Mass of asymptotically anti-de Sitter hairy spacetimes

[1]Andrés Anabalón, [2,3]Dumitru Astefanesei and [4,5]Cristián Martínez

[1]*Departamento de Ciencias, Facultad de Artes Liberales and*
Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez,
Av. Padre Hurtado 750, Viña del Mar, Chile
[2]*Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile*
[3]*Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, 14476 Golm, Germany*
[4]*Centro de Estudios Científicos (CECs), Casilla 1469, Valdivia, Chile*
[5]*Universidad Andrés Bello, Av. República 440, Santiago, Chile*

`andres.anabalon@uai.cl, dumitru.astefanesei@ucv.cl, martinez@cecs.cl`

Abstract

In the standard asymptotic expansion of four dimensional static asymptotically flat spacetimes, the coefficient of the first subleading term of the lapse function can be identified with the mass of the spacetime. Using the Hamiltonian formalism we show that, in asymptotically locally anti-de Sitter spacetimes endowed with a scalar field, the mass can read off in the same way only when the boundary conditions are compatible with the asymptotic realization of the anti-de Sitter symmetry. In particular, this implies that some prescriptions for computing the mass of a hairy spacetime are not suitable when the scalar field breaks the asymptotic anti-de Sitter invariance.

1 Introduction

Scalar fields play a significant role in physics. From a theoretical point of view, they are expected to be amongst the basic constituents of fundamental theories, e.g. string theory. Cosmologically, they are at the basis of inflation and dark energy models. More importantly, the first fundamental scalar particle was experimentally discovered [1, 2]. In the last years, phenomenological applications of the physics of hairy black holes have been proposed in different contexts. For example, some of these configurations have found interesting applications in condensed matter by using gauge/gravity dualities (for a review see for instance [3]). Additionally, astrophysical black holes have received a growing attention following the advent of new observational facilities and, consequently, different measurements for testing the spacetime geometry around these objects have been proposed. In particular, a cornerstone is to test the no-hair theorem from observations, i.e. whether or not the black hole at the center of our galaxy belongs to the Kerr class (see for instance [4, 5]). Therefore, *exact* hairy black hole solutions have an essential role in conjunction with adequate formalisms to determine their physical properties, such as the mass and angular momentum, even in the presence of matter fields.

Such hairy configurations were ruled out by no-hair theorems for which an asymptotically flat behavior for the gravitational field and positivity of the scalar field potential are assumed [6, 7, 8, 9, 10, 11]. However, the presence of a negative cosmological constant allows to circumvent these theorems in a physically sensible way. Indeed, a number of exact asymptotically anti-de Sitter (AdS) scalar hairy black holes have been obtained following the precursor ones in three [12, 13] and four

dimensions [14]. Recently, general classes of exact static hairy black hole solutions have been obtained [15, 16, 17, 18, 19, 20], as well as time dependent hairy black holes [21, 22], which in turn has opened the possibility of investigating their generic properties. For special values of the parameters in the moduli potential, some of these solutions can explicitly be embedded in supergravity theories [19, 23, 24].

An interesting physical effect emerges from asymptotically AdS scalar hairy solutions. Depending on its mass, the scalar field could acquire a slow fall-off at infinity. In this case, the scalar field induces a strong back reaction on the metric and, in this sense, it cannot be treated as a probe. In particular, it was shown by using the Hamiltonian formalism that the scalar field contributes to the mass of the hairy solution [13, 25, 26] (other approaches [27, 28, 29, 30, 31] have confirmed this result).

A relevant interval for the mass of the scalar field where the above effect appears is $m_{BF}^2 \leq m^2 < m_{BF}^2 + l^{-2}$, where $m_{BF}^2 = -9l^{-2}/4$ is the Breitenlohner-Freedman (BF) bound [32] and l is the AdS radius. In this range the evolution of scalar fields in AdS is well defined for any linear combination of Dirichlet and Neumann boundary conditions [33].

In the Hamiltonian formalism, the generators of the asymptotic symmetries — the conserved charges — contain a bulk term that is a linear combination of the constraints supplemented with a boundary term. The boundary term is fixed by requiring that the canonical generators have well-defined functional derivatives with respect to the canonical variables [34]. By virtue of the constraint equations, only the boundary term contributes to the charges and so, from this point of view, the Hamiltonian method is indeed suitable for a holographic interpretation. Since the charges can be computed from the boundary term only, they require just the asymptotic behaviour of the canonical variables and symmetries. Thus, the canonical generators provide the charges for all the solutions sharing the same asymptotic behaviour.

In this letter, we re-examine the notion of mass for asymptotically AdS scalar hairy configurations in the framework of General Relativity with a minimally coupled scalar field. The tool, we are going to use for computing the mass, is the Hamiltonian method of Regge and Teitelboim [34], following the results of [26].

A remarkable feature of this class of solutions is found. Once the canonical generator associated to the time translation (that corresponds from first principles to the mass of the configuration[1]) is evaluated using the equations of motions, its value coincides with the coefficient of the first subleading term of the lapse function only for boundary conditions that are compatible with the canonical realization of the local AdS symmetry at the boundary.

We would like to keep the discussion concrete and, therefore, we treat the case of a single scalar field with the conformal mass, $m^2 = -2l^{-2}$, which is in the allowed interval. In this case, both modes of the scalar are normalizable. This value of the mass is relevant for gauged supergravities in four dimensions [35] and we can explicitly apply our general results to analytic hairy black hole solutions [14, 15, 18, 23]. This mass is also interesting because it allows for subleading logarithmic branches (depending on the form of the scalar field potentials and boundary conditions [25, 26]), which needs to be treated separately. We expect that similar results should hold for scalar fields with arbitrary mass in the interval $m_{BF}^2 \leq m^2 < m_{BF}^2 + l^{-2}$ and for any dimensions.

There are different proposals in the literature, developed from other rationale, for computing the mass. It is interesting to study the conditions that enable those prescriptions to provide the right mass for the solutions analyzed here. In particular, the formula of Ashtekar-Magnon-Das (AMD) [36, 37] have been extensively used to obtain the mass of different hairy configurations [16, 18, 38, 39, 40, 41]. We explicitly show that the AMD mass matches the Hamiltonian mass of hairy configurations only for boundary conditions that are compatible with the local AdS symmetry at the boundary.

---

[1]Hereafter, we name it *Hamiltonian mass* just for remarking its origin.

2   Hamiltonian mass

Let us consider the action for a real scalar field minimally coupled to four-dimensional Einstein gravity in the presence of a cosmological constant $\Lambda = 3l^{-2}$ and a self-interaction potential $U(\phi)$

$$I[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{2\kappa} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right), \tag{1}$$

where $\kappa = 8\pi G$ is the Einstein constant. The corresponding field equations are

$$E^\mu_{\ \nu} \equiv G^\mu_{\ \nu} + \Lambda \delta^\mu_{\ \nu} - \kappa \left[\partial^\mu \phi \partial_\nu \phi - \left(\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + U(\phi) \right) \delta^\mu_{\ \nu} \right] = 0, \tag{2}$$

and

$$\Box \phi - \frac{dU(\phi)}{d\phi} = 0. \tag{3}$$

As we have mentioned before, we consider the Regge-Teitelboim approach to compute the mass of static scalar hairy asymptotically locally AdS spacetimes. A summary of this method is provided below.

The canonical generator of an asymptotic symmetry defined by the vector $\xi = (\xi^\perp, \xi^i)$ is a linear combination of the constraints $\mathcal{H}_\perp, \mathcal{H}_i$ plus a surface term $Q[\xi]$

$$H[\xi] = \int d^3x \left(\xi^\perp \mathcal{H}_\perp + \xi^i \mathcal{H}_i \right) + Q[\xi]. \tag{4}$$

The surface term is chosen so that the generator has well-defined functional derivatives [34].

In the static case the relevant asymptotic symmetry corresponds to the Killing vector ∂_t, and the mass is the conserved charge associated with this asymptotic symmetry. In the presence of the scalar field, apart from the usual gravitational term, an extra term coming from it appears in the mass. We then write the variation of the mass [26] as

$$\delta M = \delta Q(\partial_t) = \delta M_G + \delta M_\phi, \tag{5}$$

where

$$\delta M_G(\xi) = \frac{1}{2\kappa} \int dS_l G^{ijkl} (\xi^\perp \delta g_{ij;k} - \xi^\perp_{,k} \delta g_{ij}), \tag{6}$$

and

$$\delta M_\phi(\xi) = - \int dS_l \xi^\perp g^{1/2} g^{lj} \partial_j \phi \delta \phi. \tag{7}$$

Here, g_{ij} denotes the components of the 3-spatial metric, $g = \det g_{ij}$, and, as usual, we define

$$G^{ijkl} \equiv \frac{1}{2} g^{1/2} (g^{ik} g^{jl} + g^{il} g^{jk} - 2g^{ij} g^{kl}). \tag{8}$$

The variation of the mass given in (5) requires asymptotic boundary conditions to be integrated. As is expected from physical grounds, the mass is well defined after imposing suitable boundary conditions.

2.1 Non-logarithmic branch

For a self-interaction potential, whose power series expansion around $\phi = 0$ has the mass term $m^2 = -2l^{-2}$ and a vanishing cubic term, the asymptotically AdS behavior for the metric and scalar field does not contain logarithmic branches [26]. Following this reference, we consider a set of asymptotic conditions, that will be described below, and for which there exist analytic scalar black hole solutions [14, 15, 18, 23] whose asymptotic behavior belong the chosen one. The fall-off of the scalar field is

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2} + O(r^{-3}), \tag{9}$$

where α and β denote two real constants. For static metrics that match (locally) AdS at infinity, the relevant fall-off is

$$-g_{tt} = \frac{r^2}{l^2} + k - \frac{\mu}{r} + O(r^{-2}), \tag{10}$$

$$g_{mn} = r^2 h_{mn} + O(r^{-1}), \tag{11}$$

$$g_{rr} = \frac{l^2}{r^2} + \frac{al^4}{r^4} + \frac{l^5 b}{r^5} + O(r^{-6}), \tag{12}$$

where a and b are constants. Also, $h_{mn}(x^m)$ is the two-dimensional metric associated to the 'angular section' Σ_k , whose volume and curvature will be denoted by $V(\Sigma)$ and $2k$, respectively.

By evaluating the general expressions (6) and (7) for the above asymptotic boundary conditions, and considering that the boundary is at $r = \infty$, we obtain the gravitational contribution

$$\delta M_G = \frac{V(\Sigma)}{\kappa}[r\delta a + l\delta b + O(1/r)], \tag{13}$$

and scalar contribution

$$\delta M_\phi = \frac{V(\Sigma)}{l^2}[r\alpha\delta\alpha + \alpha\delta\beta + 2\beta\delta\alpha + O(1/r)]. \tag{14}$$

Thus, we have the variation of the mass

$$\delta M = \frac{V(\Sigma)}{\kappa l^2}[r(l^2\delta a + \kappa\alpha\delta\alpha) + l^3\delta b + \kappa(\alpha\delta\beta + 2\beta\delta\alpha) + O(1/r)]. \tag{15}$$

The above expression for δM is meaningful only in the case of vanishing constraints. For the asymptotic conditions considered here, $H_\perp = 0$ implies

$$\frac{k+a}{\kappa} + \frac{\alpha^2}{2l^2} = 0. \tag{16}$$

In this way, the divergent piece in (15) is removed and the asymptotic variation of the mass takes a finite value

$$\delta M = \frac{V(\Sigma)}{\kappa l^2}[l^3\delta b + \kappa(\alpha\delta\beta + 2\beta\delta\alpha)]. \tag{17}$$

To remove the variations from (17) we need to impose boundary conditions on the scalar field. In particular, the integration of (17) requires a functional relation between α and β. If we define $\beta = dW(\alpha)/d\alpha$, the mass of the spacetime is given by[2]

$$M = V(\Sigma)\left[\frac{lb}{\kappa} + \frac{1}{l^2}\left(\alpha\frac{dW(\alpha)}{d\alpha} + W(\alpha)\right)\right]. \tag{18}$$

---

[2]The mass in (18) is defined up to a constant without variation. Since in four dimensions there is no Casimir energy, this constant is zero in order to fix a vanishing mass for the locally AdS spacetime.

Indeed, we recover the result of [42] (see, also, [43] for 5-dimensional black holes). At this point, it is important to emphasize that the coefficient of the first subleading term, μ, in the expansion (10) of g_{tt} does not appear explicitly in the expression of the mass. In fact, in static spacetimes g_{tt} is the lapse function which is not a canonical variable and consequently, does not appear either in the constraints or in the surface terms. However, as we will see shortly, once we use the equations of motion the situation will change.

Now, for a *given solution* with the required asymptotics, we have additional information since not only the constraints are satisfied, but also the equations of motion. The $E_t^t - E_r^r$ combination of the Einstein-scalar field equations, which is not a constraint, is independent of the scalar field potential and yields

$$E_t^t - E_r^r = \frac{2a + 2k + \kappa\alpha^2 l^{-2}}{r^2} + \frac{-3\mu + 3bl + 4\kappa\alpha\beta l^{-2}}{r^3} + O(r^{-4}) = 0. \tag{19}$$

The first term gives the same relation as the constraint $H_\perp = 0$, but the second one provides a relation containing μ and the parameters of the asymptotic expansions of g_{rr} and the scalar field

$$bl = \mu - \frac{4}{3}\kappa\alpha\beta l^{-2}. \tag{20}$$

Then, the mass can be written as

$$M = V(\Sigma) \left[\frac{\mu}{\kappa} + \frac{1}{l^2} \left(W(\alpha) - \frac{1}{3}\alpha\frac{dW(\alpha)}{d\alpha} \right) \right]. \tag{21}$$

Therefore, there are only three situations when the mass reduces to $M = \mu V(\Sigma)\kappa^{-1}$:

- $\alpha = 0$: this is the usual Dirichlet boundary condition and ensures asymptotic AdS invariance;

- $\beta = 0$: this is the Neumann boundary condition and also ensures asymptotic AdS invariance;

- $\beta = C\alpha^2$: this boundary condition[3] corresponds to multi-trace deformations in the dual field theory [44] and is also compatible with the asymptotic AdS symmetry [26].

It is important to emphasize that the relation between the Hamiltonian mass and the parameter μ that appears in the expansion of g_{tt} will allow us to establish a clear relation with the AMD prescription.

2.2 Logarithmic branch

It is well known that a second order differential equation has two linearly independent solutions. When the ratio of the roots of the indicial equation is an integer, the solution may develop a logarithmic branch. This is exactly what happens when the scalar field saturates the BF bound, in which case the leading fall-off contains a logarithmic term [25]. However, we are interested in a scalar field with the conformal mass $m^2 = -2l^{-2}$. To obtain the logarithmic branch, a cubic term in the asymptotic expansion of the scalar field potential is necessary [26]

$$U(\phi) = -\frac{3}{l^2\kappa} - \frac{\phi^2}{l^2} + \lambda\phi^3 + O(\phi^4), \tag{22}$$

so that the fall-off of the scalar field to be considered is

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2} + \gamma\frac{\ln(r)}{r^2} + O(r^{-3}), \tag{23}$$

---

[3]The fact that under this boundary condition the contribution of the scalar field vanishes was noticed in [31] using a different approach.

and the suitable asymptotic behaviour for the metric can be expressed as

$$-g_{tt} = \frac{r^2}{l^2} + k - \frac{\mu}{r} + O(r^{-2}), \tag{24}$$

$$g_{mn} = r^2 h_{mn} + O(r^{-2}), \tag{25}$$

$$g_{rr} = \frac{l^2}{r^2} + \frac{al^4}{r^4} + \frac{l^5 c \ln(r)}{r^5} + \frac{l^5 b}{r^5} + O\left(\frac{\ln(r)^2}{r^6}\right). \tag{26}$$

Using a similar procedure as in the previous section we get

$$M = \left[\frac{lb}{\kappa} + \frac{1}{l^2}\left(\alpha \frac{dW}{d\alpha} + W(\alpha) + \alpha^3 l^2 \lambda\right)\right] V(\Sigma). \tag{27}$$

To relate the mass with the first subleading term of g_{tt} we use the combination of the Einstein equations $E_t^t - E_r^r$ which yields

$$b = \frac{\mu}{l} - \frac{2\kappa\alpha\left(\alpha^2 l^2 \lambda + 2\beta\right)}{3l^3}, \tag{28}$$

and then the mass becomes

$$M = \left[\frac{\mu}{\kappa} + \frac{1}{l^2}\left(W(\alpha) - \frac{1}{3}\alpha\frac{dW}{d\alpha} + \frac{1}{3}\alpha^3 l^2 \lambda\right)\right] V(\Sigma). \tag{29}$$

Therefore, we obtain $M = \mu V(\Sigma)\kappa^{-1}$ only for $\alpha = 0$ or

$$W(\alpha) = \alpha^3 \left[C + l^2 \lambda \ln(\alpha)\right], \tag{30}$$

which are the AdS invariant boundary conditions [26].

3 Ashtekar-Magnon-Das mass

The AMD procedure [36, 37] is particularly attractive because it can be straightforwardly applied to hairy black holes (detailed applications related black hole physics can be found in [18, 41]). The AMD conserved quantities are constructed from the electric part of the Weyl tensor. First, consider a conformally rescaled metric that is regular at the boundary

$$\tilde{g}_{\mu\nu} = \omega^2 g_{\mu\nu}, \tag{31}$$

where $g_{\mu\nu}$ is the asymptotically AdS metric of interest and ω has a zero of order one at infinity. $\tilde{g}_{\mu\nu}$ defines a conformal structure at infinity since ω is defined up to a multiplication of a regular function of the boundary coordinates. The central object of the AMD prescription is the electric part of the Weyl tensor

$$\mathcal{E}_\mu^\nu = l^2 \omega^{-1} n^\alpha n^\beta C_{\alpha\beta\mu}^\nu, \tag{32}$$

where $n_\mu = \partial_\mu \omega$ is the normal vector on the boundary and $C_{\beta\alpha\mu}^\nu$ is the Weyl tensor of $\tilde{g}_{\mu\nu}$. Note that all the objects in (32) are intended to be calculated and index-manipulated with the metric $\tilde{g}_{\mu\nu}$. The energy in both cases, with or without logarithmic branches, is

$$M_{\text{AMD}} = \frac{l}{\kappa} \int_\Sigma \mathcal{E}_{tt} d\Sigma^t = \frac{\mu V(\Sigma)}{\kappa}. \tag{33}$$

It is now clear that AMD mass matches the actual mass of the spacetime, defined by the Hamiltonian, only for AdS invariant boundary conditions.

4   Discussion

In this letter, we have computed the mass for asymptotically AdS configurations endowed with a massive minimally coupled scalar field. It has been shown that the canonical generator associated to the time translation symmetry, i.e. the mass, once evaluated using the equations of motions, coincides with the coefficient of the first subleading term of the lapse function only for boundary conditions that are compatible with the canonical realization of the local AdS symmetry at the boundary.

Additionally, we have explicitly shown that the AMD mass provides the right result, as defined by the Hamiltonian method, only for boundary conditions that preserve the conformal invariance of the boundary (and so of the dual theory)[4].

A conformal field theory (CFT) embedded in a curved spacetime background can be characterized by the trace anomaly coefficients of the stress tensor. This result applies to even dimensional CFTs because there is no trace anomaly in odd dimensions. Since within the AdS/CFT duality a CFT has a gravity dual, one expects that a dual gravitational computation will also account for the Casimir energy. Indeed, this was explicitly proven in the so called holographic renormalization [46]. From a holographic point of view, the quantum fluctuations contribute to the inertia in the boundary. However, a flaw of the AMD method is that it has low accuracy in this regard, namely it can not account for the Casimir energy. Since the dual field theory is living on a 3-dimensional sphere, there is no Casimir energy associated and so, when the boundary conditions preserve the conformal symmetry, the AMD prescription and the Hamiltonian method produce identical results. However, when the boundary conditions do not preserve the conformal symmetry these methods produce different results.

Let us comment now on the test particle motion in scalar hairy AdS spacetimes. This could be related with potentially observable effects. For the clarity of the argument, let us make the discussion quantitative. Consider the four-dimensional static asymptotically flat metric:

$$ds^2 = -\left[1 - \frac{\mu}{r} + O(r^{-2})\right]dt^2 + \frac{dr^2}{\left[1 - \frac{m}{r} + O(r^{-2})\right]} + r^2 d\Omega^2. \tag{34}$$

Note that we have parametrized differently the $O(r^{-1})$ term of g_{tt} and g_{rr}^{-1}, respectively. When there is no contribution from the matter fields, i. e. when the matter fields fall off fast enough at infinity, the Hamiltonian mass of the spacetime is

$$M = \frac{m}{2G}. \tag{35}$$

Indeed, this is the case for massive scalar fields since they are exponentially suppressed in asymptotically flat spacetimes, and consequently the field equations yield $\mu = m$. The motion of test particles on circular orbits is driven by the (mass) parameter μ, as is revealed by the expression for the rate of revolution $\omega = d\phi/dt$ in a circular orbit at radius R which, for a generic spherically symmetric spacetime, is given by

$$\omega^2 = -\frac{1}{2R}\frac{dg_{tt}}{dr}\Big|_{r=R} . \tag{36}$$

Then, for the asymptoticaly flat metric (34), we have $\omega^2 = \mu/(2R^3)$ when R is large. Therefore, this parameter can be interpreted as the gravitational mass that generates the gravitational field responsible for the test particles' motion. If one interprets the Hamiltonian mass as the inertial mass of the system, then, in agreement with the equivalence principle, it is not surprising that $m = \mu$.

---

[4]We also have an independent computation using the counterterms that supports this claim [45].

It is instructive to contrast the previous result with the circular geodesic motion on an asymptotically AdS spacetime in the presence of a massive scalar field, described by the metric

$$ds^2 = -\left[\frac{r^2}{l^2} + 1 - \frac{\mu}{r} + O(r^{-2})\right]dt^2 + \frac{dr^2}{\left[\frac{r^2}{l^2} + 1 - \frac{m(r)}{r} + O(r^{-2})\right]} + r^2 d\Omega^2, \qquad (37)$$

where $m(r)$ grows slower than r^3 [26]. In this case, the rate of revolution reduces to $l^{-2} + \mu/(2R^3) + O(R^{-4})$. As is expected, apart from the parameter μ, the motion is driven also by the cosmological constant. Considering the latter constant as a fundamental one, the motion is addressed only by μ. However, when the backreaction of scalar fields is taken into account, the mass matches $\mu/(2G)$ only for the AdS invariant boundary conditions discussed in the previous section. This suggests that an issue arises from the interpretation of μ as the gravitational mass when the boundary condition on the scalar field breaks the conformal symmetry.

One obvious extension of this work is an application to higher dimensional scalar hairy black holes [18, 19]. Also, one can study the charged hairy black holes and their extremal limits. In the extremal limit, the attractor mechanism plays the role of a no-hair theorem [47] in the sense that the moduli are fixed at the horizon and the near horizon geometry is universal. The moduli flow is interpreted as an RG flow and it will be interesting to compare the charges computed at the horizon [48, 49] with the charges computed in the boundary. In this way, one can understand better the role of the hair (scalar degrees of freedom living outside the horizon) for black hole physics.

Acknowledgments

References

[1] G. Aad *et al.* [ATLAS Collaboration], "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[3] G. T. Horowitz, "Introduction to Holographic Superconductors," Lect. Notes Phys. **828** (2011) 313 [arXiv:1002.1722 [hep-th]].

[4] C. M. Will, "Testing the general relativistic no-hair theorems using the Galactic center black hole SgrA*," Astrophys. J. **674** (2008) L25 [arXiv:0711.1677 [astro-ph]].

[5] A. E. Broderick *et al.* [Perimeter Institute for Theoretical Physics Collaboration], "Testing the No-Hair Theorem with Event Horizon Telescope Observations of Sagittarius A*," Astrophys. J. **784**, 7 (2014) [arXiv:1311.5564 [astro-ph.HE]].

[6] J. E. Chase, "Event Horizons in Static Scalar-Vacuum Space-Times," Commun. Math. Phys. **19**, 276 (1970).

[7] J. D. Bekenstein, "Nonexistence of baryon number for static black holes," Phys. Rev. D **5**, 1239 (1972).

[8] C. Teitelboim, "Nonmeasurability of the quantum numbers of a black hole," Phys. Rev. D **5**, 2941 (1972).

[9] J. D. Bekenstein, "Novel 'no scalar hair' theorem for black holes," Phys. Rev. D **51**, 6608 (1995).

[10] D. Sudarsky, "A Simple proof of a no hair theorem in Einstein Higgs theory," Class. Quant. Grav. **12** (1995) 579.

[11] M. Heusler, "No hair theorems and black holes with hair," Helv. Phys. Acta **69** (1996) 501 [gr-qc/9610019].

[12] C. Martínez and J. Zanelli, "Conformally dressed black hole in (2+1)-dimensions," Phys. Rev. D **54**, 3830 (1996) [gr-qc/9604021].

[13] M. Henneaux, C. Martínez, R. Troncoso and J. Zanelli, "Black holes and asymptotics of 2+1 gravity coupled to a scalar field," Phys. Rev. D **65**, 104007 (2002) [hep-th/0201170].

[14] C. Martínez, R. Troncoso and J. Zanelli, "Exact black hole solution with a minimally coupled scalar field," Phys. Rev. D **70**, 084035 (2004) [hep-th/0406111].

[15] A. Anabalón, "Exact Black Holes and Universality in the Backreaction of non-linear Sigma Models with a potential in (A)dS4," JHEP **1206**, 127 (2012) [arXiv:1204.2720 [hep-th]].

[16] A. Aceña, A. Anabalón and D. Astefanesei, "Exact hairy black brane solutions in AdS_5 and holographic RG flows," Phys. Rev. D **87**, no. 12, 124033 (2013) [arXiv:1211.6126 [hep-th]].

[17] A. Anabalón, D. Astefanesei and R. Mann, "Exact asymptotically flat charged hairy black holes with a dilaton potential," JHEP **1310**, 184 (2013) [arXiv:1308.1693 [hep-th]].

[18] A. Aceña, A. Anabalón, D. Astefanesei and R. Mann, "Hairy planar black holes in higher dimensions," JHEP **1401**, 153 (2014) [arXiv:1311.6065 [hep-th]].

[19] X. -H. Feng, H. Lü and Q. Wen, "Scalar Hairy Black Holes in General Dimensions," Phys. Rev. D **89**, 044014 (2014) [arXiv:1312.5374 [hep-th]].

[20] A. Anabalón and D. Astefanesei, "On attractor mechanism of AdS_4 black holes," Phys. Lett. B **727**, 568 (2013) [arXiv:1309.5863 [hep-th]].

[21] X. Zhang and H. Lü, "Exact Black Hole Formation in Asymptotically (A)dS and Flat Spacetimes," arXiv:1403.6874 [hep-th].

[22] H. Lü and X. Zhang, "Exact Collapse Solutions in D=4, N=4 Gauged Supergravity and Their Generalizations," arXiv:1404.7603 [hep-th].

[23] A. Anabalón and D. Astefanesei, "Black holes in ω-defomed gauged $N = 8$ supergravity," Phys. Lett. B **732**, 137 (2014) [arXiv:1311.7459 [hep-th]].

[24] H. Lü, Y. Pang and C. N. Pope, "An ω Deformation of Gauged STU Supergravity," JHEP **1404**, 175 (2014) [arXiv:1402.1994 [hep-th]].

[25] M. Henneaux, C. Martínez, R. Troncoso and J. Zanelli, "Asymptotically anti-de Sitter spacetimes and scalar fields with a logarithmic branch," Phys. Rev. D **70** (2004) 044034 [hep-th/0404236].

[26] M. Henneaux, C. Martínez, R. Troncoso and J. Zanelli, "Asymptotic behavior and Hamiltonian analysis of anti-de Sitter gravity coupled to scalar fields," Annals Phys. **322** (2007) 824 [hep-th/0603185].

[27] G. Barnich, "Conserved charges in gravitational theories: Contribution from scalar fields," gr-qc/0211031.

[28] J. Gegenberg, C. Martínez and R. Troncoso, "A Finite action for three-dimensional gravity with a minimally coupled scalar field," Phys. Rev. D **67**, 084007 (2003) [hep-th/0301190].

[29] T. Hertog and K. Maeda, "Black holes with scalar hair and asymptotics in N = 8 supergravity," JHEP **0407**, 051 (2004) [hep-th/0404261].

[30] M. Bañados and S. Theisen, "Scale invariant hairy black holes," Phys. Rev. D **72**, 064019 (2005) [hep-th/0506025].

[31] A. J. Amsel and D. Marolf, "Energy Bounds in Designer Gravity," Phys. Rev. D **74**, 064006 (2006) [Erratum-ibid. D **75**, 029901 (2007)] [hep-th/0605101].

[32] P. Breitenlohner and D. Z. Freedman, "Positive Energy in anti-De Sitter Backgrounds and Gauged Extended Supergravity," Phys. Lett. B **115** (1982) 197.

[33] A. Ishibashi and R. M. Wald, "Dynamics in nonglobally hyperbolic static space-times. 3. Anti-de Sitter space-time," Class. Quant. Grav. **21** (2004) 2981 [hep-th/0402184].

[34] T. Regge and C. Teitelboim, "Role of Surface Integrals in the Hamiltonian Formulation of General Relativity," Annals Phys. **88** (1974) 286.

[35] B. de Wit and H. Nicolai, "N=8 Supergravity," Nucl. Phys. B **208** (1982) 323.

[36] A. Ashtekar and A. Magnon, "Asymptotically anti-de Sitter space-times," Class. Quant. Grav. **1** (1984) L39.

[37] A. Ashtekar and S. Das, "Asymptotically Anti-de Sitter space-times: Conserved quantities," Class. Quant. Grav. **17** (2000) L17 [hep-th/9911230].

[38] H. Lü, Y. Pang and C. N. Pope, "AdS Dyonic Black Hole and its Thermodynamics," JHEP **1311** (2013) 033 [arXiv:1307.6243 [hep-th]].

[39] D. D. K. Chow and G. Compère, "Dyonic AdS black holes in maximal gauged supergravity," Phys. Rev. D **89**, 065003 (2014) [arXiv:1311.1204 [hep-th]].

[40] H. -S. Liu and H. Lü, "Scalar Charges in Asymptotic AdS Geometries," Phys. Lett. B **730**, 267 (2014) [arXiv:1401.0010 [hep-th]].

[41] W. Chen, H. Lü and C. N. Pope, "Mass of rotating black holes in gauged supergravities," Phys. Rev. D **73** (2006) 104036 [hep-th/0510081].

[42] T. Hertog and G. T. Horowitz, "Designer gravity and field theory effective potentials," Phys. Rev. Lett. **94** (2005) 221301 [hep-th/0412169].

[43] D. Astefanesei, N. Banerjee and S. Dutta, "(Un)attractor black holes in higher derivative AdS gravity," JHEP **0811**, 070 (2008) [arXiv:0806.1334 [hep-th]].

[44] E. Witten, "Multitrace operators, boundary conditions, and AdS / CFT correspondence," hep-th/0112258.

[45] A. Anabalón, D. Astefanesei, D. Choque, C. Martínez, "Holographic renormalization of hairy black holes," *to appear.*

[46] M. Henningson and K. Skenderis, "The Holographic Weyl anomaly," JHEP **9807**, 023 (1998) [hep-th/9806087].

[47] D. Astefanesei, H. Nastase, H. Yavartanoo and S. Yun, "Moduli flow and non-supersymmetric AdS attractors," JHEP **0804**, 074 (2008) [arXiv:0711.0036 [hep-th]].

[48] D. Astefanesei, N. Banerjee and S. Dutta, "Near horizon data and physical charges of extremal AdS black holes," Nucl. Phys. B **853**, 63 (2011) [arXiv:1104.4121 [hep-th]].

[49] D. Astefanesei, N. Banerjee and S. Dutta, "Moduli and electromagnetic black brane holography," JHEP **1102**, 021 (2011) [arXiv:1008.3852 [hep-th]].