J. Janicke and R. H. Lemmer, Phys. Rev. D **43** (1991) 3040.
  - [66] S. Tulin, H. -B. Yu and K. M. Zurek, Phys. Rev. D **87** (2013) 036011 [arXiv:1208.0009 [hep-ph]].
  - [67] S. Kanemura, S. Matsumoto, T. Nabeshima and H. Taniguchi, Phys. Lett. B **701** (2011) 591 [arXiv:1102.5147 [hep-ph]].