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Abstract

Radially inward particle convection is shown to be a consequence of the conservation of canonical angular momentum in an axisymmetric geometry with a prescribed radial electric field distribution. Analytic expressions as well as quantitative estimates of the particle pinch profile are obtained.

Inward convection of particles in tokamak plasmas can be much larger than that expected from Ware[1] pinch. Recently, similar inward particle drifts have been shown to exist in the W7-AS stellarator.[2] Such particle drifts directed against ambient density gradients are referred to as anomalous particle pinches.

The anomalous pinches are invariably accompanied by sizable radial electric fields directed towards the plasma center. In this Letter it is shown that the radial pinches are an inescapable consequence of the radial electric fields. The deficit of ions necessary to produce the radial electric field gives rise to a small but significant toroidal rotation of the bulk plasma column. The collisions between the rotating bulk and the trapped fraction transfer a part of this toroidal momentum to the trapped ions which are thereby subjected to the inward pinch. Both the toroidal rotation and the inward pinch are governed by the conservation of canonical angular momentum in an axisymmetric geometry. The inward pinch resembles Ware[1] pinch, albeit driven by parallel friction force between rotating and trapped ions instead of an externally applied electric field.

Let $E(r)$ be the given inward-directed radial electric field produced by a deficit of ions in the two-component electron-proton plasma. The density $\delta n(r)$ of the deficit ions is related to the radial electric field by

$$2\pi r \epsilon_0 \epsilon_\perp(r) E(r) = e \int_0^r 2\pi r \delta n(r)\, dr, \tag{1}$$

where $\epsilon_\perp$ is the perpendicular dielectric constant and the cylindrical approximation has been used. In order to create the deficit density $\delta n(r)$, the ions escaping the plasma carry away a net canonical angular momentum. The canonical angular momentum removed by a single ion is given by

$$m_i \delta v_\phi R = \frac{e}{2\pi} \delta\Phi, \tag{2}$$

where,

$$\delta\Phi(r) = 2\pi R B_\theta(r)\, \delta r. \tag{3}$$

The total contribution to the canonical angular momentum at radius r is given by

$$m_i \Delta v_\phi R = eRB_\theta(r)\Delta r \int_0^r 2\pi r \delta n(r)\, dr = RB_\theta(r) 2\pi r \epsilon_0 \epsilon_\perp(r) E(r)\, \Delta r. \tag{4}$$

This momentum is shared by the untrapped ions numbering $(1-\sqrt{2\varepsilon})2\pi r n(r)\Delta r$, which receive a net momentum per ion given by

$$m_i v_\phi(r) = \frac{2\pi r \epsilon_0 \epsilon_\perp(r) E(r) B_\theta(r)}{(1-\sqrt{2\varepsilon})2\pi r n(r)}. \tag{5}$$

In obtaining Eq.(5), the small momentum input to the electrons has been neglected. Approximating $\epsilon_\perp$ as

$$\epsilon_\perp(r) \approx \frac{\omega_{pi}^2(r)}{\omega_{ci}^2} = \frac{m_i n(r)}{\epsilon_0 B^2}, \tag{6}$$

gives the expected $\mathbf{E} \times \mathbf{B}$ toroidal drift

$$m_i v_\phi(r) = \frac{m_i E(r) B_\theta(r)}{(1-\sqrt{2\varepsilon})B^2}. \tag{7}$$

The rotating plasma transfers momentum to the trapped ions via collisions at the rate ν_{ii}. The corresponding toroidal force exerted on the trapped ions is $\mathcal{F}_\phi = \nu_{ii} m_i v_\phi$. The inward trapped-ion velocity becomes $v_{in} = \mathcal{F}_\phi / eB_\theta$. Since only the fraction $\sqrt{2\varepsilon}$ of the ions is trapped, one obtains an effective inward plasma drift given by

$$v_{in}(r) = \frac{\sqrt{2\varepsilon}\nu_{ii}(r) m_i v_\phi(r)}{eB_\theta(r)} = \frac{\sqrt{2\varepsilon}}{1-\sqrt{2\varepsilon}} \frac{\nu_{ii}(r) m_i E(r)}{eB^2}. \tag{8}$$

For $T(a) \sim 100\,\mathrm{eV}$, $B \sim 3\,\mathrm{T}$, $\nu_{ii}(a) \sim 10^5\,\mathrm{s}^{-1}$, and $E(a) \sim 2 \times 10^4\,\mathrm{Vm}^{-1}$, one obtains $v_{in}(a) \sim 5\,\mathrm{ms}^{-1}$, which falls within the range of the experimentally observed values. The corresponding toroidal rotation velocity $v_\phi(a) \sim 1400\,\mathrm{ms}^{-1}$. The small and inconsequential outward electron convection is ignored in this paper.

Both $\nu_{ii}(r)$ and $E(r)$ decrease sharply as the plasma center is approached; no significant pinch occurs near the axis. These figures, however, represent an extreme case that does not include viscous drag. Inclusion of viscous drag redistributes the toroidal rotation and would cause the plasma interior to share the toroidal momentum as well as the inward particle convection of the external layers.

At the opposite extreme, the viscous effects may be assumed to be so large that the entire plasma column rotates at the same speed. For this case, the total toroidal momentum input into the plasma contributed by the deficit ions becomes

$$\int_0^a dr \int_r^a 2\pi r_1 \delta n(r_1) e B_\theta(r_1)\, dr_1 = \int_0^a 2\pi\epsilon_0 \left[a\epsilon_\perp(a)E(a) - r\epsilon_\perp(r)E(r)\right]\, B_\theta\, dr, \tag{9}$$

where B_θ has been assumed to be constant over the plasma radius. One obtains the momentum per particle as

$$m_i v_\phi = \frac{2\pi a\epsilon_0 B_\theta \left[a\epsilon_\perp(a)E(a) - < r\epsilon_\perp E >\right]}{\int_0^a 2\pi(1-\sqrt{2\varepsilon})rn(r)\, dr} = \frac{\epsilon_0 B_\theta \left[a\epsilon_\perp(a)E(a) - < r\epsilon_\perp E >\right]}{< (1-\sqrt{2\varepsilon})rn >}, \tag{10}$$

where the expressions in the angle brackets represent the average over the plasma radius. The inward convection velocity becomes

$$v_{in}(r) = \frac{\sqrt{2\varepsilon}\nu_{ii}(r)\epsilon_0 \left[a\epsilon_\perp(a)E(a) - < r\epsilon_\perp E >\right]}{e < (1-\sqrt{2\varepsilon})rn >} = \frac{\sqrt{2\varepsilon} m_i \nu_{ii}(r) \left[an(a)E(a) - < rnE >\right]}{eB^2 < (1-\sqrt{2\varepsilon})rn >}. \tag{11}$$

For this case, both the pinch and the toroidal rotation velocities are somewhat reduced near the periphery, but are greatly enhanced towards the axis and could, in principle, give rise to peaked density profiles frequently encountered in tokamak and stellarator discharges.[2] The inward convection velocity v_{in} in Eq.(11) scales as ν_{ii}/B^2. This scaling matches the particle diffusivity scaling in accord with the experimental observations.[2,3]

The ν_{ii} values used for estimating the inward particle convection were obtained from Ref. 4, which predicts anomalously enhanced collisional contributions to plasma transport due to momentum-transfer collisions via Kirchhoff thermal radiation. The most pronounced enhancement (vis-a-vis the classical values derived from two-particle collisions using the Fokker-Planck equation) is found to occur in ν_{ee} and ν_{ie}. The inclusion of radiative effects increases the ratio of ν_{ie}/ν_{ei} to $\mathcal{O}(\sqrt{m_e/m_i})$ from its classical value of $\mathcal{O}(m_e/m_i)$. This enhancement in ν_{ie} causes the ions to diffuse out faster than the electrons, thereby creating an inward-directed radial electric field, toroidal rotation as well as inward ion convection, till a new equilibrium is established and ambipolarity is restored.

The foregoing analysis assumes that the trapped fraction equals $\sqrt{2\varepsilon}$ irrespective of the collisionality regime, includes no self-consistent transport processes, nor considers contributions due to viscosity, particle sources, recycling, neutral particles, and impurities; the estimates for v_{in} and v_ϕ have only a rough quantitative significance. The analysis, nevertheless, demonstrates the importance of the radial electric fields in causing particle convection. Additional particle convection would accompany toroidal momentum injection during neutral-beam heating.

The crucial role played by the impurity ions, particularly near the plasma periphery, cannot be overemphasized. Radiative transport theory[4] implies a large enhancement in the impurity collisions with the bulk-plasma ions, which would significantly augment both the bulk-plasma and impurity-ion convection. Work on the role of impurities in radiative plasma transport is currently in progress.

REFERENCES

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[3] For the radiative collisionality scaling $\nu_{ii} \sim B^2$ of Ref. 4, v_{in} in Eqs.(8) and (11) becomes independent of the magnetic field.

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