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Empirical Scaling Laws and Extrapolation to Helias Reactors

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Abstract: Empirical scalings of energy confinement are used to predict ignition parameters in a Helias reactor. These scaling laws are: Lackner -Gottardi scaling (LGS), International Stellarator Scaling (ISS95) and International Stellarator Scaling with W-7 data only ($ISS95_{W7}$). For comparison, tokamak scaling laws (ITER89 and ELMYH92y) are also taken into account. The results show that ISS95 yields a confinement time which is too small for ignition. LGS and $ISS95_{W7}$, however, are sufficient for ignition. An isotope factor or any improvement by H-mode confinement are not taken into account. The Helias reactor (HSR, R = 22m, a = 1.8 m, B = 4.75 T, $<\beta> = 4.4\%$, $P_{fus} = 3040$ MW) requires a confinement time $\tau_E = 1.7$ s.

1. Introduction

In the following, plasma parameters in a Helias reactor will be calculated on the basis of empirical scaling laws. Several scaling laws of energy confinement have been proposed. These are: the Lackner-Gottardi scaling (LGS)[1], the gyro-Bohm (GRB) scaling and the LHD scaling (LHD). Recently, based on experimental results from various stellarators and torsatrons, new stellarator scaling laws have been derived from the international stellarator data base (ISS)[2]. These results are listed in the following table (ISS95 and ISS$_{W7}$). The scaling law ISS$_{W7}$ is derived from Wendelstein 7-AS and Wendelstein 7-A data only. The general form of these scaling laws is a power law

$$\tau_E = \text{Const. } R^{a1} a^{a2} B^{a3} \langle n \rangle^{a4} P^{a5} \kappa^{a6} \iota^{a7}\ A^{a8}$$

with R = major radius, a = minor plasma radius, B = magnetic field, P = heating power, κ = elongation of magnetic surfaces (unity in stellarators), A = effective atomic mass, <n> = line averaged density, ι = rotational transform. The isotope factor A has not yet been confirmed in stellarators; here this factor may be used in order to test the sensitivity to parameter changes. The coefficients of the scaling laws are given in the following table. The units are: length in m, magnetic field in T, power in MW, density in 10^{20} m^{-3}, time in s.

Table 1: Exponents of empirical scaling laws

		LGS	ISS95	ISS$_{W7}$	LHD	GRB
Const		0.175	0.256	0.36	0.17	0.25
P	a5	-0.6	-0.59	-0.54	-0.58	-0.68
R	a1	1	0.65	0.74	0.75	0.6
a	a2	2	2.21	2.21	2.0	2.4
B	a3	0.8	0.83	0.73	0.84	0.8
ι	a7	0.4	0.4	0.43	0.0	0.0
<n>	a4	0.6	0.51	0.5	0.69	0.6

LHD-scaling and Gyro-Bohm scaling do not depend on the rotational tranform, however the experimental data of Wendelstein 7-AS indicate an ι–dependence and therefore support the Lackner-Gottardi scaling law in this respect.

In extrapolating these scaling laws to a stellarator reactor the proper choice of the scaling law is of great importance. The ISS95 is based on all stellarators, its database is the largest, however it does not distinguish between low shear and high shear devices. The experimental results, however, show that there is a difference between these two

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[1] K. Lackner, E. Gottardi, *Nucl. Fusion* , Vol. **30**, 1990, p. 767

[2] U. Stroth, M. Murakami, R.A. Dory, H. Yamada, S. Okamura. F. Sano, T. Obiki , Nucl. Fusion 36, 1063 (1996)

categories. The experimental confinement times in Wendelstein 7-AS are larger than predicted by the International Stellarator Scaling as can be seen in the following figure.

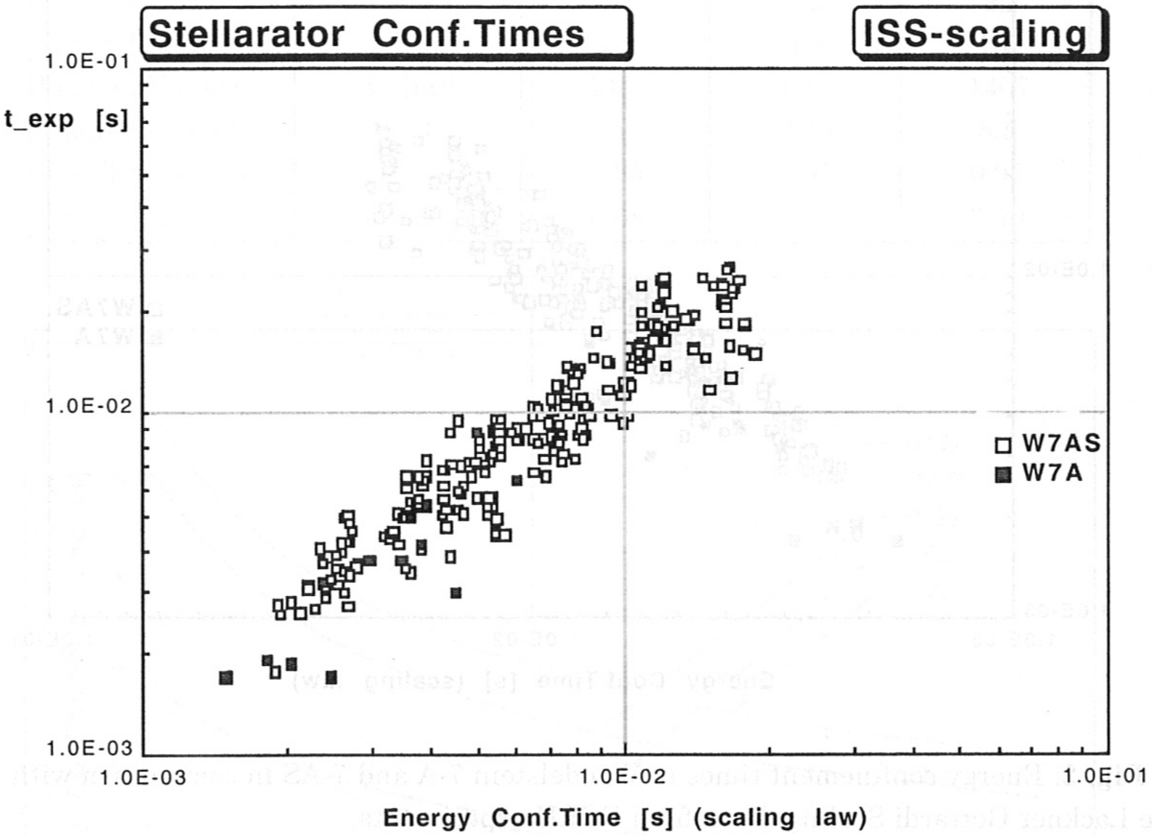

Fig. 1: Energy confinement times in Wendelstein 7-A and 7-AS in comparison with the International Stellarator Scaling ISS95. Data from ECRH experiments.

The energy confinement times in W 7-AS and W 7-A are roughly 25% larger than predicted by the ISS95 law[3]. Since the major radius in these two devices is the same, scaling with major radius is undetermined. The choice here is to use the same exponent as in LHD scaling, however it will be shown later how the results of the extrapolation depend on this coefficient.

The Helias reactor is an upgraded version of the Wendelstein 7-X experiment under construction in Greifswald. It is more closely related to low shear-configurations such as Wendelstein 7-A and Wendelstein 7-AS than to high-shear devices. Therefore extrapolating confinement times on the basis of LGS or ISS_{W7} scaling may be more appropriate than using those from the ISS95 scaling. The comparison with LGS is shown in the following figure.

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[3] The best fit to W7-AS is given by the ISS_{W7} scaling but also LGS gives a fair representation of the data

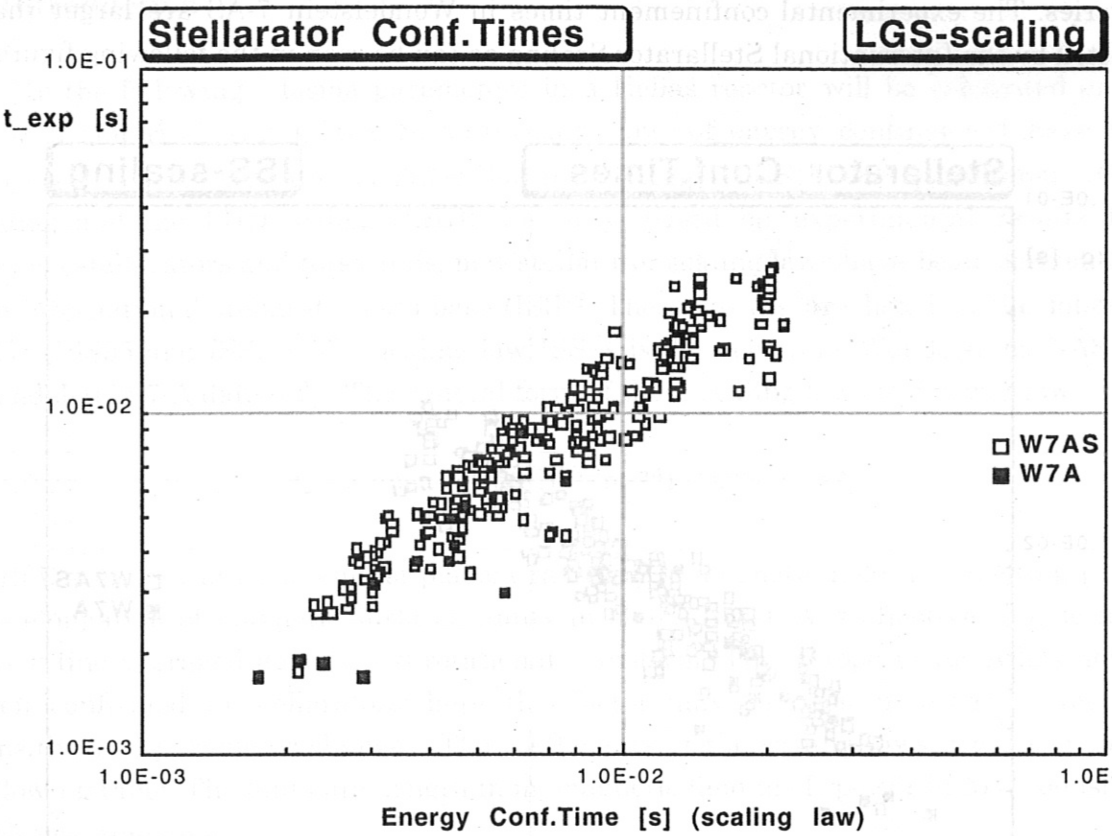

Fig. 2: Energy confinement times in Wendelstein 7-A and 7-AS in comparison with the Lackner Gottardi Scaling. Data from ECRH experiments.

A similar result as in Fig. 2 is given by the LHD-scaling and the experimental data from W 7-AS do not allow one to distinguish between these two scaling laws. An isotope factor has not yet been verified in stellarator experiments; in extrapolating towards the reactor an isotope factor is not taken into account.

A short overview on the expected confinement times in a Helias reactor is given in the following section.

2. Parameters of a Helias reactor

The Helias reactor is a straightforward extrapolation of the Wendelstein 7-X configuration, its dimensions are mainly determined by the space needed for blanket and shield and the requirements of the superconducting coils. Details are described elsewhere [4]. In the following we take the parameters of the HSR reference case which are listed in the following table.

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[4] C.D. Beidler et al., 16th IAEA Fusion Energy Conference, Montreal ,Oct. 1996, paper CN-64/G1-4

Table 2: Parameters of the Helias reactor HSR22

Device		HSR22A	HSR22B	HSR22C
Major Radius	R [m]	22	22	22
Minor Radius	a [m]	1.8	1.8	1.8
Plasma Volume	V [m³]	1407	1407	1407
Magnetic Field	B [T]	5	4.75	5.5
Rot. Transform	ι	0.95	0.95	0.95
Equiv. Current	I [MA]	3.68	3.50	3.50

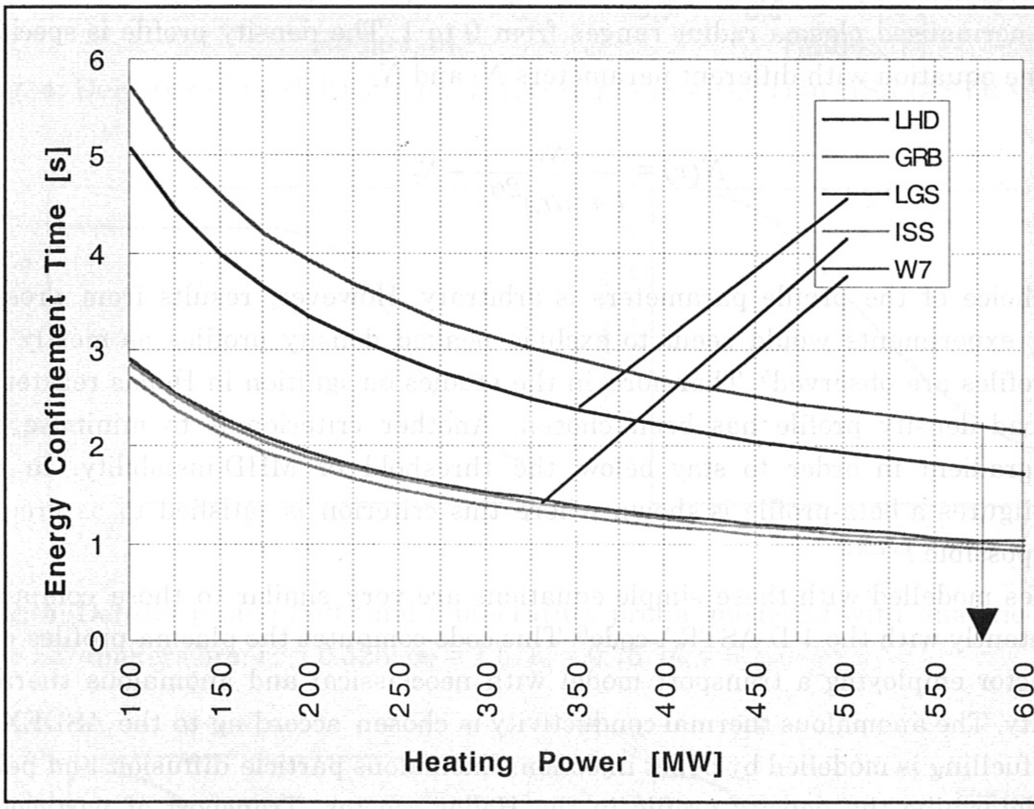

Fig. 3: Confinement times in HSR22A. R = 22 m, a = 1.8 m, B = 5 T, $n_L = 2.73 \times 10^{20}$ m^{-3}. The arrow indicates the reactor regime

This figure shows that energy confinement times in the the reactor regime (P = 600 MW) are 1 - 2 s. Scaling laws like ISS95 predict a confinement time of 1 s, LG scaling and ISS_{W7} confinement times around 2 s. As will be shown in the following analysis a confinement time of 1 s is too low to reach ignition. The required number is around 1.8 s. The heating power is the alpha-particle heating power minus the radiation losses. Under reactor conditions the heating power is in the range from 300 to 700 MW.

3. Plasma profiles

The temperature profiles in the Helias reactor are modelled by a simple analytic formula which has four free parameters: the central temperature T(0), the temperature at the boundary T(a) and two parameters r_T and α describing the width of the profile and the slope at $r = r_T$. The general shape of the temperature profile is given by the following equation

$$T(r) = \frac{T_1}{1 + \left(r/r_T\right)^{2\alpha_T}} + T_2$$

The coefficients T_1 and T_2 are determined by the condition $T_1 + T_2 = T(0)$ and $T(1) = T_a$. The normalised plasma radius ranges from 0 to 1. The density profile is specified by the same equation with different parameters N_1 and N_2

$$N(r) = \frac{N_1}{1 + \left(r/r_n\right)^{2\alpha_n}} + N_2$$

The choice of the profile parameters is arbitrary. However, results from present stellarator experiments would seem to exclude peaked density profiles as mostly flat density profiles are observed[5]. Therefore in the studies on ignition in Helias reactors a rather broad density profile has been chosen. Another criterion is to minimise the pressure gradient in order to stay below the threshold of MHD-instability. In the following figures a beta-profile is shown where this criterion is satisfied to as great a degree as possible .

The profiles modelled with these simple equations are very similar to those computed self-consistently with the 1-D ASTRA-code[6]. This code computes the plasma profiles of a Helias reactor employing a transport model with neoclassical and anomalous thermal conductivity. The anomalous thermal conductivity is chosen according to the ASDEX L-mode[7]. Refuelling is modelled by pellet injection. Anomalous particle diffusion and pellet injection determine the density profile in the Helias reactor. Transport of particles is dominated by anomalous transport; it is assumed that the particle confinement time is 3-4 times the energy confinement time. Plasma profiles resulting from these conditions are shown in Fig. 4. If the anomalous transport of particles is assumed to be smaller the neoclassical particle transport driven by temperature gradients leads to hollow density profiles. Density and temperature profiles modelled according to the results of ASTRA are given in Figs. 5. From these input data the beta profile and the profile of α-heating power can be computed (Fig. 6).

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[5] F. Wagner, U. Stroth, Plasma Physics and Contr. Fusion 35 (1993) 1321

[6] N. Karulin, Start-up scenario in a Helias Reactor, IPP-report IPP 2/337

[7] K. Lackner et al. in Plasma Physics and Contr. Fusion 31, No. 10, (1989) 1629

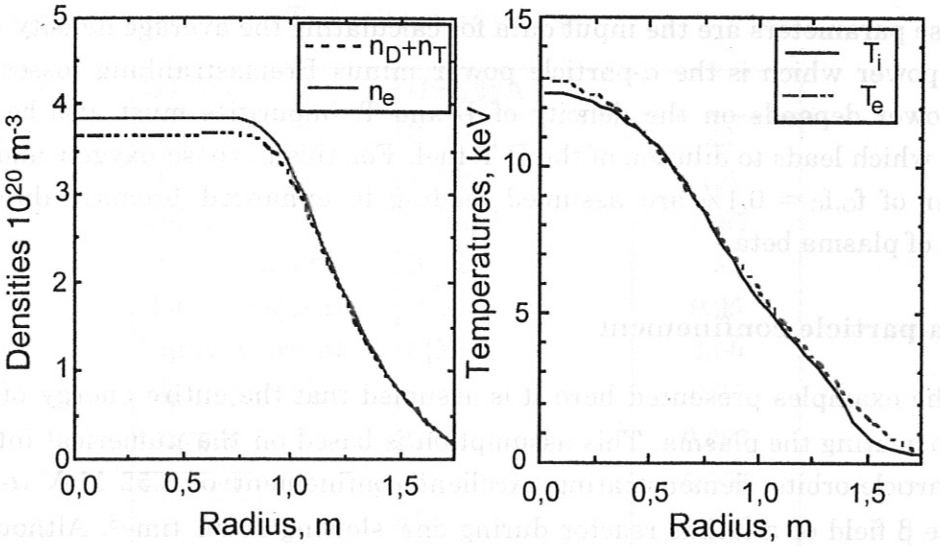

Fig. 4: Density profile (left) and temperature profiles (right) in HSR (ASTRA-code)

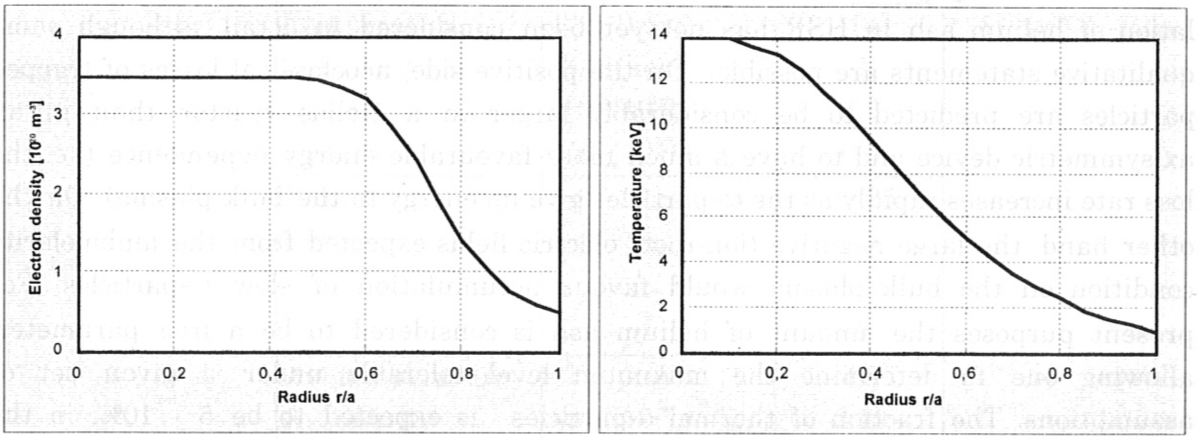

Fig. 5: Density profile (left) and temperature profile modelled with analytic equations. The parameters are: $r_T = 0.525$, $\alpha_T = 1.5$, $r_n = 0.75$, $\alpha_n = 5$.

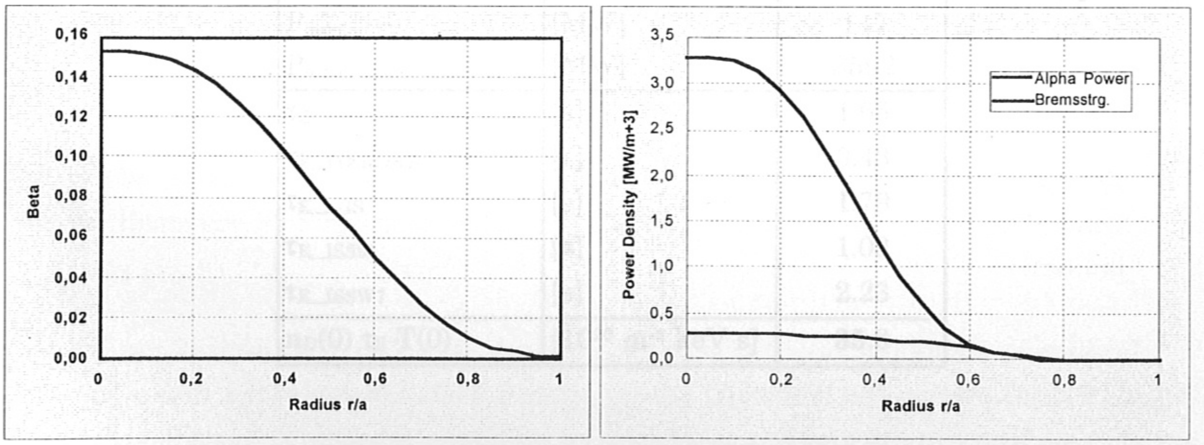

Fig. 6: Beta profile (left) and profile of α-heating power (right)

These parameters are the input data for calculating the average density and the net heating power which is the α-particle power minus Bremsstrahlung losses. Since the fusion power depends on the density of D and T, impurities must also be taken into account which leads to dilution of the D-T fuel. For this purpose oxygen and carbon on the order of $f_O, f_C = 0.1\%$ are assumed leading to enhanced bremsstrahlung and an increase of plasma beta.

4. Alpha-particle confinement

In the examples presented here it is assumed that the entire energy of α-particles goes into heating the plasma. This assumption is based on the numerical integration of single particle orbits, demonstrating excellent confinement of 3.52 MeV α-particles in the finite β field of a Helias reactor during one slowing-down time[8]. Although a small number of fast alpha particles do fall victim to the modular-coil ripple[9], these prompt losses have a negligible influence on the power balance of the reactor. The accumulation of helium ash in HSR has not yet been considered in detail, although some qualitative statements are possible. On the positive side, neoclassical losses of trapped particles are predicted to be considerably larger in a Helias reactor than in an axisymmetric device and to have a much more favourable energy dependence (i.e. the loss rate increases rapidly as the α–particles give up energy to the bulk plasma). On the other hand, the large negative (ion-root) electric fields expected from the ambipolarity condition on the bulk plasma would favour accumulation of slow α–particles. For present purposes the amount of helium ash is considered to be a free parameter, allowing one to determine the maximum level tolerable under a given set of assumptions. The fraction of thermal α-particles is expected to be 5 - 10%; in the following example $f_\alpha = 5\%$ is assumed, however the effect of higher α-particle content is also analysed. The parameters of the reference reactor HSR22A are listed in the following table.

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[8] W. Lotz, P. Merkel, J. Nührenberg and E. Strumberger, Plasma Phys. Control. Fusion 34 (1992) 1037.

[9] C.D. Beidler, G. Grieger, E. Harmeyer, F. Herrnegger, J. Kißlinger, E. Strumberger, H. Wobig andA.V. Zolotukhin, 24th European Conf. on Controlled Fusion and Plasma Physics (Berchtesgaden, Germany, 9-13 June 1997) vol 21A part IV (1997) 1681.

Table 2: Standard case: B = 5 T, R = 22 m, a = 1.8 m

Device	**HSR22A**	
Maj. radius	R [m]	**22**
Min. radius	a [m]	**1.8**
Volume	V [m³]	**1407**
Magnet. field	B [T]	**5**
Rot. transform	iota	**0.95**
Equiv. Current	I [MA]	**3.50**
Alpha-particles	f_α	**0.05**
Oxygen	f_O	**0.001**
Carbon	f_C	**0.001**
	Z_{eff}	1.186
	f_{D+T}	0.886
Eff. mass	A	**1**
Temperature	T(0) [kev]	14
Temperature	T(a) [kev]	1
Density	n(0) [10^{20} m^{-3}]	3.5
Density	n(a) [10^{20} m^{-3}]	0.5
Profile	r_T	0.525
Parameter	α_T	1.5
	r_n	0.75
	α_n	5
Av. temperature	<T> [keV]	4.9
Av. density	<n> [10^{20} m^{-3}]	2.19
Max. beta	ß(0)	0.153
Av. beta	<ß>	0.045
P_α	[MW]	718
P_{brems}	[MW]	147
P_{fusion}	[MW]	3592
τ_E	[s]	1.65
$\tau_{E\_ITERL(89)}$	[s]	0.43
τ_{E_LGS}	[s]	1.79
τ_{E_ISS95}	[s]	1.03
τ_{E_ISSW7}	[s]	2.23
$n_D(0)\ \tau_E\ T(0)$	[10^{20} m^{-3} keV s]	**35.8**

5. Ignition conditions

The confinement time required to sustain the burning plasma is 1.65 s. It is computed from the plasma energy and the available heating power, which is the α-particle power minus the bremsstrahlung. This net heating power and the line averaged electron density is used to compute the confinement times from the empirical scaling laws. As seen from table 3, confinement times are in the same range as that required. However, the LGS-time and the ISS_{W7}-time are larger than the required time, while τ_ISS95 is too small for ignition. Using the equivalent current of 3.5 MA and the ITER-L mode scaling yields a confinement time of 0.43s.

The averaged beta is 4.5 %, which is expected to be below the threshold of the ballooning mode instability. The ignition margin is defined as the ratio between the empirical confinement time and the required confinement time. In the following figure we have varied the plasma temperature while keeping the density profile fixed. As a result we obtain the ignition margin as a function of plasma beta.

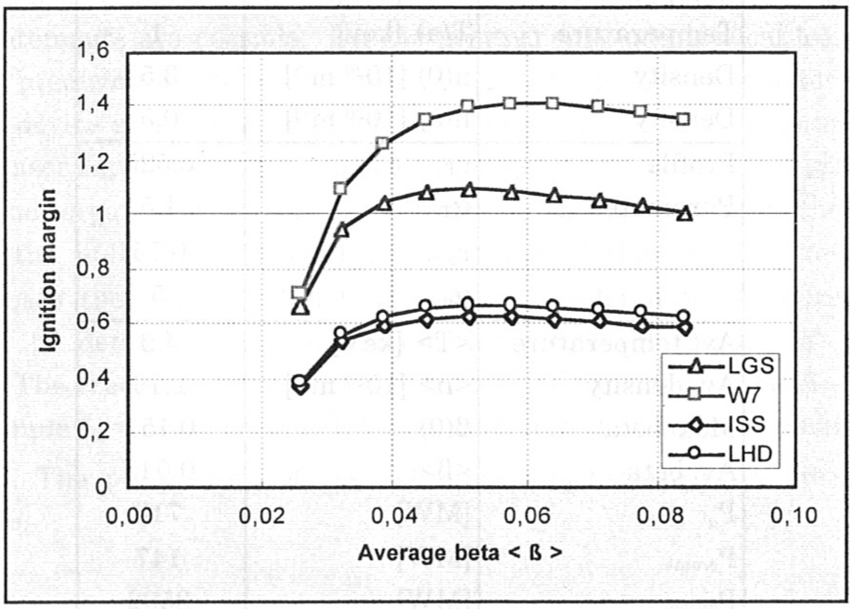

Fig. 7: Ignition margin in HSR22A. LHD-scaling, LGS (Lackner-Gottardi - scaling), ISS = stellarator scaling based on all stellarators (ISS95), W7 = ISS_{W7}. The confinement time following the ISS95 is about 40% too small for ignition.

Ignition is possible above $<\beta> = 3\%$ according to the ISS_{W7} scaling; if confinement follows the LG-scaling the threshold $<\beta>$ is around 3.5%. However the fusion output is low at these lower limits and it is necessary for economic reasons to operate the Helias-reactor at a beta value around 4.5% in order to achieve a fusion power of 3.5 GW. The fusion power as function of the averaged beta is shown in Fig. 8. .

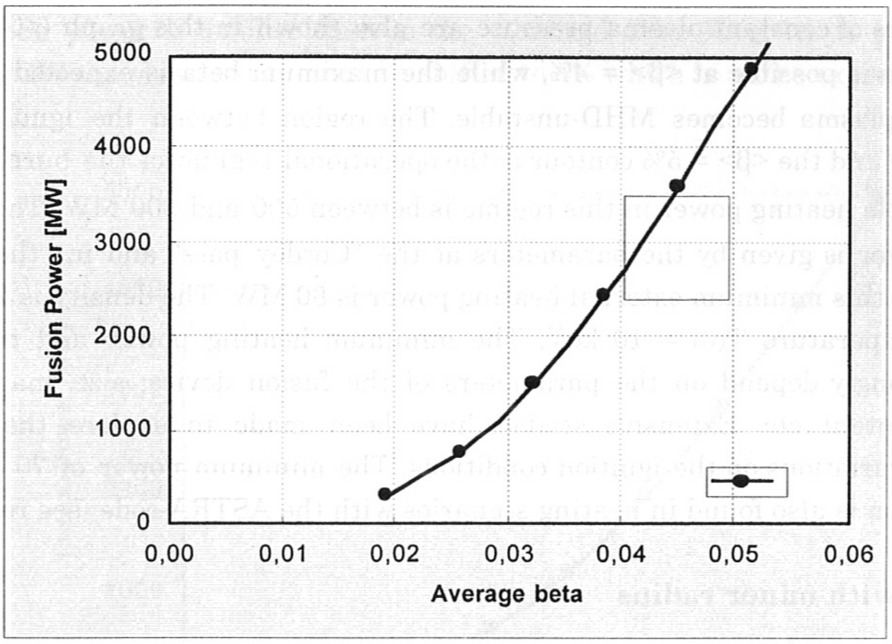

Fig. 8: Fusion power in HSR22A vs averaged plasma beta. The marked region indicates where the power reactor is expected. Fusion power is in the range of 2.5 to 3.5 GW. This regime is below the expected stability limit.

The heating power needed to reach ignition can be found by balancing the power loss to the heating power. Plasma energy and heating power are computed using the profiles specified above, the result is displayed in the following POPCON-plot which shows the contour lines of heating power.

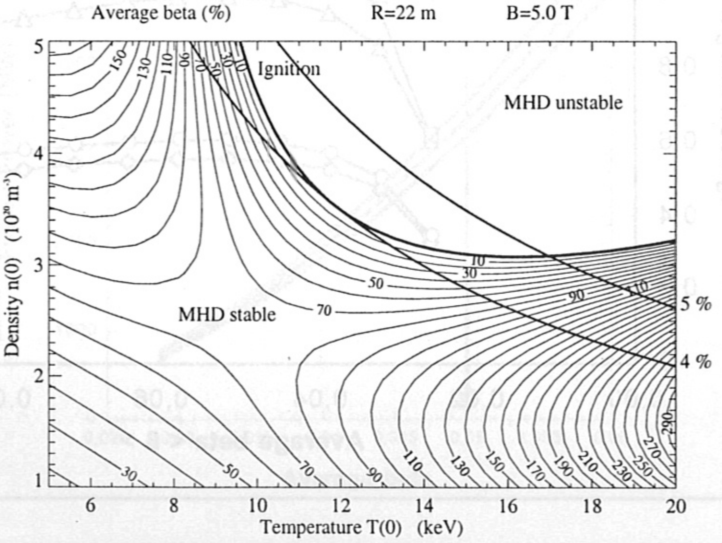

Fig. 9: POPCON plot of HSR22A standard case. The scaling of confinement in this plot is the LGS scaling. Heating power in MW

Contours of constant plasma pressure are also shown in this graph ($<\beta>$ = 4% and 5%). Ignition is possible at $<\beta>$ = 4%, while the maximum beta is expected around 5% where the plasma becomes MHD-unstable. The region between the ignition contour with $P_{ex} = 0$ and the $<\beta>$ = 5% contour is the operational regime of the burning plasma. The α-particle heating power in this regime is between 500 and 900 MW. The minimum heating power is given by the parameters at the "Cordey pass" and in the reference case of HSR this minimum external heating power is 80 MW. The density is $2.5*10^{20}$ m^{-3} and the temperature T(0) = 10 keV. The minimum heating power and the point of ignition strongly depend on the parameters of the fusion device: size, magnetic field, impurity content etc. Extensive studies have been made to explore the impact of parameter variations on the ignition conditions. The minimum power of 70 - 80 MW to reach ignition is also found in heating scenarios with the ASTRA-code (see ref. 3.).

6. Scaling with minor radius

A reduction of minor radius leads to smaller plasma volume and less fusion output. Furthermore confinement times become smaller and ignition is more difficult to reach. The following figure shows the ignition margin at a plasma radius of 1.6 m. In this case the LGS time is too small for ignition.

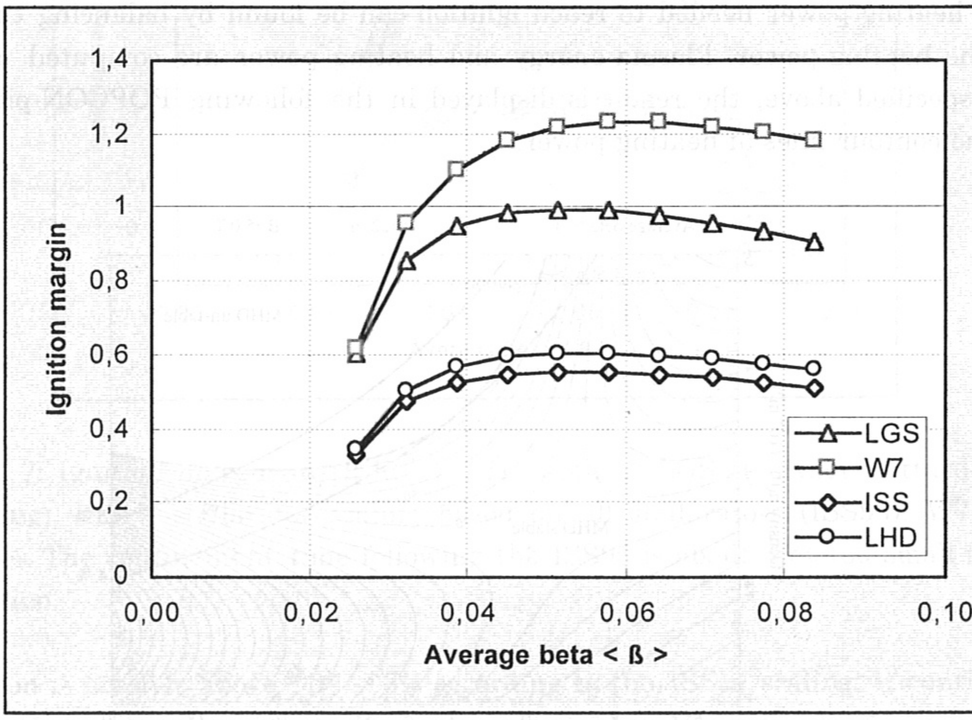

Fig. 10: Ignition margin at a reduced plasma radius of 1.6 m. LGS scaling predicts an ignition margin close to unity. The ISS_{W7} scaling yields an ignition margin higher than 1.2.

The fusion power is smaller than in the standard case. In order to get a fusion power of 3.5 GW the plasma beta must be shifted to 5% which is at the expected stability limit.

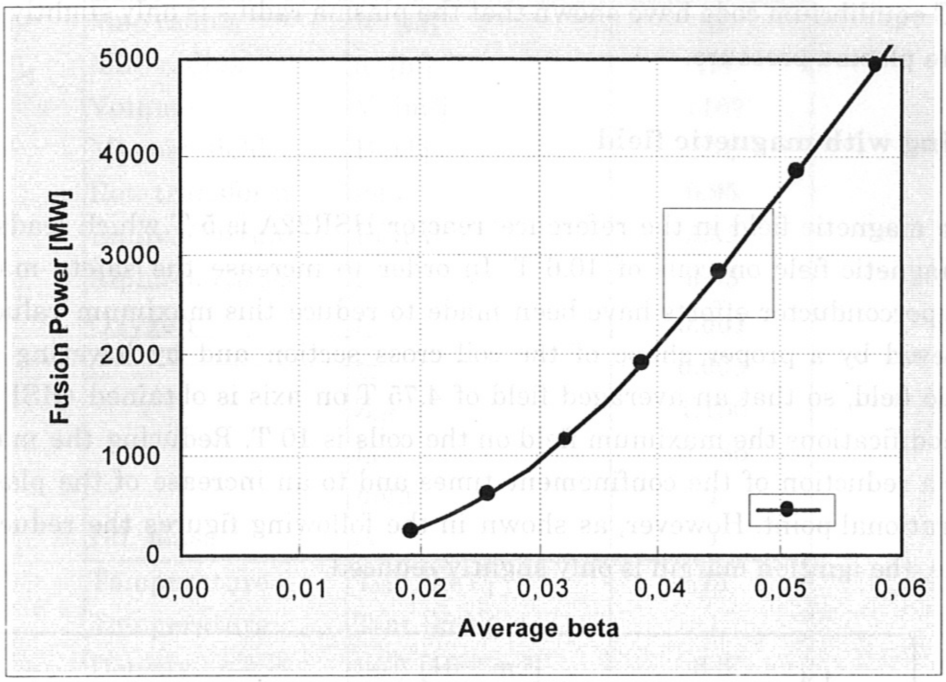

Fig. 11: Fusion power vs averaged beta. Effective plasma radius 1.6 m.

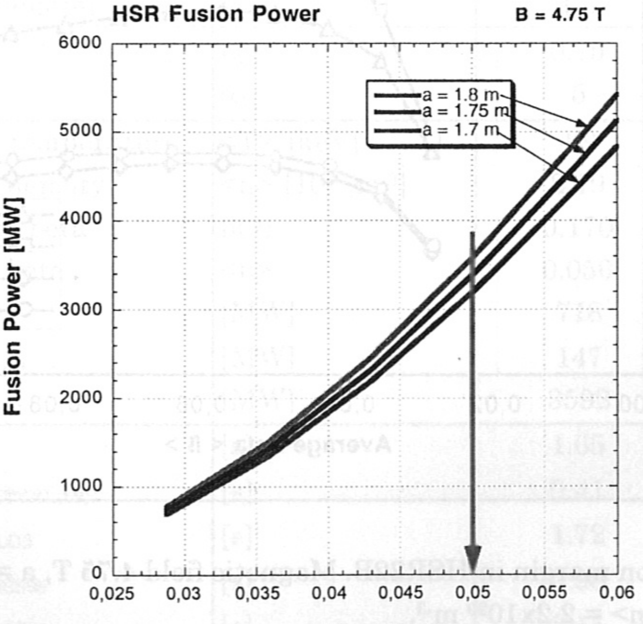

Fig. 12: Fusion power in HSR22B. Minor radius a = 1.7 - 1.8 m. In order to achieve a fusion output of 3 GW the operational regime is between <β> = 4.6% and 4.9%. The arrow indicates the expected stability limit.

A reduction of minor radius may be the effect of finite plasma pressure on the plasma equilibrium. a = 1.8 m is the averaged radius of the last magnetic surface in the vacuum magnetic field. However, first results of finite beta computations with the NEMEC equilibrium code have shown that the plasma radius is only slightly modified by the finite plasma pressure.

7. Scaling with magnetic field

The magnetic field in the reference reactor HSR22A is 5 T which leads to a maximum magnetic field on coils of 10.6 T. In order to increase the safety margin of the NbTi-superconductor efforts have been made to reduce this maximum value. This may be achieved by a proper shape of the coil cross section and by lowering the overall magnetic field, so that an averaged field of 4.75 T on axis is obtained (HSR22B). After these modifications the maximum field on the coils is 10 T. Reducing the magnetic field leads to a reduction of the confinement times and to an increase of the plasma beta at the operational point. However, as shown in the following figures the reduction of B is tolerable, the ignition margin is only slightly reduced.

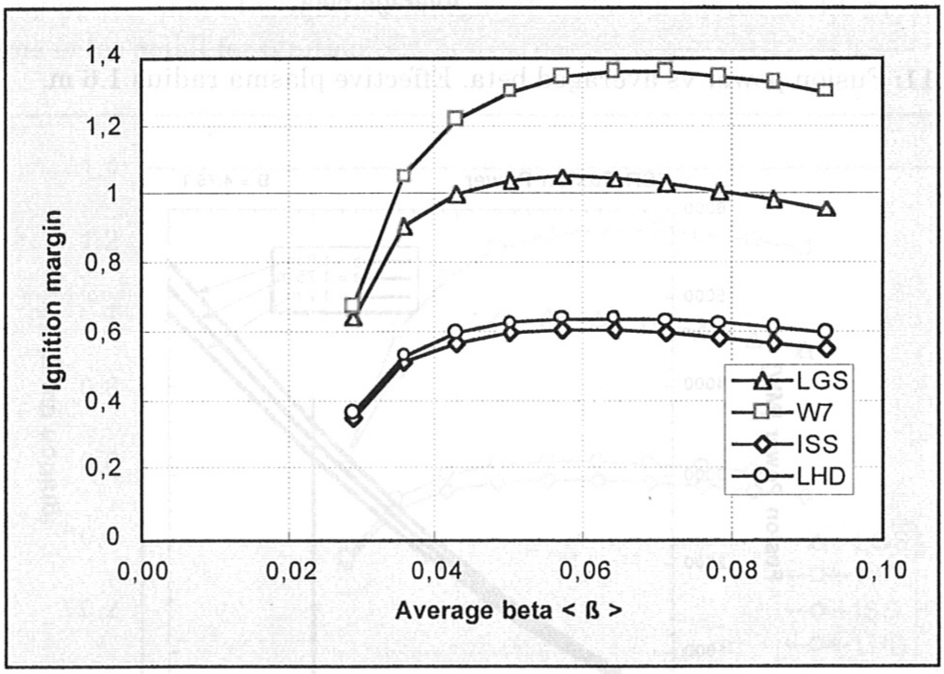

Fig. 13: Ignition margin in HSR22B. Magnetic field 4.75 T, a = 1.8 m, $n_e(0)$ = 3.5x10^{20} m^{-3}, <n> = 2.2x10^{20} m^{-3}.

Ignition according to LGS scaling is still possible above <β> = 4.3%. A list of the parameters similar to table 3 is given in the following table.

Table 3: B = 4.75 T, R = 22 m, a = 1.8 m,

Device	**HSR22B**	7
Maj. radius	R [m]	**22**
Min. radius	a [m]	**1.8**
Volume	V [m^3]	**1407**
Magnet. field	B [T]	**4.75**
Rot. transform	iota	**0.95**
Equiv. Current	I [MA]	**3.32**
Alpha-particles	f_α	**0.05**
Oxygen	f_O	**0.001**
C	f_C	**0.001**
	Z_{eff}	1.186
	f_{D+T}	0.886
	n_D/n_T	1
Eff. mass	A	1
Temperature	T(0) [keV]	14
Temperature	T(a) [keV]	1
Density	n(0) [10^{20} m^{-3}]	3.5
Density	n(a) [10^{20} m^{-3}]	0.5
Profile	r_T	0.525
Parameter	α_T	1.5
	r_n	0.75
	α_n	5
Av. temperature	<T> [keV]	4.9
Av. density	<n> [10^{20} m^{-3}]	2.19
Max. beta	ß(0)	0.170
Av. beta	<ß>	0.050
P_α	[MW]	718
P_{brems}	[MW]	147
P_{fusion}	[MW]	3592
τ_E	[s]	1.65
$\tau_{E\_ITERL(89)}$	[s]	0.41
τ_{E_LGS}	[s]	1.72
τ_{E_ISS95}	[s]	0.98
τ_{E_ISSW7}	[s]	2.15
$n_D(0)\ \tau_E\ T(0)$	[10^{20} m^{-3} keV s]	**35.8**

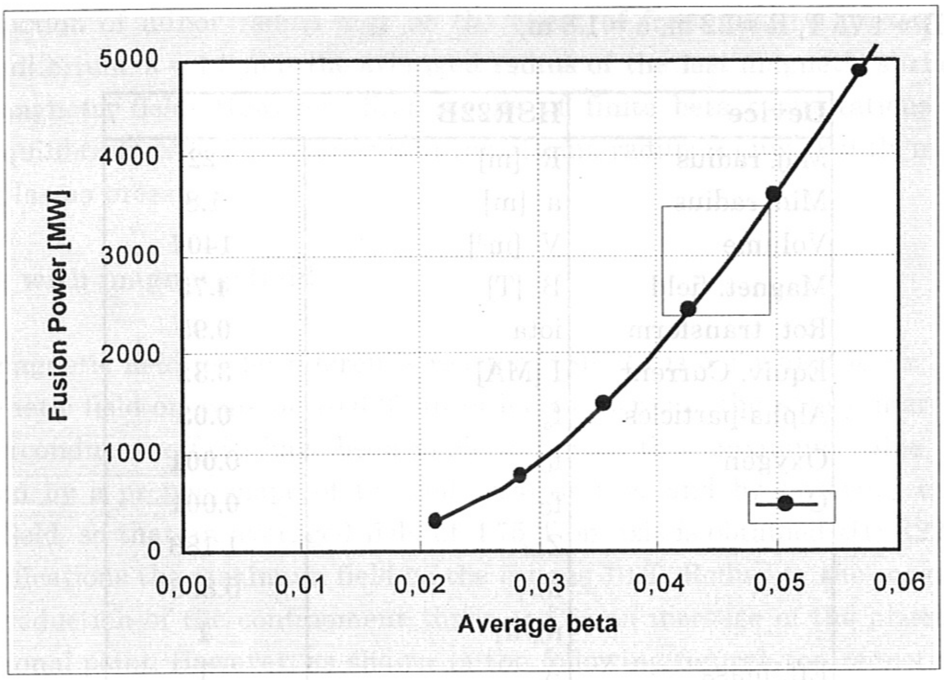

Fig. 14: Fusion power in HSR22B. Magnetic field 4.75 T, a = 1.8 m, $n_e(0) = 3.5 \times 10^{20}$ m^{-3}, $<n> = 2.2 \times 10^{20}$ m^{-3}.

In Table 2 and Table 3 the fusion power is the same: 3.59 GW. In case of the reduced magnetic field (HSR22 B, B = Berchtesgaden) the ignition margin of the LGS scaling is very close to 1, however, following the ISSW7 scaling it is 1.3. The strongest effect of the reduction of B is the increase of beta from $<\beta> = 4.5$ % to 5%. Since $<\beta> = 5$ % is close to the stability limit there is no safety margin.

In the following section the increase of the magnetic field to B = 5.5 T is investigated. This alternative is relevant if a NbSn superconductor is used instead of NbTi. The maximum magnetic field on the coils in this case is 12.2 T. Therefore the increase of the magnetic field from 5 T to 5.5 T means a step into another technology and the increase of mechanical forces on the coils by 20%. However, with respect to plasma confinement and stability the situation improves appreciably. The ignition margin will be increased as well as the distance to the stability limit.

Fig. 12 shows the plasma parameters for the case HSR22C with increased magnetic field. The averaged beta at the operational point is 3.7% and the ignition margin of the LGS scaling is 1.17. These results show that increasing the magnetic field to 5.5 T improves the physics conditions appreciably: The plasma beta is well below stability limits and confinement times are sufficiently larger than needed. This would allow one to increase the fusion output which makes the reactor more attractive from the economic point of view. The limit is the wall loading by neutrons which in the HSR standard case is around 1 MW/m² at a power output of 3 GW. An increase of the fusion power to 6 MW would push the wall load to 2 MW/m² on average which is in the range envisaged for tokamak reactors.

Table 4: Plasma parameter at B = 5.5 T (HSR22 C)

Device	**HSR22C**	
Maj. radius	R [m]	**22**
Min. radius	a [m]	**1.8**
Elongation	kappa	**1**
Volume	V [m^3]	**1407**
Magnet. field	B [T]	**5.5**
Rot. transform	iota	**0.95**
Current	I [MA]	**3.85**
Alpha-part.	f_a	**0.05**
Oxygen	f_O	**0.001**
Carbon	f_C	**0.001**
	Z_{eff}	1.186
	f_{D+T}	0.886
	n_D/n_T	**1**
Eff. mass	A	**1**
Temperature	T(0) [keV]	14
Temperature	T(a) [keV]	1
Density	n(0) [10^{20} m^{-3}]	3.5
Density	n(a) [10^{20} m^{-3}]	0.5
Profile	r_T	0.525
Parameter	α_T	1.5
	r_n	0.75
	α_n	5
Av. temperature	<T> [keV]	4.9
Av. density	<n> [m^{-3}]	2.19
Density_lim	$<n>_{lim}$ [10^{20} m^{-3}]	1.66
Max. beta	ß(0)	0.127
Av. beta	<ß>	0.037
P_α	[MW]	718
P_{brems}	[MW]	147
P_{fusion}	[MW]	3592
τ_E	[s]	1.65
$\tau_{E\_ITERL(89)}$	[s]	0.48
τ_{E_LGS}	[s]	1.93
τ_{E_ISS95}	[s]	1.11
$\tau_{E\_ ISSW7}$	[s]	2.42
$n_D(0)\ \tau_E\ T(0)$	[10^{20} m^{-3} keV s]	**35.8**

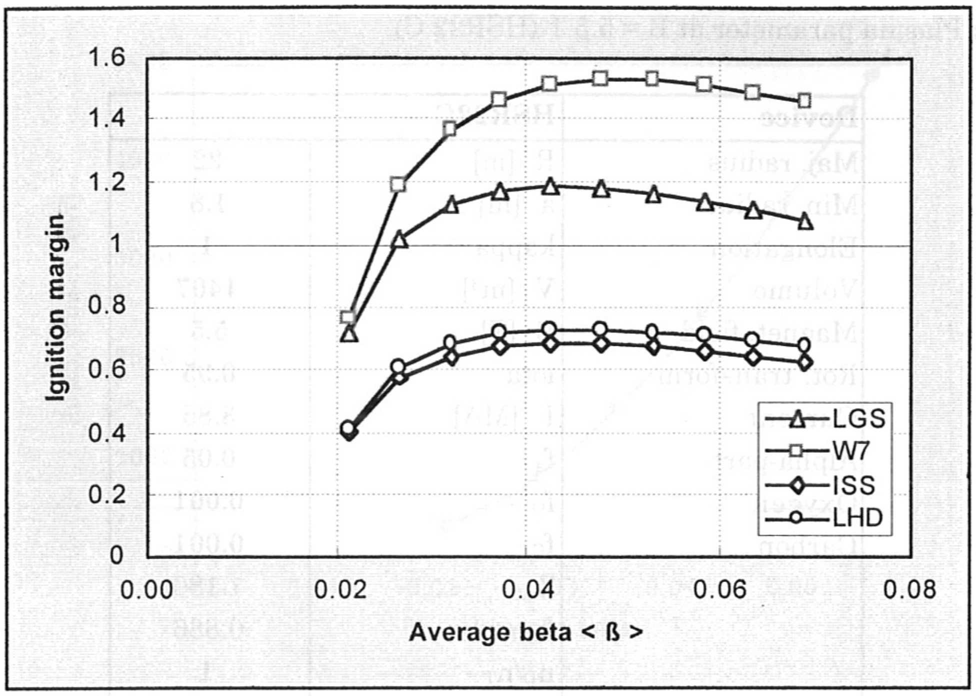

Fig. 15: Ignition margin in HSR22C, B = 5.5 T. The ignition margin according to the LG scaling may reach a maximum of 1.2 and in case of the scaling law ISS_{W7} 1.5.

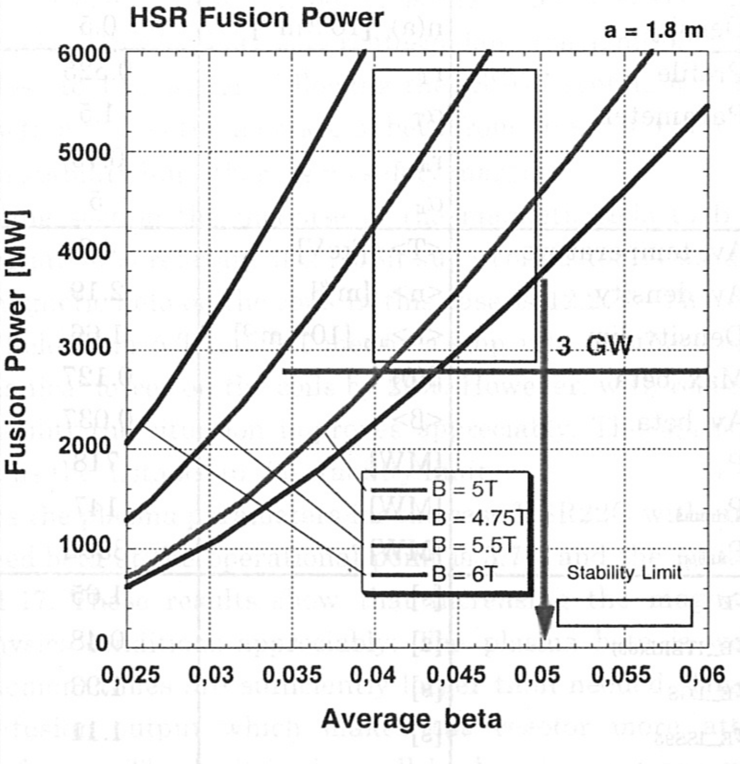

Fig. 16: Fusion output power in HSR22 vs averaged beta. The parameter of the curves is the magnetic field on axis. The marked area indicates the attractive region in which to operate a Helias power reactor.

Fig. 16 summarizes the fusion power output in HSR22 with various options of the magnetic field. Assuming a maximum wall load of 2 MW/m² on average and a magnetic field of 5.5 T an attractive regime in which to operate a Helias reactor would be at a beta value of 4 to 4.3%. Fusion power output in this regime is 4.3 to 5 GW.

8. Wendelstein 7 scaling (ISS$_{W7}$)

This scaling laws has been derived from experimental data in Wendelstein 7-A and Wendelstein 7-AS. Since the major radius is the same in both devices a dependence of the confinement time on major radius cannot be determined by these experiments. The coefficient $\alpha_R = 0.74$ used in the previous analysis has been chosen in line with the LHD scaling law. The ISS95 scaling has the exponent $\alpha_R = 0.65$. In the following we study how strongly the HSR ignition conditions depend on this coefficient.

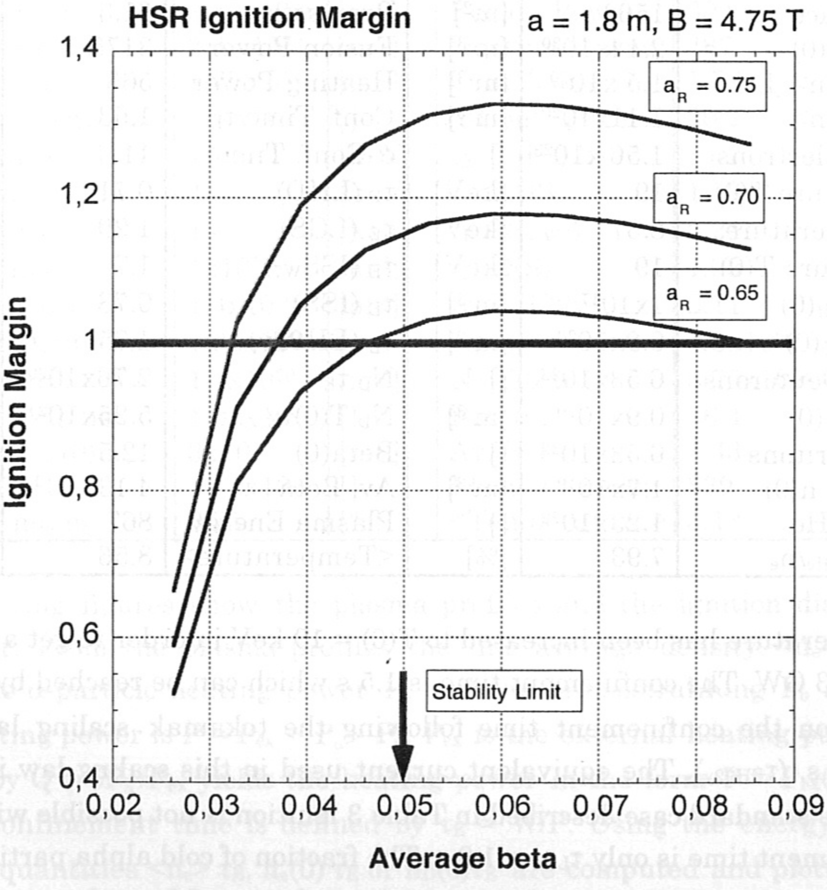

Fig. 17: Variation of exponent in major radius. $n(0) = 3.22\times10^{20}$ m^{-3} B = 4.75 T, a = 1.8 m.

The figure shows that there is a wide range of the exponent where ignition is still possible. Below $\alpha_R = 0.65$ it is difficult to reach ignition in the stable beta-regime.

In the previous computations the density was assumed to be in the order of $n(0)$ = 3- 4 x10^{20} m^{-3}. This is higher than experimental data in present stellarator experiments and therefore beyond the limits of validity of the scaling laws. For this reason it has been checked whether ignition is possible at lower densities and enhanced temperatures. The following table shows a data set where the line averaged density has been reduced to 1.5x10^{20} m^{-3}.

Table 5 (Reduced density)

Major Radius	22	[m]	Z_{eff}	1.16	[]
Minor Radius	1.8	[m]	DT_Power(0)	1.84x10^{6}	[MW m^{-3}]
Elongation	1.0	[]	DT_Power	635	[MW]
Iota(0)	1	[]	Neu._Power	2539	[MW]
Equiv.Current	3.68	[MA]	Neutrons	1.13x10^{21}	[s^{-1}]
Toroidal. Current	0	[MA]	T_burnup	487	[g/d]
Equiv. Iota	0	[]	D_burnup	324	[g/d]
Plasma Volume	1407	[m^3]	N_Power_FW	1.08	[MWm^{-2}]
Magnetic Field	5.0	[T]	Bremsstr. (0)	1.24x10^{5}	[MW m^{-3}]
Plasma Surface	1563	[m^2]	Bremsstr.	61.3	[MW]
El. Density n(0)	2.14x10^{20}	[m^{-3}]	Fusion Power	3175	[MW]
El. Density <n>_L	1.5 x10^{20}	[m^{-3}]	Heating Power	565	[MW]
El. Density <n>	1.11x10^{20}	[m^{-3}]	Conf. Time τ_E	1.53	[s]
Number of Electrons	1.56 x10^{23}	[]	α-Conf. Time	11	[s]
El. Temperature T(0)	19	[keV]	τ_E (LHD)	0.71	[s]
Av. El. Temperature	8.37	[keV]	τ_E (LGS)	1.29	[s]
H_Temperature T(0)	19	[keV]	τ_E (ISS_{W7})	1.7	[s]
H_Density $n_H(0)$	1x10^{17}	[m^{-3}]	τ_E (ISS)	0.78	[s]
D_Density $n_D(0)$	0.9x10^{20}	[m^{-3}]	τ_E (EH92y)	1.75	[s]
Number of Deuterons	6.53x10^{22}	[]	N_D τ_E	2.76x10^{20}	[sm^{-3}]
T_Density $n_T(0)$	0.9x10^{20}	[m^{-3}]	N_D T(0)τ_E	5.25x10^{21}	[skeVm^{-3}]
Number of Tritons	6.53x10^{22}	[]	Beta(0)	12.5	[%]
4He_Density n(0)	1.7x10^{19}	[m^{-3}]	Av. Beta	4.13	[%]
Number of 4He	1.23x10^{22}	[]	Plasma Energy	867	[MJ]
α-Fraction n_{He}/n_e	7.93	[%]	<Temperature>	8.56	[keV]

The temperature has been increased to T(0) = 19 keV in order to get a fusion power of more than 3 GW. The confinement time is 1.5 s which can be reached by τ_{ISSW7} (1.7 s). For comparison the confinement time following the tokamak scaling laws (ELMY-H mode) is 1.75 s. (τ_{EH92y}). The equivalent current used in this scaling law is 3.68 MA. In contrast to the standard case described in Table 3 ignition is not possible with LG-scaling as the confinement time is only τ_{LGS} = 1.3 s. The fraction of cold alpha particle is 8%, this number is consistent with a confinement time of 11 s. More than 10% cold alpha particles would be prohibitive to ignition.

9. Self-consistent data set

In the following we consider a set of plasma profiles modelled with the equations listed above and compute the $n\tau$-diagrams and the $n\tau T$-diagrams. The data are listed in the following table. The profile parameters are slightly different from those in Table 3.

Table 6 (HSR22B)

Major Radius	22	[m]	Z_{eff}	1.181	[]
Minor Radius	1.8	[m]	DT_Power(0)	2.66×10^{6}	[MW m^{-3}]
Elongation	1.0	[]	DT_Power	608	[MW]
Iota(0)	1	[]	Neu._Power	2434	[MW]
Equiv.Current	3.5	[MA]	Neutrons	1.1×10^{21}	[s^{-1}]
Toroidal. Current	0	[MA]	Tritium	466	[g/d]
Equiv. Iota	0	[]	Deuterium	311	[g/d]
Plasma Volume	1407	[m^3]	N_Power_FW	1.04	[MWm^{-2}]
Magnetic Field	4.75	[T]	Bremsstr. (0)	2.8×10^{5}	[MW m^{-3}]
Plasma Surface	1563	[m^2]	Bremsstr.	123	[MW]
El. Density n(0)	3.43×10^{20}	[m^{-3}]	Fusion Power	3044	[MW]
El. Density <n>_L	2.4×10^{20}	[m^{-3}]	Heating Power	487	[MW]
El. Density <n>	1.77×10^{20}	[m^{-3}]	Conf. Time τ_E	1.72	[s]
Number of Electrons	2.5×10^{23}	[]	α-Conf. Time	20.2	[s]
El. Temperature T(0)	14	[keV]	τ_E (LHD)	1.02	[s]
Av. El. Temperature	4.63	[keV]	τ_E (LGS)	1.79	[s]
H_Temperature T(0)	14	[keV]	τ_E (ISS_{W7})	2.24	[s]
H_Density $n_H(0)$	1×10^{18}	[m^{-3}]	τ_E (ISS)	1.03	[s]
D_Density $n_D(0)$	1.4×10^{20}	[m^{-3}]	τ_E (EH92y)	2.11	[s]
Number of Deuterons	1.02×10^{23}	[]	$N_D\ \tau_E$	4.8×10^{20}	[sm^{-3}]
T_Density $n_T(0)$	1.4×10^{20}	[m^{-3}]	$N_D\ T(0)\tau_E$	6.75×10^{21}	[$skeVm^{-3}$]
Number of Tritons	1.02×10^{23}	[]	Beta(0)	16.4	[%]
4He_Density n(0)	3×10^{19}	[m^{-3}]	Av. Beta	4.43	[%]
Number of 4He	1.18×10^{22}	[]	Plasma Energy	839	[MJ]
α-Fraction n_{He}/n_e	8.75	[%]	<Temperature>	5.18	[keV]

The following figures show the plasma profiles and the ignition diagrams of the Helias reactor. From the plasma profiles the line average density $<n_e>$, the plasma energy W, the α-particle heating power P_α and the bremsstrahlung P_b are computed. The total heating power is $P = P_{ex} + P_\alpha - P_b$. P_{ex} is the external heating power. Defining the factor Q by $Q = 5P_\alpha/P_{ex}$ yields the heating power in the form $P = P_{ex}(Q/(5+Q)) - P_b$. The energy confinement time is defined by $\tau_E = W/P$. Using the energy confinement time various quantities $<n_e>\ \tau_E$, $n_e(0)\ \tau_E$ or $n_D(o)\tau_E$ are computed and plotted versus the peak temperature or the averaged temperature. The averaged temperature is defined by the plasma energy $<T> = W/(3*V*<n_e>)$, $<n>$ is the line-averaged density. This definition does not distinguish between electron and ion temperatures. The ignition parameters can be plotted either against peak temperature or averaged temperature, all plots describe the same physics.

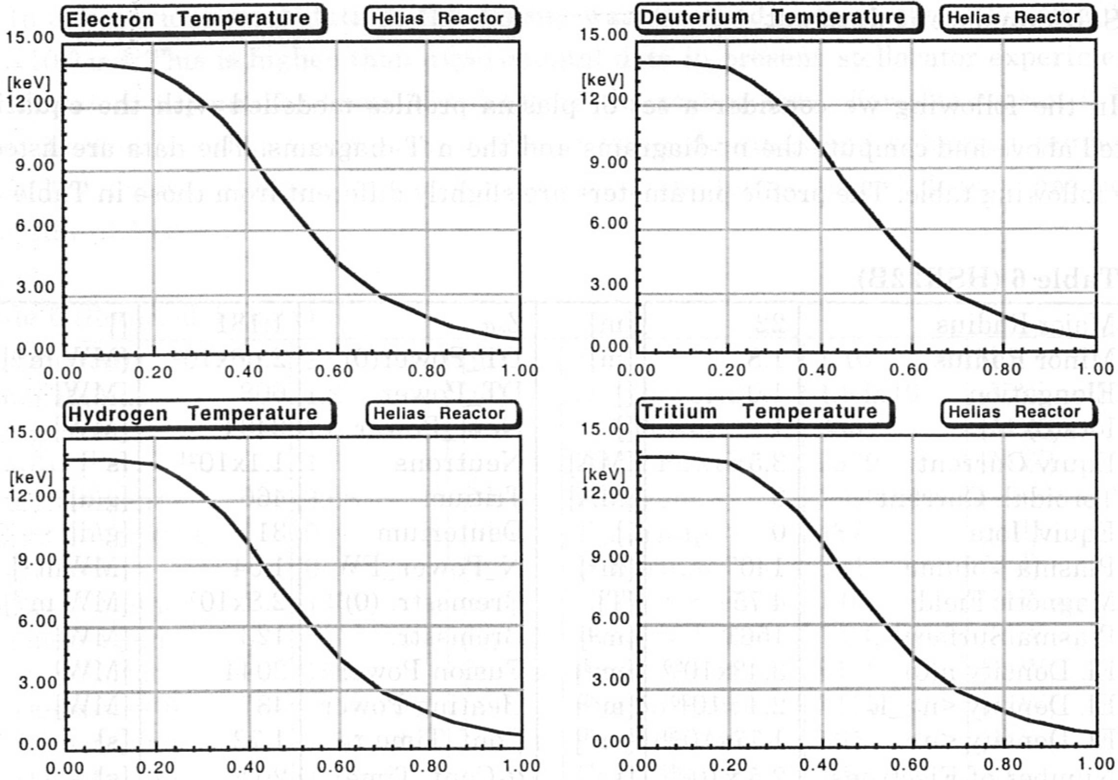

Fig. 18: Temperature profiles in HSR22, B = 4.75 T, $\alpha_T = 2, r_T = 0.5$.

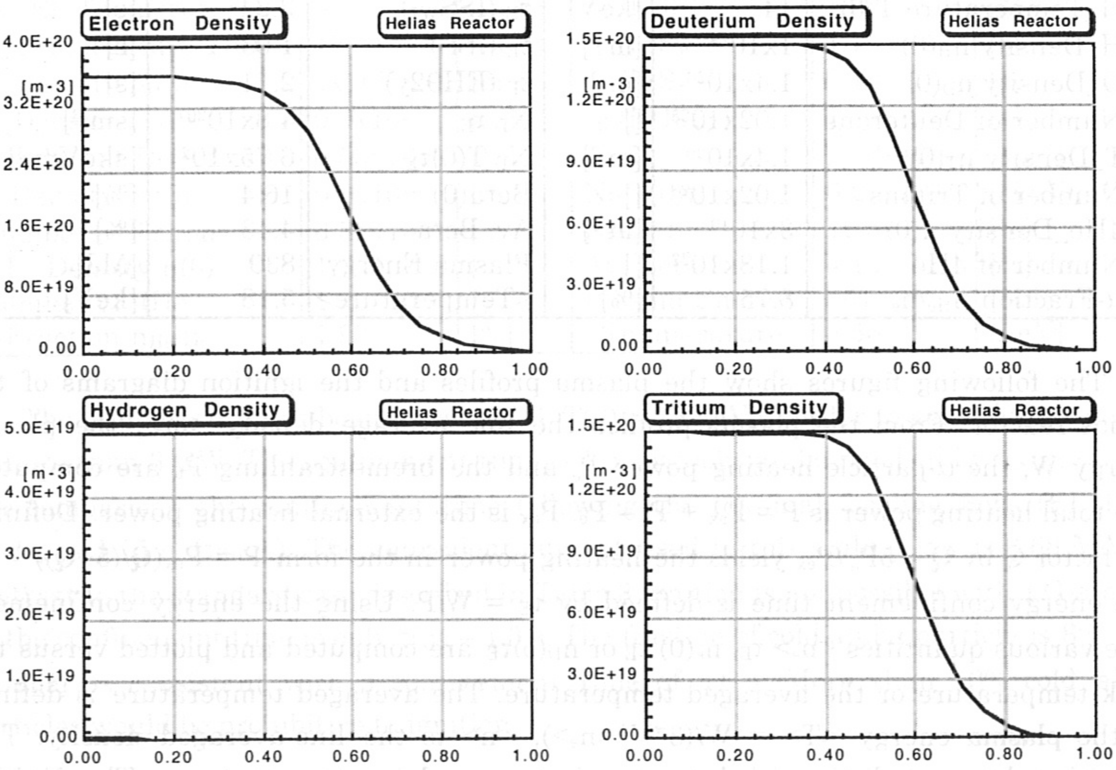

Fig. 19: Density profiles in HSR22, B = 4.75 T, $r_n = 0.7$, $\alpha_n = 5$

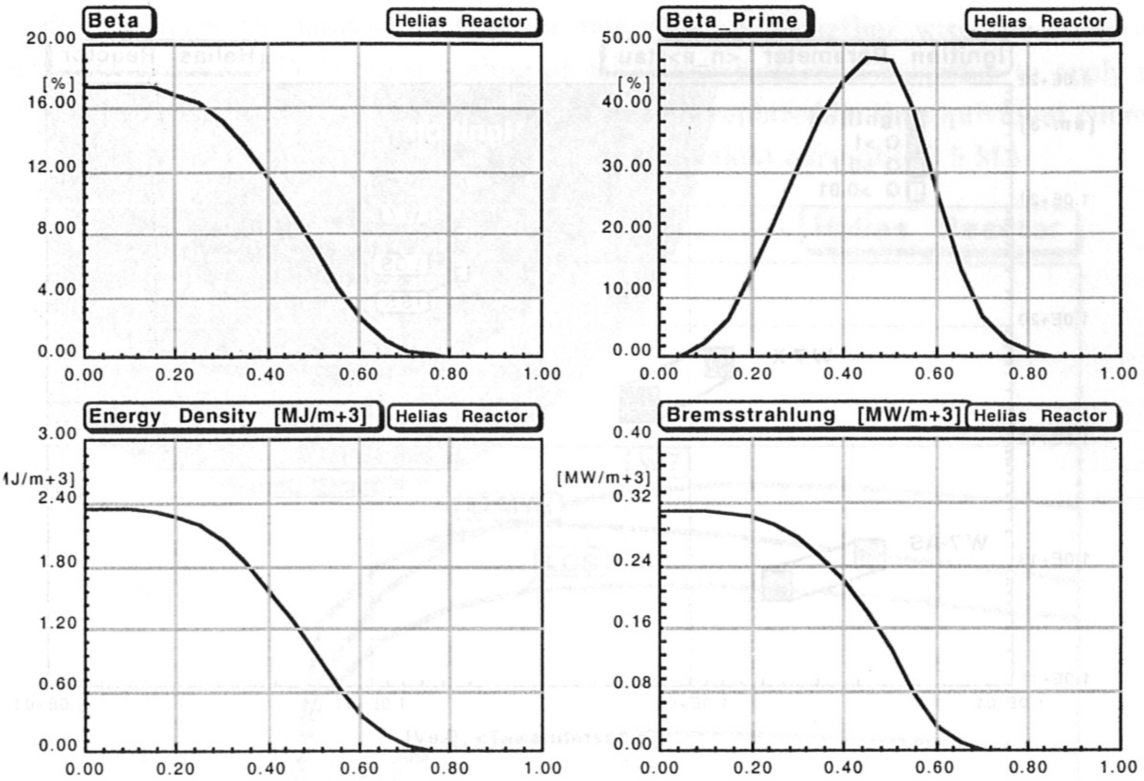

Fig. 20: Beta profile in HSR22, B = 4.75 T.

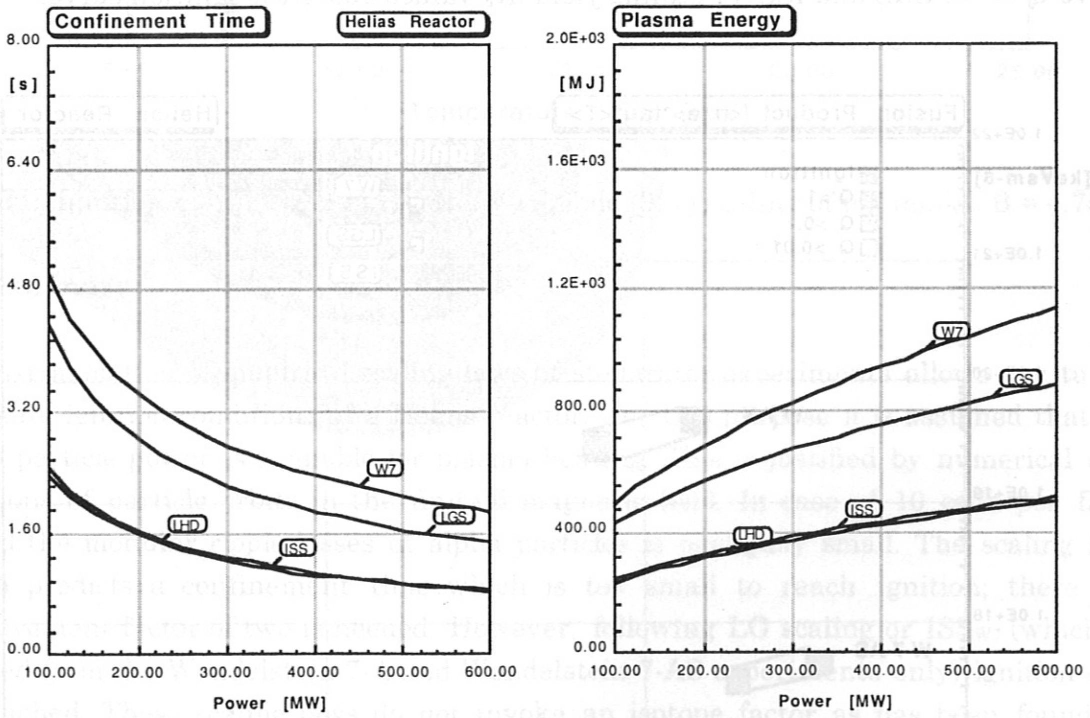

Fig. 21: Confinement times and plasma energy in HSR22B, B = 4.75 T.

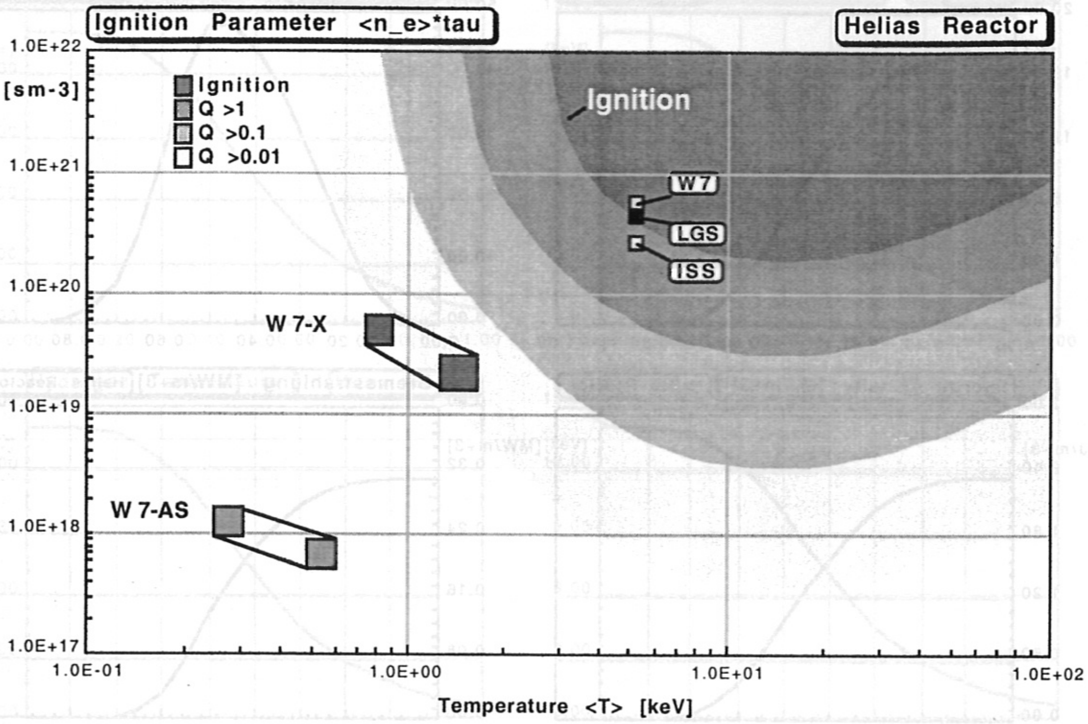

Fig. 22: <n>τ_E-diagram of HSR. The nτ_E value marked by ISS is below the ignition curve Q --> ∞. LGS and ISSW7 scaling yield nτ_E values above the ignition curve.

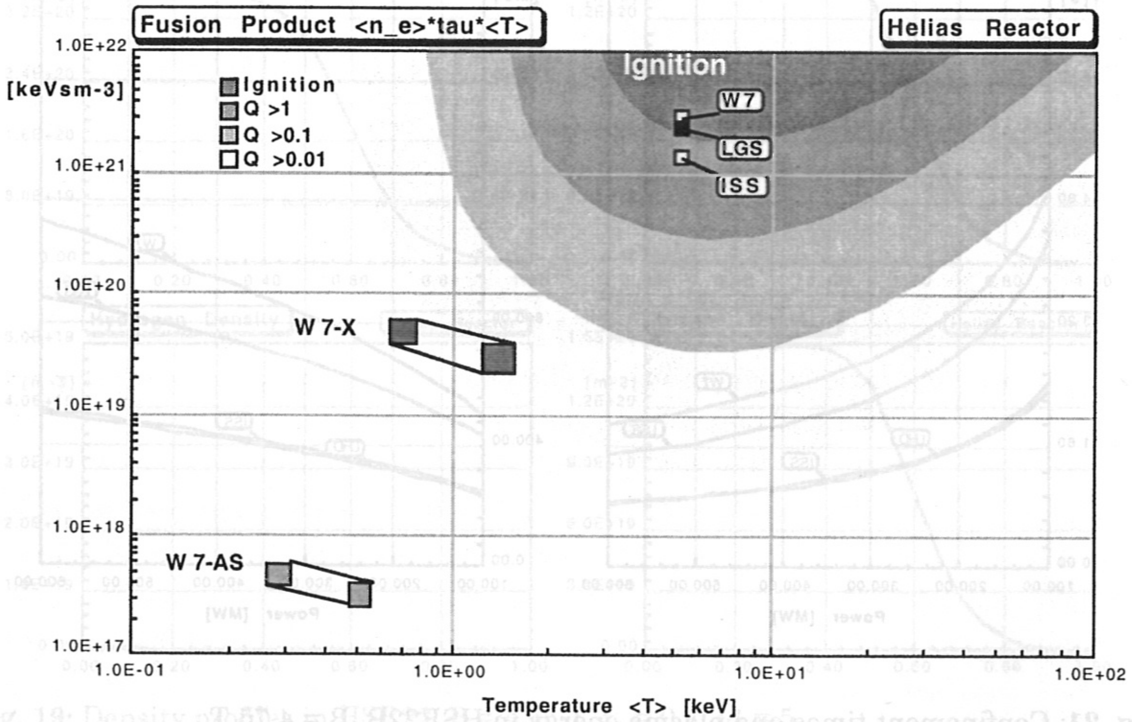

Fig. 23: Fusion product in HSR. Projected data of W 7-X (B = 3T , P = 5 - 20 MW)

Fig. 24 shows the ignition margin for this data set. Together with the stellarator scaling laws the ELMY-H-mode scaling of tokamaks is shown. In order to apply this scaling law to stellarators the rotational transform is replaced by the equivalent current. In the Helias reactor (iota = 1.0, B = 4.75 T) the equivalent current is 3.5 MA.

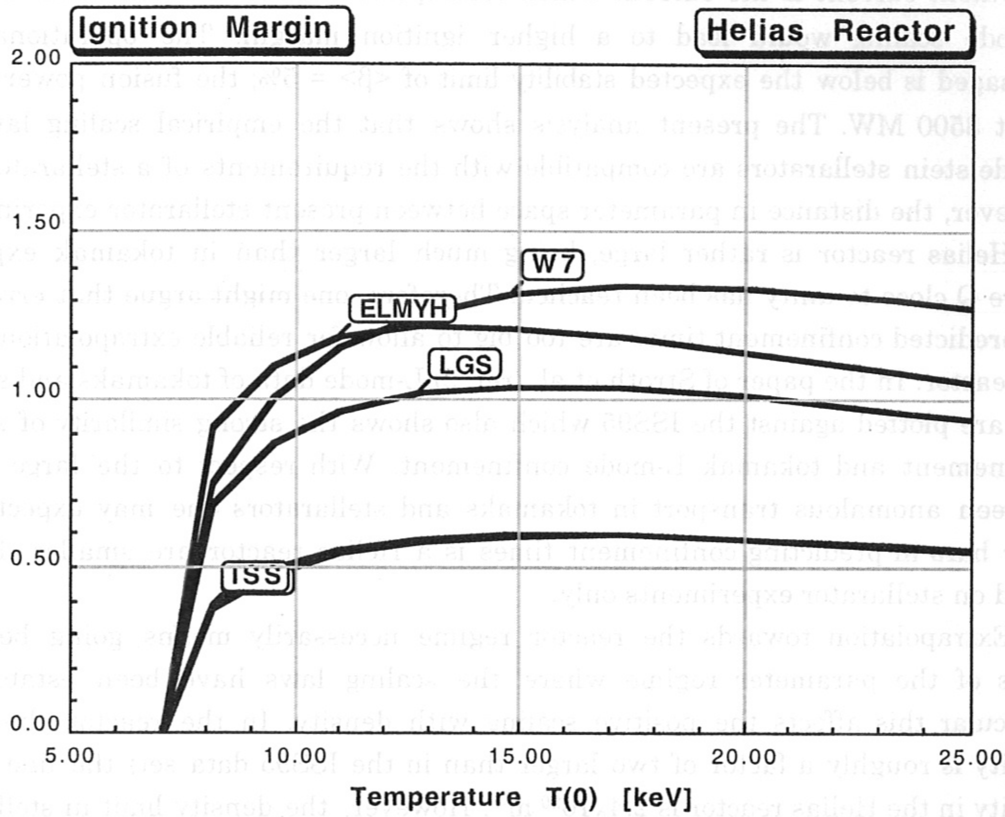

Fig. 24: Ignition margin. ELMYH = Elmy-H-mode (92y) scaling of tokamaks. B = 4.75 T

10. Summary

Extrapolation of empirical scaling laws of stellarator experiments allows one to investigate ignition conditions of a Helias reactor. For this purpose it is assumed that all alpha particle power is available for plasma heating. This is justified by numerical calculations of particle orbits in the finite-ß magnetic field. In case of 10 coils per field period the modular ripple losses of alpha particles is negligibly small. The scaling law ISS95 predicts a confinement time which is too small to reach ignition; there an improvement factor of two is needed. However, following LG scaling or ISS_{W7} (which is derived from the Wendelstein 7-A and Wendelstein 7-AS experiments only) ignition can be reached. These scaling laws do not invoke an isotope factor as has been found in toka-mak experiments. Although LGS is derived for L-mode confinement in tokamaks it is sufficient for ignition in the Helias reactor. The reason is mainly the dependence on density ($\tau_E \sim n^{0.6}$) and the dependence on the rotational transform ($\tau_E \sim \iota^{0.4}$). High rotational transform in the Helias reactor improves the confinement and since there is no disruptive density limit in stellarators the scaling with density leads to an increase of confinement times. Furthermore, the tokamak scaling laws applied to the Helias reactor also yield ignition (Elmy-H and H-mode scaling). Fig. 20 shows the Elmy-H-mode scaling where the toroidal current is replaced by the equivalent current of 3.5 MA. The equivalent current is the current which corresponds to the rotational transform $\iota = 1$. H-mode scaling would lead to a higher ignition margin. The operational regime envisaged is below the expected stability limit of $<\beta> = 5\%$; the fusion power output is about 3500 MW. The present analysis shows that the empirical scaling laws of the Wendelstein stellarators are compatible with the requirements of a stellarator reactor. However, the distance in parameter space between present stellarator experiments and the Helias reactor is rather large, being much larger than in tokamak experiments where Q close to unity has been reached. Therefore, one might argue that error bars of the predicted confinement times are too big to allow for reliable extrapolations towards the reactor. In the paper of Stroth et al. (ref. 2) L-mode data of tokamaks and stellarator data are plotted against the ISS95 which also shows the strong similarity of stellarator confinement and tokamak L-mode confinement. With respect to the large similarity between anomalous transport in tokamaks and stellarators one may expect that the error bars in predicting confinement times is a Helias reactor are smaller than those based on stellarator experiments only.

Extrapolation towards the reactor regime necessarily means going beyond the limits of the parameter regime where the scaling laws have been established. In particular this affects the positive scaling with density. In the reactor the assumed density is roughly a factor of two larger than in the ISS95 data set; the line averaged density in the Helias reactor is 2.4×10^{20} m^{-3}. However, the density limit in stellarators is mainly caused by impurity radiation and therefore depends on wall conditioning and impurity control, which indicates the uncertainty in density scaling of τ_E. As shown in Table 5 ignition at lower densities than envisaged in the standard case is still possible if confinement is determined by the scaling law ISS$_{W7}$ or the tokamak Elmy-H-mode scaling. The plasma parameters in this case are $T(0) = 19$ keV and $<n>\_L = 1.5 \times 10^{20}$ m^{-3}, which are close to those envisaged for ITER. Furthermore the dependence on rotational transform goes beyond the present parameter regime. In the regime below $\iota = 0.5$ the positive scaling of τ_E with ι is established, however in the Helias reactor ι is equal to unity which is a factor of two larger than in Wendelstein 7-AS. Larger experiments in the future (Wendelstein 7-X and LHD) will improve the experimental data base of stellarator confinement appreciably and will lead to a better prediction of the reactor confinement times.