IPP-Report

Max-Planck-Institut für Plasmaphysik

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IPP III/234

September 1998

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A statistical description is developed for self-sustained subcritical turbulence of interchange mode in inhomogeneous plasmas. Langevin equation for a dressed test mode, in which coherent and incoherent nonlinear interactions are kept as renormalized drag and random noise, is solved. Fluctuation level, decorrelation rate, auto and cross-correlation functions are obtained for submarginal strong turbulence, as nonlinear functions of non-equilibrium parameters like gradient. Extended Einstein relation and fluctuation-dissipation theorem are described as statistical relations.

PACS Index Category: 52.35.Ra, 52.25.Fi, 52.35.-g, 52.25.Gj

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The strong turbulence in high temperature plasmas has attracted attentions motivated by the fusion research or by the observation on extra-terrestrial phenomena like solar flare. The study of such turbulence is an important issue of physics: the problems of statistical physics for systems far from thermal equilibrium remain quite open, in contrast to those near thermal equilibrium in which the principles that govern fluctuations (i.e., equipartition of energy, Einstein relation, fluctuation-dissipation (FD) theorem, etc.) are established [1].

The strong plasma turbulence is governed by the $V \nabla V$ Lagrange-nonlinearity, which is universal to various fluid systems. In the past, turbulence spectrum has been discussed by choosing the distribution of the Gibbsian thermal equilibrium, where attention is focused on the conservation property of the $V \nabla V$ term [2,3]. In this framework, nonequilibrium property is not incorporated. For the study of strongly unstable plasmas, nonlinear theory has been developed and applied to experiments, based upon the methodology of clump and two-point correlation functions [4,5]. Another approach, to solve the renormalized nonlinear dispersion relations by the method of dressed-test mode, was proposed in ref.[6]. These have given (at least partly successfully) understanding of the anomalous transport and improved confinement in toroidal plasmas. Nevertheless, it is still far from clarifying the physics of strong turbulence in plasmas. Within the statistical theory of turbulence, one of the most successful methods is the one based on the DIA (direct interaction approximation) and RCM (random coupling model) [7]. Extension to the two-scale DIA for nonequilibrium situations is also made [8]. By using the DIA and RCM methods, a Langevin equation for turbulence was derived. Various kinds of closure models to formulate the Langevin equation have been tested for plasmas and the validity of models has been examined through a comparison with numerical simulations [9,10].

In this article, a statistical description is developed for the self-sustained strong turbulence which is caused by the subcritical excitation of interchange mode. Langevin equation for the dressed test mode, in which coherent drag term and incoherent random term are kept, is solved. The level and decorrelation rate of turbulence and the auto and cross-correlation functions are derived. Thus the extended Einstein relation and FD-theorem are explicitly described by the nonequilibrium-parameter that characterizes the gradient of the system. The method proposed here provides one way to explore the physics of far-nonequilibrium systems with strong instabilities.

We consider a slab plasma which is inhomogeneous in the x-direction and is sustained in a sheared magnetic field. The magnetic field is given as $\boldsymbol{B} = B_0(0, sx, 1)$ with $B_0(x) = (1 + \Omega' x + \cdots)B_0$. In this system, the current-diffusive interchange mode (CDIM) can be subcritically excited [11], which is of our interest. The reduced set of equations for the electrostatic potential ϕ, current J and pressure p are employed[12]. The electron inertia effect is kept. The classical resistivity is neglected. Three field equations are: equation of motion: $(\partial/\partial t)\Delta_\perp\phi + [\phi, \Delta_\perp\phi] = \nabla_{||}J + (\Omega' \times \hat{\boldsymbol{b}})\cdot\nabla p + \mu_{vc}\Delta_\perp^2\phi$, Ohm's law: $\partial\Psi/\partial t = -\nabla_{||}\phi - \xi^{-1}(\partial J/\partial t + [\phi, J] + \mu_{ec}\Delta_\perp J)$ and energy balance equation: $\partial p/\partial t + [\phi, p] = \chi_c\Delta_\perp p$. The bracket $[f,g]$ denotes the Poisson bracket, $[f, g] = (\nabla f \times \nabla g)\cdot\boldsymbol{b}$, $(\boldsymbol{b} = \boldsymbol{B}_0/B_0)$, $\Delta_\perp = \nabla_\perp^2$, Ω' is the average curvature of the magnetic field, Ψ is the vector potential, and $1/\xi$ denotes the finite electron inertia, $1/\xi = (\delta/a)^2$, δ being the collisionless skin depth. The transport coefficients μ_{vc}, μ_{ec}, χ_c are the ion viscosity, the electron viscosity and the thermal diffusivity, respectively. The suffix c denotes the contributions from thermal fluctuations (collisional diffusion). Length, time, static potential and pressure are normalized to the global plasma size a, the Alfven transit time $\tau_{Ap} = a/v_{Ap}$, $B_0^2 R/2a\mu_0$ and $av_{Ap}B_0$, respectively (see [11] for details). In this letter, the electrostatic approximation is employed and the nonlinear terms of the form $[\Psi, \cdots]$ are neglected. The CDIM has a quasi-2D nature, $|\nabla_{||}^2| << |\nabla_\perp^2|$; nevertheless, the small but finite $\nabla_{||}$ is essential.

Langevin equation is derived by use of the renormalization and random coupling model (RCM). The basic set of equations has the form $\partial \boldsymbol{f}/\partial t + \mathcal{L}^{(0)}\boldsymbol{f} = \textit{Nonlinear Terms}$, where $\mathcal{L}^{(0)}$ denotes the linear operator and $\boldsymbol{f}^T = (\phi, J, p)$. We consider a test mode f_k and treat the nonlinear terms as follows: The coherent contribution of the nonlinearity on f_k is renormalized, by using the direct interaction approximation (DIA), to the nonlinear transfer rate $(\gamma_{j,\,k})$ in the k-space. The incoherent part (rest of Lagrangean nonlinearity) is considered as a random noise according to RCM. (The system which has numerous degrees of freedom and many positive Lyapunov exponents is considered.) The test mode (dressed-test mode) is shielded by the turbulence effect and is subject to random force of other modes. By following these two procedures, a Langevin equation for a dressed test mode is obtained as

$$\partial \boldsymbol{f}/\partial t + \mathcal{L}\boldsymbol{f} = \tilde{\boldsymbol{S}} \tag{1}$$

with $\tilde{\boldsymbol{S}}_k^T = (\tilde{S}_{i,k})$ and $\mathcal{L}_{ij,\,k} = \mathcal{L}_{ij,\,k}^{(0)} + \gamma_{i,\,k}\delta_{ij}$ is the renormalized operator and the effective transfer rates are expressed as $\gamma_{i,\,k} = \sum_{\Delta} M_{i,\,kpq} M_{i,\,qkp}^{*} \theta_{qkp}^{*} \left|\tilde{f}_{1,p}^{2}\right|$, the random noise is given by use of the Gaussian white noise term $w(t)$ as $\tilde{S}_{i,\,k} = w(t)\sum_{\Delta} M_{i,\,kpq}\sqrt{\theta_{kpq}}\,\zeta_{1,\,p}\zeta_{i,\,q}$ and summation Δ means the constraint $\boldsymbol{k} + \boldsymbol{p} + \boldsymbol{q} = 0$. ($\delta_{ij}$ is the Kronecker's delta. In this letter, suffix $i,\ j = 1,2,3$ denotes the i-th or j-th field, and $k,\ p,\ q$ descries wave numbers. Suffix kpq is omitted unless confusion is caused.) In these expressions, the nonlinear interaction matrix is given as, e.g., $M_{1,\,kpq} = ((\boldsymbol{p}\times\boldsymbol{q})\cdot\boldsymbol{b})(p_\perp^2 - q_\perp^2)k_\perp^{-2}$ or $M_{(2,\,3),\,kpq} = (\boldsymbol{p}\times\boldsymbol{q})\cdot\boldsymbol{b}$, and the propagator satisfies the relation $(\partial/\partial t + \mathcal{L}(k) + c.p.)\theta_{kpq} = 1$. The term $\zeta_{j,\,p}$ in a random noise represents the j-th field of q-component; therefore their correlation functions satisfy the relation $\langle\zeta_i\zeta_j\rangle = \langle f_i f_j\rangle$ and $\langle\zeta_{i,\,p}\zeta_{j,\,q}\rangle \propto \delta_{pq}$ [7,9, 13]. In the calculation of the nonlinear transfer rate, it is assumed that 1) the contribution from the shorter wave-length components to the test mode plays the dominant role and that 2) the time rates (γ_j), $(\gamma_v, \gamma_e, \gamma_p)$, are expressed as the diffusion coefficients $\mu_v k_\perp^2$, $\mu_e k_\perp^2$, $\chi k_\perp^2$. This system has a strong instability source due to the presence of inhomogeneities, and the product of pressure gradient and magnetic field inhomogeneity, $G_0 = \Omega' p_0'$, denotes the driving parameter (p_0 being the equilibrium pressure profile). This system also describes the submarginal turbulence, where the nonlinear transfer rate can be an origin of the instability [6]. Solving Eq.(1), we shall determine the nonlinear decorrelation rate and transfer rate of fluctuations, simultaneously.

In order to solve the Langevin equation (1), we introduce an ansatz of large number of random modes, N. Renormalized terms γ_j in $\mathcal{L}$ is the sum of contributions from N components, so that its relative variation in time is $O(N^{-1/2})$ times smaller in comparison with that of f_k. Therefore, in solving the rapidly fluctuating time evolution of f_k, $\mathcal{L}$ is approximated as constant in time in the limit of $N \to \infty$. Then, Eq.(1) is solved by use of the Laplace transformation. The solution is formally given as

$$f(t) = \sum_m exp\,(-\lambda_m t) f(0) + \int_0^t exp\,[-\mathcal{L}(t-\tau)]\tilde{S}(\tau)\, d\tau \qquad (2)$$

where λ_m ($m = 1,2,3$ and $\lambda_1 < \lambda_2 < \lambda_3$) represents the eigenvalue of the non-normal matrix $\mathcal{L}$. The eigenvalue is determined by the nonlinear dispersion relation:

$$Det(\lambda \boldsymbol{I} + \mathcal{L}) = 0 \qquad (3)$$

and $\boldsymbol{I}$ is a unit tensor. The eigenvalue λ_1 corresponds to the branch of CDIM which drives strong turbulence, and others (λ_2, λ_3) denote highly-stable branches. Since Eq.(3) is a third order equation of λ, one can also write as $Det(\lambda \boldsymbol{I} + \mathcal{L}) = k_\perp^2(\lambda + \lambda_1)(\lambda + \lambda_2)(\lambda + \lambda_3)$. This equation (3) provides an explicit relation between λ_1, γ_j and global parameters such as G_0. The matrix $exp\,[-\mathcal{L}(t-\tau)]$ is explicitly written as

$$(exp[-\mathcal{L}(t-\tau)])_{ij} = A_{ij} exp(-\lambda_1(t-\tau)) + A_{ij}^{(2)} exp(-\lambda_2(t-\tau)) + A_{ij}^{(3)} exp(-\lambda_3(t-\tau)) \qquad (4)$$

where the elements of matrix $\boldsymbol{A}$ are given as $A_{11} = (\gamma_e - \lambda_1)(\gamma_p - \lambda_1)d$, $A_{12} = ik_{\parallel}(\gamma_p - \lambda_1)d$, $A_{13} = ik_y\Omega'(\gamma_e - \lambda_1)d$, $A_{21} = -\xi A_{12}$, $A_{22} = -\xi k_{\parallel}^2 d(\lambda_1 - \gamma_p)/(\lambda_1 - \gamma_e)$, $A_{23} = \xi k_{\parallel} k_y \Omega' d$, $A_{31} = p_0' A_{13}/\Omega'$,

$A_{32} = -p_0' A_{23}/\xi\Omega'$, $A_{33} = G_0 k_y^2 d(\lambda_1 - \gamma_e)/(\lambda_1 - \gamma_p)$ and $d = 1/(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)k_\perp^2$. Elements $A_{ij}^{(2,3)}$ are also obtained in a similar way, but is not reproduced here.

According to the standard procedure, the statistical average is calculated. For this purpose, the initial condition in Eq.(2) is unimportant and is neglected. We write

$$f_i(t) f_j(t) = \int_0^t d\tau \int_0^t d\tau' \{exp[-\mathcal{L}(t-\tau)]\tilde{S}(\tau)\}_i \{exp[-\mathcal{L}(t-\tau')]\tilde{S}(\tau')\}_j \qquad (5)$$

where the relation for $\tilde{S}$ and Eq.(4) should be substituted. After the long time average, we have

$$\langle f_i f_j \rangle = \frac{1}{2\lambda_1} A \sigma A^{T*} \qquad \sigma_{ij} = \langle \tilde{S}_i \tilde{S}_j \rangle \qquad (6)$$

In deriving Eq.(6), rapidly-decaying terms (the second and third terms in the right hand side of Eq.(4)) are neglected without loosing the generality. Equation (6) relates the fluctuation level, the correlation rate and random noise; in other words it corresponds to the Einstein-relation in the thermodynamics. Terms $\sigma_{ij} = \langle \tilde{S}_i \tilde{S}_j \rangle$ could be given in terms of correlation functions $\langle \zeta_i \zeta_j \rangle$. The average $\langle \zeta_{i,\,p} \zeta_{j,\,q} \zeta_{i,\,p'} \zeta_{j',\,q'} \rangle$ ($p + q = k$, $p' + q' = k$) is decomposed as $\langle \zeta_{i,p} \zeta_{j,q} \zeta_{i,p'} \zeta_{j',q'} \rangle = \langle \zeta_{i,p}^2 \rangle \langle \zeta_{j,q} \zeta_{j',q} \rangle \delta_{pp'} \delta_{qq'} + \langle \zeta_{i,p} \zeta_{j',p} \rangle \langle \zeta_{i,q} \zeta_{j,q} \rangle \delta_{pq'} \delta_{qp'}$ based on the random coupling approximation. This yields relations $\langle S_1 S_1 \rangle = 2\sum_q M_{1,\,kpq}^2 \theta_{kpq} \langle \zeta_{1,p}^2 \rangle \langle \zeta_{1,q}^2 \rangle$, $\langle S_1 S_2 \rangle = 2\sum_q M_{1,\,kpq} M_{2,\,kpq} \theta_{kpq} \langle \zeta_{1,p}^2 \rangle \langle \zeta_{1,q} \zeta_{2,q} \rangle$, $\langle S_1 S_3 \rangle = 2\sum_q M_{1,\,kpq} M_{3,\,kpq} \theta_{kpq} \langle \zeta_{1,p}^2 \rangle \langle \zeta_{1,q} \zeta_{3,q} \rangle$, $\langle S_2 S_2 \rangle = \sum_q M_{2,\,kpq} M_{2,\,kpq} \theta_{kpq} \{ \langle \zeta_{1,p}^2 \rangle \langle \zeta_{2,q}^2 \rangle + \langle \zeta_{1,p} \zeta_{2,p} \rangle \langle \zeta_{1,q} \zeta_{2,q} \rangle \}$, $\langle S_3 S_3 \rangle = \sum_q M_{3,\,kpq}^2 \theta_{kpq} \{ \langle \zeta_{1,p}^2 \rangle \langle \zeta_{3,q}^2 \rangle + \langle \zeta_{1,p} \zeta_{3,p} \rangle \langle \zeta_{1,q} \zeta_{3,q} \rangle \}$ and $\langle S_2 S_3 \rangle = \sum_q M_{2,\,kpq} M_{3,\,kpq} \theta_{kpq} \{ \langle \zeta_{1,p}^2 \rangle \langle \zeta_{2,q} \zeta_{3,q} \rangle + \langle \zeta_{1,p} \zeta_{3,p} \rangle \langle \zeta_{1,q} \zeta_{2,q} \rangle \}$. Substituting these expressions into Eq.(6), a closed set of equations for λ_1 and $\langle f_i f_j \rangle$

functions $\langle f_i f_j \rangle$. For the analytic transparency, we employ an approximation that the cross correlations $\langle f_i f_j \rangle$ $(i \neq j)$ are smaller than the autocorrelations $\langle f_i f_i \rangle$. By this ordering the statistical equation Eq.(6) is simplified for the reduced variables $\boldsymbol{F}_k$, $F_{i,k} = \langle f_{i,k} f_{i,k} \rangle$ $(i = 1 - 3)$, as

$$\boldsymbol{F}_k = \frac{1}{\lambda_1} \sum_p \langle f_{1,p} f_{1,p} \rangle \mathcal{R} \boldsymbol{F}_q \tag{7}$$

where the matrix $\mathcal{R}$ is given as $\mathcal{R}_{ij} = (1 + \delta_{j1}) M_{j,kpq}^2 \theta_{kpq} A_{ij} A_{ji}^*$. (The cross-correlations are derived later in relation with the induced dissipation.) Besides the trivial solution, i.e., $\langle f_i f_i \rangle = 0$, the consistent solution is obtained from Eq.(7). For the analytic estimate, we assume that the spectrum average is a smooth function, and the ratio between two moments are given by a coefficient as $(1 + \delta_{j1}) \sum_q M_{j,kpq} M_{j,kpq} \theta_{kpq} \langle \zeta_{1,p}^2 \rangle \langle \zeta_{j,q}^2 \rangle \left\{ \sum_q M_{1,kpq} M_{1,kpq}^* \theta_{qkp}^* \langle \zeta_{1,p}^2 \rangle \langle \zeta_{j,k}^2 \rangle \right\}^{-1} \simeq C_0$, i.e., $(1 + \delta_{1j}) \sum_q M_{j,kpq} M_{j,kpq} \theta_{kpq} \langle \zeta_{1,p}^2 \rangle \langle \zeta_{j,q}^2 \rangle = C_0 \gamma_v \langle \zeta_{j,k}^2 \rangle$. With this analytic estimate, Eq.(7) is simplified as $\boldsymbol{F} = \lambda_1^{-1} \hat{\mathcal{R}} \boldsymbol{F}$ with $\hat{\mathcal{R}}_{mn} \simeq C_0 \gamma_v A_{mn} A_{nm}^*$. λ_1 is determined by the secular equation

$$det\left[\lambda_1 \boldsymbol{I} - \hat{\mathcal{R}}\right] = 0 \tag{8}$$

Equation (8) together with Eq.(3) determines the decorrelation rate λ_1 and transfer rates γ_j. For instance, once the eigenvalue λ_1 is expressed in terms of γ_j by use of Eq.(8), the substitution of which into Eq.(3) provides the solution of γ_j. If γ_j is obtained, the average fluctuation amplitude $\langle f_i(t) f_i(t) \rangle$ is obtained by solving the integral equation $\gamma_v = \sum_\Delta M_{1,kpq} M_{1,qkp}^* \theta_{qkp}^* \left| \tilde{f}_{1,p}^2 \right|$ as has been performed in [14].

Explicit forms are derived in an analytic limit. Let us show the simplest result for the case of $\gamma_v \simeq \gamma_e \simeq \gamma_p$. Additional approximation, small λ_1 limit, is used. Equation (3) yields the relations $\lambda_2 \simeq \gamma_v$ and $\lambda_3 \simeq 2\gamma_v$, if $\lambda_1 \simeq 0$ is substituted, which are taken in evaluating $\hat{\mathcal{R}}_{mn}$. This simplification gives an estimate of the eigenvalue equation Eq.(8) as

$$\lambda_1/\gamma_\nu \simeq C_0/2 \tag{9}$$

Nonlinear eigenvalue equations (3) has been solved in the geometry associated with the magnetic shear [15]. In the vicinity of the marginal stability condition, nonlinear dispersion relation has been solved, providing

$$\lambda_1 \simeq (1/2)(\gamma_\nu - \gamma_0) \tag{10}$$

where $\gamma_0 = \mu_{\nu 0} k_0^2 \simeq G_0^{1/2}$ and $\mu_{\nu 0} = G_0^{3/2} s^{-2} \xi^{-1}$ and $k_0 = \xi^{1/2} s G_0^{-1/2}$ for the representative mode number. Combining Eqs.(9) and (10), we finally have the decorrelation rate and nonlinear transfer rate as

$$\lambda_1 = \frac{C_0}{2(1 - C_0)} G_0^{1/2} \quad \text{and} \quad \gamma_\nu = \frac{1}{(1 - C_0)} G_0^{1/2} \tag{11}$$

This result is the FD-theorem of the second kind in the turbulent plasma. Spectral function has been solved, once the nonlinear transfer rates are given. As was the case of [14], Eq.(11) shows only a weak dependence on k, and the spectrum of the kinetic energy of fluctuations, $E_1(k) = k_\perp \langle k_\perp^2 f_{1,k} f_{1,k} \rangle$, is deduced as

$$E_1(k) = 2(1 - C_0)^{-1} G_0 k_\perp^{-3} \tag{12}$$

for $k_\perp > k_0$. (Detail of derivation for given γ_ν is described in [14].) This is a partition law (in the energy-containing regime) for the turbulent systems, and is not given by the equi-partition law. By use of these results, various physics quantities could be derived: correlation function $\langle f_i(t) f_i(t+\tau) \rangle$ is given as $\langle f_i(t) f_i(t+\tau) \rangle = \langle f_i f_i \rangle exp\left(-\lambda_1 |\tau|\right)$, and the power spectrum $I(\omega)$ is given by the Lorentzian distribution $I(\omega) \propto \lambda_1 (\omega^2 + \lambda_1^2)^{-1}$. The test-particle diffusion coefficient is also directly calculated.

Cross correlation function is given from the off-diagonal elements of Eq.(6). The (1,2) component of Eq.(6) provides $\langle f_1 f_2 \rangle$, i.e., the dissipation term associated with the parallel electron motion. The (1,3) component gives a relation for $\langle f_1 f_3 \rangle$ as

$$Im\langle f_1 f_3 \rangle \simeq - Im\left(A_{11} A_{31}^{*}\right)\gamma_v C_0 \lambda_1^{-1} \langle f_1 f_1 \rangle \tag{13}$$

which is used to calculate the heat flux, providing an estimate $\sum_k \langle - ik_y \phi p \rangle_k \simeq \sum_k C_0 \lambda_1^{-1} k_\perp^{-2} \langle f_1 f_1 \rangle \left(- p_0'\right)$. Detailed derivation will be reported in a separate article. The result corresponds to FD-theorem of the first kind.

In summary, a new method is proposed to develop the nonlinear-nonequilibrium physics for the system with strong (nonlinear-) instability and turbulence. This method involves (i) derivation of nonlinear Langevin equation by use of renormalization and RCM, (ii) solution of nonlinear dispersion relation for the coherent part, (iii) consistency relation between the random noise and decorrelation rate based upon the statistical average, and (iv) derivation of consistent solution. The quantities such as turbulence level, decorrelation rate, auto and cross correlations are explicitly given as functions of the parameter that characterizes the nonequilibrium property. The step (iii) is an extension of the Einstein-relation, and the results are the extension of FD-theorem to far nonequilibrium systems. In this theoretical framework, the decorrelation rate and eddy-viscosity damping rate γ_v can be different. The analytic forms are derived, at the sacrifice of accuracy in numerical factor. Quantitative prediction requires numerical calculation of Eq.(6).

The nonlinear transfer rate γ_0 has been obtained in the previous work in the absence of random noise [6, 14]; in this simple treatment, the turbulent decorrelation rate could not be obtained and a statistical description was impossible. The present analysis extends the previous framework for the self-sustained turbulence including the random noise effect consistently. It confirms that the previous simple model has provided a qualitatively appropriate estimate for the nonlinear transfer rates.

The present analysis is given by choosing CDIM as one typical example, and the method itself could be applied to much wider circumstances, e.g., the problems of various instabilities, other external forces (like flow shear) or turbulence-turbulence transition.

Acknowledgements

Authors wish to thank Prof. A. Yoshizawa, Prof. A. Fukuyama and Dr. M. Yagi for useful discussions. This work is completed during the authors' stay at Max-Plank-Insitut für Plasmaphysik (IPP), which is supported by the Research-Award Programme of Alexander von Humboldt-Stiftung (AvH). Authors wish to acknowledge the hospitality of IPP and AvH. This work is partly supported by the Grant-in-Aid for Scientific Research of Ministry of Education, Science, Sports and Culture Japan.

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