DISTORTION OF NEUTRON STARS WITH A TOROIDAL MAGNETIC FIELD

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Models of rotating relativistic stars with a toroidal magnetic field have been computed for a sample of eight equations of state of cold dense matter. Non-rotating models admit important levels of magnetization and quadrupole distortion accompanied by a seemingly unlimited growth in size. Rotating models reach the mass-shedding limit at smaller angular velocities than in the non-magnetized case according to the larger circumferential equatorial radius induced by the magnetic field. Moreover, they can be classified as prolate–prolate, oblate–prolate, or oblate–oblate with respect to surface deformation and quadrupole distortion. Simple expressions for surface and quadrupole deformation are provided that are valid up to magnetar field strengths and rapid rotation.

Keywords: gravitational waves; magnetars; neutron stars.

1. Introduction

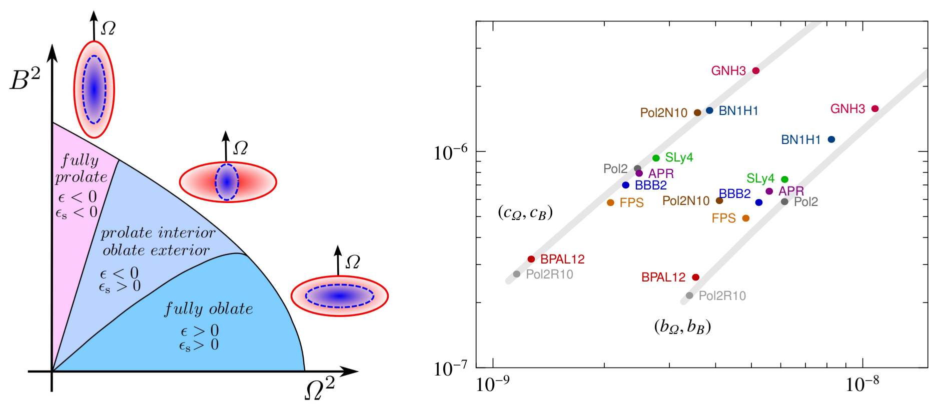
Neutron stars with a strong toroidal magnetic field have attracted increasing interest as the magnetically induced distortion of their matter distribution may lead to the quasi-periodic emission of gravitational waves,[1,2] for example, in the case of low-mass X-ray binaries (LMXBs). Moreover, strong magnetic fields are believed to power the electromagnetic activity of magnetars, which subsume both anomalous X-ray pulsars (AXPs) and soft-gamma repeaters (SGRs).[3,4] Models of relativistic stars with a toroidal magnetic field can be obtained within the standard formalism for stationary and axisymmetric relativistic stars,[5] since the electromagnetic stress–energy tensor then satisfies the same compatibility condition[6] as the stress–energy tensor of an unmagnetized perfect fluid in purely rotational motion. Based on this finding, numerical models of relativistic stars with a toroidal magnetic field have emerged[7,8] whereas the poloidal case was already studied a long time ago.[9]

2. Method and results

The neutron star matter is modeled as a perfectly-conducting perfect fluid at zero temperature, described by a one-parameter equation of state (EOS). For stationary and axisymmetric models in rigid rotation as considered hereafter, the general-relativistic line element in spherical coordinates (t, r, θ, ϕ) can be chosen as

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \Phi^2 r^2 \sin^2\theta^2(\mathrm{d}\phi - N^\phi \mathrm{d}t)^2 + \Psi^2(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) \qquad (1)$$

with gravitational potentials N, N^ϕ, Ψ, and Φ that are functions of (r, θ) alone. The toroidal magnetic field must then ensure that the Lorentz force is the gradient of a scalar potential, which is the case for $B = \lambda_0\,(e + p)\,\Phi N r \sin\theta$, where e is the proper energy density of the fluid, p is the fluid pressure, and λ_0 is the magnetization parameter. The field and matter equations are derived from the perfect-fluid case[5]



(a) (b)

Fig. 1. (a) Solution space restricted to magnetized and rotating Pol2 EOS models between the non-magnetized limit and the maximum field strength limit. Three distinct classes depending on the relative strength of magnetic and centrifugal forces can be distinguished. (b) Distortion coefficients $(b_Ω, b_B)$ for the surface deformation ϵ_s and $(c_Ω, c_B)$ for the quadrupole distortion ϵ obtained by perturbing non-magnetized and non-rotating models with a gravitational mass of $M = 1.4\,M_\odot$. In addition, coefficients for a Newtonian model `Pol2N10` with $R = 10\,$km, built upon a $\gamma = 2$ polytropic EOS, and its relativistic counterpart `Pol2R10` are shown. The grey-shaded bands correspond to models of increasing circumferential radius R with a gravitational mass of $M = 1.4\,M_\odot$, built with a sequence of $\gamma = 2$ polytropic EOSs of increasing polytropic constant κ.

by taking into account additional magnetic source terms, expressed in terms of B, and the magnetic potential $\tilde{M} = \lambda_0^2/(4\pi)\,(e+p)\,\Phi^2 N^2 r^2 \sin^2\theta$, supplemented by the above relation for B and the EOS.

The numerical models have been computed by means of a multidomain and surface-adaptive pseudo-spectral code for stationary and axisymmetric relativistic stars from the LORENE[a] package, extended to the case of the toroidal magnetic field specified above, and employing its standard sample of nuclear matter EOSs.

All models built with a certain EOS have the same rest mass corresponding to a gravitational mass of $M = 1.4\,M_\odot$ in the non-rotating and non-magnetized case. For the polytropic Pol2 EOS, defined by $p = \kappa\rho^\gamma$ with the polytropic exponent $\gamma = 2$ and the rest-mass density ρ, the adopted polytropic constant $\kappa = 83$ (in units in which $c = G = M_\odot = 1$) implies a circumferential radius of $R = 12\,$km.

Non-rotating models have been obtained up to large values of λ_0 (limited only by computational resources) for all EOSs, and the surface deformation $\epsilon_s = r_e/r_p - 1$, computed from the equatorial coordinate radius r_e and the polar coordinate radius r_p, as well as the quadrupole distortion $\epsilon = -(3/2)\mathscr{I}_{zz}/I$, obtained from Thorne's quadrupole moment $\mathscr{I}_{zz}$ and the moment of inertia I, attain considerable negative values as the magnetization is increased. The dimensions of the star even appear to grow without bounds. In turn, the volume-averaged magnetic field strength $\langle B^2\rangle^{1/2}$

[a] `http://www.lorene.obspm.fr`

always falls off after attaining a maximum value of several 10^{17} G.

The solution space of magnetized and rotating models, parametrized by $\langle B^2 \rangle$ and Ω^2, has been determined for the Pol2 EOS. Its lower part up to the maximum field strength limit, beyond which $\langle B^2 \rangle$ decreases, is schematically shown in Fig. 1 (a). Since the curves of vanishing surface deformation, $\epsilon_{\rm s} = 0$, and of vanishing quadrupole distortion, $\epsilon = 0$, are different, the models can be divided into three classes for which surface deformation and quadrupole distortion are (1) both prolate, (2) oblate and prolate, or (3) both oblate, depending on the relative strength of magnetic and centrifugal forces. In the rotating case, the mass-shedding limit of a magnetized star is reduced with increasing magnetization in agreement with the condition of geodesic motion at the stellar equator since the circumferential equatorial radius is enlarged by the toroidal magnetic field.

Magnetic field strengths and angular velocities of all known magnetars are small enough that ϵ can be well approximated by a linear function of $\langle B^2 \rangle$ and Ω^2, $\epsilon = -c_B \langle B_{15}^2 \rangle + c_\Omega \Omega_0^2$, with the distortion coefficients c_B and c_Ω shown in Fig. 1 (b), adopting normalized variables $B_{15} = B/(10^{15}\,\mathrm{G})$ and $\Omega_0 = \Omega/\mathrm{s}^{-1}$. An estimate for the type II superconducting case[10] is then given by $\epsilon = -c_B \langle B_{15}^2 \rangle^{1/2} \langle B_{\rm c2,15}^2 \rangle^{1/2} + c_\Omega \Omega_0^2$ below the second critical magnetic field strength $\langle B_{\rm c2}^2 \rangle^{1/2} \simeq 7.6 \times 10^{15}$ G. Likewise, $\epsilon_{\rm s}$ can be computed by using b_B and b_Ω instead of c_B and c_Ω. The Newtonian model `Pol2N10` with $R = 10$ km and its relativistic counterpart `Pol2R10` demonstrate that relativistic effects strongly attenuate both the surface deformation induced by the toroidal magnetic field and the quadrupole deformation in general. In contrast, the rotational surface deformation is only slightly reduced since the centrifugal force is more effective at larger distances from the rotation axis where relativistic effects have already weakened.

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