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Abstract. The evolving Laser Interferometer Space Antenna (eLISA) will revolutionize our understanding of the formation and evolution of massive black holes (MBHs) along cosmic history, by probing massive black hole binaries (MBHBs) in the $10^3 - 10^7 M_\odot$ range out to redshift $z \gtrsim 10$. High signal-to-noise ratio detections of $\sim 10 - 100$ MBHB coalescences per year will allow accurate measurements of the parameters of individual MBHBs (such as their masses, spins and luminosity distance), and a deep understanding of the underlying cosmic MBH parent population. This wealth of unprecedented information can lead to breakthroughs in many areas of physics, including astrophysics, cosmology and fundamental physics. We review the current status of the field, recent progress and future challenges.

1. Introduction

The evolving Laser Interferometer Space Antenna (eLISA, [1]) is designed to be sensitive to gravitational waves (GWs) at mHz frequencies. One of the strongest sources in this frequency window are MBHBs merging throughout the Universe [2]. According to our current understanding of structure formation in a ΛCDM Universe, MBHBs frequently form along cosmic history following galaxy mergers. MBHs we see in today's galaxies are expected to be the natural end-product of a complex evolutionary path, in which black holes (BHs) seeded in proto-galaxies at high redshift grow through cosmic history via a sequence of MBHB mergers and accretion episodes [3, 4]. However, our current observational knowledge of the MBH population is limited to a small fraction of these objects: either those that are active (see e.g. [5]), or those in our neighborhood, where stellar- and gas-dynamical measurements are possible (see [6] for a review).

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eLISA will revolutionize this picture by probing MBHBs in the $10^3 - 10^7 M_\odot$ range out to redshift $z \gtrsim 10$ [2]. In the current design [1], eLISA will be capable of detecting $\sim 10 - 100$ MBHB coalescences per year and of accurately measuring the parameters of individual MBHBs, such as their masses, spins and luminosity distance. This wealth of unprecedented measurements has the potential to revolutionize many areas of physics, ranging from astrophysics to cosmology and fundamental physics. This paper summarizes contributions to the "LISA Symposium X" that were devoted to this subject.

We start in Section 2 by describing current models for MBH formation and evolution, highlighting present uncertainties due either to the lack of observations or to poor theoretical constraints. eLISA will dramatically change this situation by probing the very first coalescences of seed BHs at high redshift. The interaction of MBHs with the environment has a key role in bringing MBHBs close enough that GW emission becomes efficient. Section 3 describes recent developments on this front, focusing on observational signatures of merging MBHBs in the electromagnetic (EM) domain. Coincident detection of MBHB mergers as both GW *and* EM sources will pave the way to multimessenger astronomy, promising extraordinary advances in the understanding of accretion physics. In Section 4 we show how eLISA will open a new era of precision measurements of MBH spins. Individual spin measurements will allow us to test gravity in the strong-field regime with unprecedented accuracy; the *collective* properties of spin and mass distributions of the whole MBH population carry information on the physics of accretion flows, and in general of the intimate link between MBHs and galaxy evolution across cosmic time. Section 5 explores eLISA's potential for cosmology. GW observations can yield a direct measurement of the luminosity distance to the source. If an EM counterpart is detected, the coincident measurement of the source redshift will provide an appealing opportunity for calibration-free cosmography. Last but not least, Section 6 touches on the invaluable insights that eLISA will provide in terms of fundamental physics. Dynamical measurements of the behavior of gravity in the strong-field regime will constrain the geometry of the spacetime around BHs, telling us whether the Kerr solution actually describes these objects and potentially even yielding smoking guns of new physics beyond General Relativity (GR).

2. Birth and growth of massive black holes

There is mounting observational evidence that supermassive BHs with masses between 10^6 and $10^{10} M_\odot$ reside at the heart of nearly every galaxy [7, 8, 9]. Though MBHs are an observational certainty, nearly every aspect of their evolution – from their birth, to their fuel source, to their basic dynamics – is a matter of lively debate.

The existence of bright quasars at high redshift implies that billion solar mass BHs must be in place within their host galaxies less than a billion years after the Big Bang. This remarkable observational fact suggests that the seeds of these most massive MBHs are sown during, or even before, the formation of protogalaxies. However, the precise MBH seed formation mechanism is not known, nor is it clear that there is only one seed formation channel at play over the entire MBH mass spectrum.

Currently, the two most widely accepted MBH birth scenarios are a) the remnants of the first generation of stars (Population III: see e.g. [10, 11, 12, 13, 14]), and b) the direct collapse of pristine gas within massive dark matter halos [15, 16, 17, 18, 19, 20]. In both these models, overdensities within a gas cloud collapse as the gas radiates away energy; if the gas can cool efficiently, then it gravitationally fragments into ever smaller clouds. The mass of the final collapsed object, be it a protostellar cloud or a seed black hole, is determined by the Jeans mass of the gas that can no longer cool efficiently enough to fragment further.

In the Population III seed scenario, metal-free gas is cooled by H_2 and HD, but since these molecules are much less effective than metals in radiating away energy, the Jeans mass is likely to be larger than that of stars formed in the present era. However, one of the difficulties in this scenario is that the actual initial mass function of this first generation of stars is hotly debated, and the issue will not easily resolve itself with the current state-of-the-art simulations. Early work placed the stellar mass at $100 - 1000\,M_\odot$ [21, 22], but later work suggested that fragmentation is far more common [23, 24, 25, 26, 27]. A recent study examining the role of turbulence in Population III star formation suggested a flat initial mass function with a characteristic mass less than $100\,M_\odot$ (e.g. [28]). The newest work tends to indicate that, even though the protostellar clouds do fragment, they tend to quickly merge within a dynamical time, essentially generating the very massive Population III stars once again [29, 30]. While these theoretical advances are promising, a more thorough treatment involving radiative transfer and 3D magnetohydrodynamics is required before we will truly understand the nature of the first stars and whether they can spawn MBH seeds.

In the fiducial direct collapse model, the only coolant is atomic Hydrogen, and this requires that the gas sit in dark matter halos with virial temperatures higher than 10^4 K. To inhibit fragmentation and to suppress H_2 formation, there is an additional photon bath that heats the gas; one of the most likely candidates for this radiation field, the Lyman-Werner background, is thought to be generated by nearby Population III stars [31, 32, 33] at redshifts z $\sim 10 - 20$. Preventing efficient cooling and fragmentation in this way results in a huge Jeans mass, and the eventual seed BHs are of order $10^4 - 10^6\,M_\odot$. These direct collapse BHs are the most obvious candidates to grow into the MBHs powering high-redshift quasars, because they require no stringent assumptions on the accretion or merger history to grow to $10^9\,M_\odot$ by $z \sim 6$.

As with the Population III scenario, the main epoch of direct collapse seed formation is brief, ending as metals pollute the halo gas and the radiation field drops below a critical heating threshold as the Universe expands [34, 35]. Still, pockets of pristine and irradiated gas could remain at low redshift that could collapse into MBH seeds [36, 37]. One potential issue with this direct collapse model is that the intensity of radiation needed to prevent fragmentation is under debate, and the actual Lyman-Warner background generated at redshift $z \sim 20$ is not well-constrained, as it depends in part on the occupation fraction and initial mass function of Population III stars. Therefore, direct collapse BH formation may arguably be too rare to account for the quasar population, much less the entire MBH mass spectrum (see however [38]).

One of the great promises of a future space-based GW observatory like eLISA is that it may be able to pin down the relative efficiency of light versus MBH seed formation channels [39, 40, 41, 42]. These pathways affect the occupation fraction of MBHs in local dwarf galaxies, the existence of intermediate-mass BHs, and the scaling relations between MBHs and their host galaxies (i.e. the M–σ relation) at low masses. Studies have attempted to predict MBH-galaxy occupation fractions [43, 37] and the low-mass M–σ relation [44] for a range of seed models, but observations in this regime are very difficult, and thus constraints are difficult to obtain. Direct GW detections will elucidate much regarding the hidden MBH population and their seeding mechanism.

Turning now to MBH growth, it is tempting to think that a clearer picture emerges. The famous Soltan argument has often been invoked to claim that MBHs are fueled nearly entirely by gas during the brief quasar epoch. The logic goes like this. If we assume that quasars are powered by gas accretion onto MBHs, then we can turn the observed energy and number density of optically-bright quasars into an estimate of the gas mass accreted by MBHs during the quasar era. Happily, if we compare the mass density of accreted gas during the quasar phase (assuming a radiative efficiency of 10%) to the mass density locked up in the local MBH population, the numbers agree. This implies that 90% of MBH masses are built from gas that is accreted at the Eddington limit before redshift 2; it also implies that MBHs do not seem to accrete in a low-efficiency, or "quiet" mode, nor are there many obscured or undetected quasars within the MBH mass budget. Though this argument seems pat and iron-clad, there are many uncertainties involved. For example, converting the optically observed quasar luminosity function to a bolometric energy density is fraught with difficulty. Furthermore, it is very clear that optical quasar surveys do miss a large fraction of the real quasar population, those that are radio-loud or X-ray bright (see e.g. [45, 46]). In addition, many theoretical efforts call into question the assumption of Eddington-limited accretion, some advocating super-Eddington accretion along filaments to feed ultramassive BHs [47], and others promoting a "radio" mode of quiescent gas accretion [48, 49]. Along this line, simulations indicate that the high-redshift universe may be rife with hidden MBHs that are accreting at less than the Eddington rate [50, 51].

Despite these uncertainties, it is widely accepted that gas is the primary fuel for MBH growth. It is thought that galaxy mergers help to restock the gas reservoir around the BH as the gas is shocked and then falls toward the galactic center. Indeed one of the best explanations of the M–σ relation invokes galaxy mergers that drive gas toward the BH, which fuels it and subsequently generates a prodigious radiative "feedback" that pushes the remaining gas toward the galaxy outskirts and cuts off the MBH fuel source (see e.g. [52]). Once the gas is in the galaxy bulge potential, it slowly cools and forms new stars with high velocity dispersion [53, 54]. Though early simulations of equal-mass galaxy mergers were very promising in supporting this view, later unequal-mass mergers fail to grow the primary MBHs enough to fall on the M-σ relation [55, 51]. The problem is that most low-mass MBHs, like the one in our Milky Way, simply don't undergo equal-mass mergers at the rate needed to build the MBH and to couple this growth to the bulge in this framework.

Fear not: mergers are not the only process which may trigger MBH growth. In fact, galaxies which lack a bulge (often thought to have extremely quiescent merger histories) may host MBHs as well, and even actively growing ones. One thought is that MBHs may grow efficiently through the accretion of cold, unshocked gas (i.e. "cold flows") which enter the galaxy through filamentary accretion. Some galaxies acquire most of their gas through this process [56, 57], and MBHs may efficiently accrete this gas as well [58, 50]. Another method which may be prevalent consists of mergers of MBHs with other MBHs [59]. In the simulations presented in [50], the central MBHs in massive galaxies at $z = 4$ have built up over half of their mass through MBHB mergers. This is a regime where GW observations can verify the accuracy of these predictions, which depend on many properties of MBH seed formation (initial mass, formation redshift, and efficiency of formation). By directly collecting GW data from the growth of high-redshift MBHs, we can determine if this phenomenon is dominated by MBH mergers, or whether it is correlated with galaxy merger rates, or some combination of the two.

Overall, it is likely that there are several mechanisms to grow a MBH, depending on the assembly history of the host galaxy. The most massive MBHs are likely fueled via gas accretion from major mergers at high redshift [60], which explains the relative tightness of the M–σ relation in this regime. Lower-mass galaxies, which experience fewer major mergers, exhibit more scatter within the local scaling relations, likely because their MBHs grow through more stochastic processes such as cold flow accretion, or through secular processes like bar-driven gas inflow. The combination of these processes plus the unknown contribution of MBHB mergers leads to a myriad of possibilities for MBH growth, which will be possible to disentangle by means of low-frequency GW observations.

3. Gas accretion onto massive black hole binaries and their electromagnetic signature

According to the evolution picture emerging from Section 2, MBHB coalescences must be common events at all redshifts. These systems do not live in isolation, but they are embedded in dense galactic nuclei, surrounded by gas and stars. Therefore, the interaction of MBHBs with their environment may provide a unique opportunity to observe EM signatures as well as GWs, opening new avenues in multimessenger astronomy. Information from a simultaneous detection of EM and GWs may be useful for studying fundamental aspects of gravitational physics. For example, in some modified gravity scenarios, the propagation velocity for gravitons may differ from that of photons [61, 62]. Additionally, the measurement of the luminosity distance from the GW signal at an accuracy of $1 - 10\%$, coupled with the redshift information from the EM detection, could serve as a cosmological "standard siren" of unprecedented accuracy (better than $\sim 1\%$) [63]. Such detections may also combine accurate measurements of MBH spins and masses obtained from GW signals with EM observations to probe MBH accretion physics in great detail [64]. Since most eLISA sources will be relatively low-mass systems at high redshift, where gas-rich environments are common, we focus here on the interaction between a MBH and a putative massive circumbinary disk.

The standard picture of circumbinary accretion disks can be described as follows. Tidal torques from the binary tend to drive gas outward, clearing an evacuated cavity in the innermost region of the disk. Meanwhile, viscous torques transport angular momentum outward in the disk, allowing gas to flow inward and refill this cavity. The balance of tidal and viscous torques determines the location of the inner edge of the circumbinary disk at $r \approx 2a$, where a is the binary separation. This balance can be maintained, provided the timescale $t_{\rm gw}$ for inspiral of the binary due to GW emission is much longer than the viscous timescale of the disk, $t_{\rm visc}$. This is known as the "pre-decoupling" epoch.

To date, a number of "dual" systems in which two MBHs occupy the same galaxy but are too widely separated to be gravitationally bound have been observed [65, 66, 67, 68], as well as several candidate binary systems (see e.g. [69, 70] and references therein). Proposed EM signatures of such binaries include spatially resolving two AGN-like point sources, identifying double-peaked broad emission lines, spatial structures in radio jets, characteristic time variability in quasar emission, and characteristic features in quasar spectra.

Theoretical aspects of this problem have been studied analytically [71, 72, 73, 74, 75, 76, 77, 78, 79], often using approximate angle-averaged tidal torque formulae. While these techniques have proven very useful in highlighting qualitative features of the accretion, they tend to overestimate the barrier to accretion imposed by binary torques by imposing symmetry in the accretion flow. As EM counterparts to MBH mergers depend sensitively on the amount of gas available for accretion, it is important to understand non-axisymmetric effects which may provide a mechanism for delivering more gas to the MBHs. Indeed, 2D and 3D simulations have demonstrated that gas streams may be stripped from the inner cavity, accreting directly onto the binary. Prior numerical simulations have been performed in 2D [80, 81] and in 3D [82, 83, 84, 85, 86, 87, 88]. While 3D codes have been useful in probing the gas dynamics during the final orbits prior to MBH merger, long viscous timescales render them prohibitively costly for simulating the quasi-steady-state flow during the "pre-decoupling" epoch. For this epoch, 2D simulations such as those performed using the *DISCO* code [89] have proven quite useful. These simulations include the inner cavity in the computational domain, and use shock-capturing Godunov-type methods to evolve thin ($h/r \sim 0.03$) disks over the viscous timescales necessary to accurately capture the steady-state accretion at high resolution.

The *DISCO* code is a moving-mesh code which allows one the freedom to specify the motion of the computational cells [90]. For binary accretion, the chosen rotation profile matches the nearly Keplerian fluid motion outside the cavity, while transitioning to uniform rotation at the binary orbital frequency inside the cavity. Because grid cells move azimuthally with the fluid, advection errors are minimized, allowing for the accurate capture of the dynamics of accretion streams which penetrate into the cavity.

The quasi-steady-state solutions which the simulations relax to after several viscous timescales can be interpreted as generalizations of the Shakura-Sunyaev disk solutions [91], with the central gravitating object replaced by a binary. The initial disk configurations consist of the "middle region" Shakura-Sunyaev solution for a steady-state, geometrically thin, optically thick accretion disk, assuming a gas-pressure dominated fluid with electron scattering as the dominant opacity.

The fluid evolves according the the 2D viscous Navier-Stokes equations, assuming an α-law viscosity prescription. A Γ-law equation of state of the form $P = (\Gamma - 1)\epsilon$ is chosen, where ϵ is the internal energy density, and the adiabatic index is set to $\Gamma = 5/3$. Appropriate radiative cooling and viscous heating terms are added to the energy equation. The cooling rate for an optically thick, geometrically thin disk is $q_{cool} = 4\sigma/3\tau T^4$, where T is the mid-plane temperature, and τ is the optical depth for electron scattering ($\tau = \Sigma\sigma_T/m_p$). It is assumed that the fluid is gas-pressure dominated everywhere, ignoring radiation pressure. In each simulation the binary is chosen to have zero eccentricity.

The important findings of these simulations are the following:

(i) As shown in Fig. 1, gas enters the circumbinary cavity along accretion streams, and the interaction of these streams with the cavity wall causes the cavity to become lopsided (in agreement with e.g. [80, 83, 88, 87, 86, 92]). See [92] and [87] for a description of the mechanism driving the growth of this lopsidedness.

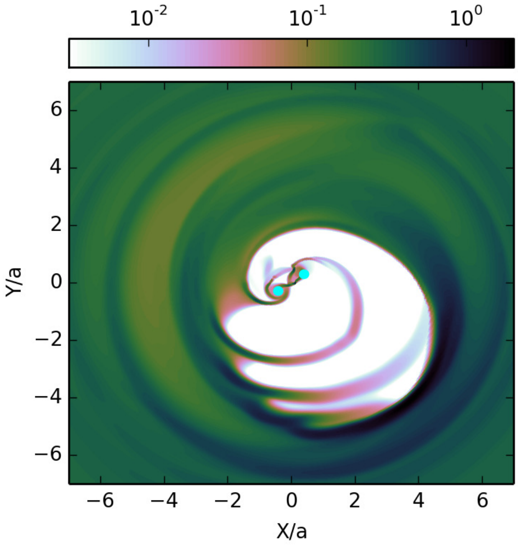

Figure 1. Snapshot of surface density Σ during quasi-steady state after $t \gtrsim t_{vis}$. Surface density is normalized by the maximum value at $t = 0$ and plotted on a logarithmic scale in the inner $\pm 6a$. Orbital motion is in the counter-clockwise direction.

(ii) These simulations support the growing consensus that the accretion rate onto the MBHs is not significantly reduced by the presence of a binary, when compared to the accretion rate onto a single MBH of the same mass [81, 87, 86, 88]. This is the case in spite of the fact that much of the inner cavity is cleared of gas by the action of the binary torques, and it is due to the effectiveness of the narrow accretion stream in delivering gas from the circumbinary disk inner edge to the individual black holes.

(iii) For each mass ratio considered, "mini-disks" surrounding each MBH are formed. In each case, the mini-disks are persistent, as their accretion timescale greatly exceeds the binary orbital timescale. For the binary mass ratios $q = 0.11$ and $q = 0.43$, the size of these mini-disks is in rough agreement with the semi-analytic predictions of [72].

(iv) Significant periodicity in the accretion rates emerges for $q \gtrsim 0.1$. At these mass ratios, the binary torques are strong enough to excite eccentricity in the inner cavity and create an overdense lump, whose interaction with the passing MBHs leads to periodicity in the accretion rate. The strongest peak in the periodograms for these cases corresponds to the orbital frequency of the lump, with many associated harmonics for the $q \gtrsim 0.43$ cases. This periodicity may constitute a unique observational signature of MBHBs.

(v) For each case considered, the accretion rate onto the secondary is sufficiently large relative to that of the primary, so that the mass ratio q is increasing. Similar results have been found previously in SPH calculations [82, 83, 93, 88]. As MBHs are expected to gain a significant fraction of their mass through gas accretion [94, 95, 96], this suggests a mechanism which may bias the distribution of binaries near merger toward higher mass ratios.

(vi) The emission associated with the shock heating in the accretion streams is sufficient to bring the emission from within the cavity above that of a disk around a single MBH. The peak in dL/dr which appears at $r/a \approx 5$ corresponds to the shock heating of gas in the stream which is not directly accreted, but rather impacts the cavity wall as seen along the lower-right edge of the cavity in Fig. 1, leading to the thin strip of bright emission. Scaled to a $10^8 M_\odot$ binary with a separation near decoupling at $a/M = 100$, this enhancement is significant in soft and hard X-rays (see Fig. 2). This X-ray enhancement has been predicted by [97], who estimated the characteristic frequency of mini-disk "hot spot" emission by estimating the amount of energy released due to shock heating when accretion streams impact the minidisks. Instruments sensitive to X-ray emission from AGN such as XMM-Newton[2], NuSTAR[3], the upcoming eROSITA[4] all-sky survey, and the proposed ATHENA[5] X-ray observatory may be sensitive to these signatures.

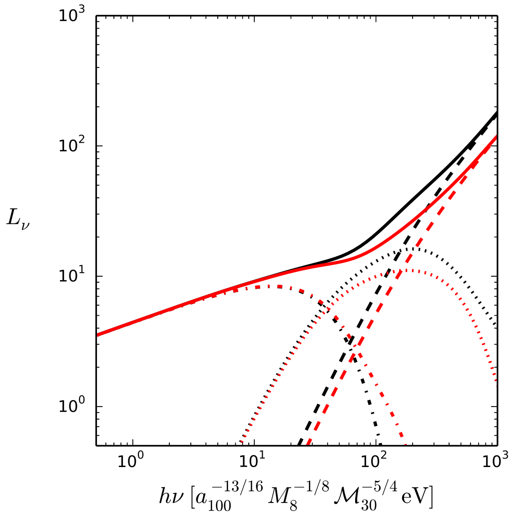

Figure 2. Thermal spectra computed from simulation snapshot at $t \gtrsim t_{vis}$. Red curves are calculated from disk data $\approx 3t_{bin}$ after that of black curves. The full spectrum is represented by solid lines, the component arising only from minidisk regions within distance $d < 0.5a$ of either MBH is represented by dashed lines, the "cavity" emission is represented by dotted lines, and the emission from the "outer region" is represented by dashed-dotted lines. We have introduced the scaling parameters $a_{100} \equiv a/100M$, $M_8 \equiv M/10^8 M_\odot$, and $\mathcal{M}_{30} \equiv \mathcal{M}_a/30$.

While there remain no confirmed observations of EM counterparts for MBHBs, simulations have already provided tantalizing evidence that significant gas can accrete onto such a binary, even at separations in the eLISA band. A number of distinguishing observational signatures of such systems have been proposed, including directly resolving two AGN-like point sources, searching for offset emission lines, spatial structures in radio jets, characteristic time variability in quasar emission, and characteristic features in quasar spectra. As we enter the age of GW astronomy the possibility of a simultaneous measurement of both gravitational and EM radiation from a merging MBHB appears increasingly likely.

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[2] http://sci.esa.int/xmm-newton/
[3] http://www.nustar.caltech.edu/
[4] http://www.mpe.mpg.de/erosita/
[5] http://www.the-athena-x-ray-observatory.eu/

4. Massive black hole spins as gravitational and cosmological probes

In GR, MBHs are described by the Kerr metric and are completely characterized by three "charges" (or "hairs"), the mass M, the spin S and an electric charge Q, with the spin satisfying the Kerr bound $a = cS/(GM^2) \leq 1$ (if the spin is larger than this bound, the Kerr metric does not describe a BH but a naked singularity). Also, the charge Q is usually expected to be negligible for astrophysical BHs, due to the presence of plasma that quickly neutralizes it, and also because of quantum effects such as Schwinger pair production or vacuum breakdown mechanisms producing cascades of electron-positron pairs near the BH horizon. BHs in gravity theories different from GR are still characterized by the mass and spin, but may also present additional charges that might provide a way to test gravity in strong-field regimes (cf. Section 6 for a related discussion).

The effect of the mass in GR is qualitatively similar to Newtonian gravity, i.e. it sources an attractive force decaying as GM/r^2 for $r \gg GM/c^2$ (r being the distance from the BH). Because of the long-range character of the force it sources, the mass can be estimated rather easily with EM observations, e.g. (in the case of MBHs) by observations of the nuclear dynamics of stars and gas, fitting of the spectral energy distribution of galaxies, spectroscopic single epoch measurements and reverberation mapping. EM measurements of MBH masses are typically accurate only within $20 - 50\%$ (e.g. even for our own SgrA*, the mass can only be estimated to within $\sim 10\%$), but still allowed discovering correlations with galactic properties, hinting at a symbiotic co-evolution of MBHs with their galactic hosts [98, 99, 100, 101, 102].

Measuring BH spins is more complicated, because they do not enter the dynamics at Newtonian order, but only at 1.5 post-Newtonian order, i.e. they change the Newtonian equations of motion only by corrections $\sim \mathcal{O}(v/c)^3$, where v is the characteristic velocity of the gas and/or stars surrounding the MBH. This results in significant effects only very close to the event horizon. The most promising EM technique to estimate the spins of MBHs is through the spectra of relativistically broadened Kα iron lines (cf. [103, 104] for recent reviews on this topic). Indeed, X-ray observatories such as XMM-Newton and Suzaku have by now measured significant samples of spins, and more constraints are becoming available from NuSTAR's hard X-ray data [105, 106, 107]. The masses and spins of MBHs in binary systems will be measured very accurately by eLISA, with errors $\delta M \lesssim 0.1\%$ and $\delta a \lesssim 0.01$ [108, 2]. These GW measurements are very clean compared to EM ones, because they are not affected by the systematics usually present in EM data (which are due to poor understanding of the dissipative gas physics). This is because GWs are emitted in the latest stages of a binary system's evolution, when the effect of the gas on the orbital dynamics is typically negligible [109, 110].

More specifically, eLISA will detect GWs from MBHBs both during the long post-Newtonian inspiral, which radiates in the eLISA band for separations less than about 200 gravitational radii for binaries with total mass $\sim 10^5 - 10^6 M_\odot$ (i.e. those to which eLISA is most sensitive), and during the final plunge, merger and ringdown of the system. This will allow testing a plethora of strong-field general-relativistic effects, namely:

(i) The spin-orbit coupling (also known as "frame dragging") [111, 112], which causes the BH spins to precess during the inspiral, thus producing amplitude modulations in the emitted gravitational waveforms [112, 113, 114, 115, 116], which increase the signal-to-noise ratio and improve the estimation of the source parameters (and in particular the sky localization). The precessional dynamics can be very rich: for example, to a first approximation (and on short timescales) the spins precess around the binary's total angular momentum, but in certain configurations they can undergo a more complicated "transitional precession" [112] (whereby spins almost anti-aligned with the orbital angular momentum change their orientation on a very short timescale at some point during the inspiral), or get locked in secularly stable resonant configurations that tend to align or anti-align the spins [117, 118, 119, 120];

(ii) The spin-spin coupling [121, 122], which appears at higher post-Newtonian order than the spin-orbit coupling and thus modifies the spin precession in the later inspiral, causing additional modulations in the gravitational waveforms and improving parameter estimation;

(iii) The non-linear relations between the final mass, spin and recoil velocity of the BH remnant forming from the merger, and the masses and spin vectors of the binary's components at large separations. These relations have been studied in detail with fully general-relativistic numerical simulations (see e.g. [123, 124, 125] for reviews on this topic), whose results can be extrapolated to generic binary configurations by exploiting knowledge of the test-particle, self-force and post-Newtonian dynamics [126, 127, 128, 129, 130, 131, 132, 133];

(iv) The quasi-normal mode ringing of the BH remnant (see [134] for a review), whose frequencies and decay times are functions of the remnant's mass and spin alone if GR is correct. Measuring these frequencies therefore constitutes a genuine strong-field test of the gravity theory (cf. Section 6).

Besides testing the strong-field general-relativistic dynamics through these effects, eLISA measurements of the spins will also provide useful information on the cosmological evolution of MBHs. In fact, although the impact of gas on the dynamics of MBHBs is negligible once they enter the eLISA band [109, 110], radiatively efficient accretion of gas is known to be the main driver of the evolution of the masses and spins on cosmological timescales [94, 135, 136, 137]. Also, because eLISA will detect GWs from MBHBs at redshift $z \gtrsim 10$, mass and spin measurements will give precious information about the high-redshift formation of the first generation of BH seeds, from which present-day MBHs are believed to descend.

Early studies [41] in this direction highlighted for instance that eLISA mass measurements will be able to discriminate with high confidence between a scenario in which MBHs evolve from "heavy" ($\sim 10^4 - 10^5 M_\odot$) seeds forming at $z = 10 - 15$ from the collapse of protogalactic disks, and a "light-seed" scenario in which the seeds form at $z \sim 20$ from the collapse of Population III stars into BHs with masses $\sim 50 - 300 M_\odot$, shedding light on the nature of the first seed BHs forming in the young universe (cf. Section 2). As for the spin evolution, [138] (see also [139, 140, 141, 142]) showed that eLISA should be able to tell a coherent accretion scenario (where gas accretion always happens on prograde orbits on the equatorial plane) from a chaotic scenario (where the MBH captures clouds that are isotropically distributed around it with randomly oriented angular momenta). Clearly, the first scenario predicts almost extremal spins (because accretion is always prograde), while the second predicts small spins $a \lesssim 0.3$, because the angular momentum transferred by the clouds tends to cancel out on long timescales.

Two ingredients were missing from these early attempts, namely *(a)* the strong-field spin-orbit coupling reviewed above, and *(b)* a more realistic connection between the properties of the accretion flow and those of the host galaxy. Regarding *(a)*, [143, 144, 145, 146] showed that the interaction between the spin-orbit coupling and the viscous stresses active inside an off-equatorial, geometrically thin accretion disk results in a quick alignment between the MBH spin $\boldsymbol{S}$ and the disk's orbital angular momentum $\boldsymbol{L}$ (a phenomenon known as "Bardeen-Petterson effect"), provided that $|\boldsymbol{L}| > 2|\boldsymbol{S}|$. Because this alignment takes place on a timescale much shorter than the accretion timescale, accretion will be essentially coherent if a MBH is hosted in a gas-rich galactic nucleus (where it is more likely that the condition $|\boldsymbol{L}| > 2|\boldsymbol{S}|$ will be satisfied), while in gas-poor nuclei accretion will be more likely to resemble the chaotic accretion scenario outlined above. The Bardeen-Petterson effect is also expected to be important for the orientation of the spins of the merging binaries detectable with eLISA. Indeed, binaries forming in gas-rich nuclei will likely have almost aligned spins, and thus produce lower recoil velocities for the merger remnant, which is therefore unlikely to escape from the host galaxy [147].

An investigation of the impact of effects *(a)* on the cosmological evolution of the MBH spins was first performed by [148], which also accounted for *(b)* by simulating the co-evolution between the MBHs and their host galaxies with a semi-analytical galaxy formation model, including both the evolution of dark-matter (via merger trees) and the evolution of baryonic structures (intergalactic and interstellar media, galactic disks, star-forming spheroids, as well as MBHs with their accretion disks). Ref. [148] also obtained predictions (in principle testable with eLISA) for the fraction of MBH mergers in gas-rich environments (and thus with aligned spins) as opposed to ones in gas-poor environments (and thus with misaligned spins). However, in spite of its sophistication, the model of [148] still assumed that the accretion flow onto the MBH had vanishing average angular momentum in gas-poor regimes (i.e. whenever the condition $|\boldsymbol{L}| > 2|\boldsymbol{S}|$ is not satisfied). This assumption is an idealization, as galaxies do have a non-zero angular momentum, and one would therefore expect the average angular momentum $\bar{L}$ of the clouds accreting onto the MBH to be non-zero. Allowing for $\bar{L} \neq 0$ can indeed have important consequences for the spin evolution of MBHs, as shown in [149].

Ref. [149] left $\bar{L}$ as a free parameter, because a first-principle calculation would be extremely challenging: galaxy formation simulations are not yet capable of resolving the MBH's sphere of influence and the accretion disk, and in any case it is far from clear whether such simulations have full control of the subgrid/dissipative physics at small scales (see however [150] for a recent attempt to extract the angular momentum of the clouds from a hydrodynamical simulation, with resolution up to 10 pc.) Alternatively, a bold attempt can be made at trying to connect $\bar{L}$ to measurements of the velocity dispersion v/σ of the gas and stars in galaxies (e.g. if v/σ were zero, it is clear that accretion would be perfectly isotropic, i.e. $\bar{L} = 0$). Ref. [151] adopted this approach and used the semi-analytical galaxy formation model of [148] with the spin-evolution model of [149], connecting $\bar{L}$ to measurements of v/σ of the gas and stellar components in various galaxy morphologies. While still debatable because these measurements are currently only available at distances of $\gtrsim 100$ pc from the MBH (and thus far from the accretion disk and the MBH's sphere of influence), this approach allowed the authors of [151] to produce testable predictions for the spin distribution in competing models for the "isotropy" $\bar{L}$ of the accretion flow. In particular, three models were considered: (A) one connecting $\bar{L}$ to the velocity dispersion of the gaseous component of galaxies; (B) one connecting $\bar{L}$ to the velocity dispersion of the stellar component; and (C) a hybrid model. A comparison with existing iron-Kα measurements of MBH spins shows that model (A) is ruled out quite convincingly, while both models (B) and (C) are in agreement with observations (with marginally significant evidence in favor of the hybrid model (C)). Ref. [151] also showed that the idealized accretion prescriptions discussed above (namely purely coherent and purely chaotic accretion, as well as the original model of [148]) are disfavored by existing iron-Kα measurements. Clearly, eLISA's measurements of the spins of merging binaries will allow discriminating between these competing models with much higher significance, thus providing a way to test the properties of the accretion flow onto MBHs with unprecedented accuracy.

5. Cosmography with space-based detectors

The geometry, large-scale structure and dynamics of the Universe can be inferred with precision if we can accurately measure the distance and redshift to sources distributed throughout the Universe. This is because the luminosity distance $D_{\rm L}$ to a source as a function of its redshift z depends on the geometry of the Universe and a number of cosmological parameters, such as the Hubble parameter, relative fractions of density in dark energy, dark matter, baryonic matter, etcetera [152]. Redshift is very well measured by spectroscopic methods, except for sources at low redshifts ($z \ll 1$), where peculiar velocities due to the gradient of the local gravitational potential could be comparable to cosmological expansion. The real challenge, however, is to accurately measure distances to cosmological sources.

Precision cosmography is enabled by sources whose intrinsic luminosity L can be deduced by some observed property of a source, e.g. its time variability, so that one can infer the luminosity distance D_L from its apparent luminosity F, namely $D_L = \sqrt{L/4\pi F}$. In 1986 Schutz pointed out how GW observations of inspiralling compact binaries could provide an astronomer's ideal standard candle [153]. Since then there has been quite a lot of work in trying to understand how observations of mergers involving MBHs by LISA (and eLISA) could be used to measure cosmological parameters [154, 155, 156, 157]. Here we will briefly review the basic idea, challenges posed by observations and some solutions. The current perspective on the problem is that eLISA's application for cosmography will be limited by weak gravitational lensing, and precision measurements of cosmological parameters are only possible if eLISA observes several tens of sources during its lifetime. In this regard, ground-based detectors are likely to be more useful for cosmology as they are expected to observe a very large number of sources, which helps to mitigate problems posed by weak lensing [158].

5.1. Self-calibrating standard sirens

Inspiralling compact binaries are often referred to as self-calibrating standard sirens [63]. The word "siren" is used to indicate that GWs are more akin to sound waves than EM waves, and eLISA is a detector that "listens" to its sources. They are "self-calibrating" as no other calibration process is required to measure the distance to an inspiralling binary other than the description of its dynamics by GR [153]. This favorable situation should be contrasted with the astrophysical modelling of sources and construction of a cosmic distance ladder, that are required to calibrate the distance to cosmological sources [152].

The response of eLISA to GWs from the inspiral phase of a coalescing binary depends on its distance D_L from the Earth, sky position (θ, φ), component masses (m_1, m_2) and their spins, orbital eccentricity and orbital angular momentum $(\vec{S}_1, \vec{S}_2, e, \vec{L})$ (all at some fiducial time) [63]. In order to measure the luminosity distance to the source it is necessary to disentangle all the other parameters. The shape of a signal that spends a sufficiently long time (say several months) in the eLISA band depends quite sensitively on its intrinsic parameters (masses, spins and eccentricity), and the motion of eLISA with respect to the source induces amplitude and phase modulations in the signal that depend on the position of the source and the orientation of its orbital angular momentum. By fitting the data with precomputed templates that depend on the different parameters of the signal one can resolve all the source parameters. Significant correlations between the luminosity distance and other parameters (most notably the orbital inclination) corrupt the accuracy with which we can estimate the distance. Even so, due to the large signal-to-noise ratios expected from supermassive BH binaries, eLISA should be able to measure distances at $z \sim 1$ to better than 0.1%-1% accuracy [63, 156, 108].

Schutz also pointed out that although GW observations can measure distances, they are not able to measure the source redshift. Recent work, however, has shown that at least in the case of binary neutron stars, and quite possibly also for neutron star-BH binaries, it should be possible to also measure the source's redshift from the effect of tides on the waveform phasing [159, 160]. The intrinsic mass $M_{\rm int}$ of the neutron star will be imprinted in the tidal effects, and once the intrinsic mass is known the redshift can be inferred from the fact that the observed mass $M_{\rm obs}$ is given by $M_{\rm obs} = (1+z)M_{\rm int}$.

For MBHBs, however, there is no way to infer the source's redshift from GW observations alone. Since eLISA will not be sensitive to merging neutron star binaries (at least not at cosmological distances), we have to rely on EM follow-up of GW events to infer the source's redshift. Thus, it was argued that there is great synergy in multi-messenger observations of MBH binaries (cf. Section 3). GW observations would provide distance measurements, while EM observations would provide redshift, and the two observations together would be a new tool for cosmology.

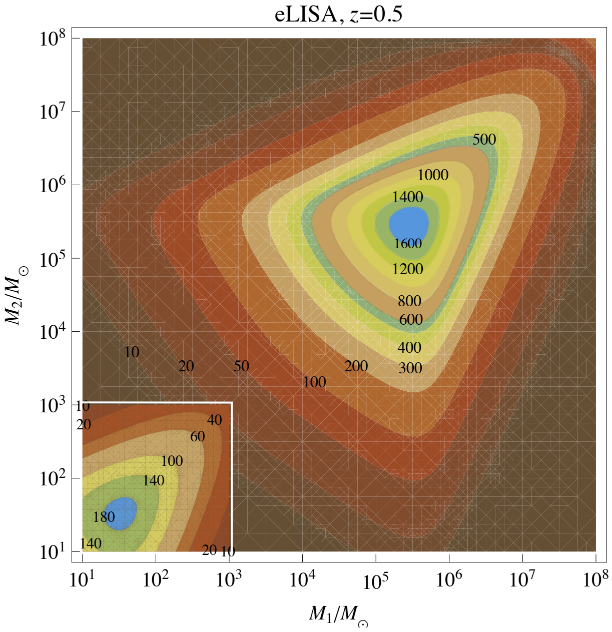

Figure 3. Signal-to-noise ratio (SNR) of the inspiral phase of coalescing BH binaries as a function of the component masses of binary sources at $z = 0.5$ Signals are assumed to last for a year before they merge in the eLISA band. The large SNR helps in measuring the distance with great accuracy in spite of correlations between other signal parameters. The inset shows the SNR for Einstein Telescope (ET)—a third generation underground GW detector. Although eLISA and ET are sensitive to different range of masses, there are some sources that could be observed by both eLISA and ET.

5.2. Challenges in using eLISA for cosmography

At first it was thought that eLISA will be a powerful new tool for cosmography. However, it soon became clear that significant challenges are posed by different aspects of the measurement, making the problem quite hard [63, 161]. In the following we will discuss the most important challenges and what solutions, if any, have been found in confronting them.

Localizing sources. GW interferometers like eLISA are quadrupole antennas with very good sky coverage [162]. They are able to observe better than 2π steradians of the sky at any one time, but they are generally not very good at resolving the sky position of the source. The angular resolution of eLISA for sources that use only the dominant quadrupole harmonic of the signal at twice the orbital frequency is $\sim$ 10's of square degrees [63]. At a redshift of $z \sim 0.5$ (the relevant redshift where one can expect to detect one or two merging MBHBs each year) a sky patch of that size contains thousands of galaxies, and therefore there seems to be no hope of identifying the host galaxy to measure the source's redshift. Moore and Hellings [163, 164] first realized that if one includes in the search templates higher-order signal harmonics, in addition to the dominant harmonic at twice the orbital frequency, the angular resolution can get sensibly smaller. More complete analyses including higher harmonics [156] and/or spin-induced precessional effects [114] showed that sources are resolved to $\sim$ 1 square degree. This angular resolution is much smaller than before – not small enough to contain just the host galaxy, but perhaps small enough to contain the (not yet fully virialized) galaxy cluster to which the host belongs. Moreover, higher harmonics also help break the degeneracy between the inclination angle and the distance, reducing the fractional error on distance measurement by a factor of 2 to 5 [156].

The Task Force that was appointed to systematically explore how well LISA might determine source parameters and constrain dark energy concluded that each year LISA could observe a few events for which distance can be measured to within 1% *and* the source can be localized to within one square degree – accuracies that are sufficient to pin down the dark energy equation-of-state parameter w at the level of one percent [108]. Although the improvement in sky resolution brought through higher-order harmonics and spin effects is sufficient to measure the host redshift, it would be far better if sources could be identified by the EM counterpart that the merger event might produce, which we will discuss next.

Electromagnetic counterparts. Quite a lot of work has gone on in understanding the EM counterparts produced by a merger event. A MBHB merger by itself does not produce any EM radiation. The environment in which a MBHB merges consists of hot diffuse gas, circumbinary accretion disks and stars. As highlighted in Section 3, cold gas forming a putative circumbinary disk can stream toward the two MBHs and form a dense, small disk surviving at the moment of the MBHB coalescence. Numerical simulations of MBHBs seem to suggest that the merger event could shock and heat the surrounding gas particles to high temperatures, leading to bright EM counterparts in coincidence to the coalescence. If the immediate vicinity of the binary is devoided of gas at coalescence, this sort of radiation can be expected within months to years of merger, hence monitoring the sky position of merger could indeed be interesting from an astrophysical point of view as well as for cosmography (see [165] for a comprehensive review of EM counterparts to coalescing MBHBs).

Weak gravitational lensing. The space between a GW source and eLISA is not empty. Small-scale inhomogeneities that are ever present in the path of a signal from a source to eLISA render the signal brighter or dimmer, depending on the integrated effect of the inhomogeneities along a given path. This is called weak gravitational lensing, and all cosmological sources are subject to this phenomenon. Weak lensing cannot be detected accurately, and there is no way to fully correct for its effect on particular sources. Although weak lensing can significantly affect individual detections, the effect averages out when considering a large population of sources. At redshift $z \sim 1$, systematic errors in the measurement of distance due to weak lensing are typically $\sim$ 2-5%, an order of magnitude larger than statistical errors for an average signal at this distance [161].

It is possible to correct for the systematics to some extent, but the best possible technique (in fact a combination of different observations) can decrease the error by a factor of 2 [166, 167]. Hence weak lensing will severely limit how well distances can be inferred, and it is not possible to make a meaningful measurement of dark energy with a single source. If eLISA manages to accumulate $\gtrsim 30$ events at $z < 2$ during its lifetime (which implies MBHB merger rates at the upper end of what is currently estimated and/or a mission lifetime $\gtrsim 5$ years) then it should be possible to measure w to an accuracy of $\sim 4\%$ [157].

5.3. Outlook

The foregoing summary is largely based on studies that were conducted in the context of LISA. Future studies should repeat these investigations in the context of eLISA to have a concrete evaluation of its science potential. It would be useful to study what optimizations can be made within the existing mission framework to maximize the science potential. Estimates of event rates in the context of eLISA, which has significantly better sensitivity than LISA at higher frequencies (at the expense of poorer sensitivity at lower frequencies) would be useful, as higher rates are obviously the cleanest way to defeat the effect of weak lensing. Studies on how weak lensing biases can be further reduced would be useful, as also more reliable estimates of the EM radiation that would be produced after merger. In particular, it is important to study the extent to which gas and disks could affect the dynamics of the binary, and whether this could bias the estimation of signal parameters.

Two concepts beyond eLISA have been studied: Big Bang Observer [168] in the US and Deci-Hertz Gravitational Wave Observatory (DECIGO) [169] in Japan. These ambitious projects with more than an order of magnitude improvement in strain sensitivity will obviously not have the limitations of LISA or eLISA for cosmography (see e.g. [170, 171]). The problem here might be one of data analysis. The large number of sources they might detect could pose problems with unambiguous estimation of signal and source parameters. It is important to study whether our knowledge of the waveforms predicted by GR is good enough to cleanly disentangle the signals.

6. Black hole mergers as probes of strong-field gravity

Einstein's GR is certainly one of the most elegant and successful physical theories, and so far it has passed all experimental tests with flying colors [172]. However there are theoretical and experimental reasons suggesting that the theory must be modified at some level. From a theoretical point of view, GR is a purely classical theory, and power counting arguments indicate that it is not renormalizable in the standard field-theory sense; however, one can build a renormalizable theory by adding quadratic curvature terms – i.e., high-energy/high-curvature corrections – to the Einstein-Hilbert action [173]. Furthermore, high-energy (ultraviolet) corrections seem necessary to avoid singularities that are inevitable in classical GR, as shown by the Hawking-Penrose singularity theorems [174]. From an observational point of view, cosmological measurements are usually interpreted as providing evidence for dark matter and a nonzero cosmological constant ("dark energy"). This interpretation poses serious conceptual issues, including the cosmological constant problem ("why is the observed value of the cosmological constant so small in Planck units?") and the coincidence problem ("why is the energy density of the cosmological constant so close to the present matter density?"). No dynamical solution of the cosmological constant problem is possible within GR [175]. It seems reasonable that ultraviolet corrections to GR would inevitably "leak" down to cosmological scales, showing up as low-energy (infrared) corrections.

The arguments summarized above suggest that Einstein's theory of gravity should be modified at both low and high energies, but it is not easy to introduce modifications to GR that respect these requirements without facing additional problems. Einstein's theory is the unique interacting theory of a Lorentz-invariant massless helicity-2 particle [176], and therefore new physics in the gravitational sector must introduce additional degrees of freedom. Any additional degrees of freedom must modify the theory at both low and high energies *while being consistent with GR in the intermediate-energy regime*, i.e. at length scales between $\sim 1\,\mu$m and about one Astronomical Unit, where the theory is extremely well tested. Intermediate-energy constraints include laboratory experiments, Solar System experiments (that verify the Einstein Equivalence Principle to remarkable accuracy, and force parametrized post-Newtonian parameters such as β and γ to be extremely close to their GR values) and binary pulsar experiments (that place stringent bounds on popular extensions of GR such as scalar-tensor theories, Lorentz-violating theories and TeVeS): see [172, 177] for reviews.

eLISA is arguably the best strong-gravity laboratory one could hope for, as it will probe the strong-field dynamics of GR out to cosmological distances to levels unachievable by binary pulsars[6], Earth-based interferometers or Pulsar Timing Arrays (see [179, 180] for reviews). As we will argue below, space-based GW interferometers could provide crucial hints about the nature of the compact objects at galactic centers and about the nature of gravity itself.

It is useful to classify "tests of strong field gravity" as belonging to two – qualitatively very different – categories:
(1) External tests: can laboratory experiments, astrophysical observations or future GW measurements determine whether GR is the correct theory of gravity? To frame this question in terms of hypothesis testing, one would like to have a valid opponent to GR. What constitutes a "valid opponent" is a matter of taste. For our purpose (i.e., tests of strong-field gravity with BH mergers) it should be a cosmologically viable fundamental theory with a well-posed initial value formulation, and field equations that follow from an action principle. Furthermore, the theory should be simple enough to allow calculations of (say) BH solutions and GW emission. There are countless attempts to modify GR [181], but all modifications must introduce some sort of screening mechanism in order to be viable at intermediate energies. Screening mechanisms

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[6] One of the most extraordinary laboratories for strong-gravity tests is PSR J0348+0432 [178]. Even this binary system, which is highly relativistic for binary-pulsar standards, has an orbital velocity $v \simeq 2\times10^{-3}c$, much smaller than the orbital velocities $v \approx c/3$ of an astrophysical BH binary near merger.

include chameleons, symmetrons, dilatons, MOND-like dynamics, the Vainshtein mechanism, etcetera [182]. Since we don't have a full theory of quantum gravity, an effective field-theory approach is often invoked when constructing phenomenological alternatives to GR [183, 184]. For example, one can start with the most generic four-dimensional theories of gravity including quadratic curvature invariants generically coupled to a single scalar field ϕ:

$$S = \int \frac{d^4x\,\sqrt{-g}}{16\pi}\Big[R - 2\nabla_a\phi\nabla^a\phi - V(\phi) + f_1(\phi)R^2 + f_2(\phi)R_{ab}R^{ab} + f_3(\phi)R_{abcd}R^{abcd} + f_4(\phi){}^*RR\Big] + S_{\rm mat}\left[\gamma(\phi)g_{\mu\nu},\,\Psi_{\rm mat}\right]\,, \qquad (1)$$

where $\Psi_{\rm mat}$ collectively denotes matter fields, $V(\phi)$ is the scalar self-potential, $f_i(\phi)$ are generic coupling functions, the Chern-Simons term ${}^*RR \equiv \frac{1}{2}R_{abcd}\epsilon^{baef}R^{cd}{}_{ef}$, ϵ^{abcd} is the Levi-Civita tensor, and in the matter action $S_{\rm mat}$ we allow for a nonminimal coupling that violates the (weak) equivalence principle. Not all of these theories are acceptable: for example, to avoid higher-order derivatives in the equations of motion one must generally assume the couplings to be small and treat the theory as an effective field theory (the equations are second-order in the strong-coupling limit only if the quadratic invariants enter in the special "Gauss-Bonnet" combination). The action above may seem complicated, but it actually represents a very restricted class of theories, and calculations of gravitational radiation have been performed only in very specific subcases, such as scalar-tensor theories with specific choices of the potential and couplings [185, 186, 187, 188] and some forms of quadratic gravity [189, 190]. The bottom line is that there are very few "serious" alternatives to GR (in the sense that they are well posed, follow from a Lagrangian, make sensible predictions...) and even fewer for which GW calculations have been carried out. For these theories, GW observations usually yield constraints that are comparable to, and often better than, binary pulsar and Solar System bounds [179, 180]. *(2) Internal tests: is GR "internally" consistent with astrophysical observations?* One of the most striking predictions of GR is the existence of BHs. Astronomers commonly believe that the compact objects that harbor galactic centers are the BHs of GR, but this "BH paradigm" rests on somewhat shaky foundations. Evidence that these objects possess event horizons (or more correctly, apparent horizons) rather than solid surfaces usually rests on plausibility arguments based on accretion physics [191, 192], that leave room for some skepticism[7] [193]. It is also important to stress that, strictly speaking, any tests that probe the Kerr *metric* alone (such as tests based on matter accretion or ray-tracing of photon trajectories) are of little value as internal tests of GR. The reason is that most alternative theories (including generic scalar-tensor theories [194] and a large class of higher-curvature theories [195]) admit the Kerr metric as a solution, and the theories that don't (e.g. Einstein-dilaton Gauss-Bonnet [196], Dynamical Chern-Simons [197], and Lorentz-violating gravity [198, 199, 200, 201]) predict BH solutions that differ from GR by amounts that should be astrophysically unmeasurable (see, however, Refs. [202, 203] for a family of BH solutions whose deviations from the Kerr metric may be important). Many "quasi-Kerr metrics" that have been proposed in this context should be viewed as unnatural strawmen: they often have serious pathologies [204], and they are therefore unacceptable even for the limited scope of parametrizing deviations from the Kerr metric [205].

These considerations imply that the only way to unambiguously verify that the compact objects in galactic centers are actually Kerr BHs is **via their GW dynamics**, especially in the strong-field merger/ringdown phase [206, 207, 208]. This is why space-based GW

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[7] From a theorist's point of view, one of the most convincing arguments in favor of the BH paradigm is that the alternatives are either unstable (as in the case of dense star clusters, fermion stars or naked singularities), unnatural (e.g. "exotic" matter violating some of the energy conditions), contrived (such as gravastars), implausible as the end-point of collapse in astrophysical settings (boson stars) or nearly indistinguishable from Kerr (this is the case for BH solutions in alternative theories with coupling parameters that are reasonable from a fundamental physics point of view).

observations hold great promise to constrain strong-field gravity [179]. Their qualitative advantage over Earth-based detectors is simple to understand. The fundamental oscillation mode of a nonrotating BH has frequency $f \simeq 1.2 \times 10^{-2}(10^6 M_\odot/M)$ Hz. This frequency lies exactly in the "bucket" of eLISA's noise power spectral density for "light" BHs of mass $M \sim 10^6 M_\odot$, that were presumably the building blocks of the large BHs we see at galactic centers today. Advanced LIGO, by contrast, has maximum sensitivity at $f \sim 10^2$ Hz, i.e. for intermediate-mass BHs of mass $M \sim 10^2 M_\odot$, whose very existence is still highly uncertain [209]. These oscillation modes are called "quasinormal" modes, because they are damped by GW emission. The no-hair theorem stated in Section 4 implies that, in GR, the frequencies and damping times of all QNMs depend only on the BH mass M and spin a. A measurement of the dominant mode's frequency and damping time yields both M and a; the measurement of *any* other frequency and/or damping time can then be used to verify that the BH formed as a result of the merger is indeed a Kerr BH, as predicted by GR. The feasibility of this (internal) test of GR depends on the measurability of QNM frequencies/damping times and on our ability to resolve modes. Both measurability and resolvability scale like $1/\rho$, where $\rho \sim h/S_n$ is the signal-to-noise ratio of the merger event [206, 207]. The maximum sensitivity S_n of Earth-based and space-based detectors is comparable in order of magnitude, but (as we saw above) space-based detectors target sources that are $\sim 10^4$ times more massive. The GW amplitude $h \sim \sqrt{\epsilon_{\rm rd} M}$, where $\epsilon_{\rm rd} \sim 10^{-2}(4\eta)^2$ [210] is a "ringdown efficiency" and $\eta \equiv m_1 m_2/(m_1+m_2)^2 \in [0,\, 0.25]$ is the so-called symmetric mass ratio. The punchline of this argument is that $\rho \sim \sqrt{M}$, so *no-hair theorem tests with space-based observations of BH binary mergers are typically* $\sim 10^2$ *stronger than Earth-based GW tests.*

In summary, BH mergers are extraordinary (local) probes of the no-hair theorem that are potentially detectable by eLISA throughout the entire Universe. Each merger event allows us to do much more than this: it can give us tests of consistency of the post-merger Kerr remnant with the pre-merger binary dynamics [211, 212, 213], allow us to test the area theorem [214], and perhaps even allow us to peer into the nonlinear dynamics responsible for the coupling of different quasinormal modes [215]. Even more interestingly, each BH binary merger can be thought of as a local probe of whether *strong-field* GR is valid *at the redshift z at which the merger occurred.* This is an incredible opportunity for tests of GR, because it would verify that Einstein's gravity accurately describes BHs at least out to redshift z. This would place even more stringent constraints on the viable modifications of GR. This idea has one drawback: it requires (ideally) the determination of the merging binary's redshift z, or at the very least the determination of a *lower bound* on the source redshift. As described in Section 5, source localization and distance determination are intimately related in a LISA-like mission, and they get significantly better for a three-arm mission [108]. In conclusion, a three-arm mission would make a big difference to test the no-hair theorem at cosmological redshift - and thus to place tight constraints on the strong-field behavior of the theory at distances where cosmological observations are relevant.

Another aspect worth emphasizing is that, unlike Solar System tests (that only probe the "static", quasi-Newtonian behavior of gravitational fields) and binary pulsars (that essentially measure the energy flux predicted by a given theory of gravity, and not much else in terms of the dynamics of the gravitational field) the *direct* observation of GWs can probe the number of polarization states as well as the propagation properties of GWs, as encoded in their dispersion relation. Tests of the dispersion relation place constraints on a putative nonzero graviton mass, and they get better, as expected, for sources at cosmological distance. In fact it has been shown that eLISA can constrain the mass of the graviton to a level that is ~ 4 orders of magnitude better than current Solar System constraints [216, 217]. Quite naturally, constraints on any given alternative theory of gravity get better with multiple observations because of the improved statistics. Calculations in the case of a hypothetical massive graviton show that the improvement is better than the naive Poisson-statistic expectation of $\sqrt{N}$, where N is the number of events, because the louder events “carry more weight” in determining the combined bound [218].

Last but not least, the discovery space of eLISA is potentially enormous. The potential of eLISA for cosmology was discussed in Section 5; here we will focus on the discovery space related to MBH observations. Binary pulsars are already constraining phenomena like “spontaneous scalarization” to levels that are comparable to what eLISA could do by observing neutron stars spiraling into MBHs [217]. However, one can imagine scenarios where modified gravity would produce smoking-gun signatures of deviations from GR that would be observable by eLISA, and completely invisible to binary-pulsar tests. The simplest case study are extreme mass-ratio inspirals in scalar-tensor theory. If the scalar has a mass, Kerr BHs become vulnerable to the so-called “black-hole bomb” instability: superradiance can amplify incident waves, the mass of the scalar acts like a mirror that reflects amplified waves back onto the BH, and therefore the superresonant amplification of incident waves can grow without bound [219]. This instability could have striking effects, such as the existence of “floating orbits” [220] at which the inspiralling body could stall, emitting essentially monochromatic radiation – a “GW laser”! Similar effects could occur if the energy conditions are violated [221, 222], so eLISA could hint at exotic physics responsible for possible violations of the energy conditions. Finally, if the objects at galactic centers were not BHs but (say) gravastars or boson stars, the oscillation modes of these exotic objects would inevitably be excited by orbiting bodies, leaving characteristic signatures in the energy flux that are potentially detectable by eLISA [223, 224].

Opening new observational windows on the Universe inevitably reveals more than we anticipated. We should be ready for surprises.

7. Conclusions

We have reviewed our current understanding of MBH formation and evolution. The mysteries that surround the birth and growth of these cosmic monsters are enormous, and they are intimately related to the growth of structure in our Universe. Observations of gravitational radiation from MBH mergers – especially in combination with EM counterparts – have the potential to measure spins to levels unachievable by other means, clarify the role of accretion in MBH growth, and constrain fundamental physics in unprecedented ways. An eLISA-like mission will be a spectacular, unrivalled laboratory for fundamental physics and astrophysics.

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