

# Modelling the Magnetic Field Configuration of Neutron Stars

Riccardo Ciolfi \*

Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, 14476 Potsdam, Germany

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The properties of the extremely strong magnetic fields of neutron stars affect in a unique way their evolution and the associated phenomenology. Due to the lack of constraints from direct observations, our understanding of the magnetic field configuration in neutron star interiors depends on the progress in theoretical modelling. Here we discuss the effort in building models of magnetized neutron stars focussing on some of the recent results. In particular, we comment on the instability of purely poloidal and purely toroidal magnetic field configurations and on the evidence in favour of the so-called twisted-torus solutions. We conclude with an outlook on the present status of the field and future directions.

## 1 Introduction

One of the most important features characterizing neutron stars (NSs) is their extremely strong magnetic field, reaching surface strengths of  $\sim 10^{13}$  G for ordinary pulsars and up to  $\sim 10^{15}$  G in the case of *magnetars* (Duncan & Thompson 1992; Mereghetti 2008). NS interiors harbor even stronger fields, possibly by one order of magnitude or more. Such extreme magnetic fields are closely related to the observational signatures which distinguish NSs from other astrophysical sources, including, e.g., the continuous pulsar radiation and the spectacular flaring activity of magnetars. Moreover, magnetic fields are responsible for structural deformations associated with gravitational wave (GW) emission (Bonazzola & Gourgoulhon 1996; Cutler 2002) potentially detectable in the near future by ground-based interferometers such as Advanced Virgo and Advanced LIGO (Accadia et al. 2011; Harry et al. 2010).

The pivotal role played by magnetic fields in the physics and astrophysics of NSs raises a challenging but also crucial question for which a satisfactory answer is still missing: *what is the internal magnetic field configuration of a neutron star?* Properties like the amount of magnetic energy stored inside a NS and its geometrical distribution are well-known to affect its evolutionary path and consequently its emission properties. Clear examples come from recent magneto-thermal evolution studies (Viganò et al. 2013 and references therein), where it is shown how a different strength and distribution of magnetic fields (and hence electric currents) in the stellar interior leads to a very different phenomenology, potentially explaining most of the differences between the various classes of NSs (e.g. it determines the presence or absence of bursting activity). To consider a more specific example, knowing the internal magnetic field energy and geometry is essential in order to understand the mechanism behind the rare and extremely ener-

getic *giant flares* of magnetars and to predict their occurrence rate. Additionally, it is necessary to carry out NS asteroseismology from the quasi-periodic oscillations observed in the aftermath of these events (e.g. Gabler et al. 2012 and references therein). As we further discuss in the following (Section 3.1), the magnetically-induced GW emission mechanism mentioned above also depends sensitively on the internal magnetic field geometry.

While the external manifestation of NS magnetic fields can be constrained by direct observations, the internal configuration remains essentially unknown and we can only rely on a collection of indications mostly based on evolutionary or energetic considerations. In this context, our understanding can only be improved with the aid of theoretical modelling, by considering the widest range of possible “realistic” configurations (i.e., matching all the basic expectations) and predicting their imprints on NS observational properties. This makes it possible to validate or exclude the different models that have been proposed, guiding us towards a more and more precise picture of the magnetic fields hidden inside neutron stars.

At birth, NSs are hot and completely fluid, highly convective and differentially rotating. In the very dynamical first stage of their evolution, magnetic fields can be redistributed and significantly amplified by several mechanisms including dynamo action, magnetic winding and possibly shear instabilities like the magnetorotational instability. In a short timescale, of the order of seconds to minutes, convection and differential rotation are damped and magnetic fields start rearranging towards a magnetohydrodynamic (MHD) equilibrium. During this second stage, the temperature experiences a rapid decrease and within few hours it drops below  $10^9 - 10^{10}$  K, leading to the formation of a solid crust and the likely onset of superfluid/supeconductive phases in the stellar matter. This sets the beginning of a third stage, in which the magnetized NS evolves through dissipative processes on much longer

\* e-mail: riccardo.ciolfi@aei.mpg.de

timescales ( $\tau_{diss} \gtrsim 10^3 - 10^6$  years, depending on the magnetic field strength).

In the intermediate stage, the star is still fluid and well-described by ideal MHD (i.e., in the limit of infinite electric conductivity). Magnetic field rearrangement proceeds on the Alfvén timescale, which is inversely proportional to the average field strength and typically lies in the range  $\tau_A \sim 0.01 - 10$  s. Since this timescale is much shorter than a few hours, the magnetic field has enough time to reach a stable<sup>1</sup> MHD equilibrium before the crust forms and superfluid transitions take place. If this does not happen, the ongoing hydro-magnetic readjustment will efficiently dissipate most of the initial magnetic energy. The observation of long-lived magnetic fields excludes the second possibility and is commonly taken as an indication that the field should have reached already a stable MHD equilibrium at the time of crust formation.

In this paper we focus our interest on such an equilibrium. Even if the long-term magneto-thermal evolution of the NS will slowly alter the magnetic field energy and geometry, the initial<sup>2</sup> equilibrium configuration is extremely important as it dictates the overall energy budget and other fundamental properties which will remain almost unchanged on timescales  $\lesssim \tau_{diss}$ , such as the magnetic helicity and the relative contribution of the toroidal (azimuthal) and poloidal (meridional) components of the field. Therefore, the initial magnetic field represents not only a key input for long-term evolution studies, but also an important element for any model describing observed NS processes in which magnetic fields play an important role (e.g., magnetar flares). Moreover, there are potentially observable processes associated with the early evolution of NSs happening on timescales  $\ll \tau_{diss}$  which carry the direct imprint of the initial field (e.g. GW emission from newly-born NSs).

The existence of an initial equilibrium allows for a theoretical description based on stationary configurations and relatively simple physics (pure fluid matter, uniform rotation, ideal MHD, no superfluid/superconducting phases, and so on); furthermore, the requirement of stability over several Alfvén timescales considerably reduces the space of possible configurations. The effort in building models of magnetized NSs and the investigation of their properties (including stability) has already made important progress in this direction, although so far no solution has been proven to represent a fully viable description of the initial NS magnetic field.

In Sections 2 and 3, we give a brief overview of this work, from the investigation of the simplest field geometries (already shown to be unstable) to the so-called *twisted-torus* configuration, which is presently considered a strong candidate. As an application, we also discuss the continuous GW emission of magnetized NSs associated with the structural deformations induced by magnetic fields, its dependence on the internal field configuration, and the prospects of detection (Section 3.1). We

conclude with a perspective on open questions and future directions (Section 4).

## 2 Purely poloidal and purely toroidal magnetic field configurations

The growing effort to build equilibrium models of magnetized NSs started with the early work of Chandrasekhar & Fermi (1953) and continued with a large number of analytical and numerical studies mostly based on the assumptions of ideal MHD and pure-fluid matter, well-suited to describe the conditions occurring before the formation of a solid crust and the onset of superfluidity.

Until the last decade, most attention was devoted to the simplest magnetic field geometries, consisting of either a purely poloidal or a purely toroidal field (e.g. Bocquet et al. 1995; Frieben & Rezzolla 2012). However, already from the early analytical work on nonrotating magnetized stars (Markey & Tayler 1973; Tayler 1973; Wright 1973), the stability of these simple geometries was strongly questioned because of the so-called Tayler (or kink) instability, which is expected to act on Alfvén timescales in both the purely poloidal and the purely toroidal case. Recently, the unstable nature of these configurations was confirmed in Newtonian numerical simulations, both in the linear regime (Lander & Jones 2011a,b) and, for main-sequence stars, via nonlinear simulations (Braithwaite 2006, 2007). The analogous system has also been studied by means of nonlinear MHD simulations in general relativity (Ciolfi et al. 2011; Ciolfi & Rezzolla 2012; Kiuchi et al. 2008, 2011; Lasky et al. 2011, 2012; see also Ciolfi 2014), further verifying the presence of a hydrodynamic instability and validating the basic expectations concerning its onset.

In Fig. 1 we show an example of a three-dimensional general relativistic MHD simulation taken from Ciolfi & Rezzolla (2012), illustrating the instability of a purely poloidal field. For this geometry, the Tayler instability is expected to develop in the vicinity of the neutral line (in the closed field-line region), where the exponential growth of the initial perturbations is accompanied by the generation of toroidal fields. Moreover, the saturation of the instability and the beginning of the nonlinear rearrangement should take place in about one Alfvén timescale, when the local strength of toroidal fields becomes comparable to the poloidal one. These predictions are all confirmed in the simulations. In addition to these confirmations, numerical investigations such as the one in the example allowed the nonlinear part of the evolution to be explored, providing hints on the favoured state of the system. Although no stable magnetic field configuration was found at the end of the simulations, the results showed that a distribution of magnetic energy in poloidal and toroidal fields close to equipartition and a significant amount of magnetic helicity emerge naturally in the evolution and thus represent likely features of a stable geometry (Ciolfi & Rezzolla 2012).

General relativistic studies of the poloidal field instability were also used as a test case to investigate the internal rearrangement scenario of magnetar giant flares. This scenario sug-

<sup>1</sup> Here we do not refer to absolute stability, but only to the stability over timescales longer than several Alfvén timescales.

<sup>2</sup> In the following, “initial” will indicate the time of crust formation and the beginning of the long-term evolution.

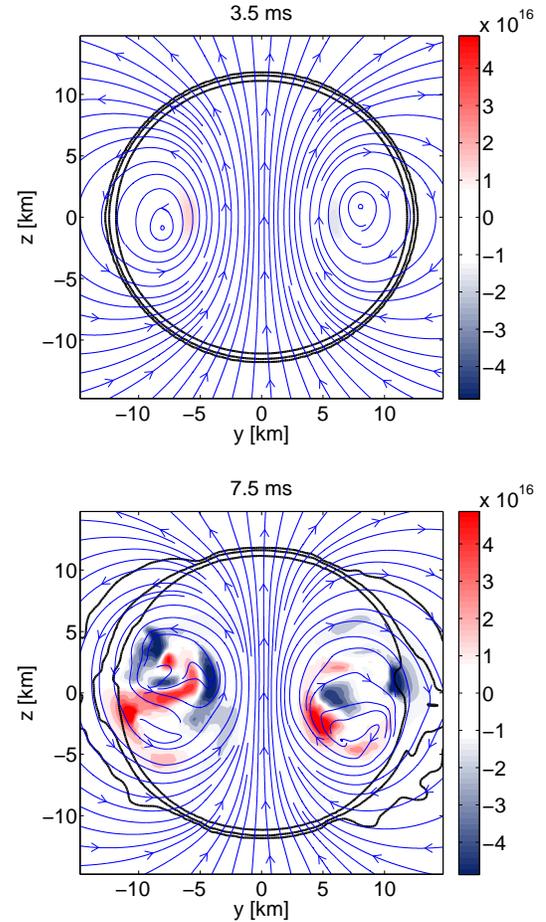
gests that the violent dynamics associated with a hydromagnetic instability might represent the mechanism that triggers a giant flare (Thompson & Duncan 1995, 2001). These studies were able to show that (i) the timescales of the observed electromagnetic emission (in particular the initial spike of the flare) are compatible with an internal rearrangement, which could also initially trigger a rapid reconfiguration of the external magnetic field in the magnetosphere (Ciolfi & Rezzolla 2012), and that (ii) the GW emission associated with a giant flare event, according to this scenario, would be hardly detectable in the near future by ground based interferometers (Ciolfi et al. 2011; Ciolfi & Rezzolla 2012; Lasky et al. 2012; Zink et al. 2012).

For both the purely poloidal and purely toroidal magnetic field configurations, the presence of rapid (uniform) rotation was long suggested to have a stabilizing effect, potentially suppressing the Tayler instability. Nevertheless, the most recent nonlinear studies found no evidence that rotation can actually stifle the development of the instability (Kiuchi et al. 2011; Lasky et al. 2012).

### 3 Mixed-field configurations: the twisted-torus geometry

The accumulated evidence on the instability of purely poloidal and purely toroidal fields strongly supports the idea that any long-lived magnetic field configuration in a NS has to consist of a mixture of poloidal and toroidal field components. In the present Section, among the possible mixed-field configurations, we devote our attention to the so-called *twisted-torus* geometry, which has recently emerged as a promising candidate for NS interiors. It consists of an axisymmetric mixed field where the poloidal component extends throughout the entire star and to the exterior, while the toroidal one is confined inside the star, in the torus-shaped region where the poloidal field lines are closed (cf. Fig. 2).

Compared to other mixed-field configurations, the twisted-torus has the important feature of having an external poloidal field (compatible with the observations) while maintaining both the poloidal and toroidal components continuous at the stellar surface and thus avoiding the need for surface currents (as, e.g., in Colaiuda et al. 2008). Moreover, a twisted-torus magnetic field appears natural in terms of the poloidal and toroidal field instabilities: the poloidal instability takes place in the closed-line region and produces a stabilizing toroidal component in that region (see Section 2), while the toroidal field instability occurs near the symmetry axis and produces a poloidal field there (see, e.g., Fig. 5 in Lander & Jones 2012). Apart from these general arguments, important indications in favour of the twisted-torus configuration came from the Newtonian simulations performed by Braithwaite & Nordlund (2006), where this geometry emerged as the final outcome of the evolution of initial random fields in a nonrotating fluid star. Those simulations were adapted to study main-sequence stars, while the equivalent evidence for NSs and in general relativity is still missing; nevertheless, this result triggered a growing



**Fig. 1** Meridional view of the instability of a purely poloidal field in a magnetized NS with an initial polar magnetic field strength of  $6.5 \times 10^{16}$  G. Poloidal field lines are shown together with the color-coded toroidal magnetic field strength; also reported are three rest-mass isodensity contours near the stellar surface, corresponding to  $(0.02, 0.2, 2) \times 10^{13} \text{ g/cm}^3$ . The two frames illustrate respectively the saturation of the instability, which sets the beginning of the non-linear evolution, and the most violent phase of the field rearrangement, when the instability has fully developed. In this case the Alfvén timescale is  $\sim 3$  ms.

interest and in the last decade a number of studies were devoted to building twisted-torus models of magnetized NSs both in Newtonian (Fujisawa et al. 2012; Glampedakis et al. 2012; Lander & Jones 2009, 2012; Tomimura & Eriguchi 2005; Yoshida & Eriguchi 2006) and general-relativistic frameworks (Ciolfi et al. 2009, 2010; Pili et al. 2014).

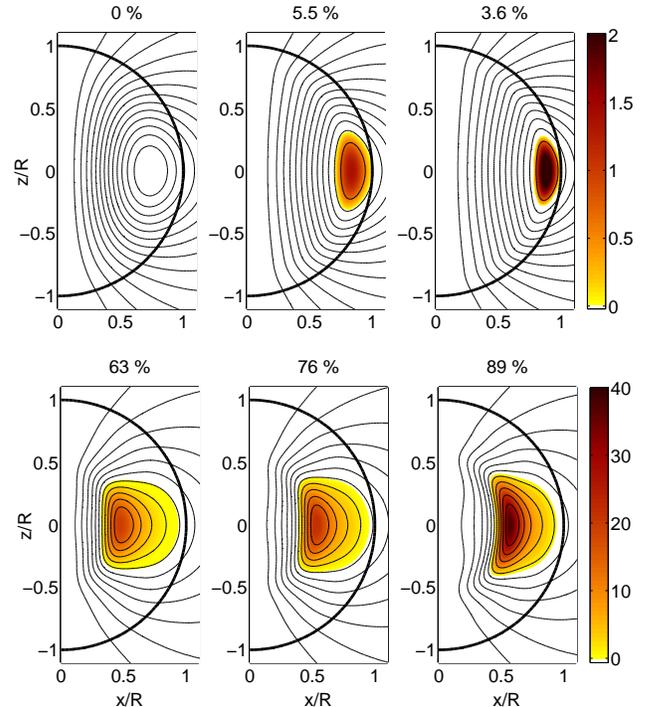
All of the twisted-torus models mentioned above gave rather similar results in terms of the possible configuration of magnetic fields, despite the different approaches adopted. Among the common findings, all of these models share the apparently unavoidable restriction to *poloidal-dominated* geometries, with an upper limit of  $\sim 10\%$  for the toroidal-to-poloidal magnetic field energy ratio inside the star (unless surface currents and discontinuous magnetic fields are included, as

in Fujisawa & Eriguchi 2013). For diverse reasons, this limitation is far more serious than it may appear. First, a much higher toroidal-field content is expected from the formation process of magnetized NSs, as a result of strong differential rotation in the nascent NS (Bonanno et al. 2003; Thompson & Duncan 1993). Second, all evidence is that poloidal-dominated geometries are unstable on Alfvén timescales (Braithwaite 2009; Lander & Jones 2012) as well as purely poloidal geometries, and hence twisted-torus configurations of this kind may not be realistic. Moreover, higher toroidal-field energies are needed to explain the magneto-thermal evolution of highly magnetized NSs, their bursting activity and their pulse profiles (Pons & Perna 2011).

The restriction to poloidal-dominated solutions has recently been overcome in Ciolfi & Rezzolla (2013), where it has been shown that this limitation is due to the overly simple prescription for the azimuthal currents adopted in all the previous models and that employing a different, more elaborate prescription allows for any toroidal energy content. We illustrate this result in Fig. 2, where we show selected examples among the models presented in Ciolfi & Rezzolla (2013). The top panels refer to twisted-torus configurations obtained with the simple and commonly-adopted prescription. Specifically, we consider one given model and increase from left to right the local strength of the toroidal field. In this way, electric currents get more intense in the outer layers of the NS, the neutral point is moved outwards and the region of closed-field lines shrinks. While the toroidal field strength keeps increasing, the toroidal magnetic energy content reaches a maximum (in this case  $\sim 5.5\%$ ) and then starts decreasing again. Although this shrinking effect cannot be avoided completely, the novel prescription proposed in Ciolfi & Rezzolla (2013) is designed to reduce this effect and at the same time to allow for larger closed-line regions. The resulting maximum toroidal energy content can be much higher. The bottom panels of Fig. 2 show configurations with up to  $\sim 90\%$  in toroidal energy (i.e. *toroidal-dominated*), proving the effectiveness of the suggested recipe.

Being able to build twisted-torus equilibria with any toroidal field energy content significantly expands the space of known solutions, including models that are more realistic compared to the previous poloidal-dominated ones. In particular, configurations with a relatively high magnetic energy in the toroidal field are potentially stable over several Alfvén timescales and could thus represent a viable description of the internal magnetic field of NSs. Future nonlinear studies, along the lines of what has been done for the purely poloidal and purely toroidal geometries (see Section 2), will possibly provide the missing evidence.

An additional finding of Ciolfi & Rezzolla (2013) is that for a fixed polar magnetic field strength, a higher relative content of toroidal field energy ( $> 10\%$ ) implies in general a much higher total (poloidal and toroidal) magnetic energy inside the star. This means that NSs (and in particular highly magnetized NSs) can harbor internal magnetic fields that are significantly



**Fig. 2** *Top panels:* Meridional view of twisted-torus magnetic field configurations obtained with the simple commonly-adopted current prescription (see Ciolfi & Rezzolla 2013 for details). Shown in colour is the toroidal-field strength in units of the polar value.  $R$  is the stellar radius. The labels on top of each panel show the toroidal-field energy content relative to the total internal magnetic energy ( $E_{\text{tor}}/E_{\text{m}}^{\text{int}}$ ). *Bottom panels:* The same as above for example configurations obtained with the novel prescription (cf. Section 2 of Ciolfi & Rezzolla 2013).

stronger than commonly expected<sup>3</sup>, with a considerable impact on their electromagnetic and GW emission properties (see the following Section).

### 3.1 Continuous GW emission from magnetized NSs

In the present Section we discuss the GW emission associated with the magnetically-induced NS deformations, which represents an ideal example to illustrate the potential impact of the internal magnetic field configuration on the emission properties of a magnetized NS.

Magnetic fields alter the NS density and pressure distributions inducing deformations that can be quantified in terms of the quadrupolar ellipticity  $\epsilon_Q \equiv Q/I$ , where  $Q$  is the mass-energy quadrupole moment and scales as the magnetic energy (i.e.  $\propto B^2$ ), while  $I$  is the mean value of the stellar moment of inertia. Poloidal fields deform a nonrotating star towards an oblate shape (equatorial radius larger than polar radius), corresponding to positive  $\epsilon_Q$ , whereas toroidal fields have the opposite effect; therefore, the sign of the deformation is reflected

<sup>3</sup> This can be seen, for instance, in Fig. 2, by comparing the color scale of the top and bottom panels, denoting the toroidal field strength.

in the toroidal-to-poloidal energy ratio. In general, a magnetized NS that rotates around an axis misaligned with respect to the magnetic axis and having  $\epsilon_Q \neq 0$ , will emit a continuous GW signal with amplitude  $h \propto |\epsilon_Q| I \Omega^2 / d$ , where  $\Omega$  is the angular velocity and  $d$  the source distance from the observer (Bonazzola & Gourgoulhon 1996).

This emission is most effective for newly-born highly magnetized NSs, having at the same time the highest spins and the strongest internal magnetic fields. In this case, the spin down rate is very high and the relevant timescale for GW emission can be extremely short compared to the long-term dissipative evolution taking place after the crust formation. As a consequence, the emitted GWs carry the direct imprint of the initial magnetic field configuration. Moreover, since the resulting signal changes significantly over the time of detection, it has an intermediate nature between a continuous signal and a long transient.<sup>4</sup>

Quadrupolar deformations of highly magnetized NSs have been computed for different magnetic field geometries, including twisted-torus models (Ciolfi et al. 2010; Ciolfi & Rezzolla 2013; Haskell et al. 2014; Lander & Jones 2009). In particular, for poloidal-dominated twisted-torus configurations and magnetar-like polar field strengths ( $\sim 10^{15}$  G), the ellipticities are always positive (oblate shape) and lie in the range  $10^{-6} - 10^{-5}$  (Ciolfi et al. 2010; Lander & Jones 2009). Simple estimates (Gualtieri et al. 2011) suggest that newly-born NSs with such deformations and initial spin periods of the order of  $\sim$ ms should be marginally detectable up to distances of the order of  $\sim 10$  kpc (i.e., within our Galaxy) by the advanced ground-based interferometers Virgo and LIGO, and up to  $\sim 0.1 - 1$  Mpc by future generation detectors such as the Einstein Telescope (Punturo et al. 2010). The main limitation for the actual detection of these GW signals comes from the relatively low birth rate of highly magnetized NSs, which is limited to only a few per century in our Galaxy.

The prospects of detection may increase significantly for twisted-torus models with a higher toroidal field energy content, which can harbor much higher internal magnetic energies. For instance, models with 50% of the internal magnetic energy in toroidal fields are found to have up to a factor of  $\sim 5 - 10$  larger ellipticities (Ciolfi & Rezzolla 2013; Haskell et al. 2014). Since the GW amplitude is proportional to  $\epsilon_Q$  and inversely proportional to the source distance, an ellipticity larger by a given factor  $\chi$  will enlarge by the same factor the maximum distance at which a highly magnetized newly-born NS can be detected. If we take, for instance,  $\chi \sim 10$ , the typical distance of detectability for the Einstein Telescope can expand up to the Virgo Cluster scale ( $\sim 20$  Mpc), with a corresponding event rate of more than one per year.

In addition, magnetized NSs with a toroidal energy content  $\gtrsim 50\%$  are deformed into a prolate shape (negative elliptic-

ity). In this case, a ‘spin-flip’ mechanism driven by internal viscosity may occur, leading to an increase in the angle between the spin and magnetic axes, towards a nearly orthogonal configuration (Cutler 2002; Jones 1975). If this mechanism is effective within the spin-down timescale, the GW emission is further enhanced, leading to very optimistic prospects of detection (Stella et al. 2005).

## 4 Outlook

The effort devoted to build models of magnetized NSs and to study how magnetic fields affect their observational properties is essential in order to improve our understanding of the physics and astrophysics of these objects. As discussed in the previous Sections, important constraints on the unknown internal magnetic field configuration of NSs can be obtained by looking for equilibrium solutions that represent a viable description of the NS magnetic field at the time of crust formation, shortly after NS birth and before the long-term dissipative evolution takes place. Viable solutions have to satisfy stringent requirements, including stability over timescales much longer than the Alfvén timescale, which already allowed the exclusion of purely poloidal and purely toroidal geometries.

Presently, twisted-torus configurations with a substantial amount of magnetic energy in the toroidal component appear as promising candidates for stability, although a proper confirmation is still missing. Future studies will provide a final answer on the viability of twisted-torus magnetic fields and possibly suggest alternative solutions to this long-standing open issue. State-of-the-art nonlinear simulations of magnetized NSs in general relativity will play an important role in this investigation.

In order to improve the realism of the simulations, various aspects need to be refined, including a proper treatment of magnetic fields in the magnetosphere surrounding the star and the use of realistic equations of state. Moreover, stable stratification due to composition gradients is an additional element to be taken into account that might represent a necessary ingredient for stability (Mitchell et al. 2013; Reisenegger 2009).

As a final note, the onset of superconductivity in newly-born NSs with internal magnetic fields below  $\sim 10^{16}$  G (i.e., ordinary NSs and possibly some magnetars) can significantly alter the equilibrium of the system (Lander 2014). This might happen on a timescale comparable to the formation of a solid crust – causing a rapid readjustment of the initial magnetic field configuration – or on much longer timescales, producing notable effects during the long-term evolution of the NS. This is an important aspect and deserves further investigation.

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<sup>4</sup> The GW signal produced by older NSs is genuinely continuous and has the advantage that it does not rely on the relatively low rate of birth of highly magnetized NSs within reach. On the other hand, it is much weaker and only relevant for known pulsars or when considering the GW background produced by the entire population of magnetized NSs (Marassi et al. 2011; Regimbau & de Freitas Pacheco 2006).

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