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Abstract. Basic physics of the anomalous particle pinch is explored by analysing a new empirical scaling law $v_{in}/D \propto Z_{eff}\, n_0$ with n_0 being the neutral particle density and by comparison with quasilinear and nonlinear theory. The scaling is found to result from the dissipation of perpendicular fluctuating electron and ion momentum. Electron-ion and charge exchange collisions are the dissipative processes of the perpendicular electron and ion dynamics, respectively. While momentum dissipation is a key process for the anomalous inward drift, the much stronger energy dissipation due to parallel electron heat conduction or electron Landau damping is responsible for diffusion. The results support a temperature gradient driven anomalous inward flux.

Quasilinear theories for the anomalous diffusion coefficient D and the anomalous inward drift velocity v_{in} [1, 2] usually conflict with experimental transport coefficients and are often disproved by turbulence simulations including nonlinear terms. Empirical expressions for v_{in} have been published, for instance, in Refs [3] and [4]. Recently, empirical scalings for v_{in}/D were successfully developed by applying a transport analysis, correlation and simulation (TCS) technique [5]. First, a scaling $v_{in}/D \propto Z_{eff}$ that explains the electron density profile peaking observed with impurity puffing was discovered [6]. Then, the scaling relation

$$\frac{v_{in}(x)}{D(x)} = F_0\, Z_{eff}(x)\, n_0(x)\, \frac{2x}{\rho_w x_s^2}\ \left(m^{-1}\right) \tag{1}$$

with $F_0 = 1.29 \times 10^{-15}$, n_0 in m^{-3} and ρ_w in m was found [5]. The importance of developing empirical scaling laws is underscored by the fact that neither the Z_{eff} nor the n_0 dependence was predicted by theory. In Eq. (1), the effective radius of a flux surface ρ normalized to the effective radius of the wall contour ρ_w is denoted by x and x_s marks the separatrix. The scaling works both in the bulk plasma and in the edge region [5]. It explains the strong rise of v_{in}/D with radius near the separatrix observed in all confinement regimes and the flattening of density profiles with rising line average density. Neutral particle penetration was found to be crucial. The empirical scaling further accounts for the small phase angles measured between the oscillating electron density and the neutral particle influx in gas oscillation experiments whereas other scalings independent of n_0 do not. Moreover, Eq. (1) explains by an increase in the neutral density background the strong density profile peaking and rise of v_{in}/D observed during and after pellet injection. Other successful applications of Eq. (1) were reported in Ref. [5]. In summary, the empirical v_{in}/D scaling is capable of accounting for a number of hitherto unexplained experimental results from different fields, which underlines its fundamental character. Quasilinear and nonlinear models of the anomalous inward drift developed so far fail to explain the above experimental findings and the scaling law given in Eq. (1). The main objective of this Report is to identify the processes underlying the empirical scaling and to reveal basic elements of the anomalous inward drift.

It is well known that the nonadiabatic (dissipative) part of the fluctuating density $\tilde{n}$ is responsible for the anomalous particle flux

$$\Gamma = <\tilde{n}\tilde{v}_E> = \sum_k \tilde{n}_k \tilde{\varphi}_k \frac{k_\theta}{B} sin\alpha_{n\varphi} = -D\frac{dn}{d\rho} - nv_{in} \tag{2}$$

The brackets denote the correlation of $\tilde{n}$ with the fluctuating $\mathbf{E} \times \mathbf{B}$ drift velocity $\tilde{v}_E$. Further quantities are the fluctuating potential $\tilde{\varphi}$, the poloidal wave number k_θ and the phase angle $\alpha_{n\varphi}$ between $\tilde{n}$ and $\tilde{\varphi}$, caused by dissipation. In principle, dissipative contributions both due to electrons and due to ions are possible. Quasineutrality results in $\tilde{n}_e = \tilde{n}_i = \tilde{n}$ and ambipolarity $\Gamma_e = \Gamma_i$.

The parameter dependences of the empirical v_{in}/D scaling can provide information about basic processes of the anomalous inward drift. From the scaling with Z_{eff} it is inferred that v_{in}/D is proportional to the collision rate of fluctuating electrons with deuterons and impurity ions [5]. The n_0 dependence of v_{in} suggests that processes with rates proportional to the neutral particle density, i.e. ionisation, excitation and charge exchange of deuterium atoms, have to be considered [5]. It is well known that dissipative processes are crucial for the anomalous transport. The impact of possible processes on v_{in}/D is therefore tested by evaluating the associated energy and momentum dissipation rates and by comparing the results with the rates due to competing processes.

In quasilinear fluid models [2], particle transport is derived from the electron energy equation with $n\nabla_{\|}\tilde{\mathrm{v}}_{\|}$ substituted with the help of the continuity equation. We thus include the ionisation rate in the continuity equation and the energy losses due to ionisation and radiation in the electron energy equation. For evaluations, data from a simulated H-mode discharge No. 7978 of ASDEX Upgrade ($R = 1.65\ m, a = 0.5\ m, \kappa = 1.6$) with $\bar{n}_e = 7.6 \times 10^{19}\ m^{-3}$, $I_p = 1.2\ MA$, $B_t = 2.5\ T$ and $P_{NI} = 5.0\ MW\ \ (D^0 \rightarrow D^+)$ [5] are used. At $t = 3\ s$ and $x = 0.77$ ($r = 0.445\ m$ close to the pedestal radius) one has $T_e = 802\ eV$, $T_i = 859\ eV$, $n = 6.0 \times 10^{19}\ m^{-3}$, $n_0 = 6.0 \times 10^{14}\ m^{-3}$ and $q = 2.56$. The rate coefficients for resonant charge exchange between deuterons and deuterium atoms and for electron impact ionisation are $<\sigma_{cx}v_{rel}> = 8.3 \times 10^{-14}\ m^3s^{-1}$ and $<\sigma_i v_e> = 2.4 \times 10^{-14}\ m^3s^{-1}$, respectively. Taking the average poloidal wave number $\bar{k}_\theta \simeq 10^2\ \ m^{-1}$ corresponding to $\bar{k}_\theta\rho_s = 0.16$ and $\tilde{n}/n \simeq e\tilde{\varphi}/T_e = 0.02$ results in $\tilde{v}_E = -i\bar{k}_\theta\tilde{\varphi}/B = 641.6\ ms^{-1}$, where $\rho_s = c_s/\omega_{ci}$ with $c_s = (T_e/m_i)^{1/2}$ and $\omega_{ci} = eB/m_i$. These values are consistent with measurements [7] and $\tilde{n}/n$ agrees with the 'mixing length' estimate $\tilde{n}/n \approx 1/(\bar{k}_\rho L_n)$ with $\bar{k}_\rho \simeq 2 \times 10^2\, m^{-1}$ and the density gradient length $L_n = 0.314\ m$. For the electron drift frequency one obtains $\omega_{*e} = T_e\bar{k}_\theta/(eBL_n) = 10^5\ s^{-1}$. Evaluating the ionisation and radiation terms shows that they are negligibly small compared with the main dissipation rate due to parallel electron heat conduction. Calculating the effect of ionisation on the quasilinear particle flux at $x = 0.77$ and normalizing to the dominant term yields a contribution of $<\sigma_i v_e> n_0/\omega_{*e} \simeq 1.5 \times 10^{-4}$.

By contrast, momentum transfer due to charge exchange of fluctuating deuterons with deuterium atoms is regarded as a process relevant for the anomalous inward drift [5]. The parallel ion momentum balance is examined first. Under collision-dominated conditions, the parallel neutral friction force at $x = 0.77$ is found to be 7% of the parallel viscous force. In the collisionless case, one obtains $m_i n\left(\mu_{\|}k_{\|}^2 + \nu_{cx} + \nu_{ion}\right)\tilde{\mathrm{u}}_{\|}$ with parallel viscosity $\mu_{\|} = T_i/(m_i\omega_{*e})$ (for ion Landau damping), charge exchange frequency $\nu_{cx} = <\sigma_{cx}v_{rel}> n_0$

and ionisation frequency $\nu_{ion} =< \sigma_i v_e > n_0$. For $k_{\|} = 1/(qR)$, the neutral friction force only amounts to 0.3% of the viscous force. It is thus concluded that parallel ion dynamics are governed by parallel viscosity. Here, neutral friction cannot introduce an n_0 dependence into v_{in}/D, because the competing dissipation process is much stronger. The study should thus rather focus on dissipative contributions by charge exchange to the perpendicular momentum balance of the ions [5].

Setting $n_e = n_i = n$, the perpendicular electron and ion momentum equations read [8]

$$0 = -\nabla_{\perp}\tilde{p}_e + en\Big(\tilde{\mathbf{E}}_{\perp} + \tilde{\mathbf{v}}_{\perp} \times \mathbf{B}\Big) + \tilde{\mathbf{S}}_{ei} \tag{3}$$

$$m_i n(\frac{\partial}{\partial t} + \tilde{\mathbf{u}}_{\perp} \cdot \nabla)\tilde{\mathbf{u}}_{\perp} = -\nabla_{\perp}\tilde{p}_i + en\Big(\tilde{\mathbf{E}}_{\perp} + \tilde{\mathbf{u}}_{\perp} \times \mathbf{B}\Big) + \tilde{\mathbf{S}}_{i0} - \tilde{\mathbf{S}}_{ei} \tag{4}$$

with

$$\tilde{\mathbf{S}}_{ei} = -\nu_{ei} n m_e(\tilde{\mathbf{v}}_{\perp} - \tilde{\mathbf{u}}_{\perp}) = -\nu_{ei} n m_e(\tilde{\mathbf{v}}_{de} - \tilde{\mathbf{u}}_{di}) \tag{5}$$

$$\tilde{\mathbf{S}}_{i0} = -(\nu_{cx} + \nu_{ion}) n m_i \tilde{\mathbf{u}}_{\perp} \tag{6}$$

Here, $\tilde{\mathbf{v}}_{\perp}$ and $\tilde{\mathbf{u}}_{\perp}$ are the fluctuating electron and ion velocities $\perp \mathbf{B}$, $\tilde{\mathbf{S}}_{ei}$ is the dissipation rate of fluctuating electron momentum due to electron-ion collisions with frequency ν_{ei} and $\tilde{\mathbf{S}}_{i0}$ is the dissipation rate of fluctuating ion momentum due to charge exchange and ionisation. From Eqs (3) and (4) one obtains $\tilde{\mathbf{v}}_{\perp} = \tilde{\mathbf{v}}_E + \tilde{\mathbf{v}}_{de}$ and $\tilde{\mathbf{u}}_{\perp} = \tilde{\mathbf{v}}_E + \tilde{\mathbf{u}}_{di}$, where $\tilde{\mathbf{v}}_{de}$ and $\tilde{\mathbf{u}}_{di}$ are the electron and ion diamagnetic drift velocities, respectively.

In the case of a background momentum, the dissipative contribution due to neutrals can be assessed by comparing the charge exchange and ion-ion collision rates, yielding a negligibly small dissipation by neutrals. For the fluctuating momentum, however, this procedure is wrong. As all ions fluctuate with $\tilde{\mathbf{u}}_{\perp}$, like-particle collisions cannot dissipate fluctuating perpendicular momentum. The ion-ion collisions do not contribute, although their collision rate at $x = 0.77$ exceeds the charge exchange rate by about a factor of 20. Electron-electron collisions do not dissipate fluctuating momentum either in spite of $\nu_{ee} \simeq \nu_{ei}$, because all electrons fluctuate with $\tilde{\mathbf{v}}_{\perp}$. It is thus concluded that electron-ion collisions and charge exchange collisions represent the dissipative processes in perpendicular electron and ion dynamics, respectively. There are no competing dissipative processes for the fluctuating momentum in this case. Data from the above H-mode discharge are used to evaluate the ratio $\tilde{S}_{ei}/\tilde{S}_{i0}$ at $x = 0.77$. By setting $|\tilde{v}_{de}| \approx |\tilde{u}_{di}| \approx |\tilde{v}_E|$ one obtains $\tilde{S}_{ei}/\tilde{S}_{i0} \approx 0.4$, i.e. the dissipation rates for electron and ion momentum are about equal. It is clear that the electrons undergo pitch angle scattering with the ions. Owing to charge exchange neutrals carry off fluctuating ion momentum which after charge exchange or ionisation is dissipated by ion-ion collisions. It is inferred that dissipation of perpendicular fluctuating momentum is a basic element of the anomalous particle pinch and that electron-ion and charge exchange collisions can cause the v_{in}/D scaling given in Eq. (1).

From the result that Z_{eff} and n_0 enter the empirical scaling as factors it is concluded that the corresponding electron and ion momentum dissipation processses simultaneously affect v_{in}/D. In the case of no impurities, one thus obtains $v_{in}/D \propto \nu_{ei}\nu_{cx}$. It is expected that the dependences of $\tilde{n}_e/n$ on ν_{ei} and $\tilde{n}_i/n$ on ν_{cx} are combined in one expression by quasineutrality,

$\tilde{n}_e = \tilde{n}_i$. Moreover, the scaling of v_{in}/D with Z_{eff} and n_0 means that the momentum dissipation processes act on the inward drift velocity only and not on the diffusion coefficient.

It is interesting to compare these results with quasilinear ion mixing mode transport considered for electron dissipation associated with passing electrons [2] and for dissipative trapped electrons [9], with other fluid and kinetic models and with turbulence simulations. The most important dissipation processes of these models are parallel electron heat conduction in the fluid case and parallel electron Landau damping in the kinetic case. They enter the electron energy equation $\parallel$ **B** (see Eq. (8)) or the drift kinetic equation. As the quasilinear particle fluxes predicted by these models have the right order of magnitude, it is concluded that these dissipation processes are sufficiently strong and responsible for the measured phase angles $\alpha_{n\varphi}$ [7]. Moreover, the quasilinear models and nonlinear simulations [10, 11] predict a temperature gradient driven particle pinch, i.e. an inward flux caused by the off-diagonal ∇T_e dependent term in the transport matrix equation. Compared with these fluxes, the quasilinear particle fluxes due to the dissipation rates $\tilde{S}_{ei}$ and $\tilde{S}_{i0}$ are negligibly small. Evaluating the contribution due to $\tilde{S}_{i0}$ at $x = 0.77$, for instance, and normalizing to the dominant term yields $(\nu_{cx} + \nu_{ion})/\omega_{*e} \simeq 7 \times 10^{-4}$. The corresponding value at the separatrix is about 10^{-3}. We thus conclude that the dissipation due to electron-ion and charge exchange collisions cannot replace the much stronger energy dissipation due to parallel electron heat conduction or electron Landau damping that determines the magnitude of D, v_{in} and Γ. As a consequence, a temperature gradient driven inward flux must also be adopted.

The nonlinear electron continuity and energy equations are

$$\frac{\partial \tilde{n}_e}{\partial t} + \tilde{\mathbf{v}}_E \cdot \nabla \tilde{n}_e + \tilde{\mathbf{v}}_E \cdot \nabla n + n \nabla_\parallel \tilde{\mathbf{v}}_\parallel = 0 \tag{7}$$

$$\frac{3}{2} n \left(\frac{\partial \tilde{T}_e}{\partial t} + \tilde{\mathbf{v}}_E \cdot \nabla \tilde{T}_e + \tilde{\mathbf{v}}_E \cdot \nabla T_e\right) + n T_e \nabla_\parallel \tilde{\mathbf{v}}_\parallel = \nabla_\parallel \left(\kappa_{e\parallel} \nabla_\parallel \tilde{T}_e\right) \tag{8}$$

where $\kappa_{e\parallel}$ is the parallel electron heat conductivity. In the quasilinear fluid description [2], the nonlinear $\tilde{\mathbf{v}}_E \cdot \nabla \tilde{n}_e$ and $\tilde{\mathbf{v}}_E \cdot \nabla \tilde{T}_e$ terms have been omitted. Particle transport is derived from Eq. (8) with $n \nabla_\parallel \tilde{\mathbf{v}}_\parallel$ substituted with the help of the continuity equation. The coefficients D and v_{in} result from the linear $\tilde{\mathbf{v}}_E \cdot \nabla n$ and $\tilde{\mathbf{v}}_E \cdot \nabla T_e$ terms, respectively, which are non-dissipative. It is obvious that perpendicular electron dynamics play an important role in anomalous particle transport. The anomalous flux predicted by quasilinear models depends on $n\omega_{*e}(1 - \eta_e 3/2)$ in the fluid case [2] and on $n\omega_{*e}\left[1 - \eta_e(3/2 - v^2/v_{te}^2)\right]$ in the kinetic case [9, 12] with $\eta_e = L_n/L_{T_e} = dlnT_e/dlnn$ and $v_{te} = (2T_e/m_e)^{1/2}$. The fluid model yields

$$\Gamma = nD \frac{eB}{T_e \bar{k}_\theta} \omega_{*e} \left(1 - \frac{3}{2}\eta_e\right) = -D \frac{dn}{d\rho} - nD \frac{3}{2} \frac{1}{L_{T_e}} \tag{9}$$

which by applying Eq. (2) leads to $v_{in}/D = 3/(2L_{T_e})$. As dependences on Z_{eff} and n_0 can only enter both D and v_{in}, they would not appear in v_{in}/D. Thus, the specific processes affecting the inward drift only are incompatible with the predictions by quasilinear fluid and kinetic models and cannot be described in the frame of quasilinear theory. Instead, a nonlinear treatment and turbulence simulations are required. This conclusion is supported by the fact that the simple linear $1/L_{T_e}$ dependence of v_{in}/D, anticipated by quasilinear models, is not found in the experiment [5] and in turbulence simulations. Evaluating the nonlinear $\tilde{\mathbf{v}}_E \cdot \nabla \tilde{T}_e$

term in Eq. (8) shows that it is strong and equal to the linear $\tilde{\mathrm{v}}_E \cdot \nabla T_e$ contribution because of $\left|\nabla \tilde{T}_e\right| \simeq \bar{k}_\rho \tilde{T}_e \simeq T_e/L_{T_e}$. Analogously, the nonlinear $\tilde{\mathrm{v}}_E \cdot \nabla \tilde{n}_e$ term in Eq. (7) matches the linear $\tilde{\mathrm{v}}_E \cdot \nabla n$ term because of $|\nabla \tilde{n}_e| \simeq \bar{k}_\rho \tilde{n}_e \simeq n/L_n$ which is identical to the mixing length relation. Thus, in the nonlinear case $\tilde{v}_E$ is multiplied with different fluctuating quantities, $\tilde{T}_e$ and $\tilde{n}_e$, so that processes affecting the inward drift only become possible. It is concluded that dissipation of perpendicular fluctuating momentum by electron-ion and charge exchange collisions plays a key role for the anomalous particle pinch while the much stronger energy dissipation is crucial for anomalous diffusion. According to quasilinear theory, the dissipative factor in D is given by the energy dissipation rate normalized to the fluctuating energy. Analogously, it is expected that a dissipative factor in v_{in} is given by the dissipation rate of perpendicular fluctuating momentum normalized to this momentum. Up to now, the dissipative processes in the perpendicular electron and ion dynamics are missing in theoretical models. Turbulence simulations can be made more realistic by including these processes.

It was stressed in Ref. [5] that the space dependence of v_{in}/D is mainly caused by n_0 and less by the Z_{eff} and x scalings. The sum of rate coefficients $< \sigma_{cx} v_{rel} > + < \sigma_i v_e >$ is identified as a new scaling factor which is almost spatially constant in the bulk. This suggests an extension of the scaling given in Eq. (1) to

$$\frac{v_{in}(x)}{D(x)} \simeq G_0(< \sigma_{cx} v_{rel} > + < \sigma_i v_e >) Z_{eff}(x) n_0(x) \frac{2x}{\rho_w x_s^2} \quad (m^{-1}) \tag{10}$$

with $G_0 = 1.21 \times 10^{-2}$. In addition, the parameter η_e should enter the scaling, since the anomalous inward flux is temperature gradient driven. The observed strong rise of v_{in}/D with radius near the edge, attributed to $n_0(x)$, cannot result from a nonlinear η_e dependence, because η_e is only a weak function of radius. A problem with the factor ν_{ei} is that it introduces density and temperature dependences, not found in the experiment. They seem to be cancelled out by a further factor that should be present according to a dimensional study of the scaling.

Remarkably, the anomalous particle pinch occurs in all collisionality and confinement regimes. This means that the presence of trapped particles or a special type of drift wave turbulence cannot be prerequisites of the anomalous inward flux. Its universal character is, however, compatible with the dissipation of fluctuating momentum due to electron-ion and charge exchange collisions which takes place in all collisionality and confinement regimes. For the same reason, the anomalous particle pinch should also occur in other devices than tokamaks, e.g. in stellarators.

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