

# How Does Prospect Theory Reflect Heuristics' Probability Sensitivity in Risky Choice?

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## Abstract

Two prominent approaches to describing how people make decisions between risky options are *algebraic models* and *heuristics*. The two approaches are based on fundamentally different algorithms and are thus usually treated as antithetical, suggesting that they may be incommensurable. Using cumulative prospect theory (CPT; Tversky & Kahneman, 1992) as an illustrative case of an algebraic model, we demonstrate how algebraic models and heuristics can mutually inform each other. Specifically, we highlight that CPT describes decisions in terms of psychophysical characteristics, such as diminishing sensitivity to probabilities, and we show that this holds even when the underlying process is heuristic in nature. Our results suggest that algebraic models and heuristics might offer complementary rather than rival modeling frameworks and highlight the potential role of heuristic principles in information processing for prominent descriptive constructs in risky choice.

**Keywords:** cumulative prospect theory; probability sensitivity; computational modeling; heuristics; risky choice.

## Introduction

How can risky decision making—in which people have to choose between options offering different outcomes with certain probabilities—best be modeled? Two prominent approaches in decision research are algebraic models and heuristics (e.g., Brandstätter, Gigerenzer, & Hertwig, 2006; Payne, 1973; Payne, Bettman, & Johnson, 1993). Algebraic models follow the principle of expectation maximization and use an algorithm that integrates (some function of) probability and outcome information multiplicatively to describe people's risky choices. Arguably the most prominent model in this tradition is *cumulative prospect theory* (CPT; Tversky & Kahneman, 1992). According to CPT, options are evaluated independently of each other. The model invokes psychophysical constructs such as probability sensitivity and loss aversion to account for characteristic phenomena in choice, and quantifies them using adjustable parameters. Heuristics, by contrast, are based on simple principles of information processing, such as

sequential and limited search, dimensional comparison, and aspiration levels; in contrast to algebraic models, heuristics often go without integrating information, and ignore part of the information (e.g., Payne et al., 1993; Thorngate, 1980). Models of heuristics for risky choice include the semiorder rule (Luce, 1956), the similarity heuristic (Leland, 1994; Rubinstein, 1988), elimination-by-aspects (Tversky, 1972), and the priority heuristic (Brandstätter et al., 2006).

Algebraic models and heuristics are often treated as antithetical (e.g., Brandstätter et al., 2006; Payne, 1973; Svenson, 1979). As pointed out by Lopes (1995), however, this opposition may be unnecessary: "Some models focus on the algebraic pattern of people's risk preferences, others on the content of their choice processes [models of heuristics]. Although one might suppose that these two kinds of accounts are alternate ways of describing the same thing—indeed, that one kind of model might eventually be reducible to the other—the approaches have tended to be disjoint" (p. 177). To date, however, the relationship between algebraic models and heuristics has yet to be elaborated.

To close that gap, we use CPT (Tversky & Kahneman, 1992) as an illustrative case highlighting that algebraic models offer a tool for describing characteristics of the decision process in psychophysical terms; here, we focus on the sensitivity to differences in probabilities. We argue that diminished sensitivity to probability information—as captured in CPT's weighting functions—can result from lexicographic and noncompensatory processing of heuristics. As such, CPT may offer a useful framework to represent heuristic decision making in terms of established constructs such as sensitivity to probability information. Conversely, as heuristics are explicit with regard to the information-processing steps underlying a decision, elaborating the relationship between heuristics and CPT might contribute to a better understanding of the cognitive mechanisms potentially underlying the characteristic shapes of CPT's weighting and value functions (cf. Hogarth & Einhorn, 1990). Overall, we thus suggest that the relationship between the algebraic and heuristic models of risky choice is complementary rather than adversarial.

In the following, we first briefly describe CPT's parametric framework and how its weighting functions reflect sensitivity to probability information. Second, we elaborate for one specific heuristic, the priority heuristic (Brandstätter et al., 2006), how heuristic choices may be reflected in CPT's parameters. Specifically, we take advantage of the fact that the degree to which the priority heuristic attends to probability information depends on the choice environment; using computer simulations, we examine how this translates into differences in CPT's weighting function.

### Probability Sensitivity in CPT

CPT assumes that decisions are made to maximize expected return. More specifically, choices between risky options are based on a person's subjective valuation of these options and then maximization. In CPT, the overall valuation  $V$  of an option  $A$  is defined as

$$V(A) = \sum_{j=1}^k v(x_j) \pi_j^- + \sum_{i=k+1}^n v(x_i) \pi_i^+ . \quad (1)$$

$v(x)$  is the value function, describing how an objective outcome  $x$  is translated into a subjective value, and  $\pi^+$  ( $\pi^-$ ) is the weight given to a positive (negative) outcome  $x$  (Tversky & Kahneman, 1992) and depends on the probability of the outcome.

CPT assumes a rank-dependent transformation of the outcomes' probabilities into decision weights. Specifically, with outcomes  $x_1 \leq \dots \leq x_k \leq 0 \leq x_{k+1} \leq \dots \leq x_n$ , the weight  $\pi^+$  ( $\pi^-$ ) given to a positive (negative) outcome  $x$  is the difference between the probability of receiving an outcome at least as good (bad) as  $x$  and the probability of receiving an outcome better (worse) than  $x$ :

$$\begin{aligned} \pi_i^+ &= w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \quad \text{for } k < i < n \\ \pi_j^- &= w^-(p_1 + \dots + p_j) - w^-(p_1 + \dots + p_{j-1}) \quad \text{for } 1 < j \leq k . \end{aligned} \quad (2)$$

$w^+$  and  $w^-$  are the probability weighting functions for gains and losses, respectively. They are assumed to have an inverse S-shaped curvature. Different types of weighting functions have been proposed (for an overview, see Stott, 2006). We use the following two-parameter version that separates the curvature of the weighting function from its elevation (e.g., Goldstein & Einhorn, 1987; Gonzalez & Wu, 1999):

$$\begin{aligned} w^+(p) &= \frac{\delta^+ p^{\gamma^+}}{\delta^+ p^{\gamma^+} + (1-p)^{\gamma^+}} \\ w^-(p) &= \frac{\delta^- p^{\gamma^-}}{\delta^- p^{\gamma^-} + (1-p)^{\gamma^-}} \end{aligned} \quad (3)$$

The parameters  $\gamma^+$  and  $\gamma^-$  (both varying between 0 and 1) govern the amount of curvature of the function in the gain and loss domains, respectively, and indicate how sensitive decisions are to differences in probabilities (with smaller values of  $\gamma < 1$  resulting in more S-shaped weighting functions, reflecting lower sensitivity to

differences in probabilities). The elevation of the weighting functions for gains and losses is controlled by the parameters  $\delta^+$  and  $\delta^-$  (both  $> 0$ ), respectively.

CPT has repeatedly been shown to be highly successful in describing risky choices between monetary gamble problems (e.g., Glöckner & Pachur, 2012; but see Birnbaum, 2004; Brandstätter et al., 2006). As a description of the underlying cognitive process, however, CPT's implied algebraic calculus and its commitment to a multiplicative framework have not been unchallenged (e.g., Brandstätter et al., 2006; Lopes, 1995). One such challenge has been put forth by proponents of heuristics. We turn to this modeling approach next.

### Probability Sensitivity Resulting From Heuristic Information Processing

In contrast to the integrative approach of CPT, heuristics often ignore part of the information and do not integrate information. They are based on simple principles of information processing, such as sequential and limited search, dimensional comparison, and aspiration levels (e.g., Payne et al., 1993; Thorngate, 1980). Lexicographic strategies, for instance, proceed through several dimensions sequentially and stop at the first dimension that enables a decision to be made (Fishburn, 1974; Gigerenzer, Todd, & the ABC Research Group, 1999; Thorngate, 1980). The priority heuristic (Brandstätter et al., 2006), which is related to lexicographic semi-orders (Luce, 1956; Tversky, 1969), belongs to this class. Its architecture is based on established principles of bounded rationality (e.g., Simon, 1955), such as sequential search, stopping rules, and aspiration levels, and it assumes that probabilities and outcomes are compared between gambles, rather than integrated within gambles (as assumed by CPT). For choices between two-outcome gambles involving gains, the priority heuristic entails the following steps:

**1. Priority rule.** Go through dimensions in the order of minimum gain, probability of minimum gain, and maximum gain.

**2. Stopping rule.** Stop examination if the minimum gains differ by 1/10 (or more) of the maximum gain; otherwise, stop examination if probabilities differ by 1/10 (or more) of the probability scale.

**3. Decision rule.** Choose the gamble with the more attractive gain (probability).

(For losses, "gains" are replaced by "losses"; for mixed gambles, "gains" are replaced by "outcomes.")

Due to its stopping rule, the priority heuristic considers probability information depending on the structure of a gamble problem. The heuristic first examines the (minimum) outcomes of the gambles. If this reason discriminates, then probabilities will not be examined. If, however, the outcomes fail to discriminate, probabilities will be examined. That is, the priority heuristic attends to probability information only when the minimum outcomes do not differ. The heuristic's

sensitivity to probability information is thus dependent on the structure of the choice environment.

### Heuristics' Probability Sensitivity as Captured in CPT's Parametric Framework

These two approaches to model risky choice—CPT and heuristics—are based on fundamentally different algorithms. Whereas CPT considers all outcome and all probability information, the priority heuristic considers the reasons sequentially, and stops information search as soon as a reason discriminates. Moreover, although CPT may be a relatively flexible model due to its several adjustable parameters (e.g., Gonzalez & Wu, 1999), it still has important constraints: both the value and the weighting function are restricted to be monotonic, the value function is concave for gains and convex for losses, and the weighting function is constrained to have an inverse S-shaped curvature. Can CPT, given these constraints and given its starkly different algorithmic structure, nevertheless accommodate choices generated by the priority heuristic and accurately reflect the degree to which the heuristic attends to probability information?

In addressing this question, we strive to contribute to a better understanding of the relationship between algebraic models and heuristics. One crucial aspect of our argument is that diminished sensitivity to probability information may be due not only to psychophysical regularities in magnitude evaluation, but also to the limited attention that a heuristic devotes to probabilities. More specifically, the weighting function's  $\gamma$  parameter (Equation 3), which reflects sensitivity to probabilities, should differ systematically as a function of whether the heuristic makes a choice based on the first reason (outcome) or the second reason (probability). The less frequently the priority heuristic considers probabilities in a set of gamble problems, the lower the resulting value of the  $\gamma$  parameter should be. Slovic and Lichtenstein (1968) made a similar proposal more than 40 years ago, suggesting that “increases [in] the saliency of the money dimensions and decreases [in] the relative importance of the probabilities” should lead to “relatively flat [i.e., more strongly S-shaped] subjective probability functions” (p. 16). Next, we test this suggestion using a computer simulation.

### Computer Simulation

We created three sets of two-outcome gamble problems, each including 180 randomly generated problems with similar expected values: 60 gain, 60 loss, and 60 mixed problems (cf. Rieskamp, 2008). Across the three sets, we varied the percentage of problems in which the minimum gains (losses) discriminated between the gambles (i.e., that differed by at least 10% of the highest gain or loss). In the first set, the minimum gains (losses) discriminated in 75% of the cases, and the priority heuristic therefore only proceeded to the second reason—the probability of the minimum gains (losses)—in 25% of the cases; in the

second set, the minimum gains (losses) discriminated in 50% of the gamble problems, and the heuristic therefore proceeded to the probability information in the remaining 50% of the cases; in the third set, the minimum gains (losses) differed in only 25% of the cases, and the heuristic therefore proceeded to the probabilities in 75% of the cases. The gambles were constructed such that if the heuristic proceeded to the probability information, this reason always discriminated. We predicted that CPT's probability sensitivity parameter  $\gamma$  fitted to the priority heuristic's choices would increase across the problem sets.

We simulated the choices of the priority heuristic in all three problem sets and subsequently fitted CPT's weighting functions and value functions, respectively, to these choices, separately for each set. Our implementation of CPT had six adjustable parameters:  $\alpha$  ( $= \beta$ ) and  $\lambda$  for the value function,  $\gamma$ ,  $\delta^+$  and  $\delta^-$  for the weighting function, and  $\phi$  for the choice rule necessary to derive predicted choice probabilities (see below).<sup>1</sup> To reflect CPT's main assumptions (e.g., an inversely S-shaped probability weighting function, a concave value function for gains, and a convex value function for losses; see Tversky & Kahneman, 1992), in the parameter estimation procedure the parameter values were restricted as follows (see Rieskamp, 2008):  $0 < \alpha \leq 1$ ;  $0 < \lambda \leq 5$ ;  $0 < \gamma \leq 1$ ;  $0 < \delta^\pm \leq 4$ ;  $0 < \phi \leq 5$ . The deviation between CPT's predictions and the heuristic's choices was quantified using the likelihood measure  $G^2$  (e.g., Sokal & Rohlf, 1994), with a smaller  $G^2$  indicating a better fit:

$$G^2 = -2 \sum_{i=1}^N \ln[f_i(y|\theta)] \quad (4)$$

where  $N$  refers to the total number of choices, and  $f(y|\theta)$  refers to the probability with which CPT, given a particular set of parameter values  $\theta$ , predicts an individual choice  $y$ . If gamble A was chosen, then  $f(y|\theta) = p_i(A,B)$ , where  $p_i(A,B)$  is the predicted probability that gamble A is chosen over gamble B; if gamble B was chosen, then  $f(y|\theta) = 1 - p_i(A,B)$ . To determine  $p_i(A,B)$ , we applied an exponential version of Luce's (1956) choice rule (also known as softmax):

$$p_i(A,B) = \frac{e^{\phi V(A)}}{e^{\phi V(A)} + e^{\phi V(B)}} \quad (5)$$

where  $V(A)$  and  $V(B)$  represent the subjective valuation of the gambles A and B according to CPT, and  $\phi > 0$  specifies how sensitively the predicted choice probability reacts to differences between the gambles' subjective valuations  $V(A)$  and  $V(B)$ , with higher values indicating higher sensitivity. In the fitting procedure, we first

<sup>1</sup> We set  $\alpha = \beta$ , as Nilsson, Rieskamp, and Wagenmakers (2011) have shown that estimating separate exponents of the value function for gains and losses (i.e.,  $\alpha$  and  $\beta$ ) can lead to unreliable estimates of  $\lambda$  (see also Wakker, 2010). We set  $\gamma^+ = \gamma^-$ , as the priority heuristic treats probabilities equally across gains and losses.

Table 1: Parameter estimates obtained when fitting CPT to the choices of the priority heuristic where the decision was made on the first reason (minimum gain/loss) in 25%, 50%, or 75% of the cases, respectively, and on the second reason (probability of the minimum gain/loss) otherwise.

% Decisions on first reason	Parameter estimates						
	$\gamma$	$\delta^+$	$\delta^-$	$\alpha$	$\lambda$	$\phi$	$G^2$
75%	0.32	0.01	0.15	1.0	2.06	0.34	84.25
50%	0.50	0.05	0.49	1.0	1.30	0.15	153.45
25%	1.00	0.13	0.45	1.0	1.18	0.21	108.54

Note.  $G^2$  assuming random choice is 249.53.

implemented a grid search to identify the parameter values that minimize  $G^2$ ; the 20 best-fitting combinations of grid values were then used as starting points for subsequent optimization using the simplex method (Nelder & Mead, 1965), as implemented in MATLAB.

## Results

Table 1 shows the best-fitting CPT parameters when fitted to the simulated choices of the priority heuristic in gamble problems where the decision was made on the minimum gain (loss) in 75%, 50%, or 25% of the cases, respectively, and on the probability of the minimum gain (loss) otherwise. As can be seen—and as predicted—the probability sensitivity parameter  $\gamma$  increased across the sets; it was lowest in the set where the priority heuristic decided on the first reason (minimum outcome) in the majority of cases and it was highest in the set where the heuristic decided on the second reason (probability of the minimum outcome) in the majority of cases. In other

words, CPT accurately reflected the different degrees to which the priority heuristic attended to probability information across the three sets.

Panels A and B of Figure 1 plot the weighting functions based on the best-fitting parameters, separately for the gain and loss domains. Irrespective of domain, for choices that only considered probabilities in 25% of the cases, the weighting function was most strongly S-shaped, indicating low sensitivity to probability information; for choices that considered probabilities in half of the cases, it was comparatively less S-shaped; and for choices that considered probabilities in 75% of the cases, it was least S-shaped. (Note that the differences in shapes of the weighting function between the gain and loss domains were due to differences in the elevation parameters; i.e.,  $\delta^+$  and  $\delta^-$ ).

The best-fitting parameter values of the priority heuristic's choices in the three problem sets are summarized in the parameter profiles in Panel C in Figure 1.

Interestingly, CPT did not fit equally well to the choices across the three problem sets (see  $G^2$  in Table 1). Specifically, the fit was best when most choices (75%) were made on the first reason, worsened when 50% considered probabilities, and improved again when only 25% of the choices were made on the first reason. CPT is thus apparently better able to fit choice sets where a substantial proportion of choices stop examination on the same reason than when choices are based on different reasons (as in the 50% choice set).

## Discussion

CPT models decisions based on a compensatory algorithm where outcomes and probabilities are integrated multiplicatively and summed up separately within each option. The priority heuristic, in contrast, models decisions based on sequential information

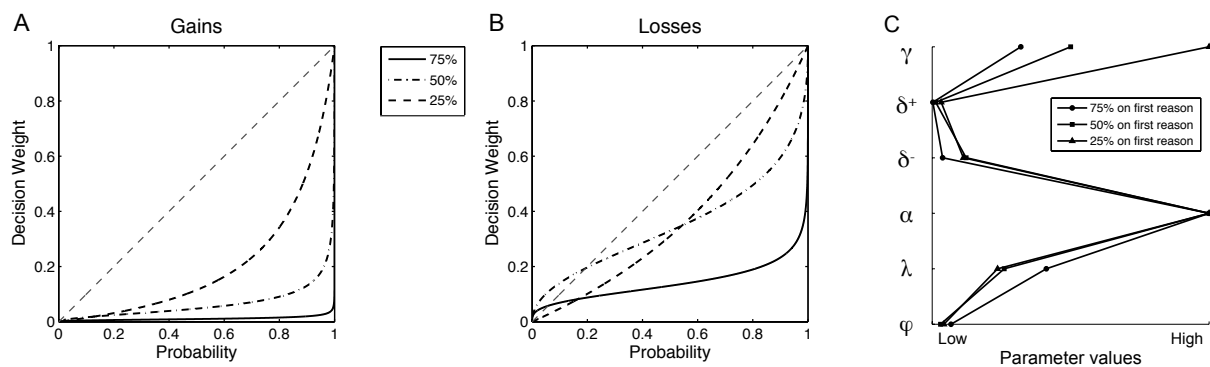


Figure 1: Panels A (gains) and B (losses) plot the weighting functions obtained when fitting cumulative prospect theory to the choices of the priority heuristic in gambles where the decision was made on the minimum gain (loss) in 75%, 50%, or 25%, respectively, and on the probability of the minimum gain (loss) otherwise. Panel C shows the parameter profiles of CPT's value and weighting function parameters fitted to the choices of the priority heuristic in the three gamble sets (as the parameters differed in their scale, they were normalized for this graph; for the exact parameter values, see Table 1).

processing and compares outcomes and probabilities between the options. Despite these stark differences, our result is that CPT is able to represent choices generated by the priority heuristic in a psychologically meaningful manner: the weighting function's curvature reflects differences in the heuristic's sensitivity to probability information between the three choice environments that differed in how frequently choices were decided based on the probability dimension.

Taken together, our results thus demonstrate that although CPT is based on a different algorithmic architecture than heuristics, its parametric framework might offer a useful tool for characterizing heuristic processes in terms of prominent descriptive constructs such as probability sensitivity (for a discussion of other constructs of CPT, such as risk aversion, loss aversion, and outcome sensitivity, see Suter, Pachur, & Hertwig, 2013a). Conversely, the integration of the two approaches might enable hypotheses to be derived about the processes generating the characteristic shapes of CPT's functions.

Our finding that specific values of CPT's  $\gamma$  parameter can reflect the processing steps of a lexicographic heuristic—that is, whether probability information was called upon or not—has important implications for the use of CPT in empirical investigations of risky choice: CPT's parameters might help to identify the interaction of a heuristic with its environment; moreover, they might help to identify the use of different heuristics by different individuals within the same environment, or of different heuristics by the same individual across different environments.

The demonstrated relationship between CPT, the information processing architecture of a heuristic, and the structure of the environment could explain apparent inconsistencies in empirical investigations (see also Hertwig & Gigerenzer, 2011)—for instance, why the same person's sensitivity to probability information appears low at some times and high at others. Such observations of variability need not mean that CPT's parameters cannot be measured reliably, or that different heuristics are used. They could arise from the interaction of a heuristic's lexicographic architecture with various choice environments. If decision problems are constructed such that a user of the priority heuristic is always able to terminate search after examining the options' minimum outcomes, the person's probability sensitivity will appear low. If they are constructed such that the same person must always move beyond the minimum outcomes and examine their probabilities, the person will seem to be highly sensitive to probabilities.

Relatedly, the elaborated relationship between CPT and heuristic processing not only allows the interactions of a heuristic to be tracked across different environments, but it may also allow differences in strategy selection between individuals within the same environment to be identified. It therefore suggests an alternative interpretation of the observed link between CPT's

parameters and variables that influence risky choice, such as gender. Fehr-Duda, De Gennaro, and Schubert (2006), for instance, concluded that women tend to be less sensitive to probability changes than men (see also Booij & van de Kuilen, 2009). To the extent that CPT reflects differences in terms of probability sensitivity also between strategies, this finding might indicate that men and women rely on different strategies that differ with regard to their probability sensitivity.

Moreover, CPT's parameters might not only reveal differences between individuals, but also within an individual. For instance, a decision maker might use different strategies for different contexts. In a study on the difference between affect-rich and affect-poor risky choice, Suter, Pachur, and Hertwig (2013b) found that people's choices in affect-rich tasks were consistent with a more strongly inverse S-shaped weighting function relative to choices in affect-poor tasks. However, in a model comparison, they found that in affect-rich choices the majority of the participants were better described by the minimax heuristic, a choice strategy that neglects probabilities and only decides based on the minimum outcomes, than by CPT; in affect-poor tasks, in contrast, participants were better described by a strategy that is sensitive to probabilities. Thus, the differences apparent on the weighting function could indicate the selection of a different strategy. Similarly, Abdellaoui, Diecidue, and Öncüler (2011) reported that, relative to lotteries with immediate outcomes, people's responses to lotteries with delayed outcomes are consistent with a less inverse S-shaped curvature (indicating higher probability sensitivity). The authors hypothesized that this difference might be due to a decreased anticipated emotional reaction the more delayed lotteries are (cf. Rottenstreich & Hsee, 2001). Again, the impact observed on the weighting function might thus reflect the use of different strategies.

Thus, rather than merely describing contextual or individual differences in prospect theory's concepts, such as differences in probability sensitivity, one could go one step further and use differences on CPT's parameters to hypothesize about individual differences in terms of information processing and strategy use. By better understanding how information processing as embodied in heuristics manifests in CPT's parameters, we can gain a more cognitive perspective on CPT and its parametric framework (for an ecological account of the shape of CPT's functions, see Stewart, Chater, & Brown, 2006).

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