

# Thermal Noise Reduction and Absorption Optimisation via Multi-Material Coatings

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## Abstract.

Future gravitational wave detectors (GWDs) such as Advanced LIGO upgrades and the Einstein Telescope are planned to operate at cryogenic temperatures using crystalline silicon (cSi) test-mass mirrors at an operation wavelength of 1550 nm. The reduction in temperature in principle provides a direct reduction in coating thermal noise, but the presently used coating stacks which are composed of silica (SiO<sub>2</sub>) and tantala (Ta<sub>2</sub>O<sub>5</sub>) show cryogenic loss peaks which results in less thermal noise improvement than might be expected. Due to low mechanical loss at low temperature amorphous silicon (aSi) is a very promising candidate material for dielectric mirror coatings and could replace Ta<sub>2</sub>O<sub>5</sub>. Unfortunately, such a aSi/SiO<sub>2</sub> coating is not suitable for use in GWDs due to high optical absorption in aSi coatings. We explore the use of a three material based coating stack. In this multi-material design the low absorbing Ta<sub>2</sub>O<sub>5</sub> in the outermost coating layers significantly reduces the incident light power, while aSi is used only in the lower bilayers to maintain low optical absorption. Such a coating design would enable a reduction of Brownian thermal noise by 25%. We show experimentally that an optical absorption of only  $(5.3 \pm 0.4)$  ppm at 1550 nm should be achievable.

## 1. Introduction

Future gravitational wave detectors (GWDs) such as Advanced LIGO upgrades and the low frequency (LF) detector within the Einstein Telescope [1, 2] are planned to operate at cryogenic temperature to reduce thermal noise. Operation at low temperatures requires a replacement substrate material for the presently used fused silica test-mass mirrors. Showing low mechanical loss at low temperatures [3], crystalline silicon (cSi) is planned to be used at an operation wavelength of 1550 nm to keep optical absorption low [4]. Due to zero crossings of the thermal expansion coefficient  $a_{\text{th}}$  of cSi, 120 K and 20 K are interesting operation temperatures with low thermo-elastic noise. For ET an even lower operation temperature of 10 K is planned [1].

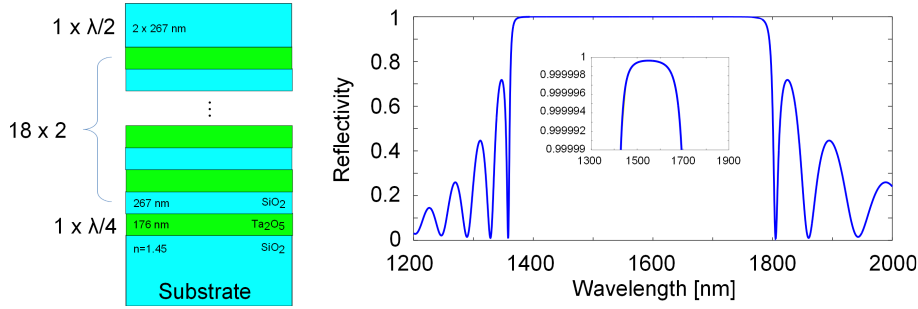
Coating thermal noise power spectral density is also proportional to the temperature of the coating, and so a reduction in temperature should in principle provide a direct reduction in thermal noise. However, the mechanical loss of many materials is temperature dependent, and cryogenic loss peaks have been identified in a doped-Ta<sub>2</sub>O<sub>5</sub>/SiO<sub>2</sub> coating stack [5] and in single layers of SiO<sub>2</sub> [8] and Ta<sub>2</sub>O<sub>5</sub> [10]. As a result of this increase in loss at low temperature, there is less thermal noise improvement than might be expected from operation at cryogenic temperature. Therefore, low mechanical loss at low temperature makes amorphous silicon (aSi) a very promising candidate material for dielectric mirror coatings and could replace Ta<sub>2</sub>O<sub>5</sub> in the presently used coatings.

A standard HR quarter-wavelength coating is composed of a stack alternating materials of differing refractive indices, where each layer has an optical thickness  $\delta = n \times t$  (where  $t$  is the geometric thickness) equal to  $\lambda/4$  for the wavelength of interest. The reflectivity depends on the number of bilayers in the coating, and on the difference in refractive index between the two materials. Thus, for a desired reflectivity, the total thickness of coating required depends on the difference in refractive index between the materials used.

Presently, stacks of alternating layers of SiO<sub>2</sub> and Ta<sub>2</sub>O<sub>5</sub> are used as highly reflective coatings for GWD test-mass mirrors. aSi has a significantly higher refractive index than Ta<sub>2</sub>O<sub>5</sub> ( $n_{\text{Si}} = 3.5$  and  $n_{\text{Ta}_2\text{O}_5} = 2.2$  at 1550 nm). Thus in comparison to a Ta<sub>2</sub>O<sub>5</sub>/SiO<sub>2</sub> coating, a coating formed from aSi/SiO<sub>2</sub> will have thinner high-index layers, and will achieve the same reflectivity with fewer bilayers. While a Ta<sub>2</sub>O<sub>5</sub>/SiO<sub>2</sub> coating stack requires 18 bilayers (plus a  $\lambda/2$  SiO<sub>2</sub> protection layer and a  $\lambda/4$  Ta<sub>2</sub>O<sub>5</sub> transitional layer) to achieve a high reflectivity with a transmission of  $T \approx 0.5$  ppm, a aSi/SiO<sub>2</sub> stack needs only 8 bilayers to achieve equivalent reflectivity. This significantly reduces the total thickness of the coating stack from 8.7  $\mu\text{m}$  to 3.7  $\mu\text{m}$  and provides a direct reduction in coating thermal noise. Furthermore, the cryogenic mechanical loss of aSi is at least a factor of 10 lower than that of Ta<sub>2</sub>O<sub>5</sub>, providing another direct reduction in coating thermal noise.

However preliminary studies of the optical absorption of a highly reflective Si/SiO<sub>2</sub> coating stack at 1550 nm have shown an absorption level of approximately 1000 ppm [11] which is substantially above the coating absorption requirement of future GWDs (which is in the order of 1 ppm). Thus optical absorption in aSi coatings would need be to significantly reduced for such an two-material arrangement containing aSi to be implemented

In this paper we explore the use of multi-material coatings and use measurements of aSi, Ta<sub>2</sub>O<sub>5</sub> and SiO<sub>2</sub> to demonstrate the optical and thermal noise performance of a three material coating. A more general theory of multi-material coatings, derived independently and in parallel to the work presented here, is given by Yam, Gras and Evans [13]. Here we investigate a three material design in which low absorption Ta<sub>2</sub>O<sub>5</sub> (combined with SiO<sub>2</sub>) is used as the high-index material in the outermost 7 bilayers (plus a  $\lambda/2$  SiO<sub>2</sub> protection layer and a  $\lambda/4$  Ta<sub>2</sub>O<sub>5</sub> transitional layer), which reflect more than 99.65 % of the incident light power, while aSi is used as the high-index material in the 5 lower bilayers, which can tolerate the higher optical absorption due to the relatively low light power present. This design should in principle allow the higher refractive index and lower mechanical loss of aSi to be exploited, without significantly reducing the optical performance of the coating stack.



**Figure 1.** Left: Schematic of a coating stack which consists of 18  $\lambda/4$  bilayers of alternating  $\text{SiO}_2$  and  $\text{Ta}_2\text{O}_5$  plus a  $\lambda/2$   $\text{SiO}_2$  protection layer on the surface and a  $\lambda/4$   $\text{Ta}_2\text{O}_5$  transition layer. The refractive indices at 1550 nm are  $n_{\text{SiO}_2} = 1.45$  and  $n_{\text{Ta}_2\text{O}_5} = 2.2$  resulting in quarter layer thicknesses of  $t_{\text{SiO}_2} = 267$  nm and  $t_{\text{Ta}_2\text{O}_5} = 176$  nm and therefore a total thickness of  $8.684 \mu\text{m}$ . Right: The transmission of this stack versus wavelength. At 1550 nm the transmission is  $T \approx 0.5$  ppm.

## 2. Model of the multi-material coating stack

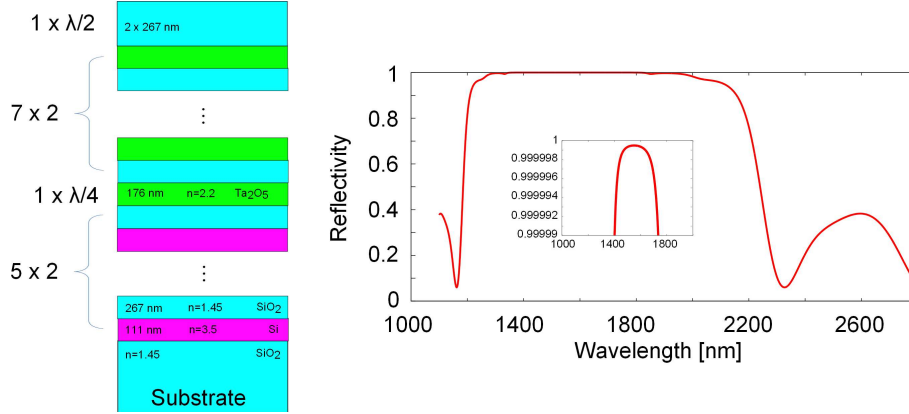
For the arm cavity end test masses (ETM) in cryogenic ET it is planned to use coating stacks of 18  $\lambda/4$  bilayers of alternating  $\text{SiO}_2$  and  $\text{Ta}_2\text{O}_5$  [1]. The refractive indices at 1550 nm are  $n_{\text{SiO}_2} = 1.45$  and  $n_{\text{Ta}_2\text{O}_5} = 2.2$ . At each layer boundary the change of refractive index causes a reflection following Fresnel's equations [14]. Using ray-transfer matrix formalism for calculating a series of reflecting boundaries, the actual reflectivity within each bilayer of the coating stack was calculated (for details see [15]). Additionally to 18  $\lambda/4$  bilayers, our model stack has a  $\lambda/2$  protection layer of  $\text{SiO}_2$  on the outside surface and a  $\lambda/4$   $\text{Ta}_2\text{O}_5$  transitional layer between substrate and the first  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  bilayer starting with  $\text{SiO}_2$ . The total reflectivity of the stack is  $T \approx 0.5$  ppm. The total thickness of the stack is  $8.684 \mu\text{m}$  of which  $5.340 \mu\text{m}$  consist of  $\text{SiO}_2$  and  $3.344 \mu\text{m}$  of  $\text{Ta}_2\text{O}_5$ . A schematic of this model stack is shown in Fig. 1, left, while Fig. 1, right, shows the reflectivity of the coating stack versus wavelength.

In a next step we create a coating stack which starts with 7 bilayers of  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  (plus a  $\lambda/2$  protection layer of  $\text{SiO}_2$  and a  $\text{Ta}_2\text{O}_5$  transitional layer) to reflect most of the laser light. Only 0.35% of laser light is transmitted by this stack. Additionally, 5 bilayers of  $\text{aSi}/\text{SiO}_2$  are used to reduce the total transmission to  $T \approx 0.5$  ppm, see Fig. 2. This reduces the number of single layers from 38 to 26 while the total thickness of the stack is reduced from  $8.684 \mu\text{m}$  to  $5.701 \mu\text{m}$  (by 34%). In Tab. 1 the total thickness of the coating stacks and of each material are listed.

## 3. Thermal noise of the coating stack

The power spectral density of the coating thermal noise is proportional to both the mechanical loss and the total thickness of the coating stack as shown in Eq. 1 [17]

$$S_x(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{1 - \sigma^2}{w_0 Y} \left\{ \phi_{\text{substrate}} + \frac{1}{\sqrt{\pi}} \frac{d}{w_0} \frac{1}{Y Y' (1 - \sigma'^2) (1 - \sigma^2)} \right. \\ \left. \times [Y'^2 (1 + \sigma)^2 (1 - 2\sigma)^2 \phi_{\parallel}] \right\}$$



**Figure 2.** Left: Schematic of the multi-material coating stack. The stack consists of 7  $\lambda/4$  bilayers of alternating  $\text{SiO}_2$  and  $\text{Ta}_2\text{O}_5$  plus a  $\lambda/2$   $\text{SiO}_2$  protection layer on the surface and a  $\lambda/4$   $\text{Ta}_2\text{O}_5$  transition layer. Below this stack another 5  $\lambda/4$  bilayers of alternating aSi and  $\text{SiO}_2$  follow. The refractive indices at 1550 nm are  $n_{\text{SiO}_2} = 1.45$ ,  $n_{\text{Ta}_2\text{O}_5} = 2.2$  and  $n_{\text{Si}} = 3.5$  resulting in quarter layer thicknesses of  $t_{\text{SiO}_2} = 267$  nm,  $t_{\text{Ta}_2\text{O}_5} = 176$  nm and  $t_{\text{Si}} = 111$  nm. This results in a total thickness of  $5.701 \mu\text{m}$  for the multi-material coating stack. Right: The transmission of the multi-material stack versus wavelength. Equivalent to the  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  stack the transmission at 1550 nm is  $T \approx 0.5$  ppm.

$$\left. \begin{aligned} &+ Y Y' \sigma' (1 + \sigma) (1 + \sigma') (1 - 2\sigma) (\phi_{\parallel} - \phi_{\perp}) \\ &+ Y^2 (1 + \sigma')^2 (1 - 2\sigma')^2 \phi_{\perp} \end{aligned} \right\}, \quad (1)$$

where  $f$  is the frequency in Hz,  $T$  is the temperature in Kelvin,  $Y$  and  $\sigma$  are the Young's modulus Poisson's ratio of the substrate,  $Y'$  and  $\sigma'$  are the Young's modulus Poisson's ratio of coating,  $\phi_{\parallel}$  and  $\phi_{\perp}$  are the mechanical loss values for the coating for strains parallel and perpendicular to the coating surface,  $d$  is the coating thickness and  $w_0$  is the laser beam waist.

To estimate the thermal noise associated with this coating stack, it is necessary to first estimate the mechanical loss of the stack. This can be calculated as a weighted (by thickness and by stiffness / Young's modulus) average of the loss of the individual coating materials as shown in Eqs 3 and 2 [18]. To calculate the loss of the 38 layer  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  coating stack and of the 26 layers the  $\text{SiO}_2/\text{Ta}_2\text{O}_5/\text{aSi}$  multi-material stack the loss values given in Tab. 2 were used.

$$Y_{\text{coating}} = \frac{Y_{\text{SiO}_2} t_{\text{SiO}_2} + Y_{\text{Ta}_2\text{O}_5} t_{\text{Ta}_2\text{O}_5} (+ Y_{\text{aSi}} t_{\text{aSi}})}{t_{\text{SiO}_2} + t_{\text{Ta}_2\text{O}_5} (+ t_{\text{aSi}})} \quad (2)$$

$$\phi_{\text{coating}} = \frac{Y_{\text{SiO}_2} t_{\text{SiO}_2} \phi_{\text{SiO}_2} + Y_{\text{Ta}_2\text{O}_5} t_{\text{Ta}_2\text{O}_5} \phi_{\text{Ta}_2\text{O}_5} (+ Y_{\text{aSi}} t_{\text{aSi}} \phi_{\text{aSi}})}{Y_{\text{coating}} t_{\text{coating}}} \quad (3)$$

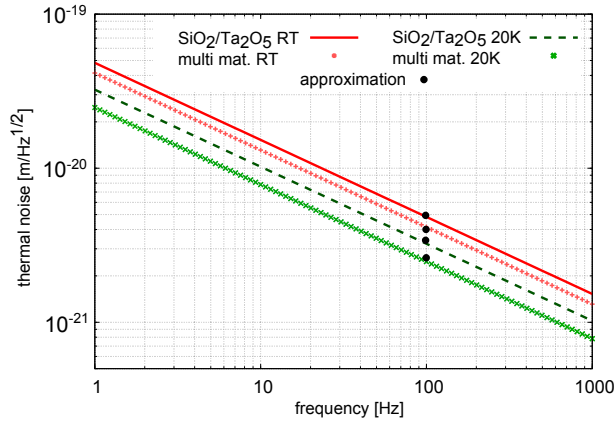
Equation 3 gives the total loss  $\phi_{\text{coating}}$  which is composed of the loss of different materials, while Eq. 2 allows the Youngs modulus of the components to be calculated [19]. The thickness per material used for the calculations can be found in Tab. 1. The results for the loss are summarized in Tab. 2. While at RT the loss for the multi-material stack is slightly higher than for the  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  stack, at lower temperatures due to the decreasing aSi loss the loss of the multi-material

**Table 1.** Thickness  $t$  for the  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  coating stack, for the multi-material stack ( $\text{SiO}_2/\text{Ta}_2\text{O}_5/\text{aSi}$ ) and for the individual materials.

	$\text{SiO}_2/\text{Ta}_2\text{O}_5$		multi mat.		
$t_{\text{total}}$ [ $\mu\text{m}$ ]	8.684		5.701		
	$\text{SiO}_2$	$\text{Ta}_2\text{O}_5$	$\text{SiO}_2$	$\text{Ta}_2\text{O}_5$	aSi
$t$ [ $\mu\text{m}$ ]	5.34	3.344	3.738	1.408	0.555
$Y$ [GPa]	72	147	140	98	96

**Table 2.** Loss  $\phi$  for  $\text{SiO}_2$ ,  $\text{Ta}_2\text{O}_5$  and aSi, and resulting loss for a  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  stack and the multi-material stack each at room temperature (RT), 120 K, 20 K and 10 K. The resulting Brownian thermal noise for the two stacks at each temperature was calculated using Eq. 1.

	loss $\phi \times 10^{-4}$					Brownian th. noise (100 Hz) $\times 10^{-21}$	
	$\text{SiO}_2$	aSi	$\text{Ta}_2\text{O}_5$	$\text{SiO}_2/\text{Ta}_2\text{O}_5$	multi mat.	$\text{SiO}_2/\text{Ta}_2\text{O}_5$	multi mat.
290 K	0.4 [6]	4.0 [7]	2.3 [6]	1.26	1.62	4.9	4.3
120 K	1.7 [8]	0.5 [7]	3.3 [9]	2.58	2.1	4.5	3.5
20 K	7.8 [8]	0.4 [7]	8.6 [9]	8.24	6.98	3.4	2.6
10 K	7 [8]	0.3 [7]	6 [9]	6.46	5.64	2.2	1.7


**Figure 3.** Brownian thermal noise for the  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  coating at room temperature (red line) and at 20 K (dashed, green line) and for the multi-material coating at room temperature (red pluses) and at 20 K (green crosses) calculated from the model used in [13]. The black dots show the corresponding results calculated with Eq. 1.

stack becomes lower. This results in a reduction of Brownian thermal noise which was calculated using Eq. 1. The results are also given in Tab. 2 and represented by the black dots in Fig 3. This approximation is in well agreement within less than 5% with the more precise model used in [13]. Figure 3 also shows the Brownian thermal noise for the  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  (lines) coating and the multi-material coating (crosses and pluses) at RT (red, upper two curves) and at 20 K (green, lower two curves) calculated from the model used in [13].

**Table 3.** Material and geometric parameters of the Corning 7980 mirror substrates and the cavity, as used for the simulations. Wavelength and temperature dependent parameters are given at 1550 nm and room temperature.

Material parameters		Ref.	Cavity geometry parameters	
index of refraction $n$	1.44	[23]	input power $P$	2, 130, 260 mW
thermo refr. coeff. $dn/dT$	$9.6 \times 10^{-6} / \text{K}$	[21]	length $L$	0.75 mm
specific heat $c$	770 J/(kg K)	[24]	beam waist $w_0$	82 $\mu\text{m}$
density $\rho$	2201 kg/m <sup>3</sup>	[21]	mirror length $D$	6.35 mm
thermal expansion $\alpha_{\text{th}}$	$0.52 \times 10^{-6} / \text{K}$	[21]	angle of incid. (AOI)	0°
thermal conductivity $k_{\text{th}}$	1.3 W/(mK)	[21]		

#### 4. Optical absorption of the Coating Stack

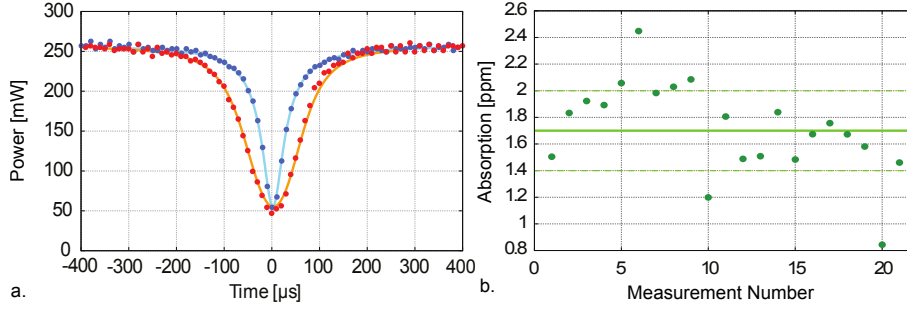
To estimate the optical absorption of this multi-material stack, we consider two experiments in which we measured the absorption of a SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> coating stack and a aSi/SiO<sub>2</sub> coating stack separately using *Photothermal Self-Phase Modulation* (PSM) [20]. In Subsec. 4.1 we present measurements of the optical absorption of a SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> coating stack at 1550 nm for the first time, while in Subsec. 4.2 a new aspect of the absorption results of a aSi/SiO<sub>2</sub> coating stack are discussed which were published in [11].

##### 4.1. SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub>

For the absorption measurement on SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> at 1550 nm a Fabry-Pérot cavity was used. The two cavity mirror substrates consisted of Corning 7980 glass [21]. The coatings were manufactured at *Advanced Thin Films* (ATF) [22] and were optimized for a finesse of 10 000 at an angle of incidence (AOI) of 0° at 1550 nm. A finesse of 10000 theoretically is reached by two identical lossless mirrors with intensity reflectivity of  $R_1=R_2=99.969\%$  (1-R=310 ppm). The transmission measured by the manufacturer on a mirror from this coating run was  $T=320$  ppm.

*4.1.1. Experimental Setup* For the Fabry-Pérot cavity setup, two identical curved mirrors  $M_1$  and  $M_2$  were clamped facing each other, separated only by a 0.75 mm viton seal. A PZT, driven by a function generator (FG), presses  $M_2$  into the viton seal and therefore changes the cavity length. Both mirrors have a convex radius of curvature (ROC) of 0.5 m resulting in a cavity waist of  $w_0 = 82 \mu\text{m}$ . The short cavity length results in a large free spectral range (FSR) of 200 GHz. The material properties and cavity geometry parameters of this setup are summarized in Tab. 3.

To linearize the mirror motion, the cavity length was modulated only in a small range around the resonance of approximately 0.3% of an FSR: this range varied slightly with the modulation frequency (the modulation voltage was constant for all measurements). The actual mirror motion for each modulation frequency and expansion and contraction of the PZT in each case was calibrated using side bands imprinted on the laser signal via an electro optical modulator (EOM) before entering the cavity, for details see [25]. The resonance peaks for the absorption measurement were detected with a photo detector (PD) in reflection of input mirror  $M_1$ . The reflected beam was separated from the incoming beam at a polarizing beam splitter cube (PBS) using a  $\lambda/2$  waveplate and a Faraday Rotator.



**Figure 4.** Measurement results for tantala HR-coatings at 1550 nm: In a. an example of deformed resonance peaks is shown from which the coating absorption was calculated. For an external lengthening of the cavity the resonance peak is broadened (simulation: orange line, measurement: red dots) while for an external shortening of the cavity the peak is narrowed (simulation: light blue line, measurement: dark blue dots). b. shows the results for the absorption of the single measurements (dark-green dots). The light-green line and dashed lines mark the mean value and standard deviation of  $\alpha = (1.7 \pm 0.3)$  ppm.

*4.1.2. Results* Resonance peaks detected at fast scan frequency or low power show no thermal effect (resonance peaks identical for lengthening and shortening of the cavity). Such measurements allow a determination of the reflectivity  $R_1$  of the input mirror  $M_1$  and the round trip loss  $\tilde{R}_2 - R_1$ , assuming  $R_1 = R_2$ . A reduction of the scan frequency and a power increase cause a thermal effect (resonance peaks are different for lengthening and shortening of the cavity as shown in Fig. 4 a.). These measurement additionally allow the determination of the absorption coefficient  $M_1$ . To measure the absorption, several single measurements of reflected resonance peaks were detected, partly showing a thermal effect, partly showing no thermal effect. The measured peaks were fitted as described in [27] with the input parameters shown in Tab. 3.

Altogether 34 single measurements were performed, 21 showing a significant thermal effect. The results for the reflectivities were  $1 - R_1 = (253.73 \pm 16.4)$  ppm and  $1 - \tilde{R}_2 = (276.3 \pm 13.3)$  ppm resulting in a cavity finesse of  $F = (11853 \pm 704)$  which was slightly better than specified.

The absorption could be obtained from 21 of the measurements which were made. The results of the single measurements are shown in Fig. 4 b., the light green bar marking the mean value of the 21 single results, the dashed light green bars their standard deviation. The resulting absorption for the input mirror coating is  $\alpha = (1.7 \pm 0.3)$  ppm (0.3 ppm  $\hat{=}$  17.6%). This is a promising low result for the optical absorption of  $\text{SiO}_2/\text{Ta}_2\text{O}_5$  at 1550 nm and, considering the thicker quarter wave layers due to the longer wavelength, in good agreement with absorption values at 1064 nm. (Note that this coating stack was not specified particularly for low absorption. Therefore, further absorption reduction might be possible.)

*4.1.3. Error Propagation* For a detailed discussion of the possible error caused by the input parameters see [25], where the expected error due to inaccurate material properties is analyzed. Since the material properties are identical or at least very similar to the parameters discussed in [25], their influence on the result also is very similar. The mirror dimensions are also identical to the experiment in [25], input power and mode matching always influence the result approximately linear. The only

**Table 4.** Results for the optical absorption  $\alpha$ , input mirror reflectivity  $R_1$ , and effective output reflectivity  $\tilde{R}_2$  (includes all optical round-trip loss apart from  $(1-R_1)$ ) resulting in an upper limit for the optical absorption  $\alpha_{M_3} = R_1 - \tilde{R}_2 - 2 \times \alpha_1$  of  $M_3$ .

parameter	result s-pol	result p-pol
1- $R_1$ [ppm]	$529 \pm 20$	$4717 \pm 212$
$1 - \tilde{R}_2$ [ppm]	$2902 \pm 103$	$7882 \pm 251$
$\alpha_{M_1}$ ( $= \alpha_{M_2}$ ) [ppm]	$1035 \pm 42$	$1428 \pm 97$
$\alpha_{M_3}$ [ppm]	$303 \pm 207$	$309 + 657 / -309$

parameters which differ strongly from the other experiment are the cavity length and the beam waist. These two parameters were changed each by  $\pm 10\%$  and the results of one single measurement were recalculated. An error of  $\pm 10\%$  in the cavity length influences the result by approximately  $\pm 10\%$ , where a cavity length assumed too long reduces the necessary absorption for the present thermal effect and inverse for a cavity length assumed too short. An error in the cavity length also affects the waist. An error of  $\pm 10\%$  in the length only affects the waist by approximately  $\pm 3\%$ , which causes an error of  $\pm 6\%$  in the absorption result. The large error of about  $10\%$  for such a short cavity is realistic. Since cavity length and waist depend on each other as discussed, both errors add. The summed up error is still within the standard deviation for the absorption result.

Therefore for the results obtained from this series of measurements the standard deviation caused by statistical fluctuations of the detected resonance peaks is assumed to dominate the error caused by inaccurate input parameters.

#### 4.2. aSi/SiO<sub>2</sub>

Results as well as experimental details for the optical absorption of aSi/SiO<sub>2</sub> coatings measured with PSM were published in [11]. The coatings were produced by *Tafelmaier* using *Ion Plating* [12]. In this experiment, the cavity consisted of three mirrors. The input and output mirrors  $M_1$  and  $M_2$  were identical and reflected the laser beam at an AOI of  $42^\circ$ , while the third mirror  $M_3$  was highly reflective (transmission negligible) at an AOI of  $3^\circ$ . The optical absorption of the input mirror of the aSi/SiO<sub>2</sub> cavity was  $\alpha_{M_1, p-pol} = (1428 \pm 97)$  ppm for p-polarisation and  $\alpha_{M_1, s-pol} = (1035 \pm 42)$  ppm for s-polarisation. The difference in absorption for the two polarisations originates from the difference in reflectivity and therefore different penetration depth of the incident laser beam into the coating stack. Considering a reduction of the laser light due to SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> bilayers to  $0.35\%$ , in s-polarisation  $3.6$  ppm of the input laser light would be absorbed (and  $5$  ppm in p-polarisation) which is in the same order as the absorption of SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> coatings.

It was observed before that the optical absorption of dielectric coating stacks at a large AOI can be significantly higher than at  $0^\circ$ . (For a SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> coating stack at an AOI of  $42^\circ$ , in [27] a rather high absorption of  $23$  ppm was measured using PSM and confirmed independently with a calorimetric measurement while due to manufacturer approximately  $1$  ppm was expected as demonstrated in [28].) Therefore, here we will discuss the optical absorption of mirror  $M_3$  which reflects almost at normal incidence from the cavity-round trip loss which is a result provided by PSM in addition to the measurement of the input mirror absorption. As  $M_1$



and  $M_2$  were coated in the same coating run, identically cleaned and then glued to a closed spacer in a clean room environment their absorption and reflectivity can assumed to be identical,  $\alpha_{M_1} = \alpha_{M_2}$  and  $R_1=R_2$ . In s-pol the reflectivity of  $M_1$  was found to be  $1 - R_1 = (529 \pm 20)$  ppm. The effective reflectivity of the output mirror was  $1 - \tilde{R}_2 = (2902 \pm 103)$  ppm where  $\tilde{R}_2$  includes the transmission of  $M_2$  as well as the absorption of all three mirrors. Considering known transmission and absorptions, for  $M_3$  an upper limit for the absorption of  $R_1 - \tilde{R}_2 - 2 \times \alpha_{M_3,s-pol} = (303 \pm 207)$  ppm remains. Equivalently, for p-polarisation where the reflectivities were  $1 - R_1 = (4717 \pm 212)$  ppm and  $1 - \tilde{R}_2 = (7882 \pm 251)$  ppm the upper limit for  $M_3$  absorption is  $\alpha_{M_3,p-pol} = (309 + 657/ - 309)$  ppm. An absorption of  $(309 + 303)/2 = 306$  ppm for a highly reflective aSi/SiO<sub>2</sub> coating stack at an AOI close to 0° is an even more promising result for aSi/SiO<sub>2</sub> coatings. The absorption of the incident laser beam would be reduced to 1.1 ppm by the upper SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> layers. All values are summarized in Tab. 4.

## 5. Discussion

In our model we replaced 11 bilayers of SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> by 5 bilayers of aSi/SiO<sub>2</sub> resulting in 26 instead of 38 single layers in total. Due to the high refractive index of  $n_{Si} = 3.5$  of the aSi layers, using the multi-material stack can achieve same high reflectivity of  $T \approx 0.5$  ppm while the total thickness is reduced from 8.684  $\mu\text{m}$  to 5.701  $\mu\text{m}$  by 34 %.

Due to reducing the number of bilayers and replacing a part of the Ta<sub>2</sub>O<sub>5</sub> layers by low loss aSi, the loss of the multi-material stack at low temperatures reduces by about 20 % at 120 K and 15 % at 20 K and 10 K. The Brownian thermal noise reduces by about 15 % at RT and 25 % low temperatures.

In our model for a three material coating stack 100 % of the laser power affects the top stack which is made of SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub>. So we can expect about  $(1.7 \pm 0.3)$  ppm absorption in this part of such a coating. (The largest amount of absorption occurs in the first 2-3 bilayers of a coating stack. So the smaller number of layers compared to the experiment will not reduce the absorption significantly.) Only 0.35 % of the laser power is transmitted into the aSi/SiO<sub>2</sub> part of the stack. This reduces the absorption of the input laser power to  $0.0035 \times (1035 \pm 42)$  ppm =  $(3.6 \pm 0.1)$  ppm. Therefore we conclude that a total optical absorption of as low as  $(5.3 \pm 0.4)$  ppm can be achieved for such a three material based coating stack which is in the order of the coating absorption requirement of about 1 ppm of future GWDs.

For an AOI close to 0° from the cavity round-trip loss a lower absorption of  $0.0035 \times 306$  ppm = 1.1 ppm was concluded for aSi/SiO<sub>2</sub> coatings which is promising for further absorption reduction.

The aSi in these coating stacks was not optimized to be low absorbing. Work on improving the optical absorption of such aSi layers provides the perspective to replace more Ta<sub>2</sub>O<sub>5</sub> layers by aSi to further reduce Brownian thermal noise. In our design so far the SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> also was not explicitly optimized to have low absorption. Absorptions of less than 0.7 ppm at 1064 nm [29] suggest the possibility of further reducing the SiO<sub>2</sub>/Ta<sub>2</sub>O<sub>5</sub> absorption also at 1550 nm.

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