Center for Lifespan Psychology

Max-Planck-Institut für Bildungsforschung
Max Planck Institute for Human Development

Statistical Power in Structural Equation Modeling

Timo von Oertzen[1], Christopher Hertzog[2], Paolo Ghisletta[3], & Ulman Lindenberger[1]

[1]Max Planck Institute for Human Development, [2]Georgia Institute of Technology, Atlanta, USA, [3]University of Geneva, Switzerland

Introduction

Empirical studies are costly. Therefore, statistical power, defined as the probability to find a statistically reliable effect, is crucial for study design. Structural Equation Modeling (SEM) enables researchers to investigate random effects, such as variances and covariances, in latent variables. However, little is known about the power to detect random effects in SEM.

We study the power to detect random effects in SEM using Monte Carlo simulations, approximation methods, and the theory of power equivalence (PE). Using PE, power can be computed faster, and informed choices between models of identical power that differ in indicator reliability, number of observations, number of indicators, and other design factors can be made. This permits researchers to find the most resource-efficient model.

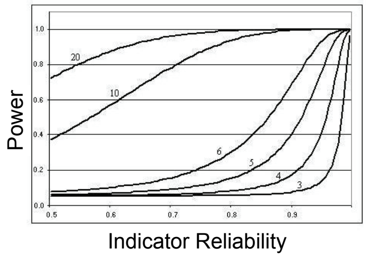
In addition, the project also uses SEM simulations as a conceptual tool to explore assumptions about change and selection in lifespan development.

Simulation Study:
Power to Detect Individual Differences in Change

Latent Growth Curve Models (LGCM) simultaneously structure changes in means, variances, and covariances. Using approximation methods, we investigated the power to detect covariance in change in a bivariate LGCM (Hertzog et al. 2006). Power was surprisingly low throughout for reliabilities below 0.9 and less than 4 to 5 measurement occasions.

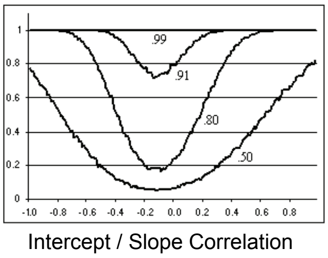
In another study (Hertzog et al., in press), we used Monte Carlo simulations to investigate the power for detecting slope variance using different statistics. Power again was low and depended on the specific significance test and the covariance between slope and intercept.

(A) **Power to Detect Covariance in Change**



N = 500. Lines refer to occasions of measurement. From Hertzog, et al., 2006.

(B) **Power to Detect Variance of Change**

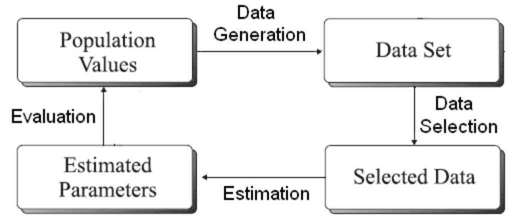


N = 500. Lines refer to indicator reliabilities. From Hertzog et al., in press.

Simulation of Change and Selection

In addition to exploring statistical power, the simulation engine developed in this project also can be used to study the effects of model misspecification, sample heterogeneity, and nonrandom attrition on model parameters and fit (von Oertzen et al., in press).

(C)



References

von Oertzen, T., Ghisletta, P., & Lindenberger, U. (in press). Simulating statistical power in latent growth curve modeling: A strategy for evaluating age-based changes in cognitive resources. In M. Crocker & J. Siekmann, (Eds.): *Resource adaptive cognitive processes*. Heidelberg: Springer Verlag.

Hertzog, C., von Oertzen, T., Ghisletta, P., & Lindenberger, U. (in press). Evaluating the Power of Latent Growth Curve Models to Detect Individual Differences in Change. *Structural Equation Modeling*.

Hertzog, C., Lindenberger, U., Ghisletta, P. & von Oertzen, T. (2006). On the power of multivariate latent growth curve model to detect correlated change. *Psychological Methods, 11*, 244-252.

Theory of Power Equivalence

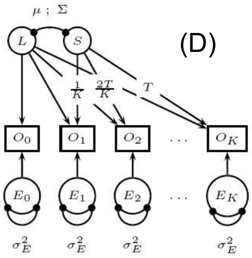
If two models with the same latent structure have the same power to detect every linear constraint between latent parameters at all significance levels, these models are called *power equivalent*.

We showed that power equivalence is a local equivalence relation, that is, if two models differ only in a submodel, and these two submodels are power equivalent, then the whole models are also power equivalent.

Using this theorem to combine many small power equivalent operations, any SEM can be reduced to a unique smallest model, for which power computations using Monte Carlo simulations are very fast; for an implementation, see www.powerequivalence.com.

Example: Same Power for Different Money

Assume a longitudinal observation of a linear process with measurement reliability 0.9 and a slope variance of 0.07 compared to an intercept variance of 100. Intercept and Slope are uncorrelated. With 11 occasions spread over 10 weeks, the power to detect a slope variance at α = .05 is 0.95; but the same power can be achieved much cheaper.



Study design with K occasions of measurement.

Occasions	Weeks	Power	Total Expenses
11	10.00	0.95	240,000 €
6	12.45	0.95	144,900 €
4	13.91	0.95	107,820 €
3	14.63	0.95	89,260 €

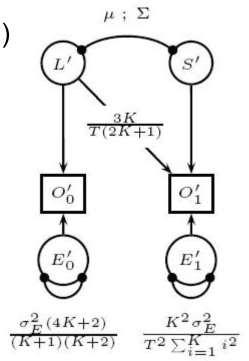
Costs at fixed power. Running costs per weak 2,000 €. Costs per measurement and participant = 200 €. N = 100.

Power Equivalent Computational Model

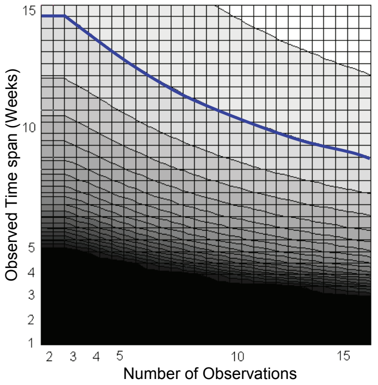
The model shown in Fig. D can be transformed into a computational model with two indicators and identical latent factors, and identical power to detect latent effects (see Fig. E). Assuming no covariance of intercept and slope, power is a constant if observation period T relates to the number K of observations as:

$$T = \sqrt{\frac{13320K^2 + 14800}{0.1(K+1)(100K^2 + 344.4K + 222.2)}}$$

(E)



Computational model that is power-equivalent to Fig. D. The K occasions of the original model are reduced to four.



Isopower lines for longitudinal data. The blue line corresponds to a power of 0.95 (see Table above). Lighter areas denote higher power.

This research is part of the project, **Formal Methods in Lifespan Psychology** (**Timo von Oertzen, Shu-Chen Li**, & **Ulman Lindenberger**). The project has three research foci. First, it studies properties of statistical methods, with an emphasis on structural equation modeling (PI: Timo von Oertzen). Second, the project develops neurocomputational models to integrate lifespan theorizing across behavioral and neuronal levels of analysis (PI: Shu-Chen Li). Third, the project has begun to explore brain signal analysis techniques (PI: Timo von Oertzen).
The research also is associated with the **Computer Science Research Circle** at the MPI for Human Development. The circle does research at the intersection between computer science and psychology, with an emphasis on algorithms and software development, brain signal analysis, and computer-controlled research environments.