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Abstract:

One of the main results of the Wendelstein 7-AS stellarator (Major radius 2 m, average plasma radius 0.18 m, magnetic field 2.5 T, low shear) is the achievement of the H-mode confinement in ECR- and NBI-heated plasmas. There is a strong dependence on the external rotational transform; the H-mode confinement can be found only in a narrow window close to $\iota = 1/2$. Viscous damping and the interaction with a neutral background are the only mechanisms proposed so far as damping mechanism, which inhibits the poloidal shear flow. Viscous damping is computed in collisional approximation and in weakly collisionless approximation (plateau regime). Large values of the poloidal viscosity are found on rational magnetic surfaces, while in the neighborhood of low order rational the viscosity is very small. Islands provide a mechanism for enhanced momentum transport in radial direction, which leads to an effective shear viscosity. In Wendelstein 7-AS islands exist on the "natural" rational surfaces with ι = 5/9, 5/10, 5/11 ... Furthermore, a next generation of islands exist on ι = 10/19, 10/21. Experimental results in Wendelstein 7-AS confirm the hypothesis that H-mode only can arise if none of these islands exists in the plasma. The regions of rotational transform predicted by this hypothesis roughly agree with those of the experiment.
The extrapolation towards Wendelstein 7-X shows that there a similar case is expected. Numerical calculations of islands are made in vacuum fields; the evolution of the islands with rising plasma pressure is unknown. Finally a qualitative model of H-mode development is discussed.

1. Introduction.

H-mode operation in Wendelstein 7-AS has been achieved with either ECRH or NBI heating and at various magnetic fields (1.25 T and 2.5T)[1,2,3]. A specific feature of this phenomenon is that the H-mode exists only in a narrow regime of the rotational transform in the very neighborhood of $\iota = 0.5$. The features of the H-mode are similar to those of a tokamak: reduction of the Hα-emission, onset of poloidal rotation and a 30% improvement of the energy confinement time, which is accompanied by a reduction of the turbulence level. With the onset of the shear flow a transport barrier arises in a 2-5 cm wide region just inside the separatrix. However, the critical dependence on the rotational transform is a particular feature of this low shear stellarator and in the following paper an attempt will be made to understand this phenomenon by investigating the specific influence of the magnetic geometry on the onset of poloidal shear flow.
In the literature several mechanisms to drive poloidal plasma rotation have been proposed:

The recoil effect of lost orbits[4,5]
The anomalous Stringer spin-up[6]
The turbulent Reynolds stresses[7]

Interaction with neutral particles, which carry momentum to the wall, and magnetic pumping or viscous damping are those mechanisms, which are able to inhibit the shear flow. At the first glance the plasma neutral interaction by charge exchange has no particular dependence on the rotational transform, however, in considering a plasma equilibrium with a frictional term in the momentum balance[8], the electric potential exhibits a strong dependence on the closure of field lines. Although this frictional model of a plasma equilibrium is non-linear and also has bifurcation properties – the relationship to Bénard convection has been investigated in Ref. 8 – the effect of inertial force has been neglected and thus classical or anomalous Stringer spin-up cannot be described in this model.
The magnetic pumping process dissipates the energy of plasma rotation and slows down any shear flow. In the collisional limit the Braginskii viscosity is the appropriate starting point and it is possible to compute the poloidal and toroidal damping coefficients in any stellarator geometry[9]. However, in the collisional limit the damping coefficients do not depends strongly on the rotational transform. This is quite understandable, since the mean free path is short compared with any length of closed magnetic field lines. The closure of field lines is a discontinuous function of the rotational transform and the

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[1] V. Erckmann et al, Phys. Rev. Lett. 70, 14 (1983) 2086
[2] F. Wagner et al. *Plasma Phys. Control. Fusion* 36 (1994) A61
[3] M. Hirsch et al. *Plasma Phys. Control. Fusion* 40 (1998) 631 - 634
[4] K. Itoh, S. Inoue-Itoh, Phys. Rev Lett. **60**, (1988) 2276
[5] K. C. Shaing, E.C. Crume, Phs. Rev. Lett. **63**, (1989) 2369
[6] A.B. Hassam, J.F. Drake, Phys. Fluids **B5**, (1993) 4022
[7] P.H. Diamond, J.B. Kim, Phys. Fluids **B3**, 1626 (1991)
[8] H. Wobig, Z. Naturforsch. **41a** (1986) 1101 1110
In this paper the friction term in the momentum balance $\nabla p = \boldsymbol{j} \times \boldsymbol{B} - \alpha \boldsymbol{v}$ de-couples pressure surfaces and magnetic surfaces and links the momentum balance to Ohm's law. At low order rational surfaces convective cells can arise, which prevent the onset of poloidal shear flow.
[9] H. Wobig, J. Kisslinger, Plasma Phys. Control. Fusion **37**, (1995) 893 - 922

length of closed field is small on low order rational surfaces and large on high order rational surfaces. Low order surfaces are those where $\iota = m/n$ has small values of m and n. In Wendelstein 7-AS the dominant low order rationals are ι = 1/2, 1/3, 2/5 and the so-called natural resonances ι = 5/9, 5/11, 5/12. Low order rational surfaces are isolated in the sense that in their very neighborhood only high order rational surfaces exist. This property has been invoked to explain that in the low shear stellarator Wendelstein 7-A this region close to low order rationals is very robust against magnetic field perturbations[10], a property, which also holds for drift orbits of passing particles[11].
Plasma parameters in the boundary region of Wendelstein 7-AS are in the plateau regime and neoclassical theory is needed to compute the viscosity. Neoclassical viscosity in non-axi-symmetric stellarators has been computed by Shaing[12], who found a resonance behavior of the viscous forces at rational values of the rotational transform. Furthermore, the viscosity has a non-linear dependence on the radial electric field, which is the origin of bifurcated solutions of the condition of ambipolarity. The combined effect of plasma-neutral interaction and viscous damping has been investigated by Talmadge et al.[13], who concluded that L–H transition will not occur if the neutral density is large enough and the local maxima in the poloidal viscosity as function of the radial electric field do not occur. The present paper follows the same analysis as in Ref. 12; however, the main interest will be focussed on the influence of the rotational transform on the viscosity and the viscous forces. The regions of strong and weak damping in Wendelstein 7-AS will be identified.
In the first part of the paper the collisional limit of the viscous forces will be discussed, the analysis starts from an ideal equilibrium and the general form of the inviscid flow tangential to magnetic surfaces. In the plateau limit the viscous forces exhibit a strong dependence on the rotational transform, which will be discussed in chapter 3. Some numerically examples will be presented on order to give a qualitative explanation of the experimental results in Wendelstein 7-AS. Finally a qualitative picture of the L–H transition in W 7-AS will be outlined.

2. Momentum balance

In a rotating plasma Lorentz forces and pressure gradient are not the only forces, which affect the momentum balance. There are inertial forces and viscous forces, which are described by the divergence of the viscous pressure tensor. If turbulence exists, the inertial forces are supplemented by the turbulent Reynolds stresses. The dominant forces are the Lorentz force and the pressure gradient and these mainly point into the radial direction. Inertial forces and viscous forces also have components parallel to the magnetic field and the balance between these forces has a strong influence on the poloidal and toroidal rotation of the plasma. In the one-fluid model the time averaged momentum balance of the plasma is

$$\rho \boldsymbol{v} \bullet \nabla \boldsymbol{v} = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} - \nabla \bullet \pi \qquad \text{Eq. 1}$$

[10] H. Wobig, Z. Naturforsch. 42a, 1054 – 1066 (1987)
[11] H. Wobig, D. Pfirsch, IPP-report III/245, see also paper at this workshop
[12] K.C. Shaing, Phys. Fluids B5 (11) (1993) 3841
[13] J.N. Talmadge, B.J. Peterson, D.T. Anderson, F.S.B. Anderson, H. Dahl, J.L. Shohet, M. Coronado, K.C. Shaing, M. Yokoyama, M. Wakatani, Proc. 15th IAEA Conf. on Contr. Fusion, Seville, 1994, Paper CN–60/A–6–I–6

where v is the macroscopic velocity of the plasma and π the viscous stress tensor. In a fluctuating plasma the inertial forces are are modified by the Reynolds stresses. The pressure is the time averaged pressure. In a turbulent plasma the velocity is the mean velocity V plus a fluctuating term δv and the intertial term is the sum of two terms depending on the mean velocity and the divergence of the Reynolds stress, which arises from the fluctuating part δv. Let us assume that an equilibrium state exists, which satisfies the balance equation (1). In such a rotating equilibrium pressure surfaces and magnetic surfaces are decoupled. Concerning axially symmetric configurations equilibria with inertial forces have been investigated by Zehrfeld and Green[14]. The issue of steady flow in stellarator equilibria has been treated by Greene et al.[15], and by Kovrizhnykh and Shchepetov[16]. Momentum balance of a rotating plasma in stellarator geometry including turbulent terms and anomalous radial losses has also been investigated by Wobig[17]. We anticipate the existence of toroidally closed pressure surfaces p = const. and average the momentum balance over the pressure surface. This provides one with the following relations parallel to the magnetic field and parallel to the plasma currents

$$< \boldsymbol{j} \bullet \rho \boldsymbol{v} \bullet \nabla \boldsymbol{v} > = - < \boldsymbol{j} \bullet \nabla p > - < \boldsymbol{j} \bullet \nabla \bullet \pi > \qquad \text{Eq. 2}$$

and

$$< \boldsymbol{B} \bullet \rho \boldsymbol{v} \bullet \nabla \boldsymbol{v} > = - < \boldsymbol{B} \bullet \nabla p > - < \boldsymbol{B} \bullet \nabla \bullet \pi > \qquad \text{Eq. 3}$$

Because of and $\nabla \bullet \boldsymbol{j} = 0$ and $\nabla \bullet \boldsymbol{B} = 0$ these relation reduce to

$$< \boldsymbol{j} \bullet \rho \boldsymbol{v} \bullet \nabla \boldsymbol{v} > = - < \boldsymbol{j} \bullet \nabla \bullet \pi > \qquad \text{Eq. 4}$$

and

$$< \boldsymbol{B} \bullet \rho \boldsymbol{v} \bullet \nabla \boldsymbol{v} > = - < \boldsymbol{B} \bullet \nabla \bullet \pi > \qquad \text{Eq. 5}$$

These relations show that the momentum balance parallel to the magnetic field and parallel to the plasma currents is governed by inertial forces and viscous forces. These equations include all processes mentioned in the introduction. The non-linear coupling of the radial plasma flow ρv to the poloidal velocity is the reason for classical or anomalous Stringer spin-up. Reynolds stresses are the result of turbulent inertial forces. The viscous stress tensor of the thermal plasma tends to slow down a macroscopic plasma rotation. This stress tensor is a linear functional of the distribution function

$$\pi = m \int \left(\boldsymbol{v} : \boldsymbol{v} - \frac{v^2}{3} \right) f(\boldsymbol{v}) d^3 \boldsymbol{v} \qquad \text{Eq. 6}$$

If there a large fraction of lost orbits the distribution function differs from a Maxwellian and the lost orbits give rise to an additional term in the stress tensor, which is not retarding but accelerating. In principle all effects mentioned above will contribute to the force balance in the rotating plasma, the main issue is to identify the dominating ones. The viscous stress tensor is zero in case of a Maxwellian. Therefore small viscous forces are expected in a collisional plasma where collisions tend to restore a Maxwellian. In the collisionless regime or in the plateau regime the deviation from a Maxwellian can be large leading to a large viscous force. Furthermore details of particle orbits have more and more influence on the distribution function and therefore the specific structure of

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[14] H.P. Zehrfeld, B.J. Green, Nucl. Fusion 12, (1972) 569

[15] J.M. Greene , J.L. Johnson, K.E. Weimer, N.K. Winsor, Phys. Fluids 14, (1971). 1258

[16] L.M. Kovrizhnykh S.V. Shchepetov, Nucl. Fusion 29, (1989), 667

[17] H. Wobig, Plasma Phys. Control. Fusion 38 (1996) 1053

the magnetic field will strongly influence the viscous forces. In the viscous damping of poloidal and toroidal shear flow the specific features of the magnetic field become apparent and therefore a unified picture of H-mode behavior in stellarator is difficult to obtain.

Ideal MHD equilibrium

In the momentum balance eqs 1 and 2 the self-consistent magnetic field has to be introduced. However, if inertial and viscous forces are small compared with the other ones one may approximate the magnetic field by the field of the ideal MHD equilibrium, which satisfies the condition

$$\boldsymbol{j} \times \boldsymbol{B} = \nabla p \qquad \text{Eq. 7}$$

On magnetic surfaces the Hamada coordinate system can be introduced, which is characterized by straight magnetic field and a Jacobian equal to unity. In this coordinate system $s,\theta\ \varphi$ the base vectors are defined by

$$\boldsymbol{e}_p = \nabla s \times \nabla \varphi \quad ; \quad \boldsymbol{e}_t = -\nabla s \times \nabla \theta \qquad \text{Eq. 8}$$

$\boldsymbol{e}_p$ is the poloidal base vector and $\boldsymbol{e}_t$ the toroidal base vector. The magnetic field can be represented by

$$\boldsymbol{B} = \chi'(s)\boldsymbol{e}_p + \psi'(s)\boldsymbol{e}_t \qquad \text{Eq. 9}$$

and the plasma current by

$$\boldsymbol{j} = J'(s)\boldsymbol{e}_p + I'(s)\boldsymbol{e}_t \qquad \text{Eq. 10}$$

χ,ψ are the poloidal and toroidal fluxes and J,I the poloidal and toroidal currents. The plasma current density can also be written in the form

$$\boldsymbol{j} = -p'(\psi)\boldsymbol{e}_p + I'(\psi)\boldsymbol{B} \qquad \text{Eq. 11}$$

which in stellarators without toroidal current reduces to a current parallel to the poloidal base vector.

Zero-order plasma flow

An arbitrary rotation in poloidal and toroidal direction can be superimposed onto this equilibrium. Since in ideal equilibrium model there is no coupling between plasma velocity and momentum balance this rotation is undetermined as long as dissipative terms and inertial forces are neglected. This lowest order rotation lies in magnetic surfaces and satisfies the equations

$$-\nabla\Phi + \boldsymbol{v}_0 \times \boldsymbol{B} = 0 \quad ; \quad \nabla \bullet \rho \boldsymbol{v}_0 = 0 \qquad \text{Eq. 12}$$

which leads to the ansatz

$$\boldsymbol{v}_0 = -E(\psi)\boldsymbol{e}_p + \Lambda(\psi)\boldsymbol{B} \qquad \text{Eq. 13}$$

The two flux functions E and Λ describe a poloidal and a parallel motion along magnetic surfaces. In this ideal model without dissipative forces there can exist a poloidal and a toroidal flow of the plasma. Using eq. 4 the plasma velocity can also be decomposed in a poloidal and a toroidal component. In next order these quantities must be computed from the balance between spin-up forces and dissipative forces (friction and viscosity).

The viscous forces (or the magnetic pumping effect) are proportional to the lowest order velocities. These surface-averaged viscous forces are

$$\begin{pmatrix} -<\boldsymbol{e}_p \bullet \nabla \bullet \pi> \\ <\boldsymbol{B} \bullet \nabla \bullet \pi> \end{pmatrix} = \begin{pmatrix} \mu_p & \mu_b \\ \mu_b & \mu_t \end{pmatrix} \begin{pmatrix} E \\ \Lambda \end{pmatrix} \qquad \text{Eq. 14}$$

The viscous tensor π is defined by

$$\pi = m\int \left(\boldsymbol{v} : \boldsymbol{v} - \frac{v^2}{3} \right) f(\boldsymbol{v}) d^3\boldsymbol{v} \qquad \text{Eq. 15}$$

which in Chew-Goldberger Low approximation yields

$$<\boldsymbol{e}_p \bullet \nabla \bullet \pi> = <\left(p_{\parallel} - p_{\perp}\right)\boldsymbol{e}_p \bullet \frac{\nabla B}{B}>$$

$$<\boldsymbol{B} \bullet \nabla \bullet \pi> = <\left(p_{\parallel} - p_{\perp}\right)\boldsymbol{B} \bullet \frac{\nabla B}{B}> \qquad \text{Eq. 16}$$

The formulation of the viscous forces given in eqs. 11 is valid in any regime of collisionality; the problem left is the computation of the anisotropic pressure, which is different in every regime of collisionality.
The parallel and the perpendicular components of the pressure tensor are defined by

$$p_{\parallel} = m\int v_{\parallel}^2 f d^3\boldsymbol{v} \quad ; \quad p_{\perp} = m\int \frac{v_{\perp}^2}{2} f d^3\boldsymbol{v} \qquad \text{Eq. 17}$$

The distribution function f must be computed by kinetic theory.

Collisional plasma

Although the boundary plasma in the Wendelstein 7-AS is in the plateau regime we consider the collisional limit first. In collision-dominated plasmas the anisotropic part of the pressure can be approximated by[18]

$$\left(p_{\parallel} - p_{\perp}\right) = -3\tau P \frac{\boldsymbol{B}}{B^2} \bullet \boldsymbol{B}\nabla \boldsymbol{v_0} \qquad \text{Eq. 18}$$

$P = nkT$ is the scalar pressure and τ is the ion-ion collision time. This leads to the averaged viscous forces in the following form

$$\begin{pmatrix} -<\boldsymbol{e}_p \bullet \nabla \bullet \pi> \\ <\boldsymbol{B} \bullet \nabla \bullet \pi> \end{pmatrix} = 3\tau P \begin{pmatrix} C_p & C_b \\ C_b & C_t \end{pmatrix} \begin{pmatrix} E \\ \Lambda \end{pmatrix} \qquad \text{Eq. 19}$$

The geometric coefficients C are given by

$$C_p = <\left(\boldsymbol{e}_p \bullet \frac{\nabla B}{B}\right)^2> \; ; \; C_t = <\left(\boldsymbol{B} \bullet \frac{\nabla B}{B}\right)^2> \; ; \; \boldsymbol{C}_b = <\left(\boldsymbol{B} \bullet \frac{\nabla B}{B}\right)\left(\boldsymbol{e}_p \bullet \frac{\nabla B}{B}\right)> \qquad \text{Eq. 20}$$

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[18] This follows from the bulk viscosity given by Braginskii (see Ref. 9)

C_p couples the toroidal velocity to the poloidal force and C_t the toroidal velocity to the toroidal viscous force. The coupling of poloidal and parallel flow is described by the coefficient C_b. In stellarators without any symmetry - neither axial symmetry nor helical symmetry - there exists a finite threshold for spin-up, which is given by

$$R_{11} = 3\iota P\left(\frac{C_b^2}{C_t} - C_p\right) \qquad \text{Eq. 21}$$

The term in brackets is purely geometrical and can be evaluated for any magnetic field. In tokamaks this threshold id zero, since the toroidal rotation is not slowed down by magnetic pumping id the toroidal is neglected. The following figure shows a comparison of this geometrical factor C_p between Wendelstein 7-AS and Wendelstein 7-X and a tokamak configuration (see Ref. 9).

In axially symmetric configurations like tokamaks a viscous damping in toroidal direction does not exist. The poloidal variation of B is determined by the 1/R-dependence. Fig. 5 shows the poloidal viscous coefficient of a circular tokamak in comparison with advanced stellarators. W 7-X and a tokamak are nearly equivalent in this respect whereas the W 7-AS device exhibits larger poloidal damping coefficients.

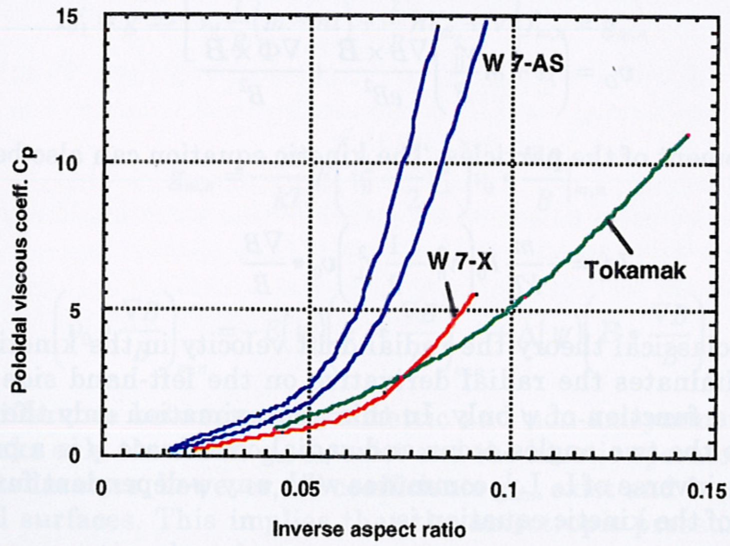

Fig. 1: Comparison with tokamak. Poloidal viscous coefficient C_p

Plateau Regime

In the Wendelstein 7-AS experiments the plasma parameters are not in the collisional regime. At the plasma boundary the mean free path is in the order of 10 to 20m, which puts the plasma into the plateau regime and in the bulk plasma we find the plasma in the collisionless regime. Viscous damping must be computed using kinetic theory.

The viscous coefficients of the collisional regime are independent of the electric field and show no particular dependence on the structure of the magnetic field. This feature changes in the plateau regime where details of particle orbits become important[19]. Furthermore, the electric field affects the particle orbits and thus modifies the viscous

[19] M.Coronado , H. Wobig, Phys. Fluids 29, (1986) 527

damping. This effect has been of interest in the context of L-H-transition. K.C. Shaing[20] has computed the poloidal and toroidal viscous forces in non-axisymmetric configurations and has studied the influence of the electric field. His main result was the existence of a local maximum of the poloidal viscous force as function of the radial electric field (or the poloidal Mach number). This phenomenon may give rise to bifurcation and thus a transition between states of different rotation velocities. In the following we consider the plateau regime and write the kinetic equation in the following form

$$Lf_1 = -\left\{ v_D \bullet \nabla F'(\psi) + F v_{\parallel} \frac{\boldsymbol{B}}{B} \bullet \nabla \left[\frac{2 v_{\parallel}}{v_{th}^2} U \right] \right\} \qquad \text{Eq. 22}$$

with

$$L = v_{\parallel} \frac{\boldsymbol{B}}{B} \bullet \nabla + v_D \bullet \nabla - C \qquad \text{Eq. 23}$$

v_D is the drift velocity and C the collision operator. U is the parallel component of the macroscopic velocity. The drift velocity is the sum of the magnetic drift and the electric drift:

$$v_D = \left(\mu + m \frac{v_{\parallel}^2}{B} \right) \frac{\nabla B \times \boldsymbol{B}}{eB^2} + \frac{\nabla \Phi \times \boldsymbol{B}}{B^2} \qquad \text{Eq. 24}$$

μ is the magnetic moment of the particles. The kinetic equation can also be written in the form

$$Lf_1 = -\frac{m}{kT} F_0 \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) v_0 \bullet \frac{\nabla B}{B} \qquad \text{Eq. 25}$$

In the standard neoclassical theory the radial drift velocity in the kinetic operator L is neglected, which eliminates the radial derivative on the left-hand side of eq. 25. The electric potential is a function of ψ only. In this approximation only the spatial derivatives with respect to the two angles occur and radial coordinate ψ is a parameter. This implies that that the inverse of L, L^{-1}, commutes with any ψ-dependent function.
The formal solution of the kinetic equation is

$$f_1 = -\frac{m}{kT} \left\{ E(\psi) L^{-1} F \omega_p + \Lambda(\psi) L^{-1} F \omega_b \right\} \qquad \text{Eq. 26}$$

with

$$\omega_p = \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) e_p \bullet \frac{\nabla B}{B} \;\; ; \;\; \omega_b = \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) \boldsymbol{B} \bullet \frac{\nabla B}{B} \qquad \text{Eq. 27}$$

and the viscous coefficients

$$\mu_p = \frac{m^2}{kT} < \int \omega_p L^{-1} F \omega_p d^3 v > \;\; ; \;\; \mu_t = \frac{m^2}{kT} < \int \omega_b L^{-1} F \omega_b d^3 v >$$

$$\mu_b = \frac{m^2}{kT} < \int \omega_p L^{-1} F \omega_b d^3 v > \qquad \text{Eq. 28}$$

[20] K.C. Shaing, Phys. Fluids 5 No. 11 (1993) , 3841-384

L^{-1} is the inverse operator of L. In the collisional regime the kinetic operator is -C and we get the results described above. In this regime the viscous coefficient do not depend on the details of the particle orbits, however, this new feature arises in the plateau or long-mean-free-path regime. In this case the result may depend strongly on the details of the magnetic field and the rotational transform.

The standard approximation in the plateau regime is a Krook collision term instead of the Landau operator

$$L \Rightarrow v_{\parallel} \frac{\mathbf{B}}{B} \bullet \nabla + \mathbf{v}_E \bullet \nabla + \nu \qquad \text{Eq. 29}$$

$\mathbf{v}_E$ is the electric drift

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \qquad \text{Eq. 30}$$

This approximation allows a Fourier transformation of the equation, which clearly shows the resonance character at rational values of the rotational transform

$$Lf_1 = g \Rightarrow \left[v_{\parallel} \frac{i}{R}(m - n\iota) + \frac{i}{R} nV_E + \nu \right] f_{m,n} = g_{m,n} \qquad \text{Eq. 31}$$

with

$$g_{m,n} = -\frac{m}{kT} F \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) \mathbf{v}_0 \bullet \frac{\nabla B}{B} \Big|_{m,n} \qquad \text{Eq. 32}$$

and

$$\left(\mathbf{v}_0 \bullet \frac{\nabla B}{B} \right)_{m,n} = -E(\psi) \left(\mathbf{e}_p \bullet \frac{\nabla B}{B} \right)_{m,n} + \Lambda(\psi) \left(\mathbf{B} \bullet \frac{\nabla B}{B} \right)_{m,n} \qquad \text{Eq. 33}$$

Here a strong difference between axisymmetric and non-axisymmetric configurations arises. In tokamaks only the terms $g_{m=0,n}$ exist and the kinetic operator has no resonance at $m-n\iota = 0$. In stellarators, however, all coefficients $g_{m,n}$ exist and L may become "small" at these rational surfaces. This implies that the anisotropic pressure and the viscous damping are large on rational surfaces.
These equations only apply to the region outside magnetic islands where toroidal magnetic surfaces exist. The surfaces close to the separatrix of magnetic islands are distorted by the neighboring islands and the Fourier series of B or $\ln B$ has large components with the periodicity of the islands. In other words: the corrugation of the magnetic surfaces in the neighborhood of islands will lead to enhanced magnetic pumping or enhanced viscous forces.
Numerical computations of collisional viscosity in the neighborhood of magnetic islands have been described by Wobig and Kisslinger[21], who confirmed the suggestion above.
The Fourier expansion of B in Hamada coordinates has the general form

$$\ln B = \sum_{l,m}^{\infty} a_{l,m} \cos(l\theta - m\varphi) + b_{l,m} \cos(l\theta + m\varphi) \qquad \text{Eq. 34}$$

which after insertion into eqs. 27 yields

[21] H. Wobig, J. Kisslinger, IPP-report IPP 2/334 Jan. 1997

$$\mu_p = A_0 \sum_{l,m}^{\infty} g_{l,m}^{(-)} a_{l,m} + g_{l,m}^{(+)} b_{l,m}$$

$$\mu_t = C_0 \sum_{l,m}^{\infty} g_{l,m}^{(-)} (l\iota - m)^2 a_{l,m} + g_{l,m}^{(+)} (l\iota + m)^2 b_{l,m}$$

$$\mu_b = B_0 \sum_{l,m}^{\infty} g_{l,m}^{(-)} (l\iota - m) a_{l,m} + g_{l,m}^{(+)} (l\iota + m) b_{l,m} \qquad \text{Eq. 35}$$

with

$$g_{l,m}^{(+)} = \frac{1}{4} \int_{-\infty}^{+\infty} \frac{\nu\left(u_{\|}^2 - \frac{1}{2}u_{\perp}^2\right)}{\nu^2 + \left(u_{\|}(l\iota + m) + lV_E\right)^2} F\left(u^2\right) u_{\perp} du_{\perp} du_{\|} \qquad \text{Eq. 36}$$

and

$$g_{l,m}^{(-)} = \frac{1}{4} \int_{-\infty}^{+\infty} \frac{\nu\left(u_{\|}^2 - \frac{1}{2}u_{\perp}^2\right)}{\nu^2 + \left(u_{\|}(l\iota - m) + lV_E\right)^2} F\left(u^2\right) u_{\perp} du_{\perp} du_{\|} \qquad \text{Eq. 37}$$

u is the normalized velocity $u = v/v_{th}$ (v_{th} is the thermal velocity), V_E is the normalized poloidal velocity $V_E = v_E R/(r v_{th})$, $v_E = E/B$ is the electric drift velocity, ν is the normalized collision frequency $\nu = R/\lambda$, λ = mean free path.
In the limit of large collision frequencies ν these coefficients scale like $1/\nu$ and we obtain the collisional limit discussed above. In the other limit of small collision frequency the integrand scales like a delta function with a peak at

$$u_{\|}(l\iota - m) + lV_E = 0 \qquad \text{Eq. 38}$$

In case of zero electric field the coefficient $g_{l,m}^{(-)}(\nu, V_E, \iota)$ is

$$g_{l,m}^{(-)}(\nu, V_E, \iota) \sim \frac{1}{|l\iota - m|} \qquad g_{l,m}^{(-)} \sim \frac{1}{|l\iota - m|} \qquad \text{Eq. 39}$$

and exhibits a singularity at rational values of the rotational transform. Finite electric fields remove this resonance and the coefficient decreases. It should by pointed out that only the viscous coefficient μ_p is affected by this resonance.

The viscous forces derived above are proportional to the lowest order flow in magnetic surfaces, derivatives of the components E and Λ do not occur. The reason is that the radial drift of the particles on the left-hand side of eq. 26 has been neglected. This term, however, becomes important for particles with small or zero parallel velocity. Drift orbits of these particles exhibit large deviations from magnetic surfaces and thus correlate the plasma flow in radial direction. This effect leads to a shear viscosity, whih in the collisional regime is small, however which may become large in the plateau or collision-less regime. A solution of the drift-kinetic equation taking into account radial drift has been presented by Hastings[22], who derived a differential equation for the radial electric field in stellarators. Standard neoclassical theory only yields algebraic equations for the electric field.

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[22] D.E. Hastings, Phys. Fluids **28**, (1985) 334

Numerical results

Numerical calculations of the collisional viscous coefficients have been presented in Ref. 20. As mentioned above the viscous damping increases in the neighborhood of magnetic islands as will be shown in the following figure

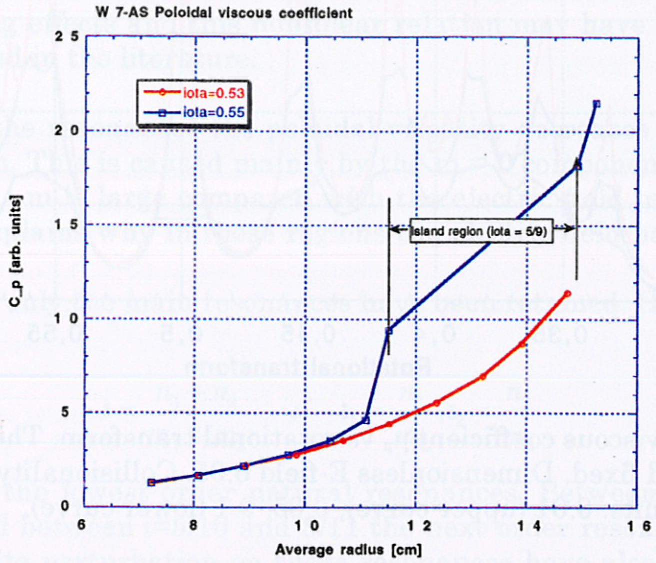

Fig. 2: Poloidal viscous damping coefficient C_p in Wendelstein 7-AS. The upper curve exhibits the effect of the 5/9 island. Inside the island the viscosity cannot be computed, since here a self-consistent equilibrium is needed. The line inside the island is meaningless. The magnetic configuration used for these computations is displayed in Fig. 8.

Although these calculations are only valid outside the islands they support the idea that magnetic islands inhibit poloidal rotation. Magnetic islands are fixed in space and they play the role of a rock in a river: the laminar flow is disturbed and slowed down. The lack of an accurate description of the equilibrium, however, in the island makes it difficult to assess the influence of the island. With growing beta the position and the size of the island may change. The computations in Fig. 2 have been made with the vacuum field.

Viscous forces in the collisional regime are the product of a geometrical factor and some plasma parameters. The geometrical factor can be easily evaluated and used for comparing various configurations. This factorization is no longer the case in the plateau regime and in addition to the Fourier spectrum of B the viscous coefficients depend on the collision frequency, the electric field and the rotational transform. For this reason the numerical calculations presented in the following will concentrate on the dependence on iota only considering the Fourier coefficients of B, the collision frequency and the electric field a free parameters. An example the coefficient μ_p as function of the rotational transform is shown in the following figure.

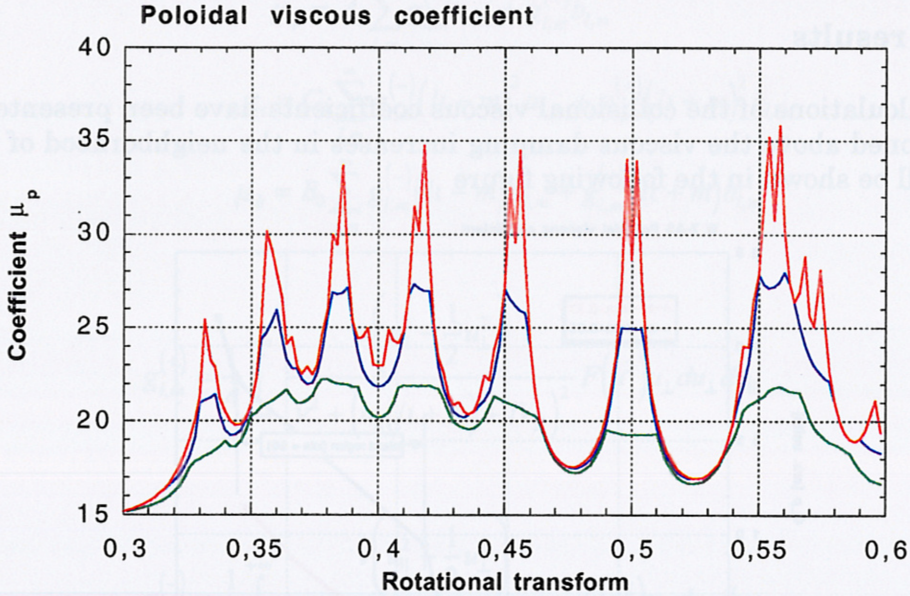

Fig. 3: Poloidal viscous coefficient μ_p vs rotational transform. The electric field is small and fixed. Dimensionless E-field 0.05. Collisionality in dimensionless units: 0.01 (upper curve), 0.05, 0.1 (lower curve).

To obtain this figure the Fourier series ln B is modeled with finite coefficients at ι = 5/n, n = 9, 10, 11, 12, 13, 14, 14 , which simulates the natural islands in W 7-AS. Islands by error fields exhibit resonances at ι = 1/2, 2/5, 1/3 etc. In that case further peaks would occur in Fig. 5. It is interesting to note that in the vicinity of ι = 1/2 a minimum of viscous damping occurs. The reason is the absence of low order rational surfaces in this region. Resonances can only arise at ι = m/l with large m,l where the Fourier coefficients of ln B are small and insignificant.

The electric field removes the resonance as shown in the next figure

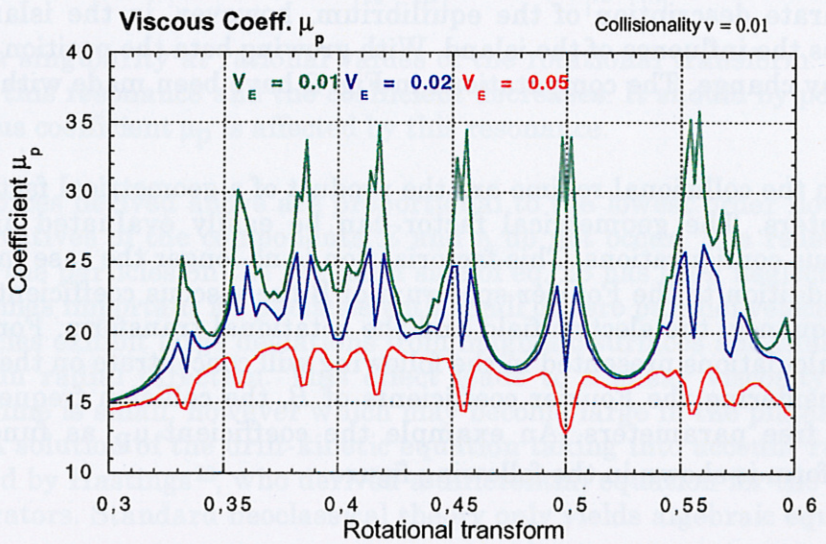

Fig. 4: Poloidal viscous coefficient μ_p vs rotational transform. The electric field increases by a factor five. The collisionality is kept fixed, ν = 0.01.

The electric field is fixed and does not depend on the rotational transform. However, it should be noticed that the E-field depends on poloidal plasma rotation. If the poloidal rotation is inhibited by large viscous damping the radial electric field is small or zero. It is expected that in the presence of magnetic islands plasma rotation is damped by large magnetic pumping, which is consistent with zero electric field. However, it cannot be excluded that also a case with large electric field and small viscous damping can be maintained close to rational surfaces. The electric field is the result of a balance between driving and damping effects and this nonlinear relation may have bifurcating solutions as has been discussed in the literature.

In regions outside the resonances the poloidal viscosity decreases slowly with growing rotational transform. This is caused mainly by the m = 0 components in eq. 38. In these regions the term $n\iota$ - m is large compared with the electric field term and the collision frequency, which explains why in these regions the electric field has little effect on the poloidal viscosity.
In the figures above only the main resonances have been retained. Higher order resonances occur at

$$\iota = \frac{n_1 + n_2}{m_1 + m_2} \quad ; \quad \iota_1 = \frac{n_1}{m_1}, \iota_2 = \frac{n_2}{m_2} \qquad \text{Eq. 40}$$

where ι_1 and ι_2 are the lowest order natural resonances. Between ι = 5/9 and 5/10 we expect ι = 10/19 and between ι=5/10 and 5/11 the next order resonance is 10/21. In the following figure finite perturbation on these resonances have also been introduced. In these calculations (Fig. 4 and Fig. 5) the Fourier coefficient have been chosen freely. As will be shown later, islands exist on these surfaces. Therefore it has to be expected that the poloidal viscosity is even larger in the neighborhood of the island, but in the island the standard kinetic theory is not applicable. Therefore, the local minima of the poloidal viscosity, which are shown in Figs. 3 and 4 must be considered with caution. In a realistic geometry with islands these minima may not exist, however a rigorous theory is lacking.

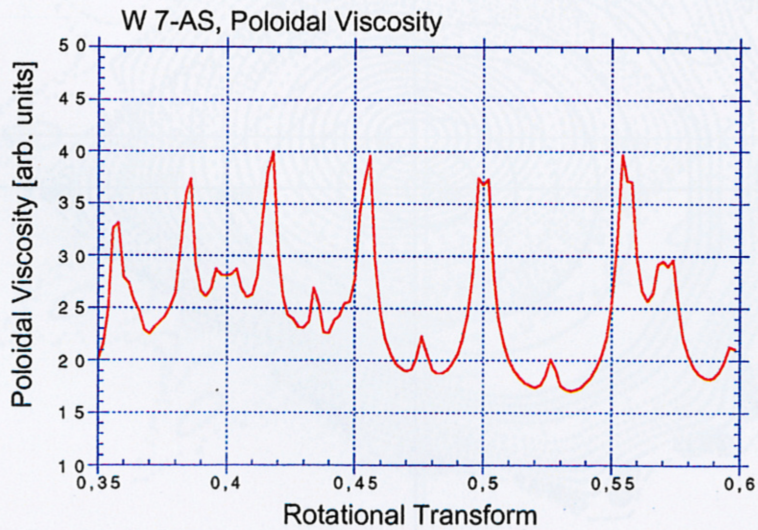

Fig. 5: Poloidal viscous coefficient μ_p vs rotational transform. Collisionality and electric field are kept fixed. Additional resonant Fourier coefficients at ι = 10/19 and 10/21 have been introduced.

3. H-mode windows in Wendelstein 7-AS

In the minima to the right and the left of $\iota = 1/2$ the viscous damping is small and nearly independent of the electric field. In this regime H-mode confinement has been observed in Wendelstein 7-AS. Plasma parameters in the boundary region of W 7-AS are in the plateau regime, therefore the present theory is applicable. Because of the low viscous damping spin-up mechanisms easily can overcome the damping and lead to plasma rotation and shear flow without a significant reaction on the damping mechanism. This has to be distinguished from the case of rational surfaces where the viscous damping is enhanced and electric shear flow cannot evolve. Therefore the electric field stays at low values and the viscous damping remains large as shown in Fig. 6. It should be noted that in case of magnetic islands on rational surfaces the theory does not apply to this region, however in the neighborhood of the low order rational surface only high order rational surfaces without islands exist and there the theory is valid.
The region above $\iota = 1/2$ shows a slightly smaller damping rate than the region below $\iota = 1/2$. This may explain why in the experiment H-mode confinement predominantly was found in this region. As pointed out above only the basic resonances at $\iota = 5/n$ were retained in the numerical calculation. It is obvious that higher order resonances occur, if the Fourier spectrum of ln B (see eq.) contains not only the main harmonics but also higher ones. Between $\iota = 5/10$ and 5/9 the next order resonance in the Farey tree is $\iota = 10/19$, there a local maximum of viscous damping is to be expected. Islands on these surfaces would enhance the damping in this region and inhibit the spin-up of shear flow.

Magnetic surfaces in Wendelstein 7-AS

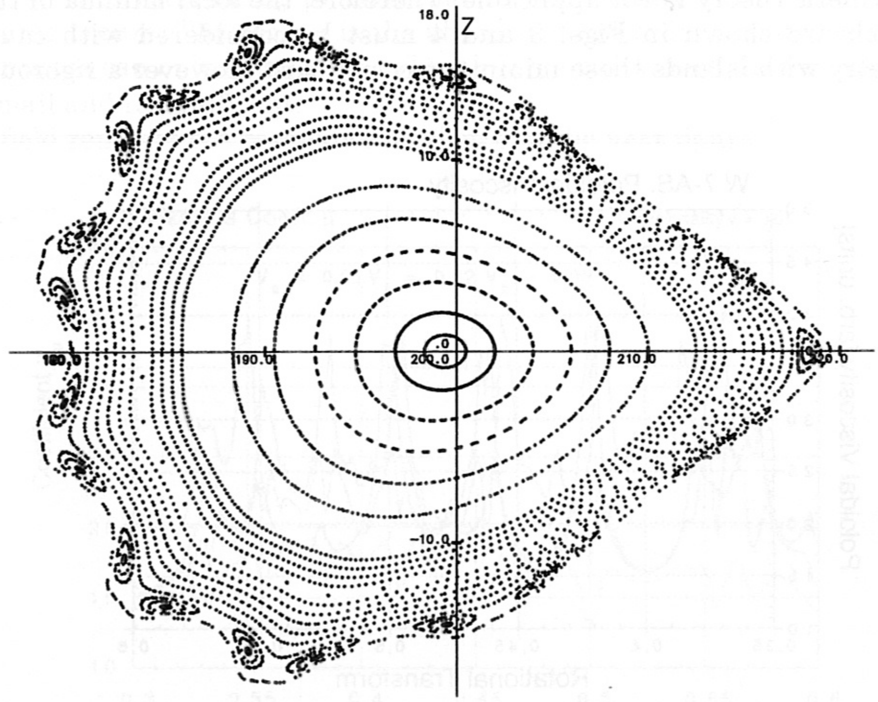

Fig. 6: Magnetic surface in Wendelstein 7-AS. Rotational transform $\iota = 10/19$ at the boundary. $I_t/I_m = 0.6351$. There are 19 islands in the boundary region. Iota in the center $\iota=0.5097$.

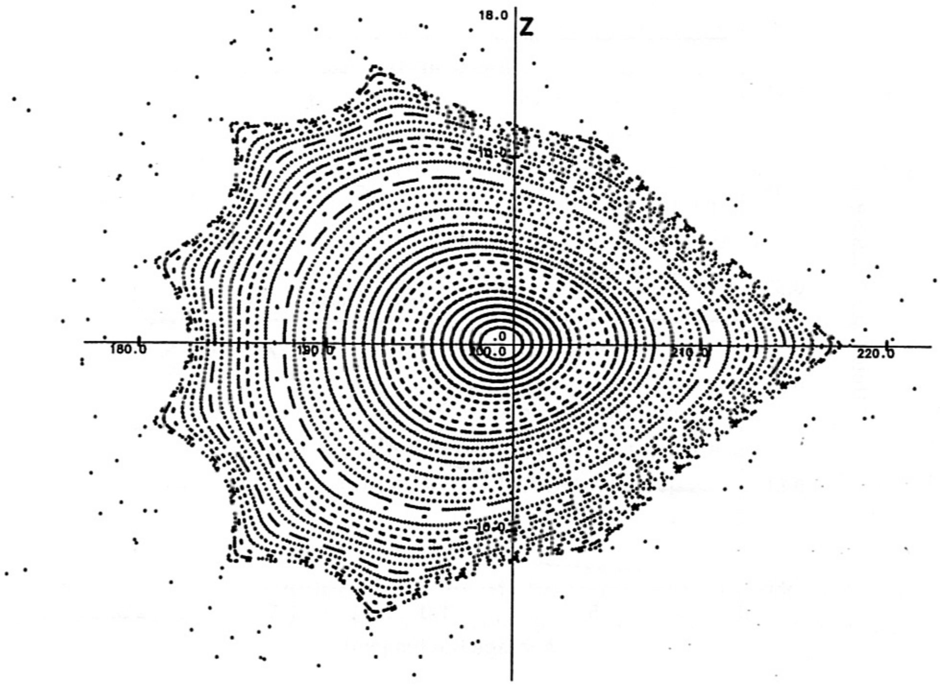

Fig. 7: Magnetic surface in Wendelstein 7AS. Iota = 5/9 at the boundary.
It/Im = 0.7320,

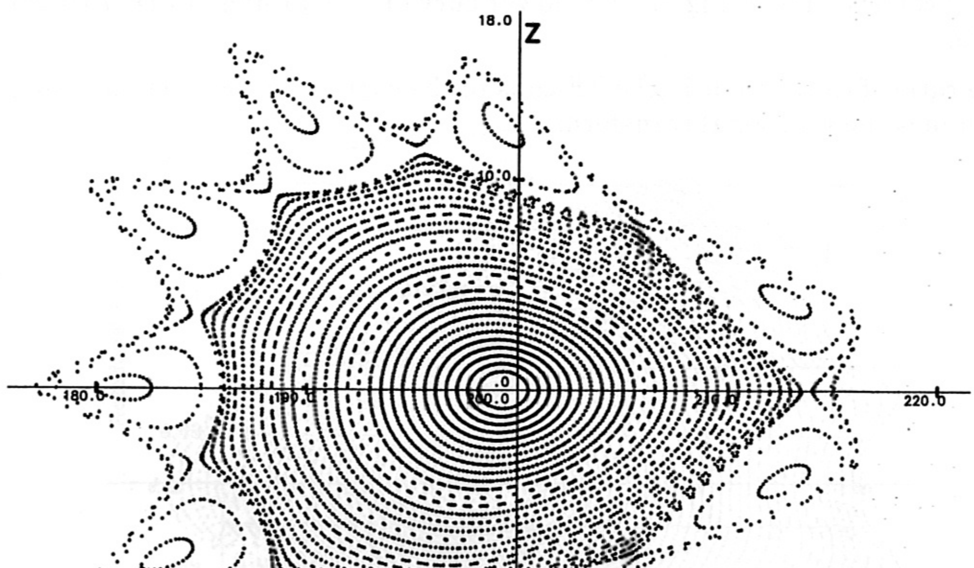

Fig. 8: Magnetic surface in Wendelstein 7AS. Iota = 5/9 at the boundary.
Iota(0)=0.5442, It/Im = 0.7883

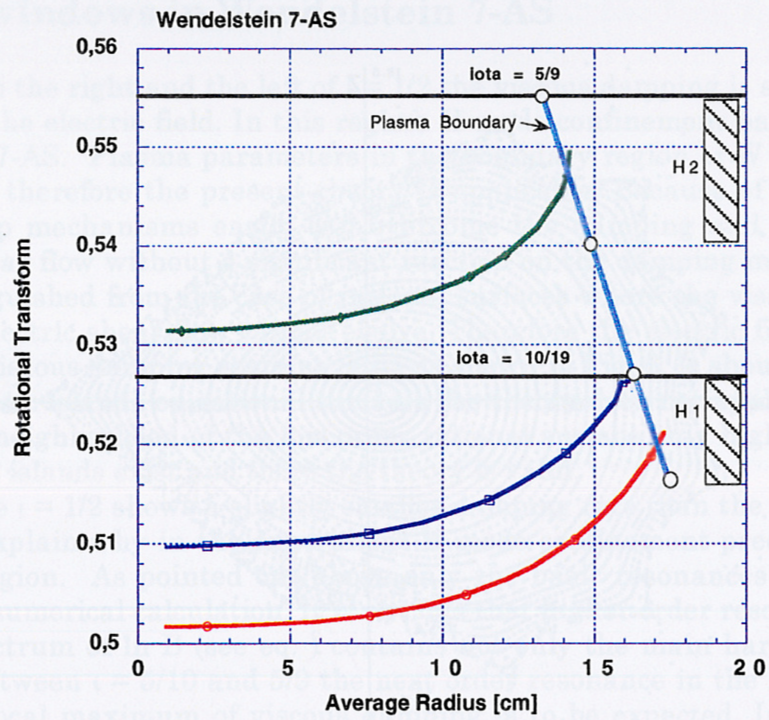

Fig. 9: Rotational transform vs average radius. Upper curve (circles, It/Im = 0.7320) is the transform of Fig. 8, curve in the middle (squares It/Im = 0.6351) corresponds to Fig. 6. The lower curve is the profile of Fig. 7 (It/Im = 0.5968).

It/Im is the ratio of the current in the TF-coils to the current in the modular coils. This ratio determines the rotational transform.

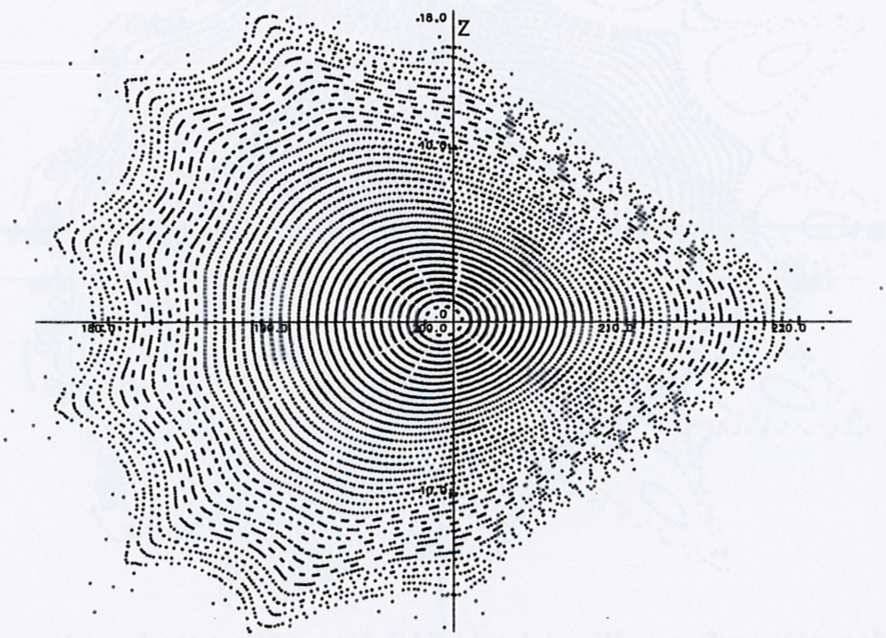

Fig. 10: Poincaré plot of magnetic surfaces in Wendelstein 7-AS. Iota(0) = 0.5017. The profile of the rotational transform is shown in Fig. 9 (lower curve). It/Im=0.5968.

H-mode confinement in Wendelstein 7-AS was observed in the narrow region above ι = 1/2 and ι below 1/2. The iota-profiles of the upper H-mode region are shown in Fig. 4. In all cases no large islands exist in the boundary region. In the center of this region, however, the 10/19 island series exists, as is shown in Fig. 1. This may be the reason why the H-mode window is divided in two parts with a center where H-mode is not achievable. The presence of these islands in the boundary can inhibit the spin-up of poloidal shear flow. Raising the iota-profile above the upper profile in Fig. 4 leads to the existence of large 5/9 islands in the boundary region, which is demonstrated in Fig. 3. This clearly will prevent spin-up of poloidal shear flow.

In summary, the following hypothesis can be established:

> Spin-up of poloidal shear flow and H-mode confinement is inhibited by enhanced poloidal viscosity in the presence of the following islands at ι = 10/21, 5/10, 10/19, 5/9, 5/11 and lower. If these islands exist somewhere inside the plasma H-mode is not possible.

The following figure displays the iota profiles of the vacuum field and the hatched areas denote those regions, where according to the previous hypothesis H-mode may be possible.

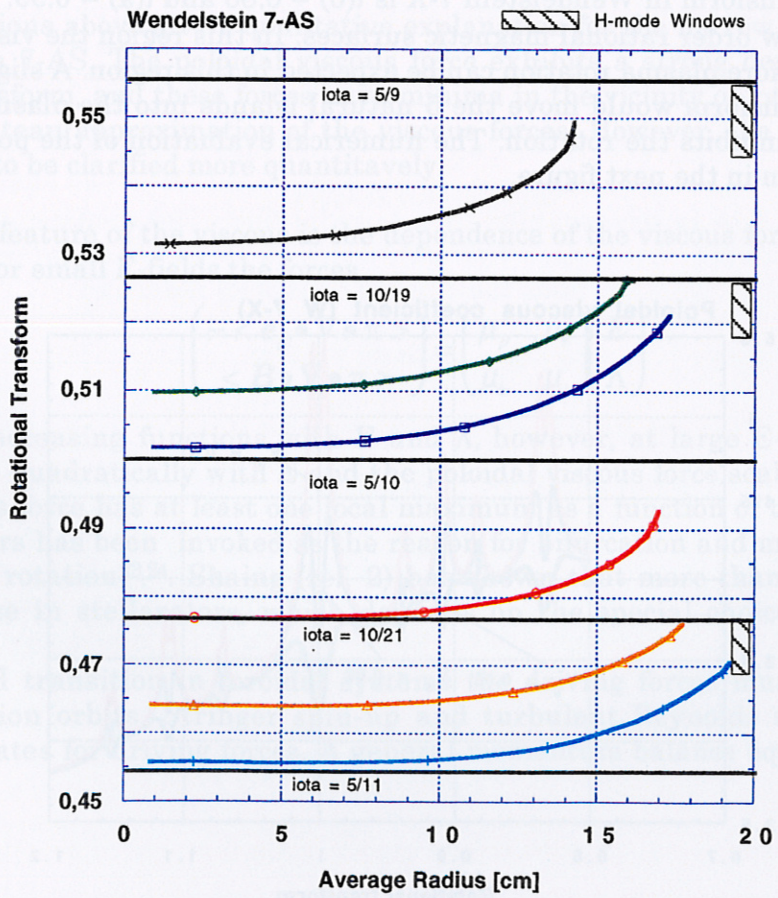

Fig. 11: Rotational transform of the vacuum field in Wendelstein 7-AS.

There is a small region just below $\iota = 1/2$ in Fig. 11, where spin-up could exist according to the hypothesis, however, the experiment does not verify this. An explanation may be that the $\iota = 1/2$ is also distorted by symmetry breaking error fields and therefore may have a strong damping effect.

The role of islands on viscous damping can be understood by following considerations. Particles transport across islands is enhanced, which has been demonstrated by Monte-Carlo calculations (ref) and particles also carry momentum across the island. Thus the rotating layers on both sides of an island are intimately correlated by the anomalous transport leading to a reduction of the velocity shear. In regions without islands radial momentum transport is provided by drift surfaces, which differ from magnetic surfaces, and by the effect of collisions. Especially transition particles, particles between being trapped and passing, with their large deviation from magnetic surfaces contribute most to the radial momentum transfer. This is the plateau approximation. In the region of islands the radial step width is determined by the islands size and the radial momentum transfer is large in regions with large islands.

4. Extrapolation to Wendelstein 7-X

The rotational transform in Wendelstein 7-X is $\iota(0) = 0.86$ and $\iota(a) = 0.99$. This is also a regime without low order rational magnetic surfaces. In this region the viscous damping is small and therefore plasma rotation can be expected in this region. A slight increase of the rotational transform would move the 5 natural islands into the plasma region and viscous damping inhibits the rotation. The numerical evaluation of the poloidal viscous coefficient is shown in the next figure.

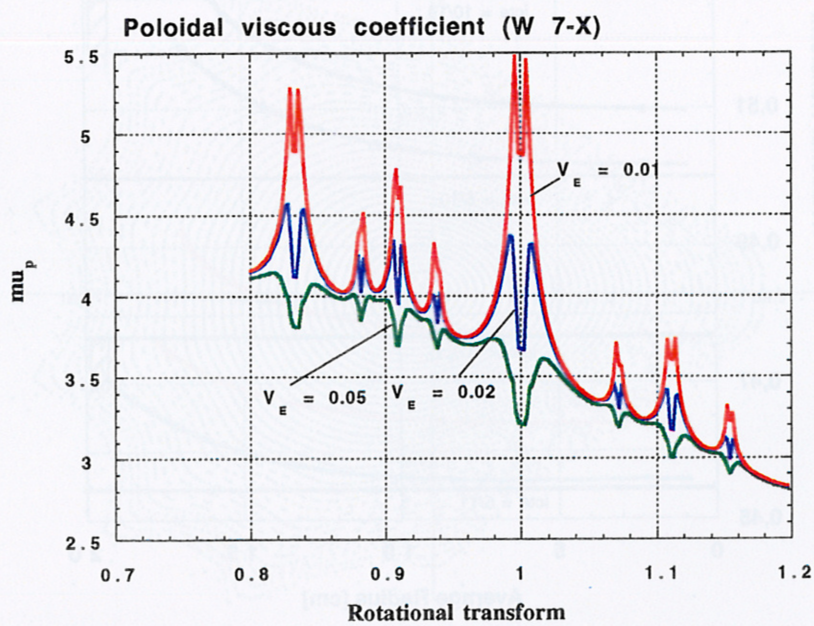

Fig. 12: Poloidal viscous coefficient μ_p vs rotational transform (Wendelstein 7-X). The electric field increases by a factor five. Collisionality is kept fixed.

This figure shows the main resonances and the largest damping aeffect at ι = 5/6 and 5/5. Higher order resonances occur at ι = 10/11, 10/9, 15/17, 15/16, 15/14, 15/13. The shear in Wendelstein 7-X is larger than in Wendelstein 7-AS, therefore the resonant surface at ι = 10/11 cannot be avoided. On the other hand, the viscosity in average is reduced by the increase of the rotational transform and the optimization effect. For this reason it is very likely that shear flow and H-mode also occurs in Wendelstein 7-X. The poloidal viscous coefficient depends on the geometric quantity

$$\boldsymbol{e}_p \bullet \frac{\nabla B}{B}$$

Minimizing this geometric term will lead to small viscous damping of poloidal rotation. To minimize this term is the main goal of the Helias geometry. In the extreme limit this term would be zero which implies that current lines are orthogonal to magnetic field lines. In Wendelstein 7-X this goal can be achieved in some regions of the magnetic surfaces which explains the reduction of the viscous coefficients compared to Wendelstein 7-AS as shown in figure 1.

5. Discussion

The considerations above give a qualitative explanation for the narrow H-mode window in Wendelstein 7-AS. The poloidal viscous force exhibits a strong dependence on the rotational transform, and these forces have minima in the vicinity of iota = 1/2, which is seen in the plateau approximation of the viscous forces. However, the role of magnetic islands needs to be clarified more quantitavely.

An interesting feature of the viscous is the dependence of the viscous forces on the radial electric field. For small E-fields the forces

$$\begin{pmatrix} -<\boldsymbol{e}_p \bullet \nabla \bullet \pi> \\ <\boldsymbol{B} \bullet \nabla \bullet \pi> \end{pmatrix} = \begin{pmatrix} \mu_p & \mu_b \\ \mu_b & \mu_t \end{pmatrix} \begin{pmatrix} E \\ \Lambda \end{pmatrix} \qquad \text{Eq. 41}$$

are linearly increasing functions with E and Λ, however, at large E-fields the coefficients decrease quadratically with E and the poloidal viscous force scales with 1/E. The poloidal viscous force has at least one local maximum as a function of the E-field which by many authors has been invoked as the reason for bifurcation and multiple solutions of the poloidal rotation[23,24]. Shaing (ref. 2) has shown that more than one local maximum may arise in stellarators, which depends on the special choice of the Fourier spectrum of B.

To explain L-H transition in toroidal systems the driving forces must be taken into account. Lost ion orbits, Stringer spin-up and turbulent Reynolds stresses are the various candidates for driving forces. A general momentum balance equation including

[23] D.E. Hastings, W.A. Houlberg, K.C. Shaing , " The ambipolar electric field in stellarators" , Nuclear Fusion **25** , No 4, (1985) 445

[24] K.C. Shaing, E.C. Crume "Bifurcation Theory of Poloidal Rotation in Tokamaks: A Model for th e L-H Transition", Phys. Rev. Lett. Vol. 63, No. 21 (1989) 2369

Stringer spin-up and turbulent Reynolds stresses has been given in[25], which includes the effect of impurities and neutral gas interaction.
In Ref. 9 a set of equation was derived which describe the transient phase of the poloidal and toroidal flow. In a stellarator without toroidal currents these equations are

$$-\rho\begin{pmatrix} <\boldsymbol{e}_p\cdot\boldsymbol{e}_p> & 0 \\ 0 & <B^2> \end{pmatrix}\begin{pmatrix} \dfrac{\partial E}{\partial t} \\ \dfrac{\partial \Lambda}{\partial t} \end{pmatrix} = \begin{pmatrix} R_{11} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} E \\ \Lambda \end{pmatrix} + \begin{pmatrix} <\left(p_{\parallel}-p_{\perp}\right)\boldsymbol{e}_p\bullet\dfrac{\nabla B}{B}> \\ -<\left(p_{\parallel}-p_{\perp}\right)\boldsymbol{B}\bullet\dfrac{\nabla B}{B}> \end{pmatrix} \qquad \text{Eq. 42}$$

which in plateau approximation is

$$-\rho\begin{pmatrix} <\boldsymbol{e}_p\cdot\boldsymbol{e}_p> & 0 \\ 0 & <B^2> \end{pmatrix}\begin{pmatrix} \dfrac{\partial E}{\partial t} \\ \dfrac{\partial \Lambda}{\partial t} \end{pmatrix} = \begin{pmatrix} R_{11} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} E \\ \Lambda \end{pmatrix} - \begin{pmatrix} \mu_p & \mu_b \\ \mu_b & \mu_t \end{pmatrix}\begin{pmatrix} E \\ \Lambda \end{pmatrix} \qquad \text{Eq. 43}$$

R_{11} is the Stringer spin-up term

$$R_{11} = <\rho u\nabla\bullet\left(\boldsymbol{e}_p\times\nabla\psi\right) \qquad \text{Eq. 44}$$

u is the parallel velocity connected to the radial plasma diffusive velocity by the equation of continuity. This relation is also valid in turbulent plasmas, where the radial particle flux is anomalous (anomalous Stringer spin-up). When the turbulence level and the radial anomalous loss are reduced by the shear flow, the anomalous Stringer spin-up also decreases. Thus the Stringer spin-up has the same general structure as viscous damping.
The driving term R_{11} only affects the poloidal component E, there is no Stringer drive in parallel direction. The mixed coefficient μ_b, however, couples the two equations together, and any poloidal flow is accompanied by a toroidal flow. The coefficient is the driving term for the bootstrap current (see Ref. 25), which in Helias configurations is close to zero. Hence in Helias configurations the spin-up equations reduce to

$$-\rho<\boldsymbol{e}_p\cdot\boldsymbol{e}_p>\frac{\partial E}{\partial t} = R_{11}E-\mu_p E \qquad \text{Eq. 45}$$

which is the equivalent equation to eq. 13 in a paper by Hassam et al. on the onset of H-mode in tokamaks[26].
In the initial phase of the spin-up the coefficients R_{11} and μ_p are independent of E and the poloidal rotation either grows exponentially or decays exponentially. Turbulent Reynolds stresses, however, provide a driving mechanism, which is finite at zero rotation and thus can sustain a small poloidal rotation even in case when viscous damping is stronger than anomalous Stringer drive. Turbulent forces would add another term RS to eq. 45 which is either constant or decreases with E if the turbulence is reduced with growing poloidal rotation (see Ref. 25).

$$-\rho<\boldsymbol{e}_p\cdot\boldsymbol{e}_p>\frac{\partial E}{\partial t} = R_{11}(E)E-\mu_p(E)E+RS(E) \qquad \text{Eq. 46}$$

This simplified equation explains qualitatively the main features of poloidal rotation.

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25 H. Wobig , " On rotation of multi-species plasmas in toroidal systems", Plasma Phys. Contr. Fusion **38** (1996) 1053-1081
26 A.B. Hassam, T.M. Antonsen, J.F. Drake, C.S.Liu, Phys. Rev. Lett. 66 (1991) 309

A driving force, which is independent of the E-field would lead to two solutions of the force balance equation where the larger one is unstable. In order to obtain two stable solutions the driving terms must decay faster than 1/E at large E-fields. A possible scenario for L-H transition could be the following:

Turbulent Reynolds stresses or lost orbits initiate a poloidal rotation of the plasma, which by viscous damping is stabilized at a low level. There is no significant reduction of the turbulent fluctuations and the anomalous transport. This is the L-mode solution. There exists a second unstable solution, which results from the balance between viscous damping, Reynolds stresses, and Stringer spin-up. If at high electric fields, when the shear flow is established, the turbulence will be reduced. As a consequence the Reynolds stresses decline and also the anomalous Stringer spin-up decreases. If this driving term decrease faster than 1/E (faster than the viscous damping forces), a third stable solution exists. This model is sketched in the following figure.

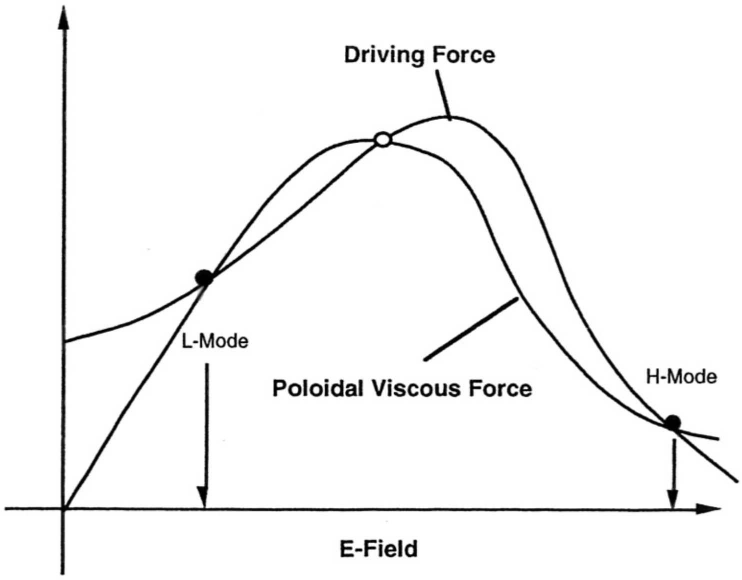

Fig. 13: Qualitative sketch of the force balance in Helias systems. The dots are the stable solutions for the radial electric field, the open circle is the unstable solution. The finite driving force at small E-fields is the result of turbulent Reynolds stresses or lost orbits. At large E the Reynolds stresses are negligible and the Stringer drive is the dominant driving force. The balance between viscous damping and Stringer drive determines the H-mode solution.

As indicated in figure 13 the curves are close together. Since the viscous damping depends on the details of the Fourier spectrum, on collisionality and rotational transform, a situation may arise where the viscous damping is larger than the driving forces except for the first stable L-mode solution. The other two solutions vanish if the viscous damping increases by a small amount. This may explain the experimental results of Wendelstein 7-AS where L-H transition is found only in a region with relatively small viscous damping. Furthermore, if the first stable solution and the unstable solution approach each other, a finite perturbation can push the system into the rotating stable solution on the right in Fig. 13.

As shown above the viscous forces are non-monotonous in the radial E-field. In order to get at least 3 solutions of the force balance the driving term must also be non-monotonous. This argues against Reynolds stresses or lost orbits as the only source of driving forces. Lost orbits are reduced by the radial E-field and therefore the recoil effect on the bulk plasma decreases monotonically with increasing E-field. The turbulent Reynolds stresses also decrease if the turbulence is reduced by the shear flow.
The hypothesis proposed here is only qualitative, in future a more quantitave verification of the model will be made.