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Abstract

This work is devoted to the study of magnetic islands and resonances of drift surfaces which can take place in the magnetic field of a Helias configuration and which affect the particle confinement properties. It is shown that due to internal magnetic field perturbations which are periodic in major and minor circumference of the torus large islands occur in the magnetic configuration and, in particular, in the drift surfaces. In contrast to external perturbations which arise from errors in the coil system and which can be kept below a tolerable level these internal perturbations may originate from the the finite beta effect or - in the time dependent case - from MHD instabilities in the plasma. In the present analysis the amplitude of these perturbations is considered as an arbitrary parameter. The paper summarizes the results of numerical computations with time independent magnetic perturbations leading to island formation in magnetic surfaces and drift surfaces of alpha particles.

1. Introduction

The stochasticity of particle motion is one of the most interesting and important aspects of plasma confinement in the toroidal fusion devices [1, 2]. In a Helias configuration island formation and stochasticity of magnetic surfaces and drift surfaces may arise from external field errors of the coil system, which, however, can be kept at a low level with a relative amplitude of $\delta B/B < 0.03$ %. Modification of the vacuum field by the finite beta plasma or MHD instabilities may lead to larger field perturbations and therefore to larger islands. Our goal here is to consider consistently the resonance behaviour of the magnetic force lines in the vacuum configuration, then in the configuration modified by the plasma pressure (finite beta case) and, at last, the resonance phenomena in the particle motion. Our main attention is focussed to the high energy particles, particularly alpha-particles with the initial energy of 3.5 MeV and the "cold" alpha-particles with an energy 10 times less 3.5 MeV (350 keV). The isolated resonances in the drift surfaces, the overlapping of the resonances in the drift surfaces and the possible stochastic processes in the particle motion under the effect of magnetic perturbations in Helias magnetic configuration [3] are the subject of our study. We would like to analyse the possible contribution of the processes mentioned above to the anomalous transport, especially of alpha-particles. Another important issue is to control the transport of alpha-particles with different energy under perturbations which depend on time, i.e. the possibility of the selective energy transport in Helias magnetic configurations.

Perturbations considered in this work do not violate the drift approximation. The attention here is focussed to perturbations which have the same periodicity along the torus as the magnetic field (period number M = 5 or multiples it).

The paper is organized in the following way. The basic equations are given in the Section 2. The resonances in the Helias configuration are discribed in the Section 3: for the magnetic force lines in the vacuum approximation, for the magnetic force lines in the finite beta case and for the drift surfaces. For the comparison the trajectories of passing particles in tokamak type field are presented (Section 4). Some conclusions which can be made from the study carried out here are given in Section 5.

2. Basic Equations

We use the guiding center equations in canonical variables [4, 5, 6] which allows us to consider the effect of an electromagnetic field on particle motion in the magnetic field both in vacuum and in finite plasma pressure cases.

We use the toroidal magnetic flux ψ as radial variable, ϑ and ζ as angular variables along the minor and major circumferences of torus, respectively, P_ϑ and P_ζ as momenta of particle motion, and $\rho_\parallel$ as the parallel gyroradius. All the quantities with a dimension of energy are normalized to $m\omega_o^2R_o$, ω_o is the gyrofrequency on axis, all the distances and lengths are normalized to the major radius of the magnetic axis R_o, the magnetic field B is normalized to its value on axis. The magnetic field perturbation is described in the following form $\delta\mathbf{B} = \nabla \times \alpha\mathbf{B}$. Φ is the electric potential. The variable $\rho_\parallel$ is connected with parallel canonical momentum ρ_c by $\rho_c = \rho_\parallel + \alpha$. The equations of motion are the following

$$\dot{\psi} = \left(g\dot{P}_\vartheta - I\dot{P}_\zeta\right)\frac{1}{\gamma}$$

$$\dot{\vartheta} = \left[\left(\mu + \rho_{\parallel}^2 B\right)\frac{\partial B}{\partial \psi} g + \rho_{\parallel} B^2 \left(\iota - \rho_c g'\right) - \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \psi} g + g \frac{\partial \Phi}{\partial \psi}\right]\frac{1}{\gamma}$$

$$\dot{\zeta} = \left[-\left(\mu + \rho_{\parallel}^2 B\right)\frac{\partial B}{\partial \psi} I + \rho_{\parallel} B^2 \left(1 + \rho_c I'\right) + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \psi} I - I \frac{\partial \Phi}{\partial \psi}\right]\frac{1}{\gamma}$$

$$\dot{\rho}_{\parallel} = \left[\left(1 + \rho_c I'\right)\dot{P}_{\zeta} + \left(\iota - \rho_c g\right)\dot{P}_{\vartheta}\right]\frac{1}{\gamma} - \left(\frac{\partial \alpha}{\partial \psi}\dot{\psi} + \frac{\partial \alpha}{\partial \vartheta}\dot{\vartheta} + \frac{\partial \alpha}{\partial \zeta}\dot{\zeta} + \frac{\partial \alpha}{\partial t}\right)$$

$$\dot{P}_{\vartheta} = -\left(\mu + \rho_{\parallel}^2 B\right)\frac{\partial B}{\partial \vartheta} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \vartheta} - \frac{\partial \Phi}{\partial \vartheta}$$

$$\dot{P}_{\zeta} = -\left(\mu + \rho_{\parallel}^2 B\right)\frac{\partial B}{\partial \zeta} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \zeta} - \frac{\partial \Phi}{\partial \zeta} \qquad (1)$$

where

$$\gamma = \rho_c\left(g I' - I g'\right) + g + I\iota \qquad (2)$$

The rotational transform ι is measured in units of 2π. The functions g, I and B^2 must by taken from a plasma equilibrium code.

3. Resonances in Helias Configuration

The particle motion and the resonance effects for the Helias configuration have been studied in number of papers [7-10]. We focus our attention to the effect of perturbing magnetic fields on particle trajectories with the aim to control the particle motion and their transport.

We considered the possible resonance phenomena in Helias configuration in the following way: in so-called vacuum approximation, i.e. for the magnetic force lines without plasma, for the magnetic force lines in the finite beta case, i.e. configuration changed by the finite plasma pressure and for the particle trajectories in the vacuum and finite beta case configurations. The magnetic field for the vacuum and finite beta cases are taken from [11-13].

3.1. The perturbation has the following form

$$\alpha_{mn} = \alpha_1 \left(r/a_{mn}\right)^m \left[\left(a_{pl} - r\right)/\left(a_{pl} - a_{mn}\right)\right]^p \sin\left(n\zeta - m\vartheta - \omega t + \delta\right) \tag{3}$$

where quantities a_{mn} and p create peak at $a_{mn}/a_{pl}=m/(m+p)$ with value α_1, a_{pl} is the plasma radius. The perturbation enters the system of equations via its spatial derivatives and the time derivatives. Here and in the system of equation the radial variable r and the flux variable ψ are connected by the following expression: $r = (2\psi)^{0.5}R_o$.

3.2. For a Helias configuration (with the M=5 magnetic field periods along the torus) three main resonances with the "wave" numbers along the major n and minor m circumferences (n=1, m=1; n=5, m=5; n=10, m=11) were studied.

Such kind of resonances may be regarded as a result of the perturbations induced from outside the plasma or induced from inside the plasma by an MHD activity. In this part of work only time-independent magnetic perturbations (ω = 0) are considered.

3.3. Perturbation with mode numbers n=1 and m=1 create a large island on a magnetic surface where the rotational transform is ι=0.94 at r_o=180 cm (Fig. 1). This occurs in the vacuum case (Fig. 1A) and in the finite beta case with $\beta(0)$=0.03 (Fig. 1B). On Fig. 1A one can see the projection of field lines in the poloidal plane and the Poincaré plot. The computated rotational transform of the perturbed magnetic force line is ι =1.

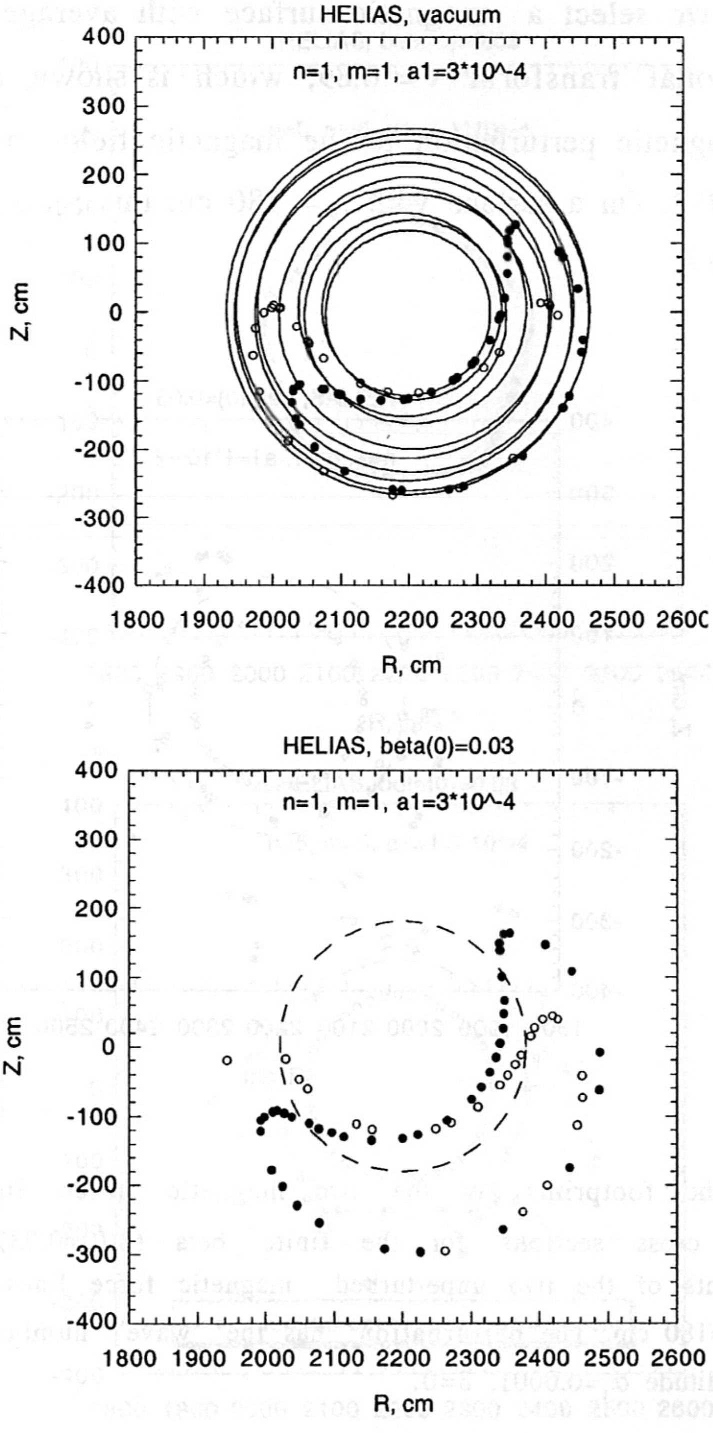


Fig. 1A(top),**1B**(bottom): The footprints of the magnetic field line in two meridional cross sections (at the beginning and at the half of the magnetic field period) for the vacuum case (A) and for the finite beta (ß(0)=0.03) case (B). The starting point of the magnetic field line is at r_0 =180 cm. The perturbation has the "wave" numbers n=1, m=1 and the amplitude α_1=0.0003, δ=0.

Further inside we select a magnetic surface with average radius r_o = 120 cm and rotational transform ι = 0.89, which is shown on Fig. 1C. The ratio of the magnetic perturbation to the magnetic field on axis is $\delta B/B_0$ = 4.5x10^{-5}. On a surface with r_o = 180 cm this ratio is $\delta B/B_0 = 7 \times 10^{-5}$.

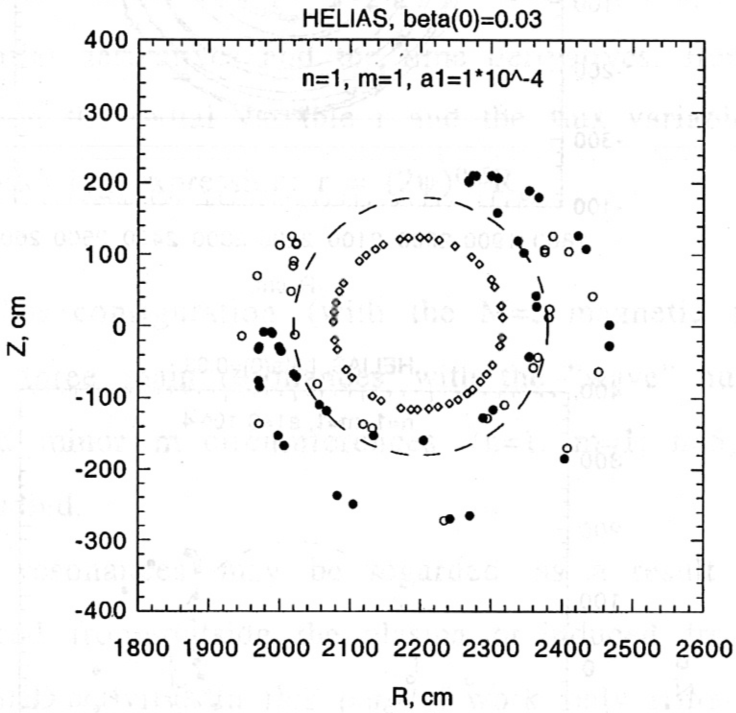
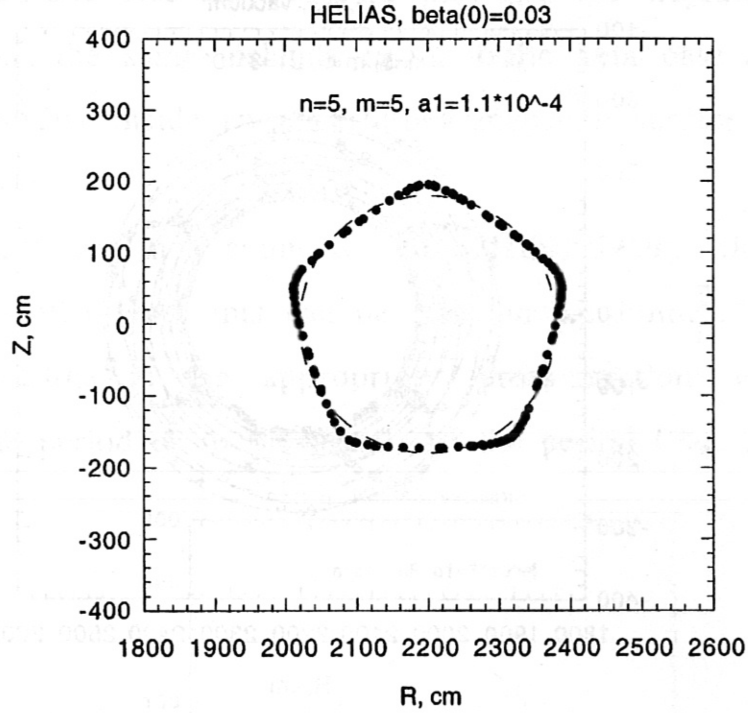


Fig.1C: The footprints of the two magnetic force lines in two meridional cross sections for the finite beta (ß(0)=0.03) case. The starting points of the two unperturbed magnetic force lines are r_0 =120 cm and r_0=180 cm. The perturbation has the "wave" numbers n=1, m=1 and the amplitude α_1=0.0001, δ=0.

3.4. Under the perturbation with n=5, m=5 a family of 5 large islands (see Fig. 2) was obtained. In the finite beta case (ß(0)=0.03) this structure arises above a threshold perturbation larger than $\delta B/B_0 = 2 \times 10^{-5}$. The magnetic surface below this threshold can be seen in Fig. 2A, and in Fig. 2B the perturbation is above this threshold.



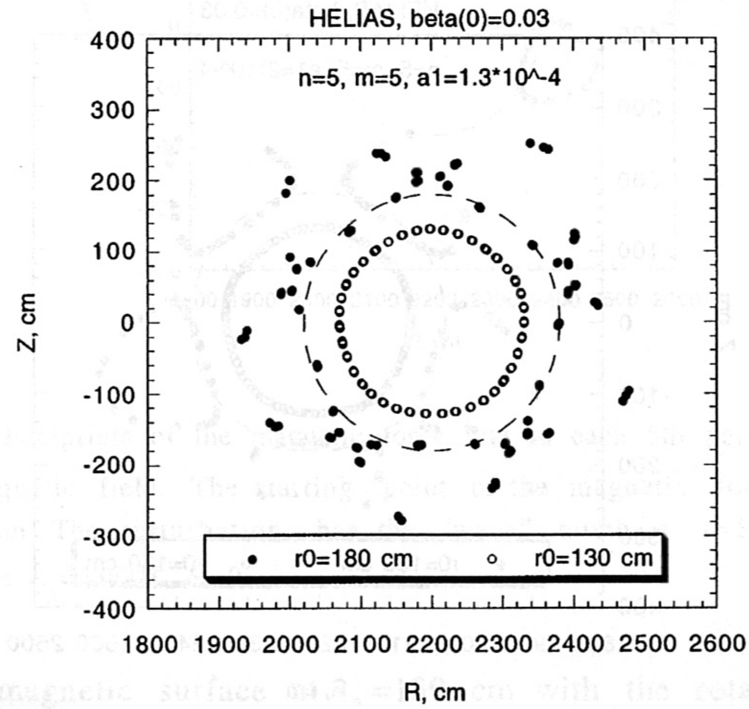
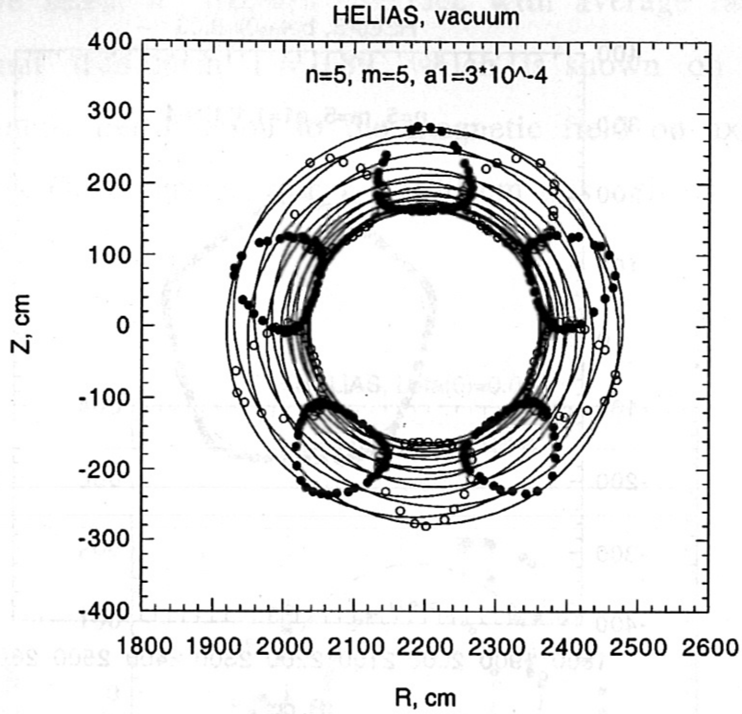


Fig. 2A(top),**2B**(bottom): The footprints of the magnetic force line in the finite beta (ß(0)=0.03) case before (A) and after bifurcation (B). The starting point of the magnetic force line is at r_0=180 cm. The perturbation has the "wave" numbers n=5, m=5 and the amplitude α_1=0.00011 (A) and α_1=0.00013 (B), δ=0.



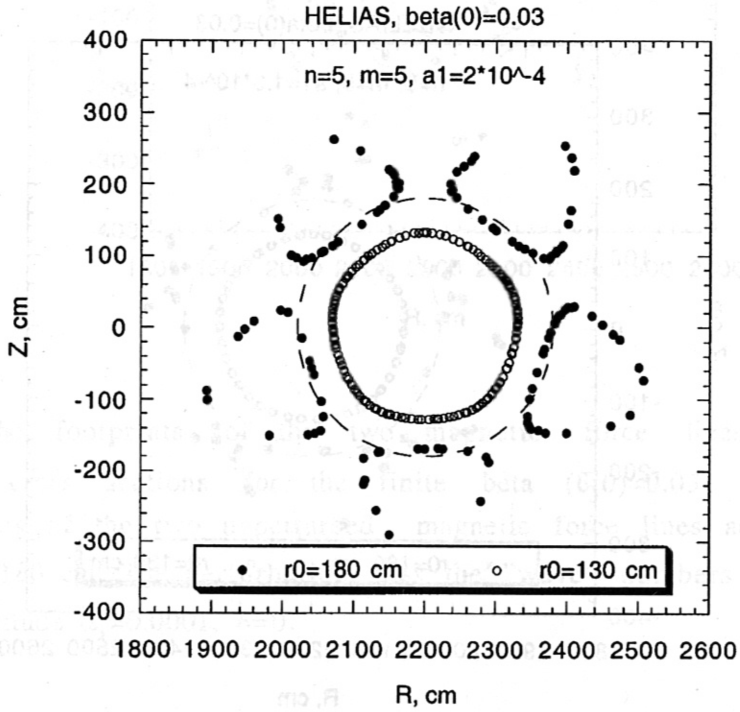


Fig. 2C(top),2D(bottom): Footprints of the perturbed magnetic force line with the starting point at r_0=180 cm in the vacuum case (C) and in the finite beta (ß(0)=0.03) case (D).

On the Fig. 2C one sees the Poincaré plot and the trajectory itself for the vacuum case, the same islands in the finite beta case are shown on the Fig. 2D. Further inside an unperturbed magnetic surface with r_o= 130 cm is also shown.

The 5 islands are not connected toroidally, every flux surface is separated from all others; this can be seen on the Poincaré plots of the magnetic force line in the appropriate cross section: either at the beginning of the period or in the middle of the period (Fig. 2E).

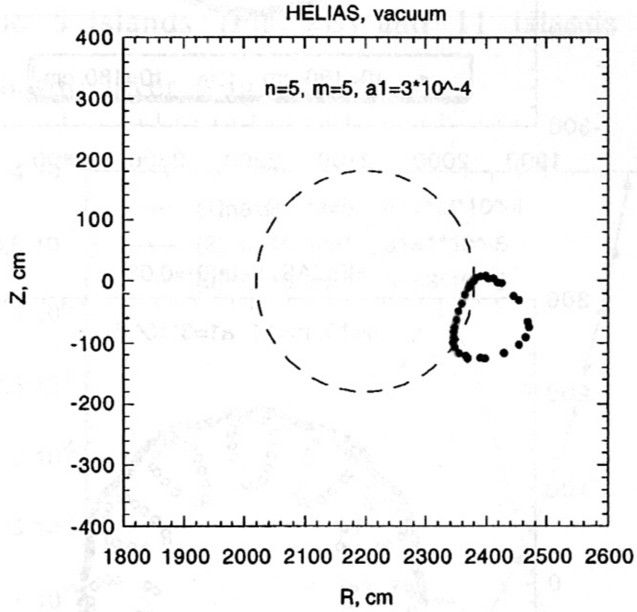
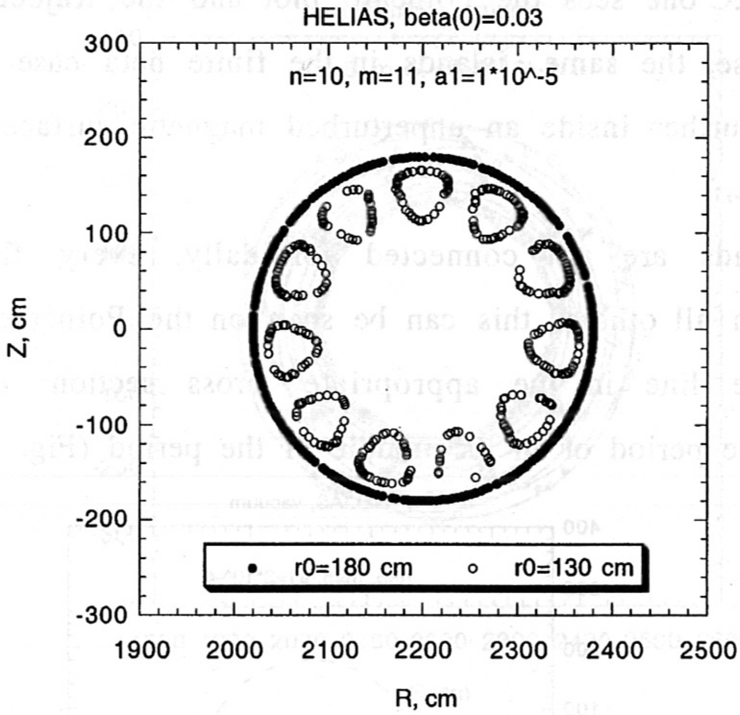


Fig. 2E: Footprints of the magnetic force line in each 5th period of the vacuum magnetic field. The starting point of the magnetic force line is at r_O =180 cm. The perturbation has the "wave" numbers n=5, m=5 and the amplitude α_1=0.0003, δ =0.

3.5. On the magnetic surface at r_o =130 cm with the rotational transform ι=0.909 an isolated resonance arises due to a single perturbation with n=10, m=11 (see Fig. 3A, 3B). The rotational transform angles of the perturbed trajectories are the following: ι=0.9423 on the surface with r_o=180 cm, and ι=0.9092 for the magnetic force line with starting point at r_o=130 cm.



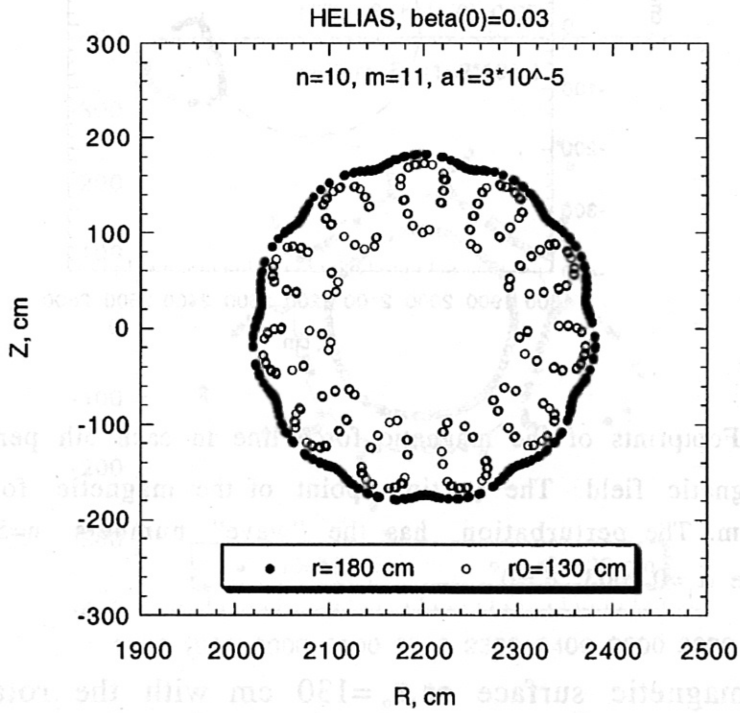


Fig. 3A(top) **3B**(bottom)**:** The footprints of the magnetic force line in one meridional cross section for the finite beta ($\beta(0)=0.03$) case. The starting point of the magnetic force line is at $r_O=180$ cm and $r_O=130$ cm. The perturbation has the "wave" numbers n=10, m=11 and the amplitude $\alpha_1=0.00001$ (A) and $\alpha_1=0.00003$ (B).

3.6. The superposition of two resonances in finite beta case ($\beta(0)$ = 0.03) (Fig. 4A, solid lines) with the maximum amplitudes $\delta B_{5,5}/B_o$=3.8 x10^{-5} (n=5, m=5, α_1=1.3x10^{-4}) and $\delta B_{10,11}/B_o$=1x10^{-5} (m=10, m=11, α_1=1x10^{-5}) leads to 5 and 11 islands on the surfaces with the radius r_o =180 cm (the rotational transform angle of the unperturbed trajectory is ι=0.94) and with r_o =130 cm (the rotational transform angle of unperturbed trajectory is ι=0.909) (Fig. 4B). Increasing the amplitude of the 10/11 perturbation to α_1=3x10^{-5} (Fig.4A, dashed line) leads to an "overlapping" of the 5 islands (Fig. 2B) and 11 islands (Fig. 3B) and a formation of a stochastic layer (Fig. 4C).

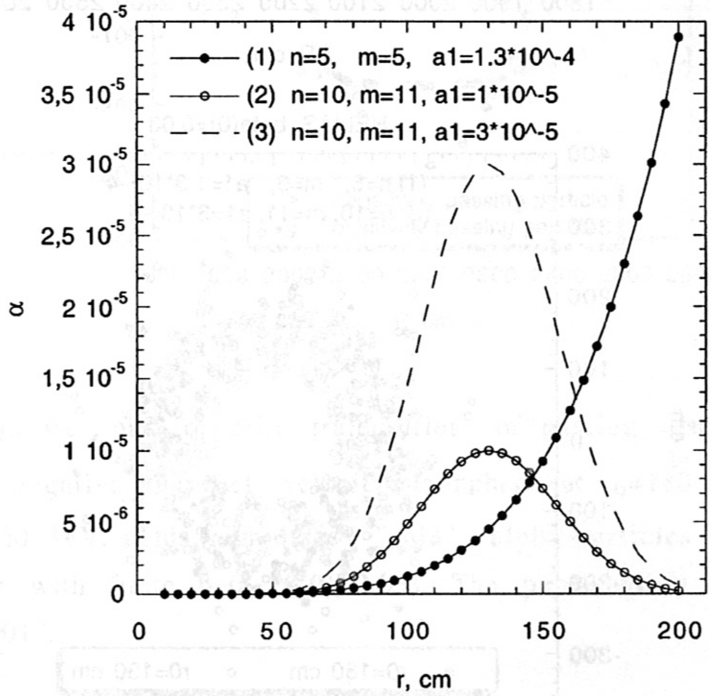


Fig. 4A: The radial profiles of the magnetic perturbatons with (1) n=5, m=5, α_1=1.3x10^{-4} ; (2) n=10, m=11, α_1=1x10^{-5};(3) n=10, m=11, α_1=3x10^{-5}.

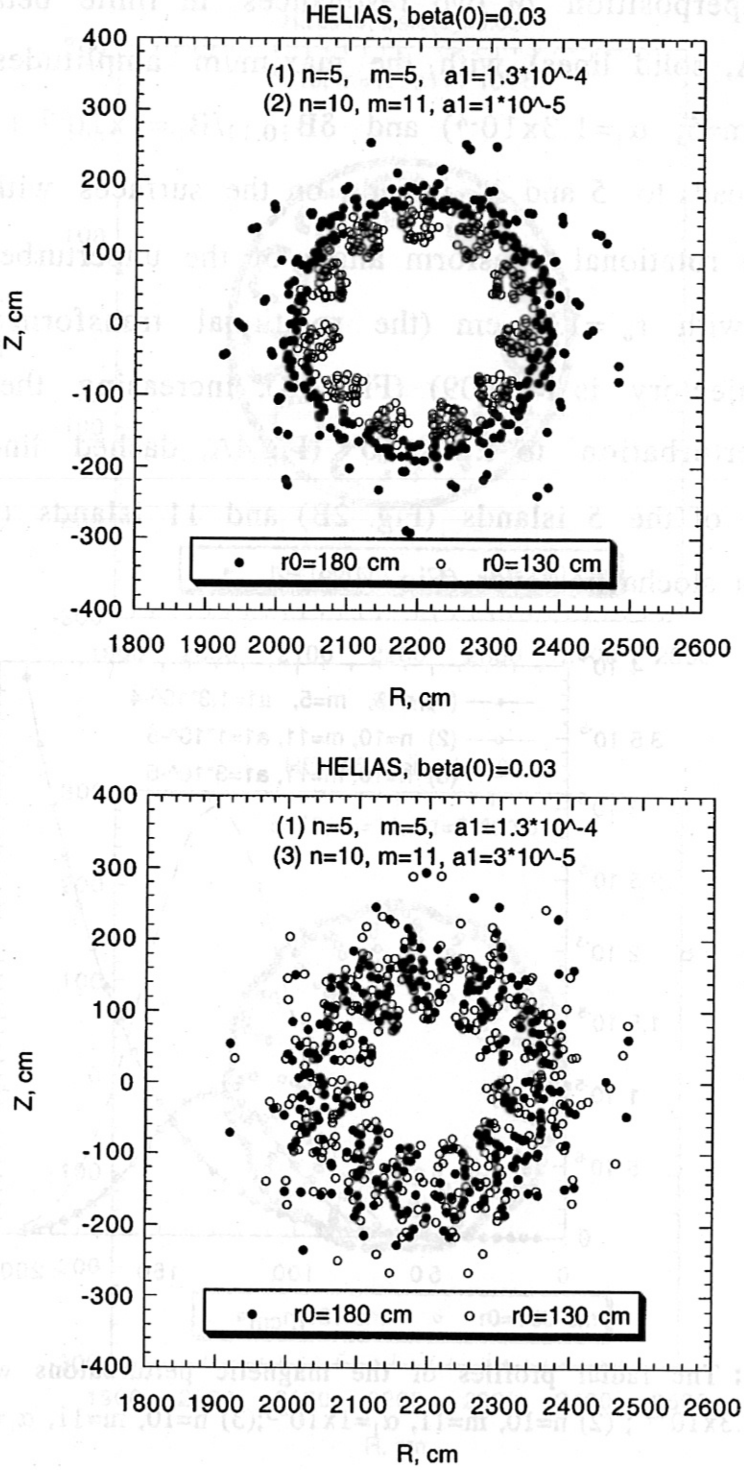


Fig. 4B(top),4C(bottom): Poincaré plot showing the superposition of two perturbations. (B) The meridional cross sections of the magnetic surfaces at the beginning of each 10th period of the magnetic field under the superposition of perturbations (1) and (2).(C) The same as (B), but under the superposition of perturbations (1) and (3).

3.7. Drift trajectories of the passing particles with the energy of "cold" alpha particles follow the 5/5 resonance structure only for one sign of the longitudinal velocity: the resonance takes place only for passing particles in positive direction. The opposite sign of the velocity gives the "pentagonal" form of the drift surfaces in the Poincaré plots (Fig. 5).

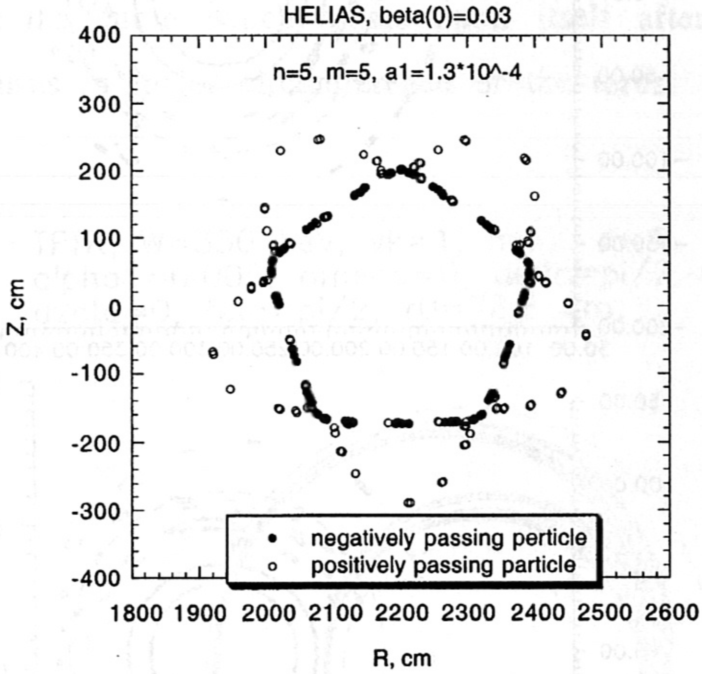


Fig. 5: Poincaré plot of drift trajectories of passing particles with positive and negative parallel velocity launched at r_0=180 cm with an energy E=350 keV. This simulates "cold" alpha-particles in Helias configurations with finite beta (ß(0)=0.03). The perturbation is n=5, m=5 and α_1 = 0.00013.

4. Stochasticity of Particle Orbits in a Tokamak Field

Following the idea of Mynick [1,2], particle orbits in a tokamak field were studied superimposing a perturbation with the "wave" numbers m=2, n=1 and an amplitude α_1=0.005. The resonance structure shows up if the phase of the perturbation coincides with the angular coordinate of the particle at start position, i.e. $\delta = \pi/2$ and $\vartheta_o = \pi/2$.

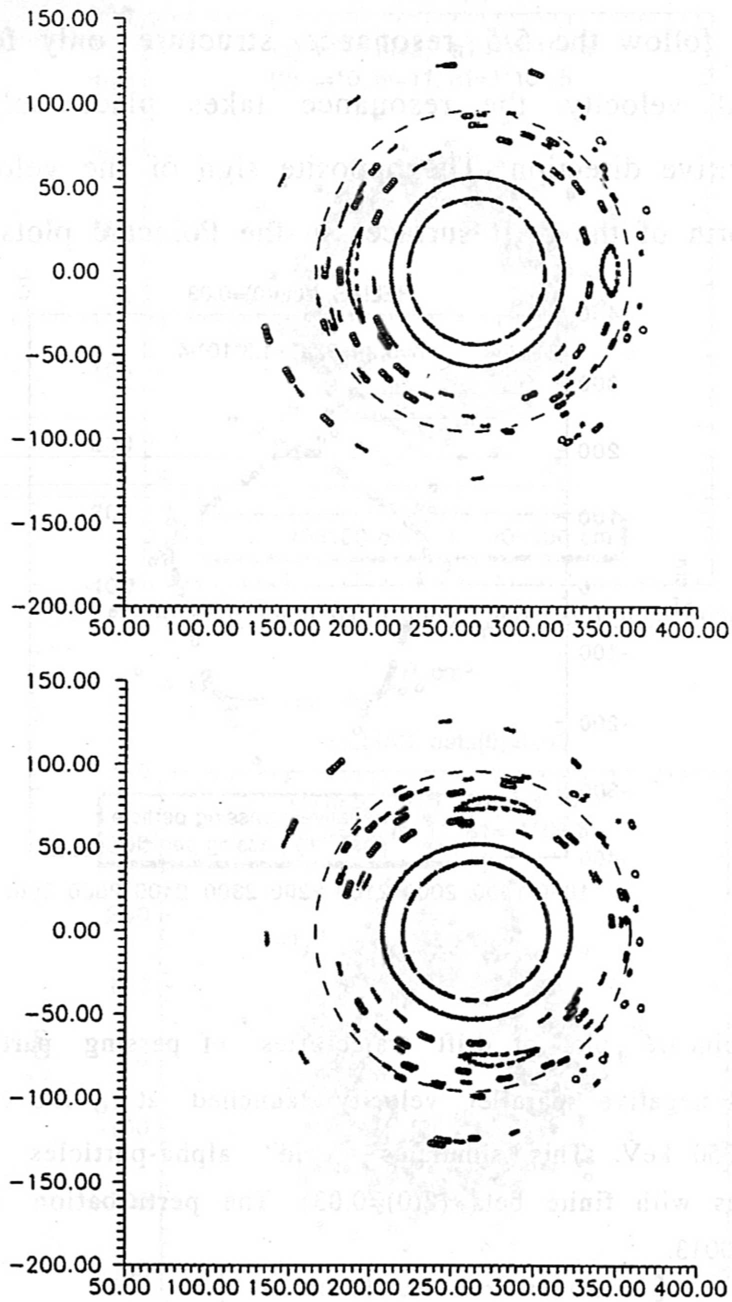


Fig. 6A,B: The projections of the particle trajectory in tokamak-type magnetic field with the parameters of TFTR on the meridional cross section taken at the beginning of the magnetic field period (A) and at the half of the magnetic field period (B). The total energy of the particle E=350 keV, the radial coordinates of the starting points are r_0=41.92 cm, 52.4 cm, 65.5 cm, 78.6 cm, 91.7 cm, the angular coordinates of the starting points are $\vartheta_0 = \pi/2$, $\zeta_0 = 0$. The perturbation has the "wave" numbers n=1, m=2 and $\alpha_1 = 0.005$, $\delta = \pi/2$.

In meridional cross sections of the drift surface at the beginning of the magnetic field period and in the center of the magnetic field period (Fig. 6A and B) one can see the islands of a particle with the starting point at r_o=78.6 cm. How close the particle orbit is to the resonance ι=1/2 can be seen from "topview" on the particle trajectory (Fig. 6C). The trajectory is rather similar to the curve which closes upon itself after one turn in minor and two turns in major circumference of the torus.

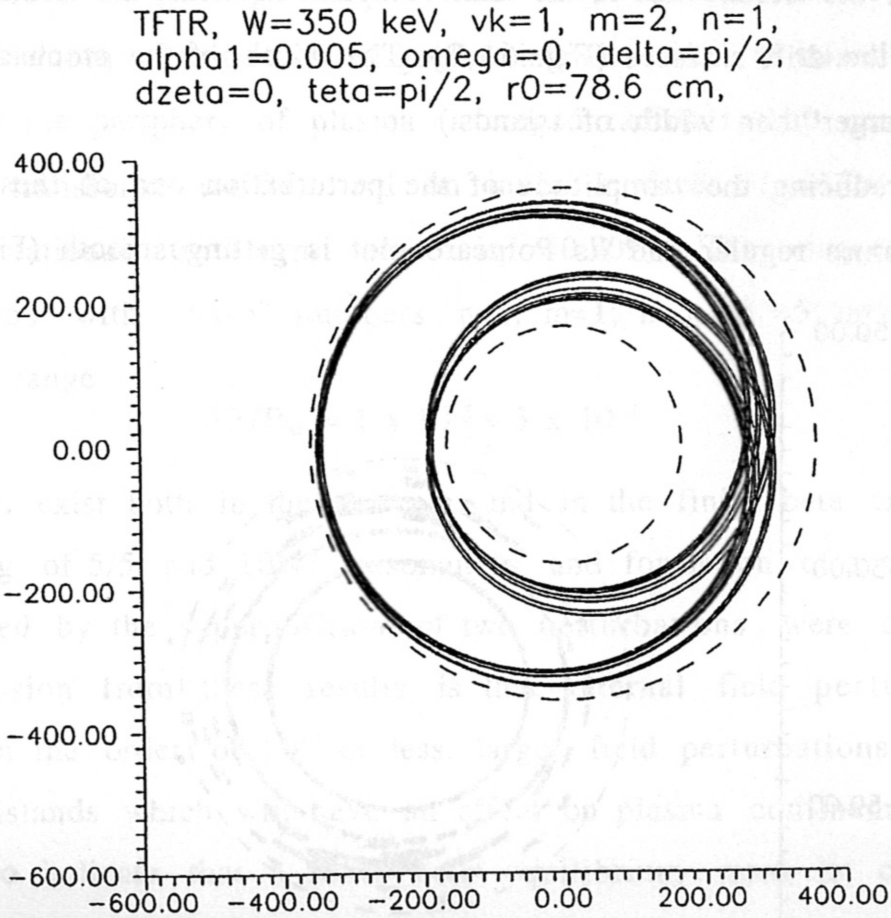


Fig. 6C: The projections of the particle trajectory in TFTR onto the equatorial plane of the torus ("topview"). The starting point is with r_0 =78.6 cm. The total energy of the particle is E=350 keV. The perturbation has the "wave" numbers n=1, m=2 and α_1=0.005, $\delta=\pi/2$.

For the particle starting at r_0=52.4 cm the meridional cross sections of the drift surfaces are smooth. The most interesting feature takes place in the orbits of particles starting radially outside the island (at the point with r_0=91.7 cm, $\vartheta_0 = \pi/2$) and radially inside to the island (at the point with r_0=65.5 cm, $\vartheta_0=\pi/2$). One can see the footprints of a particle starting radially outside the island (marked by "circles") also radially inside the island, and vice versa. From the projection onto the meridional plane of the particle trajectory with r_0=78.6 cm we can determine the radial "width" of the stochastic layer and compare it with the dimension of islands in the drift surfaces (Fig. 6A,B). The width of the stochastic layer is much larger then width of islands.

After reducing the amplitude of the perturbation one of these trajectories becomes regular and its Poincaré plot is getting smooth (Fig. 6D).

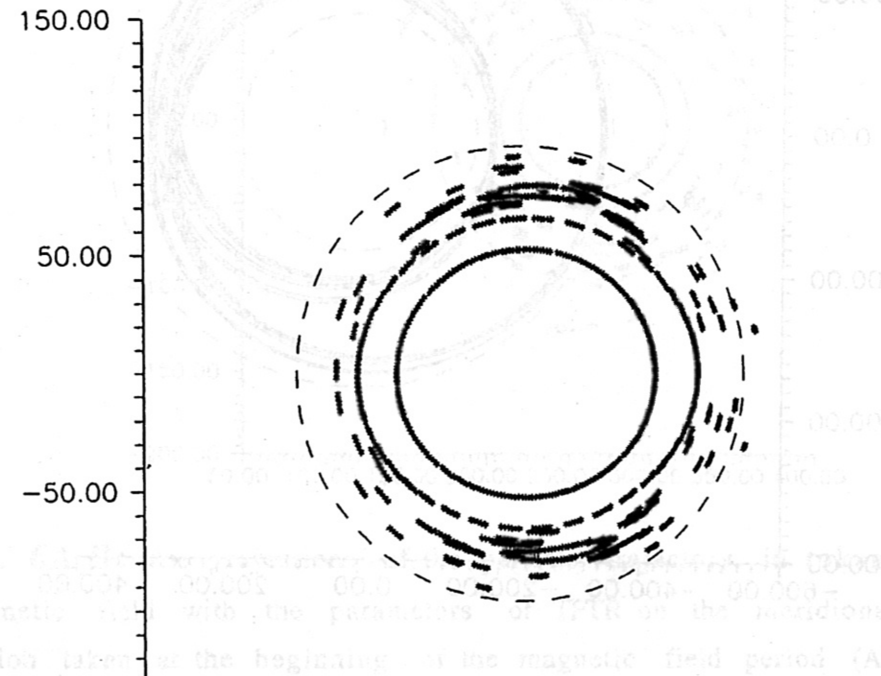


Fig. 6D: The same as at Fig. 6A,B but under the perturbation with α_1=0.0025.

5. Summary and Conclusions

5.1. As a result of internal magnetic field perturbations islands on magnetic surfaces and on drift surfaces of alpha particles were found in a Helias configuration. Such internal perturbations originate from the finite beta effect or from MHD instabilities and have to be distinguished from external field errors caused by a possible misalignment of the coil system. The amplitude of these internal perturbation is considered as a variable and it is investigated at which amplitude the effect on drift surfaces becomes significant. Magnetic islands were obtained on the surfaces at the periphery of plasma (averaged radius r_o=180 cm, original rotational transform ι=0.94), and on inner surfaces (r_o=130 cm, the original rotational transform angle is ι=0.909). The magnetic field perturbations with "wave" numbers n=1, m=1; n=5, m =5; n=10, m=11 are in the range

$$\delta B/B_o = 1 \times 10^{-5} \div 3 \times 10^{-4}$$

The islands exist both in the vacuum and in the finite beta cases. The overlapping of 5/5 and 10/11 resonances and formation of a stochastic layer caused by the superposition of two perturbations were observed. The conclusion from these results is that internal field perturbations must be in the order of 10^{-4} or less, larger field perturbations lead to magnetic islands which will have an effect on plasma confinement. The results also indicate that a finite beta equilibrium must be computed with a precision of 10^{-4} or better to give reliable results on the fine structure of magnetic surfaces.

5.2. The drift trajectories of the highly energetic (W=350 keV) and passing particles follow the resonance structure, and the Poincaré plots of drift surfaces exhibit an island structure, too. However, this effect depends on the sign of the longitudinal velocity. Islands and stochastization of drift surfaces certainly will enhance the neoclassical losses of the alpha-particles. How much the accumulation of cold alpha-particles will be reduced by this effect needs further investigation.

5.3. For comparison with a more simple magnetic configuration of the tokamak type, drift surfaces of passing particles in the TFTR configuration were studied imposing a perturbation with m=2, n=1. It was demonstrated trajectories of particles also may become stochastic. The radial deviations of the resonance trajectories from the initial surface are larger than those of non-resonance trajectories. The size of islands in the tokamak configuration can be understood as the effect of the large shear which is more than one order of magnetude larger than in a Helias configuration ($\delta\iota/\iota$ = 0.05 in Helias and $\delta\iota/\iota$ > 2 in tokamaks).

The study of passing and trapped particles under resonant perturbations, both static and time-dependent ones, and in particular the process of stochastization is under further investigation.

Acknowledgments

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