

Neutral-Beam Modulation for Heating Profile Measurements?

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IPP 4/276

May 1997



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Abstract

Modulation of the plasma heating power leads to a modulation of the electron temperature which can be measured spatially-resolved using ECE techniques. The T_e amplitude is proportional to the local power density if the modulation frequency is so high that heat conduction effects can be neglected.

If an injected neutral beam is modulated, the modulation of the resulting heating power is damped by the finite slowing-down time of the beam ions in the plasma. The damping factor is proportional to $1/lf$ if $lf > 0.5/t_{\max}$ (f = modulation frequency, l = harmonic number, t_{\max} = complete slowing-down time).

The expected modulation of the electron temperature is investigated here with special emphasis on the conditions with the stellarator experiment W7-AS.

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

1. Introduction

It has become a common technique to measure the deposition profile of ECRH power in a plasma by modulating the power with a frequency up to 10 kHz and measuring the response of the electron temperature using ECE measurements [1, 2, 3, 4]. The amplitude of the T_e modulation is determined. As demonstrated in section 2, the temperature modulation is determined by the local power density and not by heat conduction if the modulation frequency is sufficiently high.

For neutral beam injection, neither the total heating efficiency nor the profile of the power deposition has been measured so far. The power modulation will certainly be less sharp than the beam modulation because of the finite slowing-down time of the injected ions. The physics of ion slowing-down is investigated therefore in section 3. The energy transfer to the plasma electrons and its temporal development after switching the beam on or off is dealt with in section 4. In order to quantify the frequency dependence of the damping factor of the resulting power modulation, a Fourier analysis of this power is made in section 5. In section 6 the expected modulation amplitude of the electron temperature is calculated, and in section 7 the findings of this paper are discussed.

2. Temperature modulation

In a slab model, the temperature modulation $\tilde{T}(x, t)$ and the l -th Fourier component of the power modulation $\tilde{P}_l(x) = P_l(x) \sin \omega t$ can be treated in a linearized diffusion equation [5]

$$\frac{3}{2} n \partial_t \tilde{T}_l(x, t) = P_l(x) \sin \omega t + n \chi \partial_x^2 \tilde{T}_l(x, t) . \quad (2-1)$$

With the Ansatz

$$\tilde{T}_l(x, t) = T_l e^{-x/\lambda_e} \sin(\omega t - x/\lambda_e + \phi_l) , \quad (2-2)$$

thus

$$\partial_t \tilde{T}_l = \omega T_l e^{-x/\lambda_e} \cos(\omega t - x/\lambda_e + \phi_l) \quad (2-3)$$

$$\partial_x^2 \tilde{T}_l = (2/\lambda_e^2) T_l e^{-x/\lambda_e} \cos(\omega t - x/\lambda_e + \phi_l) , \quad (2-4)$$

it follows from eq. (2-1)

$$\left(\frac{3}{2} n \omega - \frac{2 n \chi}{\lambda_e^2} \right) T_l e^{-x/\lambda_e} \sin(\omega t - x/\lambda_e + \phi_l + \pi/2) = P_l \sin \omega t . \quad (2-5)$$

Introducing a linearized power profile around $x = 0$

$$P_l(x) = P_l(0) + P_l' x , \quad (2-6)$$

into eq. (2-5) gives

$$\lambda_e = \frac{P_l}{P_l'} . \quad (2-7)$$

Eq. (2-5) reveals that a measurement of the local temperature amplitude can deliver the local power density if the heat conduction term is negligible. This is fulfilled if

$$\frac{3}{2}n\omega \gg \frac{2n\chi}{\lambda_e^2}$$

or

$$1f \gg \frac{2\chi}{3\pi\lambda_e^2}. \quad (2-8)$$

\Rightarrow Modulation of the heating power in a plasma may be used to determine the power profile by measuring the resulting temperature modulation if the frequency is so high that the diffusion length within one modulation period is small compared to the profile gradient length.

Taking $\chi = 10^4 \text{ cm}^2/\text{s}$, $\lambda_e \approx 5 \text{ cm}$ for W7-AS and $\lambda_e \approx 20 \text{ cm}$ for ASDEX Upgrade, the condition for a local measurement becomes

$$1f \gg \begin{cases} 100 \text{ Hz} & \text{for W7-AS} \\ 5 \text{ Hz} & \text{for AUG.} \end{cases}$$

The power density then becomes

$$P_1 = \frac{3}{2}n\omega T_1. \quad (2-9)$$

3. Physics of ion slowing down

The energy loss of injected beam particles in a plasma can be described by

$$\frac{dE}{dt} = -(\nu^e + \nu^i) E \quad (3-1)$$

where the energy loss rates ν^e and ν^i describe the energy transfer to plasma electrons and ions. They are given e.g. in ref. [6].

In the case of neutral injection heating of a plasma, these rates can be written rather simply using the Stix-approximation [7]:

$$\nu_e \gg \nu \gg \nu_i \quad (3-2)$$

(ν = velocity of slowing-down ions, ν_e , ν_i = thermal velocities of plasma electrons and ions, respectively). Eq. (3-1) can then be integrated analytically (see Appendix I) giving

$$\frac{E}{E_c} = \left[\exp\left(\frac{2}{3} \frac{t_{\max} - t}{t_e}\right) - 1 \right]^{2/3} \quad (3-3)$$

with the critical energy

$$E_c = \left(\frac{3\sqrt{\pi}}{4} \frac{1}{m_i \sqrt{m_e}} \right)^{2/3} m T_e \quad (3-4)$$

$$= 14.8 \frac{A}{A_i^{2/3}} T_e, \quad (3-5)$$

the maximum slowing-down time

$$t_{\max} = \frac{2}{3} t_e \ln \left[\left(\frac{E_0}{E_c} \right)^{3/2} + 1 \right], \quad (3-6)$$

and the electron slowing-down time

$$t_e = \frac{1}{\nu^e} = \frac{3(4\pi\epsilon_0)^2}{8\sqrt{2\pi}} \frac{m T_e^{3/2}}{e^4 \lambda n_e \sqrt{m_e}} \quad (3-7)$$

$$= 3.18 \times 10^8 \frac{A (T_e/\text{eV})^{3/2}}{\lambda (n_e/\text{cm}^{-3})} \text{ s}. \quad (3-8)$$

The following quantities for plasma and beam, typical of W7-AS, will be used as an example

$$n_e = 1 \times 10^{14} \text{ cm}^{-3}$$

$$T_e = 500 \text{ eV}$$

$$\lambda = 15$$

$$A = 1$$

$$A_i = 2$$

$$E_0 = 50 \text{ keV}.$$

They give

$$t_e = 2.37 \text{ ms}$$

$$E_c = 7.4 \text{ keV}$$

$$E_0/E_c = 6.76$$

$$t_{\max} = 1.95 t_e = 4.61 \text{ ms}.$$

4. Energy transfer to the electrons and its temporal development

Assume an ion beam is deposited in the plasma with the power density

$$P_0 = E_0 \dot{n}. \quad (4-1)$$

Without losses, the energy distribution of the slowing-down ions will be (using eq. (AI-11))

$$\begin{aligned} f(E) dE &= \frac{\dot{n}}{-dE/dt} dE \\ &= t_e \dot{n} \frac{\sqrt{E}}{E^{3/2} + E_c^{3/2}} dE \end{aligned} \quad (4-2)$$

which is drawn in Fig. 1. The total power density given to the electrons

$$P_e = \int_0^{E_0} \nu^e E f(E) dE \quad (4-3)$$

can be calculated analytically using the distribution function (4-2). This is done in Appendix II and delivers

$$P_e = P_0 g \left(\sqrt{\frac{E_0}{E_c}} \right) \quad (4-4)$$

where the function $g(x)$ is given in Appendix II.

When the beam is *switched on* at time $t = 0$, the power transfer to electrons is smaller than given in equation (4-4) as long as the population of slowing-down ions is not filled up completely ($t < t_{\max}$). If the first ions are slowed down to $E(t)$ the instantaneous power transfer will be

$$P_e(t)|_{\text{on}} = \dot{n} \left[E_0 g \left(\sqrt{\frac{E_0}{E_c}} \right) - E(t) g \left(\sqrt{\frac{E(t)}{E_c}} \right) \right]$$

or with the abbreviations $g_0 = g(\sqrt{E_0/E_c})$ and $g_t = g(\sqrt{E(t)/E_c})$

$$P_e(t)|_{\text{on}} = \dot{n} [E_0 g_0 - E(t) g_t] . \quad (4-5)$$

When the beam is *switched off* at $t = 0$ after having been *on* for a period longer than t_{\max} , the power transfer will be

$$P_e(t)|_{\text{off}} = \dot{n} E(t) g_t . \quad (4-6)$$

The time dependence of the normalized electron heating power is shown in Fig. 2 for the case that on and off times are just equal to t_{\max} .

For practical purposes like Fourier analysis, the function g_t as used in eqs. (4-5) and (4-6) should be of a simpler form than the combination of eq. (4-4) with eq. (3-3). A simple fit to $[E(t)/E_0]g_t$ in eq. (4-6) is

$$h(t) = [h_2 (1 - t/t_{\max})^2 + h_4 (1 - t/t_{\max})^4] g_0 \quad (4-7)$$

where h_2 and h_4 are chosen such that at $t = 0$

$$\begin{aligned} h(0) &= g(t=0) \equiv g_0 \\ h'(0) &= g'(t=0) = -\frac{1}{t_e} \end{aligned}$$

which gives

$$h_2 = 1 - h_4 \quad (4-8)$$

$$h_4 = \frac{1}{2g_0} \frac{t_{\max}}{t_e} - 1 . \quad (4-9)$$

The example of section 3 gives the numbers

$$\begin{aligned} g_0 &= 0.75447 \\ h_2 &= 0.70909 \\ h_4 &= 0.29091 . \end{aligned}$$

5. Modulation of the beam and the corresponding Fourier representation of the heating power

A square wave modulated heating *power* with the period $\tau = 2\pi/\omega$ and the duty cycle d_c

$$P = \begin{cases} P_0 & \text{for } 0 < t < d_c\tau \\ 0 & \text{for } d_c\tau < t < \tau , \end{cases}$$

can be represented by the Fourier series (see Appendix III)

$$P = a_0 + \sum_{l=1}^n A_l \sin(l\omega t + \phi_l) \quad (5-1)$$

with the coefficients

$$a_0 = d_c P_0 \quad (5-2)$$

$$A_l = \frac{2 |\sin \pi l d_c|}{\pi l} P_0 \quad (5-3)$$

$$\phi_l = \frac{\pi}{2} - \pi l d_c . \quad (5-4)$$

The amplitudes of the Fourier harmonics l vary thus with $1/l$. The amplitudes of the first three harmonics are shown in Fig. 3 as a function of the duty cycle d_c . The second and third harmonics have equal amplitudes for $d_c = 0.4195$ and $d_c = 0.5805$.

If a *beam* is square modulated the resulting modulation depth of the heating power will be smaller because of the smoothing effect of the finite ion slowing-down time. We may expect that the smoothing factor A_{smooth} will strongly depend on the ratio of the Fourier frequency lf to the inverse slowing down time $1/t_{\text{max}}$. A Fourier analysis of this frequency dependence is made in Appendix III. The result is drawn in Fig. 4. It shows that A_{smooth} slowly decreases from 1 to ≈ 0.4 if the frequency lf approaches $lf|_{\text{crit}} = 1/t_{\text{max}}$. For higher frequencies, A_{smooth} decreases almost inversely with the frequency

$$A_{\text{smooth}} \approx \frac{0.4}{lf t_{\text{max}}} \quad \text{for} \quad lf > 0.5(lf)_{\text{crit}} = 1/t_{\text{max}} . \quad (5-5)$$

6. Expected modulation of the electron temperature

The modulations of temperature and power are correlated as given in eq. (2-9)

$$\tilde{T}_{e,l} = \frac{1}{3\pi} \frac{1}{lf} \frac{1}{n} P_{e,l} . \quad (6-1)$$

Leaving aside the effect of the duty cycle d_c , eq. (AIII-25) gives the modulation power

$$P_{e,l} = \frac{2}{\pi l} g_0 A_{\text{smooth}} P_0 \quad (6-2)$$

where, again, A_{smooth} is shown in Fig. 4 and discussed in the last section. For low frequencies $lf \ll (lf)_{\text{crit}} \equiv 2/t_{\text{max}}$, $A_{\text{smooth}} = 1$, and the temperature modulation is not influenced by the finite slowing-down time. With the parameters of section 3 and

$$P_0 = 0.5 \frac{\text{W}}{\text{cm}^3} ,$$

the temperature amplitude becomes

$$\tilde{T}_{e,l} = \frac{2}{3\pi^2} \frac{g_0 P_0}{nl^2 f} \quad (6-3)$$

$$= \frac{1.6 \text{ kHz}}{l^2 f} \text{ eV} . \quad (6-4)$$

If higher frequencies have to be applied, the power amplitude is smoothed according to eq. (5-5). The resulting temperature modulation becomes

$$T_{e,l} \approx \frac{0.0270}{1} \frac{1}{(lf t_{\text{max}})^2} \frac{g_0 P_0 t_{\text{max}}}{n} . \quad (6-5)$$

$$T_{e,l} \approx \frac{2.9}{1} \left(\frac{217 \text{ Hz}}{lf} \right)^2 \text{ eV} . \quad (6-6)$$

7. Discussion

The calculation of the T_e modulation in modulated beam experiments in W7-AS shows that for an interesting range of frequencies (> 100 Hz), the finite ion slowing-down time does damp the T_e amplitude. The damping, however, is not exorbitantly high if the Fourier frequency lf is kept below 500 Hz and at low harmonic numbers l .

A good spatial resolution is obtained if the ECE diagnostic can be used for the determination of the \tilde{T}_e amplitudes. This method can, however, only be used if the electron densities are low, so that $\omega_p < \omega_c$. For the maximum magnetic field strength of W7-AS (2.5 T), this density limit becomes $n_e < 6 \times 10^{13} \text{ cm}^{-3}$. To avoid ECE-beam deflection the limit should even be put lower at $n_e < 5 \times 10^{13} \text{ cm}^{-3}$ [8].

If measurements at higher densities are done, soft-X diagnostics can be used [9], perhaps in a similar way as was done by Laqua [10]. The disadvantage of this method is that it only delivers line integrals of the bremsstrahlung radiation. Local radiation intensities have to be determined by an Abel conversion. The variation of n_e and T_e along a line of sight causes t_{\max} to vary and with it not only the local smoothing factor A_{smooth} but also the relative phase shift (AIII-27). The integration along a line with varying phase shifts gives an additional damping of the measured Fourier amplitude.

Several additional issues should be mentioned:

- The treatment has been done for a single species beam. If a real modulated beam experiment has to be evaluated, it must be taken into account that the beams contain certain fractions of half and third-energy particles. Their slowing-down times are slightly shorter and their branching ratio into electron heating is smaller than with full-energy ions.
- The equipartition time between plasma electrons and ions is comparable to the ion slowing-down time. The reaction of the plasma ions on T_e has to be included in the energy balance equation (2-1).
- For modulated ICRH experiments, a similar frequency dependence of the resulting T_e modulation is expected, because the time constants for energy exchange between resonantly heated ions and plasma are similar to NBI.
- The measurements of $\tilde{T}_{i,1}(r)$ could give supplementary information to the measurement of $\tilde{T}_{e,1}(r)$. The T_i diagnostics available nowadays, however, (mainly CX-analyzers) don't have the necessary time resolution [11].

Appendix I: Ion slowing down

The energy loss of injected beam particles to electrons or ions in a plasma is given in ref. [6]

$$\left. \frac{dE}{dt} \right|_{\beta} = -\nu_E^{\beta} E \quad (\text{AI-1})$$

$$\nu_E^{\beta} = 2 \left(\frac{m}{m_{\beta}} M - M' \right) \nu_0^{\beta} \quad (\text{AI-2})$$

$$\nu_0^{\beta} = \frac{\pi \sqrt{2}}{(4\pi\epsilon_0)^2} \frac{e^2 e_{\beta}^2 \lambda_{\beta} n_{\beta}}{\sqrt{m} E^{3/2}} \quad (\text{AI-3})$$

$$\begin{aligned} M(x) &= \Phi(\sqrt{x}) - \frac{2}{\sqrt{\pi}} \sqrt{x} e^{-x} \\ &= \frac{2}{\sqrt{\pi}} \int_0^x dt \sqrt{t} e^{-t} \end{aligned} \quad (\text{AI-4})$$

$$x = \frac{E}{T_{\beta}} \frac{m_{\beta}}{m} \quad (\text{AI-5})$$

e , m , v , E = charge, mass, velocity and energy of injected beam particles, respectively, the index β refers to plasma ions or electrons, λ_{β} = Coulomb logarithm.

Using the Stix-approximation [7]

$$v_e \gg v \gg v_i, \quad (\text{AI-6})$$

the factors in eq. (AI-2) become

$$2 \left(M \frac{m}{m_i} - M' \right) \rightarrow 2 \frac{m}{m_i} \quad (\text{AI-7})$$

$$2 \left(M \frac{m}{m_e} - M' \right) \rightarrow \frac{8}{3\sqrt{\pi}} \sqrt{\frac{m_e}{m}} \left(\frac{E}{T_e} \right)^{3/2}. \quad (\text{AI-8})$$

Limiting ourselves to hydrogen isotopes for beam and plasma, and setting the Coulomb logarithms $\lambda_i = \lambda_e = \lambda$, we get from equation (AI-3)

$$\nu_0^e = \nu_0^i = \nu_0 = \frac{\pi \sqrt{2}}{(4\pi\epsilon_0)^2} \frac{e^4 \lambda n_e}{\sqrt{m} E^{3/2}}. \quad (\text{AI-9})$$

The total energy loss becomes (A , A_i = atomic mass numbers of beam and plasma, respectively)

$$\frac{dE}{dt} = -(\nu^e + \nu^i) E \quad (\text{AI-10})$$

$$\begin{aligned} &= -2 \left[\frac{4}{3\sqrt{\pi}} \sqrt{\frac{m_e}{m}} \left(\frac{E}{T_e} \right)^{3/2} + \frac{m}{m_i} \right] \nu_0 E \\ &= -\frac{1}{t_e} \left[1 + \left(\frac{E_c}{E} \right)^{3/2} \right] E \end{aligned} \quad (\text{AI-11})$$

with

$$E_c = \left(\frac{3\sqrt{\pi}}{4} \frac{1}{m_i \sqrt{m_e}} \right)^{2/3} m T_e \quad (\text{AI-12})$$

$$= 14.8 \frac{A}{A_i^{2/3}} T_e \quad (\text{AI-13})$$

and

$$t_e = \frac{1}{\nu^e} = \frac{3(4\pi\epsilon_0)^2}{8\sqrt{2\pi}} \frac{m T_e^{3/2}}{e^4 \lambda n_e \sqrt{m_e}} \quad (\text{AI-14})$$

$$= 3.18 \times 10^8 \frac{A(T_e/\text{eV})^{3/2}}{\lambda(n_e/\text{cm}^{-3})} \text{ s} . \quad (\text{AI-15})$$

In order to follow the time evolution of the energy of an ion after the moment of its birth at time $t = 0$ with energy E_0 , we have to integrate eq. (AI-11):

$$\begin{aligned} -\frac{dt}{t_e} &= \frac{dE}{E [1 + (E_c/E)^{3/2}]} \\ -\int_0^t \frac{d\tau}{t_e} &= \int_{E_0}^E \frac{\sqrt{u} du}{u^{3/2} + E_c^{3/2}} \\ &= \int_{E_0/E_c}^{E/E_c} \frac{\sqrt{\bar{u}} d\bar{u}}{\bar{u}^{3/2} + 1} \\ -\frac{t}{t_e} &= \frac{2}{3} \ln \left(\bar{u}^{3/2} + 1 \right) \Big|_{E_0/E_c}^{E/E_c} \\ &= \frac{2}{3} \ln \frac{(E/E_c)^{3/2} + 1}{(E_0/E_c)^{3/2} + 1} \quad (\text{AI-16}) \\ \exp \left(-\frac{3t}{2t_e} \right) &= \frac{(E/E_c)^{3/2} + 1}{(E_0/E_c)^{3/2} + 1} \end{aligned}$$

$$\frac{E}{E_c} = \left\{ \left[\left(\frac{E_0}{E_c} \right)^{3/2} + 1 \right] \exp \left(-\frac{3t}{2t_e} \right) - 1 \right\}^{2/3} . \quad (\text{AI-17})$$

In this approximation, the ions are completely slowed down during the time

$$t = t_{\max} = \frac{2}{3} t_e \ln \left[\left(\frac{E_0}{E_c} \right)^{3/2} + 1 \right] . \quad (\text{AI-18})$$

After switching on or off the beam, it takes this time until the population of slowing-down ions is built up or vanishes completely, respectively.

It is convenient to rewrite $E = E(t)$ of eq. (AI-17) into

$$\frac{E}{E_c} = \left[\exp \left(\frac{3}{2} \frac{t_{\max} - t}{t_e} \right) - 1 \right]^{2/3} . \quad (\text{AI-19})$$

Appendix II: Energy transfer to the electrons

The total power density given to the plasma electrons, eq. (4-3), with the ion distribution function (4-2) becomes

$$P_e = \dot{n} \int_0^{E_0} \frac{E^{3/2}}{E^{3/2} + E_c^{3/2}} dE \quad (\text{AII-1})$$

$$= 2 \dot{n} E_c \int_0^{\sqrt{E_0/E_c}} \frac{x^4}{x^3 + 1} dx \quad \text{using} \quad x = \sqrt{\frac{E}{E_c}}, \quad (\text{AII-2})$$

and with $x_0^2 = E_0/E_c$

$$P_e = \frac{2P_0}{x_0^2} \int_0^{x_0} \frac{x^4}{x^3 + 1} dx \quad (\text{AII-3})$$

$$= P_0 g \left(\sqrt{\frac{E_0}{E_c}} \right) \quad (\text{AII-4})$$

where

$$g(x) \equiv \frac{1}{x^2} \left[x^2 + \frac{1}{3} \ln \frac{(x+1)^2}{x^2 - x + 1} - \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2x-1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \frac{\pi}{3} \right]. \quad (\text{AII-5})$$

In the limit $E \gg E_c$, the function g approaches 1 — the power is mainly deposited into the electrons. In the unrealistic limit $E \ll E_c$, the power is mainly given to the ions. Electron and ion powers are the same for

$$x_0 = 1.55 \quad \Rightarrow \quad E_0 = 2.41 E_c. \quad (\text{AII-6})$$

Appendix III: Fourier analysis of the heating power

If the heating power is modulated with the period $\tau = 2\pi/\omega$ and the duty cycle d_c

$$P = \begin{cases} P_0 & \text{for } 0 < t < d_c\tau \\ 0 & \text{for } d_c\tau < t < \tau, \end{cases}$$

the corresponding Fourier series of the power is

$$P = a_0 + \sum_{l=1}^n a_l \cos l\omega t + b_l \sin l\omega t. \quad (\text{AIII-1})$$

with the coefficients

$$a_0 = \frac{1}{\tau} \int_0^{d_c\tau} P_0 dt = d_c P_0 \quad (\text{AIII-2})$$

$$a_l = \frac{2}{\tau} \int_0^{d_c\tau} P_0 \cos l\omega t dt = \frac{P_0}{\pi l} \sin 2\pi l d_c \quad (\text{AIII-3})$$

$$b_l = \frac{2}{\tau} \int_0^{d_c\tau} P_0 \sin l\omega t dt = \frac{P_0}{\pi l} (1 - \cos 2\pi l d_c) \quad (\text{AIII-4})$$

For practical purposes, the Fourier representation

$$P = a_0 + \sum_{l=1}^n A_l \sin(l\omega t + \phi_l) \quad (\text{AIII-5})$$

seems better suited than (AIII-1) with the coefficients

$$A_l = \sqrt{a_l^2 + b_l^2} \quad (\text{AIII-6})$$

$$\text{tg}\phi_l = a_l/b_l \quad (\text{AIII-7})$$

which give

$$A_l = \frac{2 |\sin \pi l d_c|}{\pi l} P_0 \quad (\text{AIII-8})$$

$$\phi_l = \frac{\pi}{2} - \pi l d_c. \quad (\text{AIII-9})$$

If the beam is switched periodically, the electron heating power will be smoothed because of the finite ion slowing-down time. If the modulation period is larger than t_{\max} , the slowing down distribution at a certain instant is determined completely by the switching situation within the last period. If the modulation period becomes shorter than t_{\max} , more periods have to be included in the determination of the ion distribution function.

In order to simplify the mathematical representation, we will use the following definition

of the heating function during the beam "afterglow" (compare with eqs. (4-5) and (4-6))

$$g_t \equiv \begin{cases} g_0 = g \left(\sqrt{E_0/E_c} \right) & \text{for } t = 0 \\ g \left(\sqrt{E(t)/E_c} \right) & \text{for } 0 < t < t_{\max} \\ 0 & \text{for } t \geq t_{\max} , \end{cases}$$

where $E(t)/E_c$ is determined by eq. (3-3). Switching on the beam at $t = t_{\text{on}}$ gives an additional normalized heating power $p_e = P_e/(\dot{n}E_0)$ at $t \geq t_{\text{on}}$ (following eq. (4-5)) of

$$p_{e,\text{on}} = g_0 - \frac{E(t - t_{\text{on}})}{E_0} g_{t-t_{\text{on}}} , \quad (\text{AIII-10})$$

whereas switching off the beam at $t = t_{\text{off}}$ can be treated at $t \geq t_{\text{off}}$ like leaving the previous beam on and adding a negative power of

$$p_{e,\text{off}} = - \left[g_0 - \frac{E(t - t_{\text{off}})}{E_0} g_{t-t_{\text{off}}} \right] . \quad (\text{AIII-11})$$

When a pulse is on from t_{on} until t_{off} , its remaining contribution to the heating power at $t \geq t_{\text{off}}$ is

$$p_e = \frac{E(t - t_{\text{off}})}{E_0} g_{t-t_{\text{off}}} - \frac{E(t - t_{\text{on}})}{E_0} g_{t-t_{\text{on}}} . \quad (\text{AIII-12})$$

If this pulse is switched on at $t_{\text{on}} = -m\tau$ and switched off at $t_{\text{off}} = -(m - d_c)\tau$, the remaining contribution to the heating power is

$$p_{e,m} = \frac{E[t + (m - d_c)\tau]}{E_0} g_{t+(m-d_c)\tau} - \frac{E(t + m\tau)}{E_0} g_{t+m\tau} . \quad (\text{AIII-13})$$

The contribution of the beam pulse within the actual period (on at $t = 0$, off at $d_c\tau$) is

$$p_e = \begin{cases} g_0 - \frac{E(t)}{E_0} g_t & \text{for } 0 < t < d_c\tau \\ \frac{E(t - d_c\tau)}{E_0} g_{t-d_c\tau} - \frac{E(t)}{E_0} g_t & \text{for } d_c\tau < t < \tau . \end{cases}$$

If m_τ is the maximum number of beam pulses contributing to the actual period, the heating power will be

$$p_e = \begin{cases} g_0 - \frac{E(t)}{E_0} g_t + \sum_{m=1}^{m_\tau} p_{e,m} & \text{for } 0 < t < d_c\tau \\ \frac{E(t - d_c\tau)}{E_0} g_{t-d_c\tau} - \frac{E(t)}{E_0} g_t + \sum_{m=1}^{m_\tau} p_{e,m} & \text{for } d_c\tau < t < \tau . \end{cases}$$

In the Fourier analysis, an integral of the following form will appear where cosine or sine are represented by the function f :

$$\begin{aligned}
I &= \int_0^{d_c \tau} g_0 f(\omega t) dt + \int_{d_c \tau}^{\tau} \frac{E(t - d_c \tau)}{E_0} g_{t-d_c \tau} f(\omega t) dt \\
&+ \int_0^{\tau} \left\{ -\frac{E(t)}{E_0} g_t + \sum_{m=1}^{m_r} \frac{E[t + (m - d_c)\tau]}{E_0} g_{t+(m-f)\tau} - \frac{E(t + m\tau)}{E_0} g_{t+m\tau} \right\} f(\omega t) dt \\
&= g_0 \int_0^{d_c \tau} f(\omega t) dt + \int_0^{(1-d_c)\tau} \frac{E(t)}{E_0} g_t f[\omega(t + d_c \tau)] dt \\
&- \int_0^{t_{\max}} \frac{E(t)}{E_0} g_t f(\omega t) dt + \int_{(1-d_c)\tau}^{t_{\max}} \frac{E(t)}{E_0} g_t f[\omega(t + d_c \tau)] dt \\
&= g_0 \int_0^{d_c \tau} f(\omega t) dt + \int_0^{t_{\max}} \frac{E(t)}{E_0} g_t [f(\omega t + 2\pi d_c) - f(\omega t)] dt . \tag{AIII-14}
\end{aligned}$$

Using this integral, the Fourier coefficients for the electron heating power become

$$a_0 = d_c g_0 \tag{AIII-15}$$

$$\begin{aligned}
a_1 &= \frac{g_0}{\pi l} \sin 2\pi d_c + \frac{2}{\tau} \int_0^{t_{\max}} \frac{E(t)}{E_0} g_t [\cos(\omega t + 2\pi d_c) - \cos \omega t] dt \\
&= \frac{2 \sin \pi d_c}{\pi l} g_0 \left[\cos \pi d_c - \omega \int_0^{t_{\max}} \frac{E(t)}{E_0} \frac{g_t}{g_0} \sin(\omega t + \pi d_c) dt \right] \tag{AIII-16}
\end{aligned}$$

$$\begin{aligned}
b_1 &= \frac{g_0}{\pi l} (1 - \cos 2\pi d_c) + \frac{2}{\tau} \int_0^{t_{\max}} \frac{E(t)}{E_0} g_t [\sin(\omega t + 2\pi d_c) - \sin \omega t] dt \\
&= \frac{2 \sin \pi d_c}{\pi l} g_0 \left[\sin \pi d_c + \omega \int_0^{t_{\max}} \frac{E(t)}{E_0} \frac{g_t}{g_0} \cos(\omega t + \pi d_c) dt \right] . \tag{AIII-17}
\end{aligned}$$

Setting

$$\sin \gamma = \frac{1}{t_{\max} G} \int_0^{t_{\max}} \frac{E(t)}{E_0} \frac{g_t}{g_0} \sin \omega t dt \tag{AIII-18}$$

$$\cos \gamma = \frac{1}{t_{\max} G} \int_0^{t_{\max}} \frac{E(t)}{E_0} \frac{g_t}{g_0} \cos \omega t dt \tag{AIII-19}$$

with

$$G = \frac{1}{t_{\max}} \sqrt{\left(\int_0^{t_{\max}} \frac{E(t) g_t}{E_0 g_0} \cos \omega t dt \right)^2 + \left(\int_0^{t_{\max}} \frac{E(t) g_t}{E_0 g_0} \sin \omega t dt \right)^2} \quad (\text{AIII-20})$$

gives

$$\text{tg} \gamma = \frac{\int_0^{t_{\max}} E(t) g_t \sin \omega t dt}{\int_0^{t_{\max}} E(t) g_t \cos \omega t dt} \quad (\text{AIII-21})$$

and

$$\text{tg} \delta = \frac{1 - \omega t_{\max} G \sin \gamma}{\omega t_{\max} G \cos \gamma}. \quad (\text{AIII-22})$$

The Fourier coefficients become

$$a_1 = \frac{2 \sin \pi l d_c}{\pi l} g_0 [\cos \pi l d_c - \omega t_{\max} G \sin(\pi l d_c + \gamma)] \quad (\text{AIII-23})$$

$$b_1 = \frac{2 \sin \pi l d_c}{\pi l} g_0 [\sin \pi l d_c + \omega t_{\max} G \cos(\pi l d_c + \gamma)] \quad (\text{AIII-24})$$

$$A_1 = \frac{2 |\sin \pi l d_c|}{\pi l} g_0 A_{\text{smooth}} \quad (\text{AIII-25})$$

$$A_{\text{smooth}}^2 = 1 - 2 \omega t_{\max} G \sin \gamma + (\omega t_{\max} G)^2 \quad (\text{AIII-26})$$

$$\phi_1 = \delta - \pi l d_c. \quad (\text{AIII-27})$$

\Rightarrow The dependence of the amplitude (AIII-25) on the duty cycle d_c is the same as (AIII-8) which was derived for a square wave power modulation. The smoothing of the power reduces the amplitude of the Fourier components by a factor of A_{smooth} , independently of the duty cycle d_c . The phase (AIII-27) is shifted against the phase (AIII-9) by $\delta - \pi/2$. This shift is also independent of the duty cycle d_c .

Using the fit for the function g_t , the equations in (AIII-18) or (AIII-19) lead to integrals of the following kind which can be solved analytically:

$$I_1 \equiv \frac{1}{t_{\max}} \int_0^{t_{\max}} (1 - t/t_{\max})^2 \sin \omega t dt = \frac{1}{\omega t_{\max}} \left[1 - \frac{2(1 - \cos \omega t_{\max})}{(\omega t_{\max})^2} \right]$$

$$I_2 \equiv \frac{1}{t_{\max}} \int_0^{t_{\max}} (1 - t/t_{\max})^4 \sin \omega t dt = \frac{1}{\omega t_{\max}} \left[1 - \frac{12}{(\omega t_{\max})^2} + \frac{24(1 - \cos \omega t_{\max})}{(\omega t_{\max})^4} \right]$$

$$I_3 \equiv \frac{1}{t_{\max}} \int_0^{t_{\max}} (1 - t/t_{\max})^2 \cos \omega t dt = \frac{2}{(\omega t_{\max})^2} \left(1 - \frac{\sin \omega t_{\max}}{\omega t_{\max}} \right)$$

$$I_4 \equiv \frac{1}{t_{\max}} \int_0^{t_{\max}} (1 - t/t_{\max})^4 \cos \omega t \, dt = \frac{4}{(\omega t_{\max})^2} \left[1 - \frac{6}{(\omega t_{\max})^2} \left(1 - \frac{\sin \omega t_{\max}}{\omega t_{\max}} \right) \right].$$

With the fit (4-7) for $[E(t)/E_0]_{gt}$, the eqs. (AIII-18) - (AIII-21) become

$$G = \sqrt{[I_1 + h_4(I_2 - I_1)]^2 + [I_3 + h_4(I_4 - I_3)]^2} \quad (\text{AIII-28})$$

$$G \sin \gamma = I_1 + h_4(I_2 - I_1) \quad (\text{AIII-29})$$

$$G \cos \gamma = I_3 + h_4(I_4 - I_3) \quad (\text{AIII-30})$$

$$\text{tg} \gamma = \frac{I_1 + h_4(I_2 - I_1)}{I_3 + h_4(I_4 - I_3)}. \quad (\text{AIII-31})$$

The amplitude (AIII-26) becomes

$$A_{\text{smooth}}^2 = \{1 - \omega t_{\max}[I_1 + h_4(I_2 - I_1)]\}^2 + \{\omega t_{\max}[I_3 + h_4(I_4 - I_3)]\}^2. \quad (\text{AIII-32})$$

For frequencies $lf > 0.5t_{\max}$, the Fourier amplitude oscillates around

$$A_{\text{lim}} = \frac{1 + h_4}{\pi} \frac{1}{lf t_{\max}} \quad (\text{AIII-33})$$

as shown in Fig. 4a.

The integrals $I_1 \dots I_4$ are very simple expressions for integer multiples of $2lf t_{\max} = n$, and the deviations of A_{smooth} from A_{lim} are as follows:

$$A_{\text{smooth}}^2 - A_{\text{lim}}^2 = -\frac{3h_4}{(\pi lf t_{\max})^4} \left\{ 2 - h_4 \left[1 + \frac{3}{(\pi lf t_{\max})^2} \right] \right\} \quad (\text{AIII-34})$$

for $n = 2, 4, 6, \dots$

$$= \frac{1}{(\pi lf t_{\max})^4} \left\{ 1 - h_4 \left[1 + \frac{3}{(\pi lf t_{\max})^2} \right] \times \right. \\ \left. \times \left[2 + h_4 \left(2 - \frac{3}{(\pi lf t_{\max})^2} \right) \right] \right\} \quad (\text{AIII-35})$$

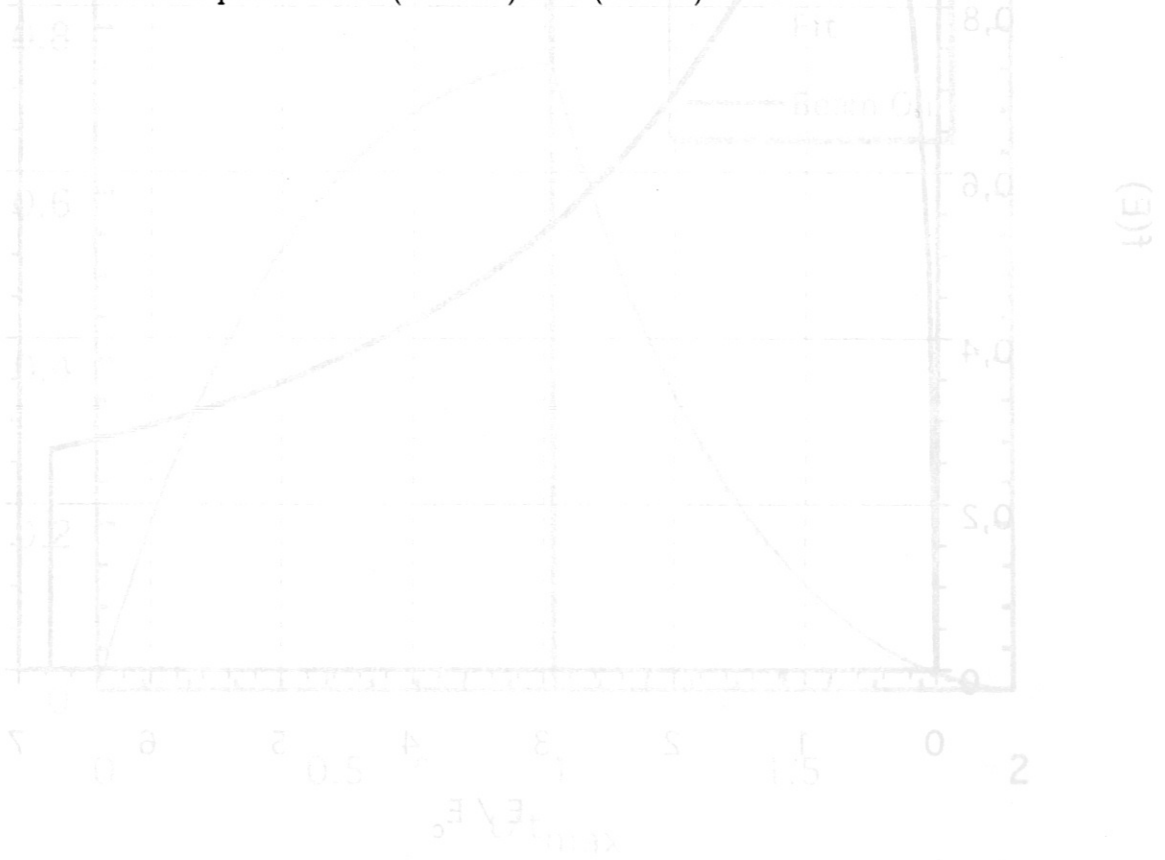
for $n = 1, 3, 5, \dots$

References:

- [1] J.M. Moret et al., Nucl. Fusion **33**, 1185 (1993)
- [2] L. Giannone et al., Nucl. Fusion **32**, 1985 (1992)
L. Giannone et al., 22nd EPS Conf. on Contr. Fusion and Plasma Phys., Bournemouth, England, 1995, IV-265
- [3] M. Peters, *Electron heat transport in current carrying and currentless thermonuclear plasmas*, Doctoral Thesis, Eindhoven, The Netherlands, 1995
- [4] M. Romé et al., Plasma Phys. Control. Fusion **39**, 117 (1997)
- [5] U. Stroth, Ringberg meeting, IPP Department of Technology, Dec. 95
- [6] David L. Book, NRL Plasma Formulary, Washington, D.C., USA
- [7] T.H. Stix, Plasma Physics **14**, 367 (1971)
- [8] H.-J. Hartfuß, private communication
- [9] A. Weller, private communication
- [10] H. Laqua, to be published in Phys.Rev.Letters
- [11] M. Kick, private information

Figure captions

- Fig. 1 Energy distribution of the injected ions (equ. (4-2)).
- Fig. 2 Time dependence of the electron heating power P_e/P_0 . The beam is switched on at $t = 0$ and switched off at $t = t_{\max}$. (Eqs. (4-5) and (4-6) combined with (3-3). The dotted line is the fit using eq. (4-7).
- Fig. 3 Relative Fourier amplitudes of the heating power in case of square wave heating pulses as a function of the duty cycle.
- Fig. 4 Decrease of the modulation amplitude of the electron heating power with rising frequency due to the finite slowing-down time of the ions. The abscissa is the product of the Fourier frequency l/τ with the slowing-down time t_{\max} (eq. AIII-26), $l =$ harmonic number, $\tau =$ modulation period of the beam.
- Fig. 4a Same as Fig. 3, but ordinate logarithmic. Also shown is the approximation (5-5).
- Fig. 5 Change of the phase shift of the electron heating power with rising frequency. Difference of phase shifts (AIII-27) and (AIII-9).



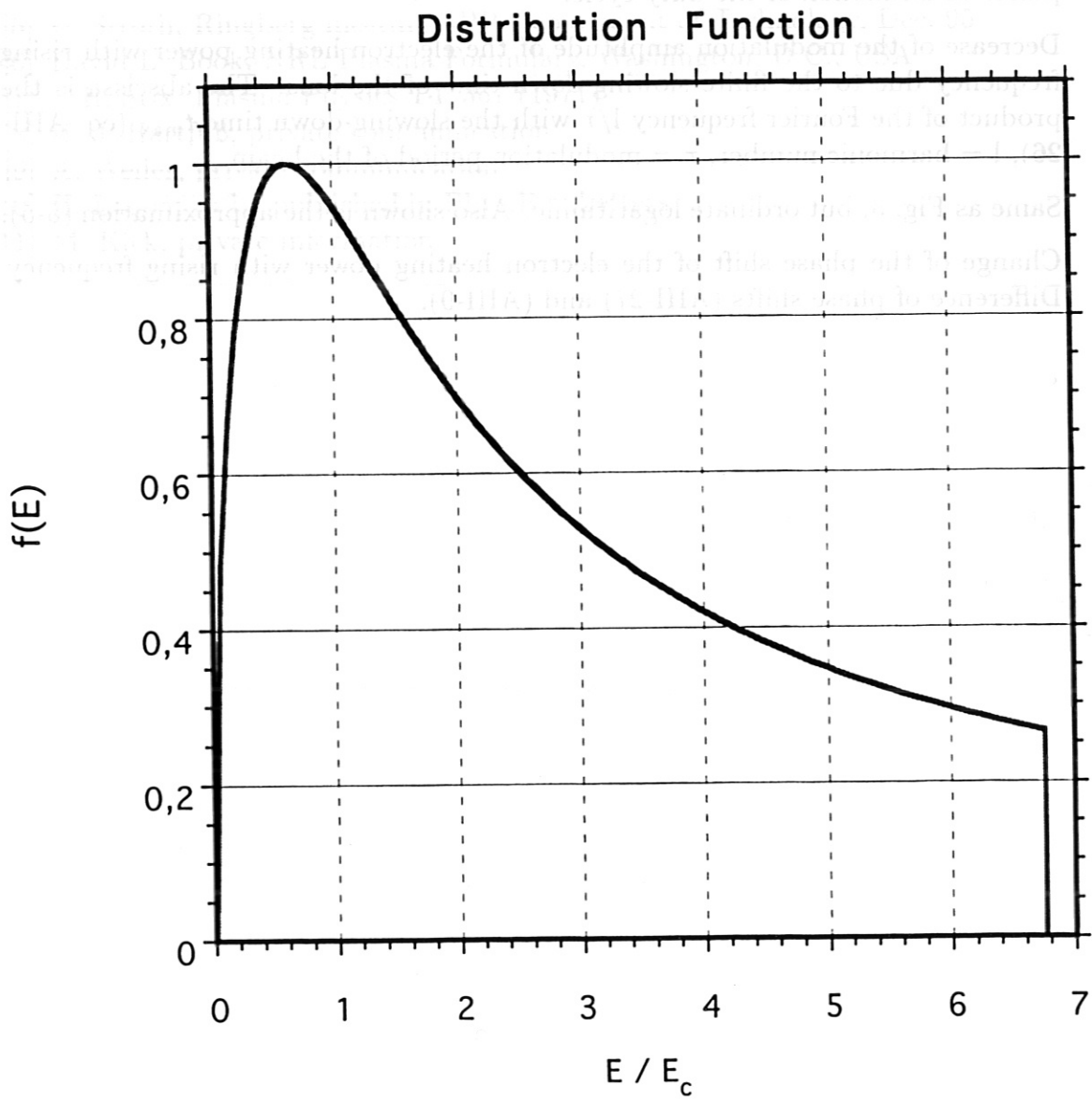


Fig. 1

Electron Heating Power

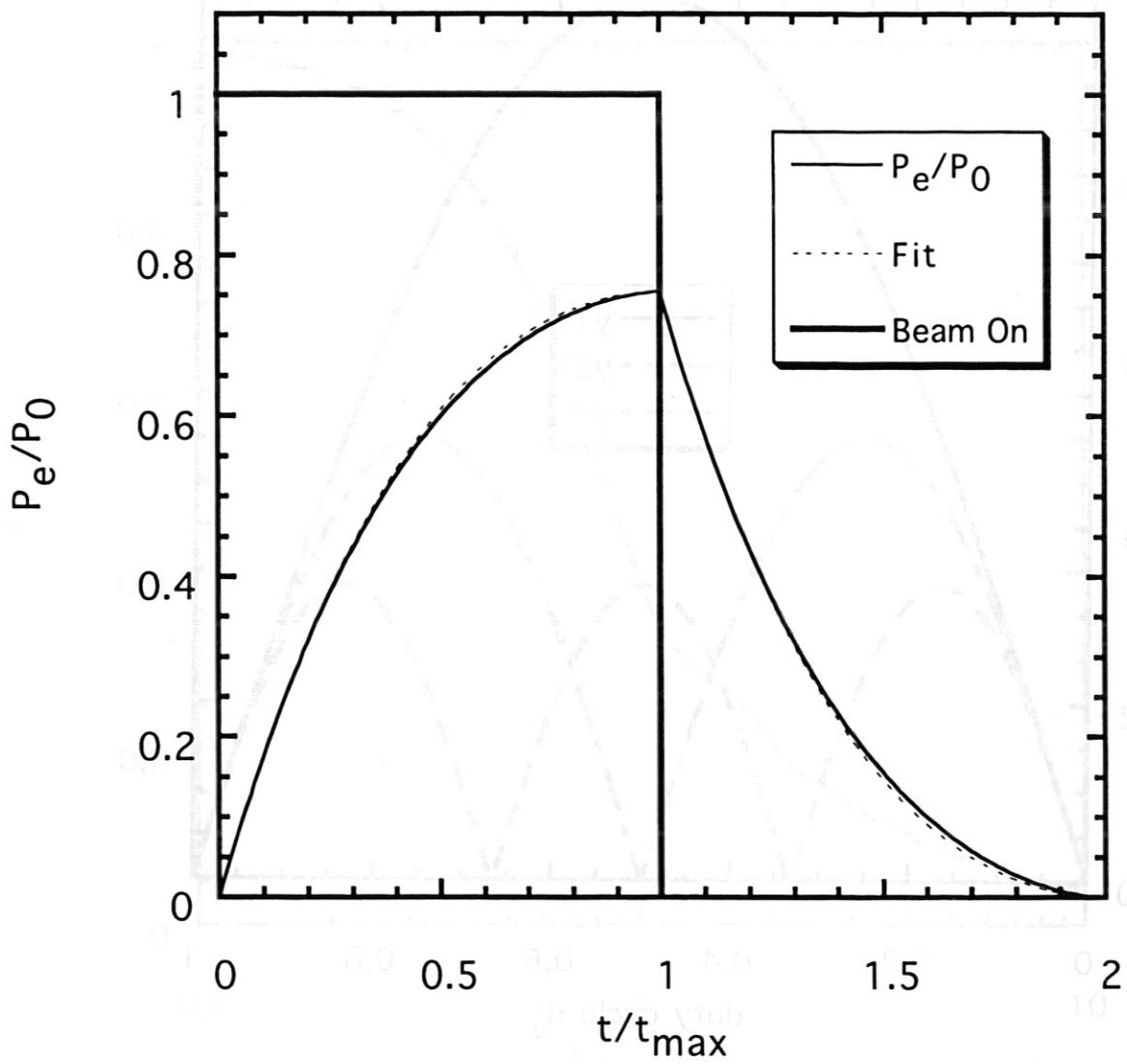


Fig. 2

Fourier Amplitude, Square wave

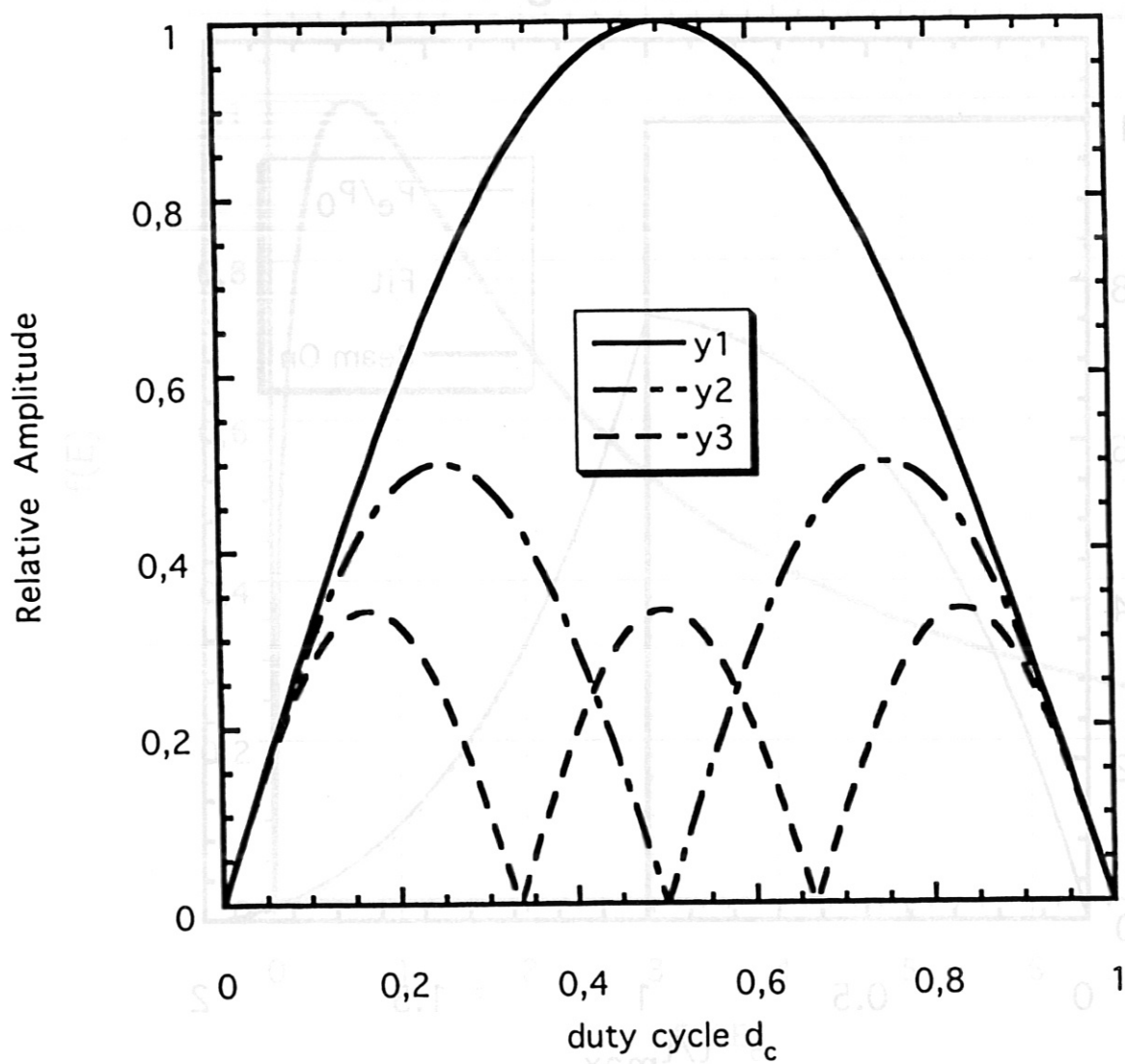


Fig. 3

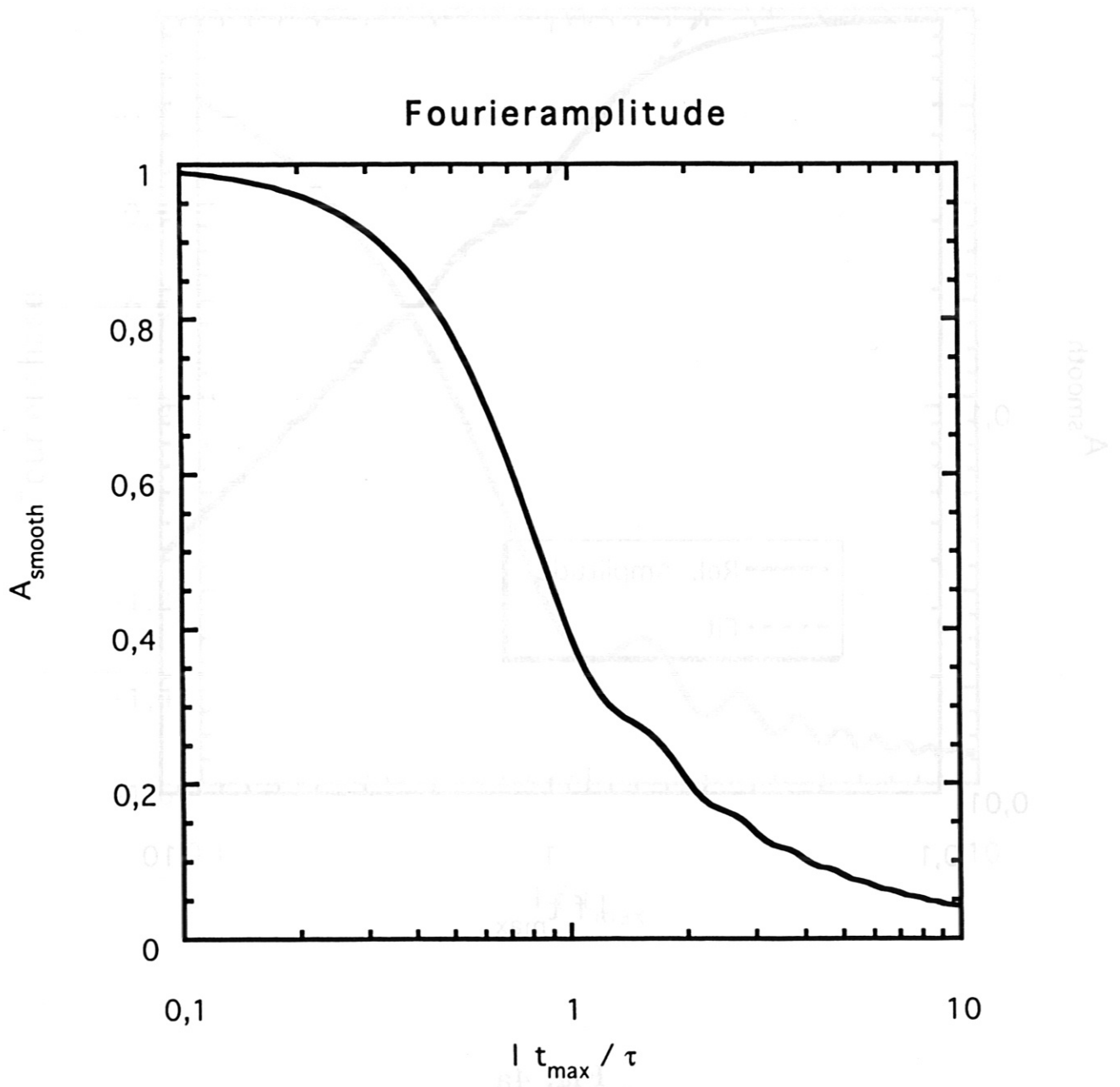


Fig. 4

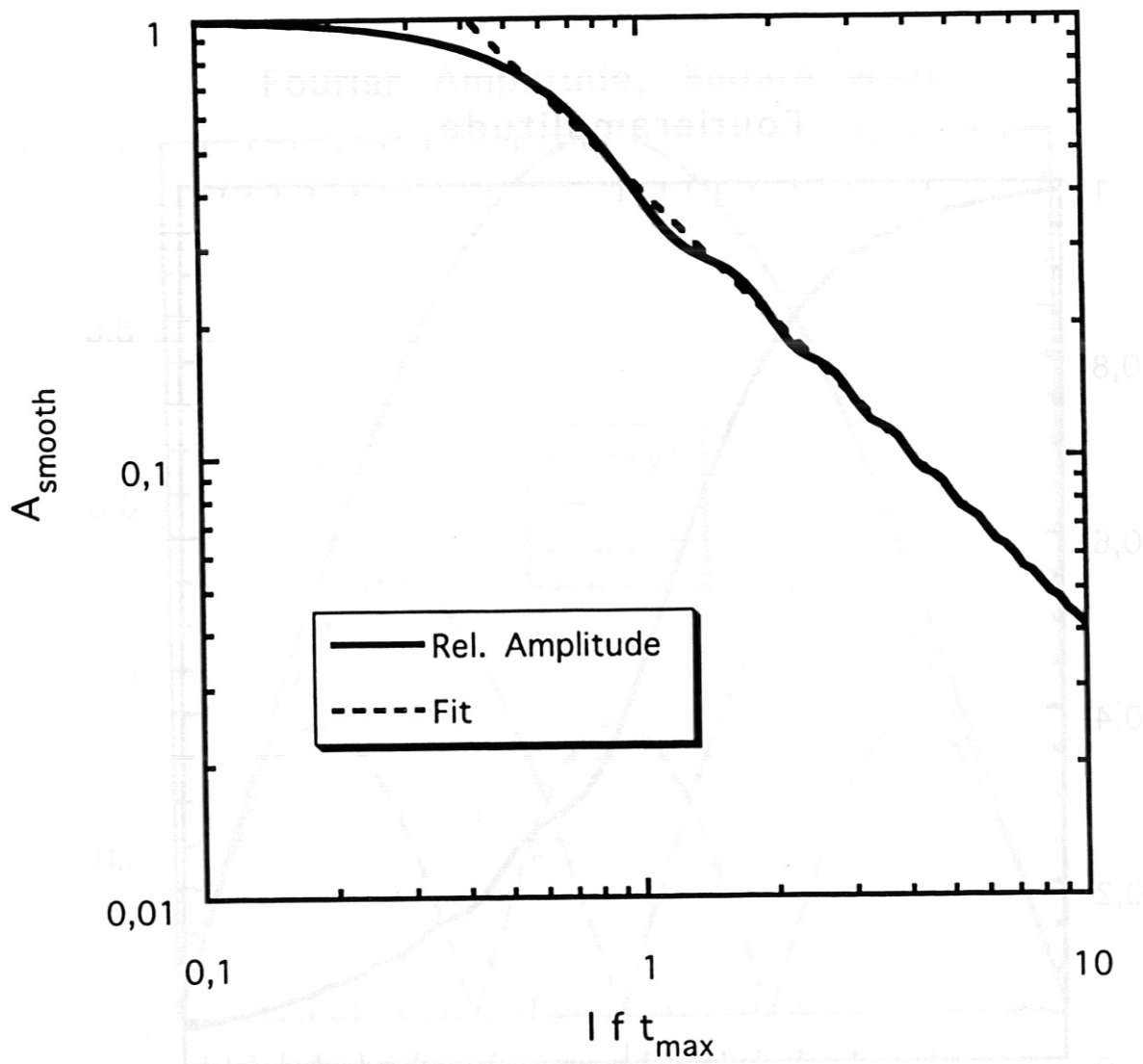


Fig. 4a

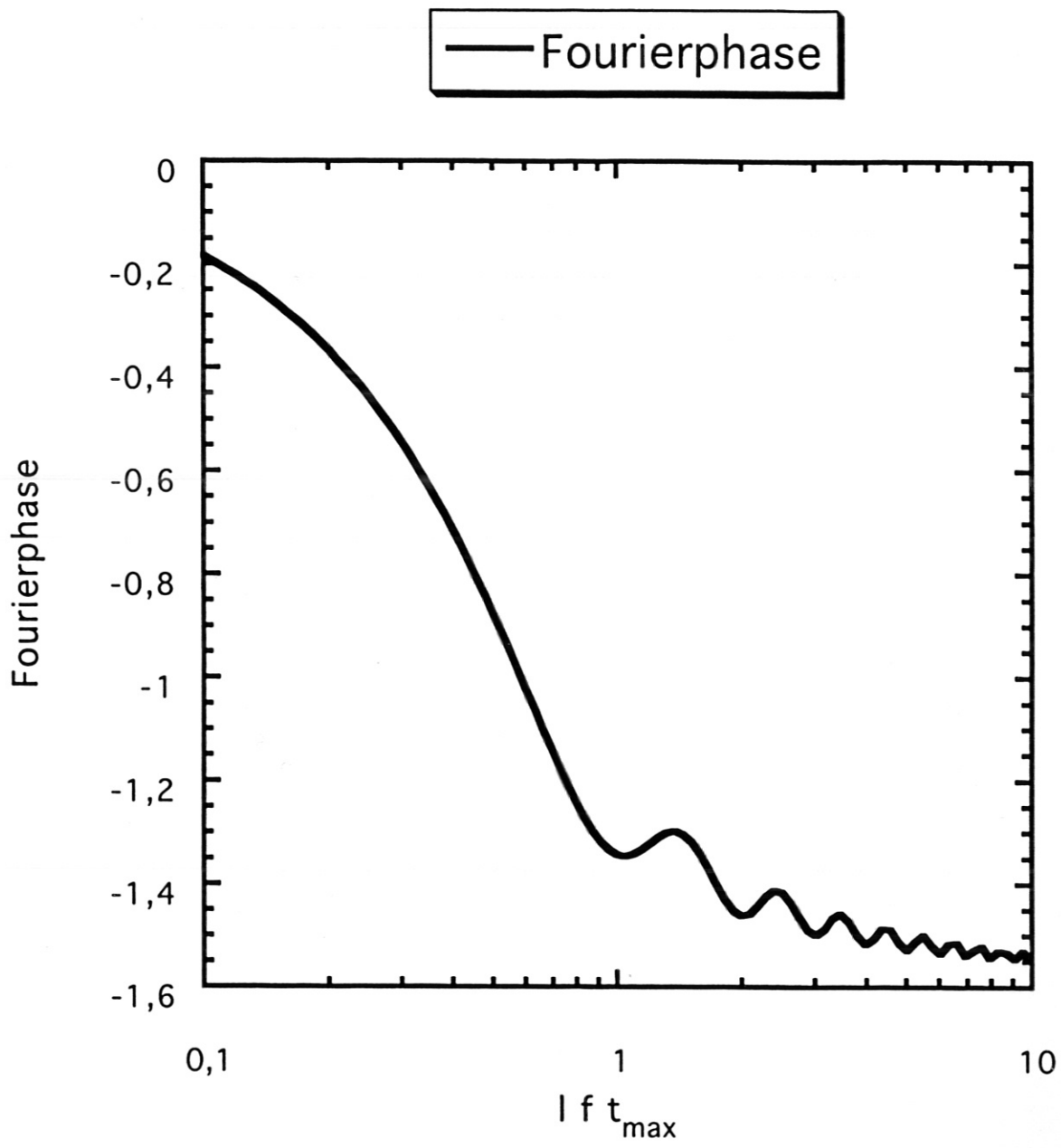


Fig. 5