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THE PHYSICS OF ION CYCLOTRON HEATING IN TOKAMAKS

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Abstract.

These lectures are an introduction to the theory of ion cyclotron heating of tokamak plasmas. In the first part we will derive the kinetic equation which describes the evolution of the ion distribution function under the effect of resonant interactions with the waves, and briefly discuss its solution. The second, somewhat longer, part will be devoted to a discussion of launching, propagation and absorption of h.f. waves in this frequency range, largely based on the exhamination of the local dispersion relation; only very briefly we will also mention the derivation and solution of differential wave equations adequate to describe wave propagation in non-uniform plasmas. It should be clear that the kinetic and wave-propagation aspects of the theory are not really independent from each other, since the distribution functions influence the coefficients of the wave equations (particularly those which describe absorption), and, viceversa, the distribution of h.f. field in the plasma is an essential ingredient of the kinetic equations describing particle-wave interactions. The coupling between wave propagation and kinetic equations is nevertheless sufficiently loose that in practice it is possible, and advantageous, to keep the two subjects separated, provided one remains aware that mutual influences always exist.

Because of the tutorial nature of the lectures and the limited time available, we will discuss mainly the physical foundations of the theory, omitting almost completely its more thechnical aspects. In particular, we could not include any detail on the extensive numerical simulations which play a most important role in the practical applications. We have, however, mentioned also some of the more subtle problems which arise because of the complicated geometry of tokamak plasmas, giving sufficient references for the interested reader to be able to deepen her or his understanding with the help of the original literature.

1 – The ion distribution function during IC heating.

1.1 – *The heating mechanism.* In its simplest form, ion cyclotron heating exploits the secular acceleration of ions gyrating in phase with the rotating component E_+ of the wave electric field when the resonance condition

$$\omega - k_\parallel v_\parallel = \Omega_{ci} \tag{1.1}$$

is satisfied, where $k_\parallel$ and $v_\parallel$ are the parallel component of the wavevector and of the ion velocity, respectively (parallel and perpendicular will always refer to the local direction of the static magnetic field $\vec{B}$). An appreciable number of ions simultaneously satisfy this condition if

$$|\omega - \Omega_{ci}| = O(|k_\parallel| v_{thi}) \tag{1.2}$$

where $v_{thi} = (2T_i/m_i)^{1/2}$ is the thermal velocity of the ions. Taking into account the horizontal variation of $B \simeq B_o\,(1 + (r/R_{tor})\cos\theta)^{-1}$ in a tokamak (where r and θ are polar coordinates in the poloidal cross-section), this defines a vertical cylindrical layer around the resonance $\omega = \Omega_{ci}$, of width

$$\Delta X_{cycl} \simeq 2n_\parallel (v_{thi}/c)\, R_{tor} \tag{1.3}$$

where $n_\parallel = ck_\parallel/\omega$, and R_{tor} the toroidal radius. ΔX_{cycl} is always a small but non-negligible fraction of the plasma radius, typically several centimeters.

1.2 – *Cyclotron heating at the fundamental.* The phase ψ between E_+ and the gyration velocity of an ion can be written

$$\psi = \psi_o + \int_{t_o}^{t} \left(\omega - k_\parallel v_\parallel - \Omega_{ci}(r,\theta)\right) dt \tag{1.4}$$

where subscript zero denotes some reference point along the orbit. As the ion moves along a magnetic field line $\theta - \theta_o = q(\phi - \phi_o)$ on the magnetic surface r ($q = R_{tor}B_\theta/rB_\phi$ is the safety factor, a measure of the pitch with which the lines of force wind around the magnetic axis), the phase becomes stationary, $d\psi/dt = 0$, at the point where (1.1) is satisfied. If the ion parallel velocity is sufficiently large so that its parallel acceleration $(\vec{\mu} \cdot \vec{\nabla})B$ can be neglected, moreover,

$$\frac{d^2\psi}{dt^2} \simeq -\frac{\partial \Omega_{ci}}{\partial \theta}\dot{\theta} \simeq \frac{\Omega_{ci}\, r \sin\theta}{qR_{tor}^2} v_\parallel \tag{1.5}$$

where we have taken into account that $\dot{\theta} \simeq \dot{\phi}/q \simeq v_\parallel / qR_{tor}$. Thus, putting the reference point at the resonance itself,

$$\psi \simeq \psi_o + \frac{1}{2}\frac{\Omega_{ci}\, r \sin\theta}{qR_{tor}^2} v_\parallel \,(t - t_o)^2 \tag{1.6}$$

It follows that the resonance duration (i.e. the time during which ψ varies by less than $\pi/2$) is

$$\tau_{Res} \simeq \left(\frac{\pi q R_{tor}^2}{r\Omega_{ci}|v_\parallel \sin\theta|} \right)^{1/2} \tag{1.7}$$

where now r and θ are the radius and poloidal angle of the point where the ion crosses the resonance. τ_{Res} is always much longer than the cyclotron period (this is the essence of the wave-particle resonance itself), although usually much shorter than the circulation time $\tau_B = qR_{tor}/v_\parallel$ of the ion in the tokamak. During the time τ_{Res} the perpendicular velocity of the ion changes from $v_\perp$ to

$$\left| v_\perp + \frac{Z_i e}{m_i} E_+ \int_{-\tau_{Res}}^{\tau_{Res}} e^{i\psi(\tau)}\, d\tau \right| \simeq \left| v_\perp + \frac{Z_i e}{m_i} E_+ e^{i\psi_o} \left(\frac{\pi q R_{tor}^2}{r\Omega_{ci}|v_\parallel \sin\theta|} \right)^{1/2} \right| \tag{1.8}$$

where the r.h. side is obtained using Eq. (1.6) around the point of stationary phase. Averaging over ψ_o (thereby killing the linear term in $\Delta(v_\perp^2)$) and taking into account that there are two transits through resonance for each connection length $2\pi q R_{tor}$, one obtains an estimate of the rate of increase of the ion perpendicular energy $K_\perp$, namely

$$\frac{dK_\perp}{dt} \simeq \frac{Z_i^2 e^2 R_{tor}}{m_i \Omega_{ci} r |\sin\theta|} |E_+|^2 \tag{1.9}$$

(for obvious geometrical reason this estimate does not hold on the magnetic axis, or when the magnetic surface is just tangent to the resonance surface. We will return on these and other details of cyclotron resonances in tokamaks later). Note that the parallel velocity $v_\parallel$ of the ion cancels out in $\dot{K}_\perp$.

1.3 – ***First harmonic heating.*** We will see in the next section, however, that in a plasma with only one ion species the component of the wave electric field gyrating in the same sense of the ions is very small at the cyclotron resonance for those waves which can be launched from the outside in tokamak geometry. One way out of the severe limitation imposed by this circumstance on the achievable heating rate is to use instead the first harmonic resonance, $\omega - k_\parallel v_\parallel = 2\Omega_{ci}$. Heating at the first harmonic is a finite Larmor radius effect. If the h.f. electric field is uniform in space the work made by the electric field on a particle at resonance has opposite sign in the two halfs of each cyclotron gyration, and the net result is zero. If $\vec{\nabla} E_+ \neq 0$, however, the compensation is not complete, and heating is possible. If the variation of E_+ over the ion Larmor radius is small, as it is usually the case, a simple Taylor expansion around the ion guiding center shows that we can expect the heating rate to be proportional to $v_\perp^2 |\vec{\nabla} E_+|^2/\Omega_{ci}^2$.

To estimate the heating rate in a toroidal magnetic field configuration we note that when finite Larmor radius effects are important the expression (1.4) for the phase $\psi =$

$\vec{k} \cdot \vec{r} - \omega t$ as seen by the particle along its orbit must be generalized by taking into account the perpendicular wavevector:

$$\psi = \psi_o + \int_{t_o}^{t} \left\{ (\omega - k_\parallel v_\parallel) + \frac{k_\perp v_\perp}{\Omega_{ci}} \sin\left(\phi - n \int_{t_o}^{t} \Omega_{ci}\, d\tau\right) \right\} dt \tag{1.10}$$

The new exponential factor in the expression for $\vec{v}_\perp(t)$ can be expanded into harmonics of the cyclotron frequency using the identity

$$e^{iz \sin\psi} = \sum_{n=-\infty}^{n=+\infty} J_n(z) e^{in\psi} \tag{1.11}$$

Modifying Eq. (1.8) accordingly, and assuming that along the particle trajectory resonance occurs only for one value of n, one obtains

$$\frac{dK_\perp}{dt} \simeq \frac{Z_i^2 e^2 R_{tor}}{m_i \Omega_{ci} r |\sin\theta|} J_{n-1}^2 \left(\frac{k_\perp v_\perp}{\Omega_c}\right) |E_+|^2 \tag{1.12}$$

For $n = 1$ to lowest order in the Larmor radius we recover (1.9); the result for $n = 2$ begins proportional to $k_\perp^2 v_\perp^2 / \Omega_{ci}^2$, as expected.

1.4 – *Quasilinear diffusion.* In most cases the fractional energy change of an ion at each crossing of the resonance is small; moreover, it is reasonable to expect phase correlations between the ion gyromotion and the wave to be destroyed (by collisions or by other mechanisms to be briefly discussed later) in a time much shorter than the time of flight τ_B between two successive transits through resonance. Under these conditions heating can be described as a random walk in velocity space, with steps $\Delta v_\perp$ given by Eq. (1.8) or its generalizations separated by a time of the order of $\tau_B/2$. The long-term evolution of the ion distribution function then obeys a kinetic equation of the form

$$\frac{\partial F_i}{\partial t} = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left\{ v_\perp D_{QL}(k_\perp, v_\perp) \frac{\partial F_i}{\partial v_\perp} \right\} + \left(\frac{\partial F_i}{\partial t}\right)_{coll} \tag{1.13}$$

where the last term is the Fokker-Planck collision operator [1], and the first describes the h.f.-driven diffusion in velocity space, known as quasilinear diffusion. The quasilinear diffusion coefficient D_{QL} is related to the average heating rate of the single particle by

$$D_{QL} = \left\langle \frac{\Delta v_\perp^2}{\Delta t} \right\rangle \simeq \frac{2}{m} \frac{dK_\perp}{dt} = D_n J_{n-1}^2 \left(\frac{k_\perp v_\perp}{\Omega_c}\right) |E_+|^2 \tag{1.14}$$

This derivation of the quasilinear kinetic equation in toroidal geometry is due to Stix [2]. Previously, the same equation was obtained [3] starting from the quasilinear diffusion coefficient of a uniform plasma [4], which is proportional to a δ-function of argument

$\omega - k_\parallel v_\parallel - n\Omega_{ci}$ (the condition for resonance in the homogeneous case), and averaging it over the Doppler-broadened resonance layer (1.3). The integration replaces the δ-function with a constant, leading to an equation essentially identical to (1.14). It goes without saying that this euristic procedure can hardly be justified in a rigorous way. Stix has shown, however, that the more realistic derivation taking into account the ion dynamics in the toroidal magnetic field leads essentially to the same equation to lowest order, i.e. as long as specific configurational effects, such as trapping of fast ions in the local mirrors of tokamaks, can be neglected. This can be understood by noting that the Bessel function factor in $D_{QL}(k_\perp, v_\perp)$, which describes the resonant component of the electric field of a plane wave as seen by a gyrating ion, should remain accurate provided that the gradient of B_o is sufficiently weak, so that the resonance lasts for a large number of cyclotron periods. As remarked above, this condition is always well satisfied. We will briefly discuss additional toroidal effects in section 1.9.

The coefficient D_n in Eq. (1.14) follows by comparison with Eq. (1.12). In practice, however, it is more convenient to express D_n in terms of the power absorbed per unit volume P_{abs}. This can be done using the power balance equation

$$P_{abs} = \frac{1}{2} m_i n_i \int v_\perp^3 \left(\frac{\partial F_i}{\partial t}\right)_{QL} d_3\vec{v}$$

$$= -2\pi m_i n_i D_n \int_{-\infty}^{+\infty} dv_\parallel \int_0^{+\infty} dv_\perp\, v_\perp^2\, J_{n-1}^2\left(\frac{k_\perp v_\perp}{\Omega_c}\right) \frac{\partial F_i}{\partial v_\perp} \tag{1.15}$$

In particular, a convenient procedure is to express D_n in terms of the 'initial' heating rate, when the distribution function is a Maxwellian:

$$P_{abs}^{lin} = 4\, D_n\, m_i\, n_i \int_0^\infty w^3\, J_{n-1}^2(\xi_i w)\, e^{-w^2}\, dw \tag{1.16}$$

where $\xi_i = k_\perp v_{thi}/\Omega_{ci}$. To lowest order in the thermal Larmor radius, then,

$$D_1 = \frac{P_{abs}^{lin}}{2\, m_i\, n_i} \qquad D_2 = \frac{P_{abs}^{lin}}{2\, m_i\, n_i} \frac{\Omega_{ci}^2}{k_\perp^2 v_{thi}^2} \tag{1.17}$$

for fundamental and first harmonic heating, respectively. The quantity P_{abs}^{lin} is directly accessible experimentally from the time-derivative of the energy content of the plasma at the beginning of the heating pulse. Note also that D_n is proportional to the power available per ion of the heated species.

1.5 – *Linearization of the collisional operator.* Due to the weak nonlinearity of the Fokker-Planck operator, it is sufficient for our purposes to regard the heated ions as test particles colliding with a thermal background plasma. This is obvious for minority heating, since it amounts to neglect collisions of the diluted minority ions among themselves. In the case of first harmonic (FH) heating the justification is that due to the low power available per particle one does not expect large 'global' deviations from thermal equilibrium.

The collision operator for the heated species can then be written [5]

$$\left.\frac{\partial F_i}{\partial t}\right)_{coll} = \sum_\beta \nu^{i/\beta}\left\{\frac{1}{v^2}\frac{\partial}{\partial v}v^2\left[\Psi(\gamma_{i\beta}v)\left(\frac{1}{2v}\frac{\partial f^i}{\partial v}+\frac{T_i}{T_\beta}f^i\right)\right]\right.$$
$$\left.+\frac{\Theta(\gamma_{i\beta}v)}{2v^3}\frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial f^i}{\partial \mu}\right)\right\} \tag{1.18}$$

where the sum extends over electrons and background ions; the second term, in which $\mu = v_\parallel/v$, describes pitch–angle scattering. Velocities are normalised to an appropriate thermal velocity $v_{th\beta} = (2T_\beta/m_\beta)^{1/2}$ (for the heated species, T_i can be chosen to be the temperature T_o before heating), and $\gamma_{i\beta} = v_{thi}/v_{th\beta}$. Moreover, with $u_\beta = \gamma_{i\beta}v$,

$$\Psi(u_\beta) = \frac{\Phi(u_\beta)}{u_\beta^2} \qquad \Theta(u_\beta) = \frac{1}{2u_\beta}\frac{d\Phi}{du_\beta} + \left(u_\beta^2 - \frac{1}{2}\right)\frac{\Phi(u_\beta)}{u_\beta^2} \tag{1.19}$$

with

$$\Phi(u_\beta) = \frac{4}{\sqrt{\pi}}\int_0^{u_\beta} u^2\, e^{-u^2} du = -\frac{2}{\sqrt{\pi}}u_\beta\, e^{-u_\beta^2} + \mathrm{Erf}\,(u_\beta) \tag{1.20}$$

The linearized FP operator conserves particles, but energy is lost to the thermal bath constituted by the background plasma: this corresponds well to the real situation and allows stationary solutions of the quasilinear equation to exist.

In the electron contribution it is sufficient to retain the leading term of the small argument expansion of $\Psi(u_e)$,

$$\Psi(u_e) \simeq k_o \frac{v_{thi}}{v_{the}} v \tag{1.21}$$

while $\Theta(u_e)$ can be neglected altogether. For the ion–ion collision terms Stix [2] has suggested the approximations

$$\Psi(u_\beta) \simeq \frac{k_o u_\beta}{1 + k_o u_\beta^3}$$
$$\Theta(u_\beta) \simeq \frac{3k_o}{2}u_\beta^2\, e^{-u_\beta^2} + \left(u_\beta^2 - \frac{1}{2}\right)\frac{k_o u_\beta}{1+k_o u_\beta^3} \tag{1.22}$$

where $k_o = 4/3\sqrt{\pi}$. The exact and approximate coefficients differ at most by a few percent.

1.6 – *Stix solution for minority heating.* A major difficulty for solving the kinetic equation (1.13) analytically is that the collisional operator is separable in spherical coordinates (in velocity space), while the natural coordinates for the quasilinear diffusion operator are cylindrical ones. If one nevertheless averages (1.13) over the pitch angle μ, one obtains an ordinary differential equation for the isotropic part F_o of the distribution function (we omit for simplicity the index referring to the heated species) which, after a trivial first integration, can be written in dimensionless form (normalizing velocities to the thermal speed)

$$\frac{1}{F_o}\frac{dF_o}{dv} = -2\frac{\Psi_\tau(v)}{\Psi_c(v) + 2v\,\bar{D}_{ql}(\xi v)}v \tag{1.23}$$

where

$$\bar{D}_{ql}(\xi v) = \frac{\bar{D}_n}{2}\int_{-1}^{+1}(1-\mu^2)\,J_n^2\left(\xi v(1-\mu^2)^{1/2}\right)d\mu \tag{1.24}$$

with $\xi = k_\perp v_{thi}/\Omega_{ci}$ and $\bar{D}_n = P_{abs}^{lin}/4v^{i/M}n_iT_i$ is the isotropic part of the diffusion coefficient.

In the case of minority heating, neglecting finite Larmor radius corrections in the quasilinear operator (i.e. approximating $J_o(\xi v)$ with unity; we will see in the next section that $\xi \ll 1$ for the waves used for IC heating in tokamaks), Eq. (1.24) can be integrated in closed form [2]. Defining

$$\alpha = \frac{\tau\epsilon_e}{\epsilon_e + D} \qquad \beta = \frac{1+\tau\epsilon_e}{1+\epsilon_e+D} - \frac{\tau\epsilon_e}{\epsilon_e+D} \qquad \gamma = k_1\frac{\epsilon_e + D}{1+\epsilon_e+D} \tag{1.25}$$

where $k_1 = (A_M/A_i)^{1/2}k_o$, $D = D_o/k_1$, $\tau = T_M/T_e$, $\epsilon_e = (n_e/Z_M^2 n_M)\left[(m_e/m_M)\tau\right]^{1/2}$, the solution can be written

$$F_o(v) = f_o\,\exp\left\{-v^2\left(\alpha + \frac{\beta}{1+q\gamma^{2/3}v^2}\right)\right\} \qquad q \simeq 0.41350 \tag{1.26}$$

This distribution can be interpreted in terms of two ion populations. The "bulk" and "tail" minority temperatures are easily recognised to be

$$T_i^{bulk} = T_M\frac{1+\epsilon_e+D}{1+\tau\epsilon_e} \simeq T_M\,(1+D)$$
$$T_i^{tail} = T_e\left(1+\frac{D}{\epsilon_e}\right) \simeq T_e\frac{D}{\epsilon_e} \tag{1.27}$$

respectively. The last approximations hold if $D \gg \epsilon_e$, $T_M/T_e = O(1)$: in this approximation T_i^{bulk} depends only on the majority temperature, while T_i^{tail} depends only on the electron temperature.

For typical tokamak parameters and moderate minority concentration D is smaller than unity even at relatively large values of the absorbed power density P_{abs}. Under these conditions the tail temperature is nevertheless much larger than that of the bulk (since $\epsilon_e \ll 1$), but the number of ions in the tail is small, so that the energy stored in the tail is only a modest fraction of the total energy in the minority. An example is shown in fig. 1. In particular, the number of ions with energy so large that $k_\perp v_\perp i/\Omega_{ci} > 1$ is very small, so that the approximations made in deriving Eq. (1.26) are well justified (except the assumption of isotropy, to which we will return below). Nevertheless some ions reach and exceed such energies, particularly in large devices where they are still well confined. To determine their number and their evolution a much more sofisticated approach is required, as briefly discussed in section 1.9.

1.7 – *Analytic solution for first harmonic heating.* No such fully explicit solution is available when finite Larmor radius (FLR) effects are included, or in the case of first harmonic heating; nevertheless the numerical quadrature of Eq. (1.23) is always trivial. It is instructive, in particular, to consider in some details the case of first harmonic heating in a single species plasma in the lowest order in the Larmor radius, $J_1^2(\xi w) \simeq \xi^2 w^2/4$. In this approximation the integral

$$2\int_0^v \frac{\Psi_\tau(v)\, v\, dv}{\Psi_c(v) + (4\bar{D}_1/15)\, v^3} \tag{1.28}$$

tends to a finite constant for $v \to \infty$, and the distribution function obtained from Eq. (1.13) is not integrable, a result which at first sight is physically unacceptable. The origin of the failure of the small Larmor radius approximation in the case of first harmonic heating is clear: as long as $k_\perp v_\perp/\Omega_{ci}$ is small the rate of energy increase of an ion is proportional to its perpendicular energy already possessed by the particle; hence the heating will run away until saturated by finite Larmor radius effects. In the lowest order approximation, therefore, the quasilinear kinetic equation does not possess a physically admissible stationary solution.

In practice, however, with a minimal modification, the approximation (1.28) can be safely used also in this case. Since all ions are heated, the power per ion is small and $\bar{D}_o$ is always much smaller than unity. It follows that $v^3\bar{D}_{ql}(v) \ll \Psi_c(v)$ up to very large energies. The value of the integral (1.28) in the limit $v \to \infty$, although finite, is therefore so large that $F_i(v)$ becomes insignificantly small well before higher-order Larmor radius effects begin to play a role. It is then sufficient to regard $F_i(v)$ as practically different from zero only over a finite interval, chosing the cutoff energy large enough for the normalization integral to have reached an essentially constant value, yet sufficiently small for higher order terms in the Larmor radius to be still negligible. On this finite interval Eq. (1.28) is perfectly adequate. This example illustrates well

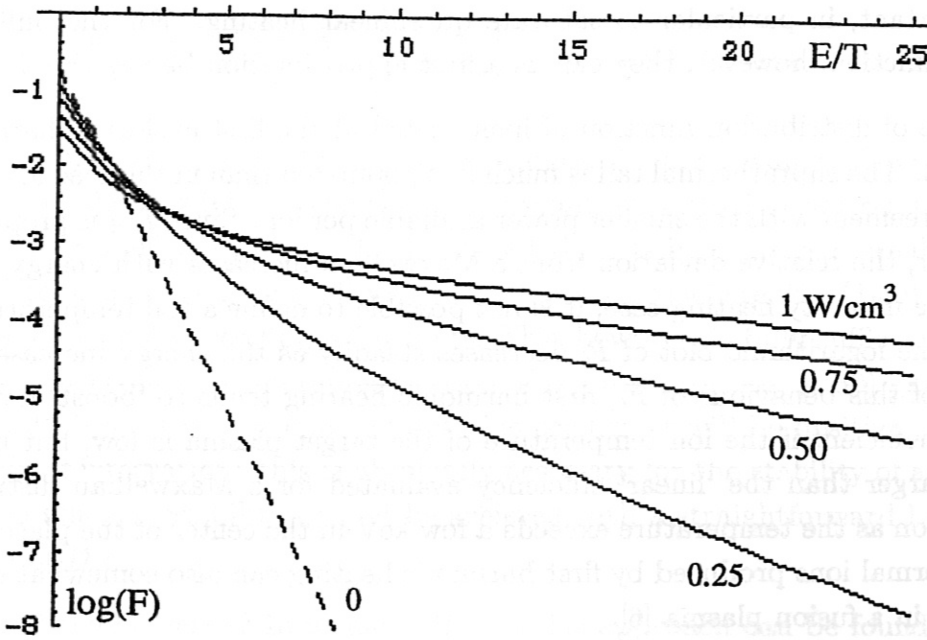

Fig. 1 - Minority distribution function in the isotropic approximation; 5% H in D, $n = 8\ 10^{13}\ cm^{-3}$, $T_e = T_H = T_D = 5$ keV.

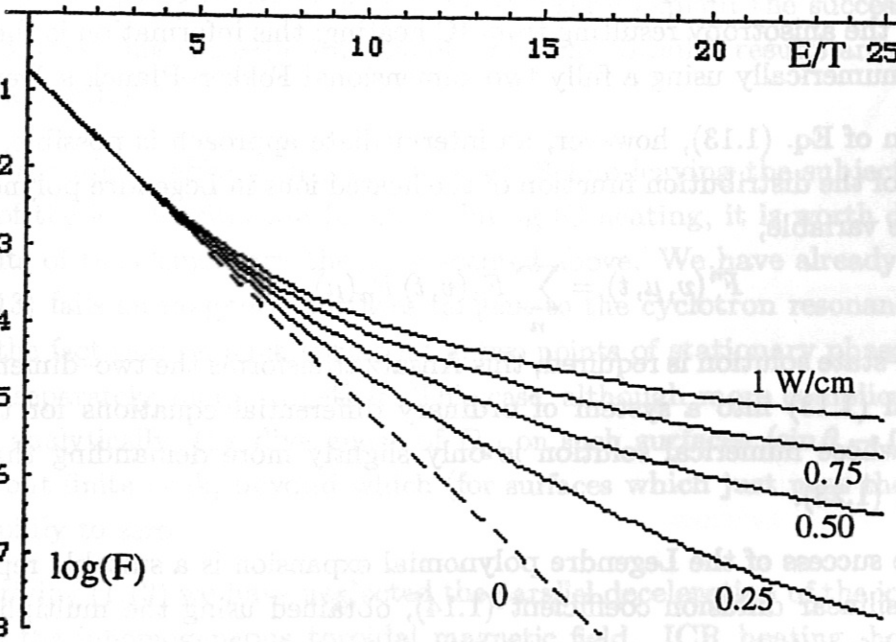

Fig. 2 - Distribution function during first harmonic heating in the isotropic approximation; Deuterium: $n_e = 8\ 10^{13} cm^{-3}$, $T_D = T_e = 5$ keV.

the role of FLR effects in the quasilinear kinetic equation. Some ions certainly reach perpendicular energies so large that such effects (and toroidicity effects discussed below) become important, in particular to saturate quasilinear heating. For the bulk of the distribution function, however, they can as a first approximation be neglected.

An example of distribution function of ions heated at the first cyclotron harmonic is shown in fig. 2. The suprathermal tail is much less populated than in the case of minority heating, in agreement with the smaller power available per ion. Since $\bar{D}_{ql}$ is proportional to v^2, however, the relative deviation from a Maxwellian increases with energy, and, in contrast to the minority heating case, it is not possible to define a tail temperature: the steepness of the logarithmic plot of F_i decreases steadily as the energy increases. As a consequence of this behaviour of F_i, first harmonic heating tends to 'boost' himself: it is relatively inefficient if the ion temperature of the target plasma is low, but becomes appreciably larger than the 'linear' efficiency evaluated for a Maxwellian distribution function as soon as the temperature exceeds a few keV in the center of the plasma. The tail of supratermal ions produced by first harmonic heating can also somewhat enhance the reactivity in a fusion plasma [6].

1.8 – *Legendre polynomials representation of the quasilinear operator.* At low to moderate power levels the predictions of the elementary one–dimensional solutions of the kinetic equation for the effective temperature of the suprathermal ion population are in good agreement with experimental observations, except for the fact that most of the energy is found in the perpendicular motion. It is surprisingly difficult to estimate analytically the anisotropy resulting from IC heating; this information is therefore usually obtained numerically using a fully two–dimensional Fokker–Planck solver [5].

For the solution of Eq. (1.13), however, an intermediate approach is possible, based on the expansion of the distribution function of the heated ions in Legendre polynomials in the pitch–angle variable,

$$F^i(v,\mu,t) = \sum_n F_n(v,t)\,P_n(\mu) \tag{1.29}$$

If only the steady–state solution is required, this Ansatz transforms the two–dimentional partial differential (1.13) into a system of ordinary differential equations for the coefficients $F_n(v)$, whose numerical solution is only slightly more demanding than the integration of Eq. (1.23).

The key for the success of the Legendre polynomial expansion is a suitable representation of the quasilinear diffusion coefficient (1.14), obtained using the multiplication theorem for Bessel functions:

$$D_{ql}(v,\mu) = D_o\,(1-\mu^2)^p \sum_{k=0}^{\infty} \mathcal{J}_k^p(\xi_\perp v)\,\mu^{2k} \tag{1.30}$$

where we have introduced the functions

$$\mathcal{J}_k^p(\xi_\perp v) = \left(\frac{\xi_\perp v}{2}\right)^k \sum_{k'=0}^{k} \frac{1}{k'!(k-k')!} J_{p+k'}(\xi_\perp v) J_{p+k-k'}(\xi_\perp v) \tag{1.31}$$

For each v Eq. (1.30) is a Taylor expansion around the exact value of D_{ql} for $\mu = 0$; at the same time, for $\mu \neq 0$ it also looks like an expansion in the Larmor radius. The series converges very rapidly for $\xi_\perp v \lesssim 1$. If $\xi_\perp v \gg 1$ there is increasing internal cancellation among successive terms; hence, although convergent everywhere, Eq. (1.30) is useful only for values of $\xi_\perp v$ not exceeding a few units. This, however, is fully sufficient in practice. The rapid convergence of the representation of D_{ql} based on Eq. (1.30) guarantees that using 5 to 10 terms the approximate diffusion coefficient will be positive everywhere in the domain of integration: this is absolutely necessary for the stability of any numerical integration scheme. Stability cannot be achieved with a straightforward Larmor radius expansion of D_{ql}.

Details of the numerical implementation of this approach can be found in [6]. Here we present (fig. 3) an example of solution for the minority heating case, for comparison with fig. 1. The distribution of perpendicular velocities is, apart from an obvious normalization factor, essentially identical with Stix analytic solution. The distribution of parallel velocities for $v_\perp = 0$ (i.e. $F(\vec{v}, \mu = 1)$), on the other hand, remains nearly Maxwellian, with a temperature close to the value T_{bulk} predicted by the analytic theory for the low-energy part of $F_o(v)$. These results explain the success of the analytic model in predicting the experimental observations. Similar results are obtained for first harmonic heating.

1.9 – *Toroidicity effects on energetic ions.* Before leaving the subject of the determination of the ion distribution function during IC heating, it is worth discussing briefly the limits of the elementary theory presented above. We have already mentioned that Eq. (1.13) fails on magnetic surfaces tangent to the cyclotron resonance layer. This is due to the fact that on such surfaces the two points of stationary phase, which we have treaded separately, merge together. This case, although more complicated, can still be treated analytically; the divergence of D_{ql} on such surfaces ($\sin\theta \to 0$) is replaced by a large but finite peak, beyond which (for surfaces which just miss the resonance) D_{ql} goes rapidly to zero.

In deriving (1.13) we have neglected the parallel deceleration of the ions by the $\vec{\mu}\cdot\vec{\nabla} B$ force in the inhomogeneous toroidal magnetic field. ICR heating, however, increases directly only the perpendicular energy of the ions, any increase in parallel energy occurring only through collisional relaxation. As the ion energy increases, its collisionality decreases, while the pitch angle is increasing, so that the ion will finally be trapped in

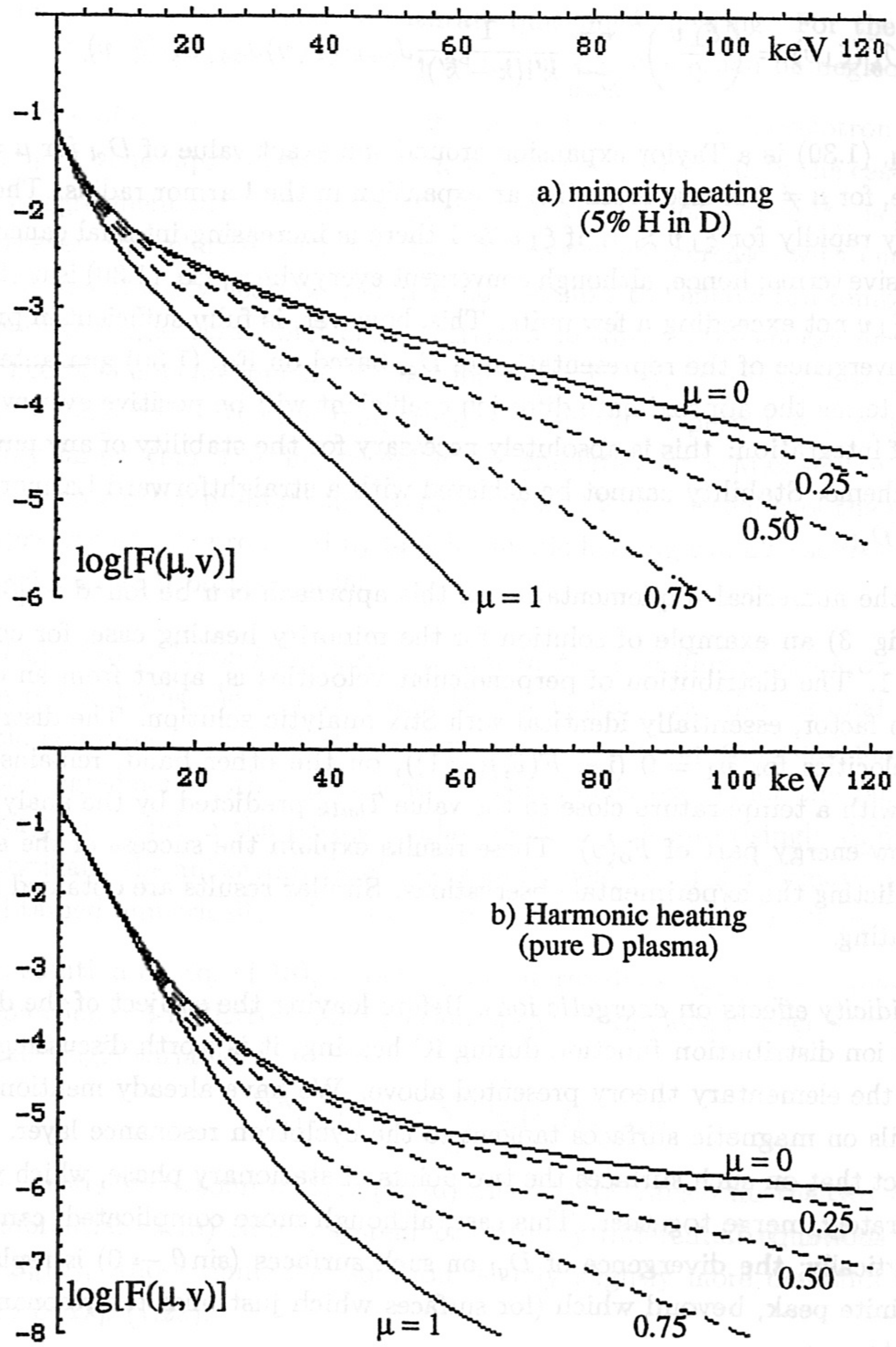

Fig. 3 - Distribution function of the ions during cyclotron heating.
$n_e = 8\ 10^{13} cm^{-3}$, $B_o = 5$ T, "background" temperature 5 keV,
linear power density 0.5 W/cm^{-3}

the toroidal magnetic well. Heating will continue only as long as the reflection point remains on the high-field side of the $\omega = n\Omega_{ci}$ layer: a more deeply trapped ion does not see the resonance until pitch-angle scattering increases sufficiently again its parallel energy. Thus ICR heating tends to accumulate energetic ions along the cone in velocity space, internal to the trapped ion region, which correspond to ions marginally missing the resonance:

$$\left.\frac{v_\perp^2}{v_\parallel^2}\right)_{eq} = \frac{B_{eq}}{B_{res} - B_{eq}} \tag{1.32}$$

(subscripts res and eq refer to the resonance position and the outer equatorial point on the magnetic surface, respectively; this equation is immediately obtained from the conservation of energy and magnetic moment outside resonance). For such ions, of course, Eq. (1.5) is not valid, and one must take into account that the quasilinear diffusion coefficient has a more complicated pitch angle dependence than predicted by Eq. (1.14); in particular, it goes to zero if $v_\perp^2/v_\parallel^2$ is larger than the r.h. side of (1.32). A simple model for D_{ql} taking these effects into account can be found in [7]; a more rigorous derivation in [8].

In large devices such as JET the energy of ions produced in this way during minority heating can reach the MeV range. Their 'banana' trajectories are so thick that a kinetic equation which assumes that they move on a magnetic surface is no longer adequate [9]. To describe the evolution of this ion population, which can be important for example for diagnostic purposes, a Montecarlo approach [10] might be more appropriate.

1.10 – *Phase randomization and superadiabaticity.* A further assumption made in deriving equation (1.13) is that the phase ψ_o at a transit through a resonance is uncorrelated with the phase at the previous transit. For most ions, this can be made plausible with the following estimate. The number of cyclotron gyrations between two resonances is $\sim \Omega_{ci}\tau_B = \Omega_{ci}qR_{tor}/v_\parallel$. Hence if collisions change $v_\parallel$ by $\Delta v_\parallel$ during τ_B, ψ_o will be changed by $\Delta\psi_o \simeq \Omega_{ci}qR_{tor}\Delta v_\parallel/v_\parallel^2$. Decorrelation is guaranteed if $\Delta\psi_o \gtrsim 1$. Since $\Delta v_\parallel/v \simeq \nu_{ii}(v_{thi}^3/v^3)\,\tau_B$, where ν_{ii} is the thermal ion-ion collision frequency, this condition can be written

$$\left(\frac{\Omega_{ci}qR_{tor}}{v_\parallel}\right)^2 \frac{\nu_{ii}}{\Omega_{ci}} \frac{v_{thi}^3}{v^2 v_\parallel} \gtrsim 1 \tag{1.33}$$

The first factor is large enough to compensate for the smallness of the other two. If the ion energy is very large, however, this condition will finally be violated.

This still does not necessarily imply the breakdown of the random phase assumption, since other mechanisms can ensure decorrelation: e. g. interactions with low frequency fluctuations, or the fact that FW has a broad spectrum in $k_\parallel$. The phase ψ_o is very sensitive to such perturbations, again because of the large numbers of cyclotron periods between two interactions. Even in the absence of such perturbations, decorrelation can also be the result of Hamiltonian stochasticity, if the amplitude of the electric field at resonance is large enough. An ion in a tokamak subiect to the IC resonance is an ideal dynamical system for the onset of Hamiltonian chaos because of the large difference in the periods of the gyration and bounce motion. This aspect of IC resonances has been investigated in [11]; the general problem of the validity of the quasilinear equation is discussed in [12].

In any case, it is true that there is an upper limit to the ion energy above which one must expect the random phase assumption to fail. Above this energy the quasilinear picture of ICR heating is no more valid, and ion heating stops; this situation is known as 'superadiabaticity'. In practice, however, the superadiabaticity limit on ICR heating is unlikely to play any role compared, for example, to losses of very energetic ions by diffusion and finite orbits effects.

2 – Launching, propagation and absorption of IC waves.

2.1 – *The dispersion relation of ion cyclotron waves.* The next question arising when investigating a h.f. heating scheme is whether a wave exists which can be launched from outside the plasma, and which is able to penetrate to the plasma core to be thermalized there. This question, or set of questions, can best be answered by examining the local dispersion relation in the plasma. This is the solubility condition of the algebraic form of Maxwell equations for a plane wave (i.e. with space and time dependence of the form $\exp\left\{-i(\omega t - \vec{k}\cdot\vec{r})\right\}$) in a homogeneous plasma having everywhere the same parameters (density, temperature, magnetic field) as the real plasma at the point of observation. These equations can be written

$$\underline{\underline{M}}_{\vec{k},\omega}\cdot\vec{E}_{\vec{k},\omega} \equiv \frac{c^2}{\omega^2}\vec{k}\times(\vec{k}\times\vec{E}_{\vec{k},\omega}) + \underline{\underline{\epsilon}}(\vec{k},\omega)\cdot\vec{E}_{\vec{k},\omega} = 0 \tag{2.1}$$

and the dispersion relation is therefore

$$H(\vec{k},\omega) = \det\left[\frac{c^2}{\omega^2}(k_i k_j - \delta_{ij}k^2) + \epsilon_{ij}(\vec{k},\omega)\right] = 0 \tag{2.2}$$

Here $\underline{\underline{\epsilon}}$ is the dielectric tensor, $\underline{\underline{\epsilon}} = \underline{\underline{I}} + 4\pi i\underline{\underline{\sigma}}/\omega$, and $\underline{\underline{\sigma}}$ the h.f. conductivity tensor. The latter can be obtained by evaluating the h.f. currents in the plasma in response to the wave field by integrating the linearized Vlasov equation [13]. The contribution of each charged species to ϵ_{ij} can be written as a series of terms resonant at each harmonics of the cyclotron frequency, with modified Bessel functions of argument $k_\perp^2 v_{th\alpha}^2/\Omega_{c\alpha}^2$ as coefficients; the series arises by expanding the wave phase as seen by the particles along their 'unperturbed' orbits just as in the previous paragraph. It is also useful to recall that the power absorbed per unit volume is proportional to the antihermitian part of $\underline{\underline{\epsilon}}$:

$$P_{abs} = \frac{\omega}{8\pi} \vec{E}^*_{\vec{k},\omega} \cdot \underline{\underline{\epsilon}}^A \cdot \vec{E}_{\vec{k},\omega} \tag{2.3}$$

It is not difficult to show that the same total power absorption is obtained by integrating this equation or Eq. (1.15) over a cyclotron resonance layer (1.3) in the tokamak.

Since equilibrium gradients are ignored when evaluating $\vec{J}^{hf}$, by symmetry $\epsilon_{ij}(\vec{k},\omega)$, and therefore also H, depend only on the parallel component $k_\parallel$ and on the modulus $k_\perp$ of the perpendicular component of the wavevector with respect to the static magnetic field (we will equivalently use the parallel and perpendicular index, $n_\parallel = ck_\parallel/\omega$ and $n_\perp = ck_\perp/\omega$). To discuss the solutions of (2.2) in a tokamak it is convenient to regard it as an equation for $k_\perp$, since the frequency and $k_\parallel$ are fixed by the generator and the antenna geometry, respectively (for $k_\parallel$ this is only a first approximation; we will return later on this point).

2.2 – *The compressional Alfvén wave.* If for the moment we neglect finite temperature effects, Eq. (2.1) can be written

$$\begin{pmatrix} S - n_\parallel^2 & -iD & n_\perp n_\parallel \\ iD & S - n_\parallel^2 - n_\perp^2 & 0 \\ n_\perp n_\parallel & 0 & P - n_\perp^2 \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{2.4}$$

where, to lowest order in the electron to ion mass ratio,

$$\begin{aligned} R &= 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} - \frac{\omega_{pe}^2}{\omega\Omega_{ce}} - \sum_i \frac{\omega_{pi}^2}{\omega(\omega+\Omega_{ci})} \simeq \sum_i \frac{\omega_{pi}^2}{\Omega_{ci}(\omega+\Omega_{ci})} \\ L &= 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} + \frac{\omega_{pe}^2}{\omega\Omega_{ce}} - \sum_i \frac{\omega_{pi}^2}{\omega(\omega-\Omega_{ci})} \simeq -\sum_i \frac{\omega_{pi}^2}{\Omega_{ci}(\omega-\Omega_{ci})} \\ P &= 1 - \frac{\omega_{pe}^2}{\omega^2} \end{aligned} \tag{2.5}$$

and $S = (R+L)/2$, $D = (R-L)/2$. In R and L (which are the components of $\underline{\underline{\epsilon}}$ when $\vec{E}_\perp$ is decomposed in rotating components $E_\pm = (E_x \pm iE_y)/2$), the contributions from the vacuum displacement current and the electron polarization drift (first and second term, respectively) are mostly negligible, while the much larger contribution from the electron $\vec{E}\times\vec{B}_o$ drift (third term, which, however, cancels out from S) can be merged with the ion contributions using charge neutrality in the form $\sum_{e,i} \omega_{p\alpha}^2/(\omega\Omega_{c\alpha}) = 0$.

It is useful to remark that in tokamak plasmas $\omega_{pe}^2/\Omega_{ce}^2$ is of order unity (typically somewhat smaller than one in the plasma center), so that in the ion cyclotron frequency range $\omega^2/\omega_{pe}^2 = O(m_e^2/m_i^2)$. We can therefore, as a first approximation, neglect the electron inertia. Then $|P| \to \infty$, and from the third line of (2.5) $E_z \to 0$: the electrons screen any h.f. field parallel parallel to the external magnetic field. We then find that (2.2) has only one solution for $k_\perp^2$, which can be written

$$n_\perp^2 = n_\perp^2)_F = -\frac{(n_\parallel^2 - R)(n_\parallel^2 - L)}{(n_\parallel^2 - S)} \tag{2.6}$$

This is the compressional Alfvén wave, or fast wave (FW) (in the russian literature also known as magnetosonic wave), which is the natural candidate for plasma heating and current drive in the ion cyclotron range of frequencies. Excluding insignificantly low densities, in a plasma with a single species of ions L and S are positive for $\omega < \Omega_{ci}$ and negative for $\omega < \Omega_{ci}$, while R is always positive (actually, larger than unity). The fast wave, therefore, is propagative only if $n_\parallel^2 < R$. The surface where the equality

$$n_\parallel^2 = R \tag{2.7}$$

is satisfied is called the 'low-density cutoff'. Waves with $n_\parallel^2 > 1$ are evanescent in vacuum, and become propagative only above this cutoff. This puts an upper limit to the values of $n_\parallel^2$ which can be efficiently launched from outside.

The name of the wave is due to the fact that the oscillating magnetic field is parallel to the static magnetic field, as it is easily shown using (2.4) and $\vec{B} = (c/\omega)\vec{k} \times \vec{E}$. Since according to (2.6) $n_\perp^2 = O(\omega_{pi}^2/\Omega_{ci}^2)$, it is also not difficult to see that the order of magnitude of the perpendicular phase and group velocity is the Alfvén speed $v_A = (\Omega_{ci}^2/\omega_{pi}^2)^{1/2}\,c$. This is true even at ion cyclotron resonances, since the divergences of L and S in numerator and denominator of (2.6) compensate each other. How this is possible can be understood by looking at the wave polarization. From the first two lines of Eq. (2.4) one finds

$$\frac{E_+}{E_-} = \frac{n_\parallel^2 - R}{n_\parallel^2 - L} \tag{2.8}$$

This shows that when $\omega \to \Omega_{ci}$ the component of the electric field which rotates with the ions vanishes, thereby allowing J_+^{hf} to remain finite, and eliminating the singularity in the behaviour of the fast wave at ion cyclotron resonances.

It is easily checked that the perpendicular wavelength of the compressional Alfvén wave is much larger than the thermal ion Larmor radius: $k_\perp^2 v_{thi}^2/\Omega_{ci}^2 = n_\perp^2 v_{thi}^2/c^2 = O(\beta_i) \ll 1$, justifying our neglect of finite Larmor radius (FLR) effects (the role of parallel dispersion near cyclotron resonance and due to the electrons will be discussed below). The only exception suggested by Eq. (2.6) is where

$$n_{\|}^2 = S \simeq -\sum_i \frac{\omega_{pi}^2}{\omega^2 - \Omega_{ci}^2} \qquad (2.9)$$

When $\omega/\Omega_{ci} \to 0$ this is the Alfvén resonance [14]. At frequencies $\omega = O(\Omega_{ci})$ in a plasma with a single ion species, on the other hand, this condition can be satisfied only at very low densities in the plasma scrape-off, where its only effect can be some parasitic absorption in front of the launching structure [15], whose discussion is outside the scope of these notes. As we will see below, on the other hand, this resonance plays a fundamental role in plasmas with more than one ion species of ions. We will also see that FLR effects are important near $\omega = 2\Omega_{ci}$, although not apparent from Eq. (2.6).

2.3 – *Power absorption at the fundamental.* The circumstance that E_+ is screened by the resonant ions themselves would seem to make impossible to heat the ions at the cyclotron fundamental. The conclusion that E_+ vanishes at $\omega = \Omega_{ci}$, of course, is not entirely correct, since when the plasma temperature is taken into account the singularity of the dielectric tensor is smoothed out by the Doppler broadening of the resonance, so that [1]:

$$\frac{\omega}{\omega - \Omega_{ci}} \quad \text{is replaced by} \quad -x_o^i\, Z(x_1^i) \qquad (2.10)$$

in L (and S), where $x_n^\alpha = (\omega - n\Omega_{c\alpha})/k_{\|} v_{th\alpha}$, and (assuming a Maxwellian ion distribution and a real argument x)

$$Z(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-u^2}}{u - x}\, du + i\sqrt{\pi}\, e^{-x^2} \qquad (2.11)$$

is the well-known Plasma Dispersion function [16]. From the behaviour of the r.h. side of (2.11) for small argument, illustrated in fig. 4 for a typical set of parameters, one concludes that at resonance $|E_+|/|E_-| = O\left(n_{\|}^2\, v_{thi}^2/c^2\right)$. If $n_{\|} \neq 0$, therefore, $|E_+|$ is small, but non zero. The fractional absorption through the resonant layer (1.3) can then be estimated by integrating the power balance equation following from the Poynting theorem and Eq. (2.3),

$$\frac{dP_X}{dX} = -\frac{\omega}{8\pi}\, \mathrm{Im}(L)\, |E_+|^2 \qquad \mathrm{Im}(L) = \sqrt{\pi} \frac{\omega_{pi}^2}{\Omega_{ci}^2}\, x_o^i\, e^{-x_1^{i\,2}} \qquad (2.12)$$

across this layer, where the incident flux P_X (here the horizontal component of the Poynting vector) is given by

$$P_X = \frac{c}{8\pi} n_\perp |E_Y|^2 \qquad (2.13)$$

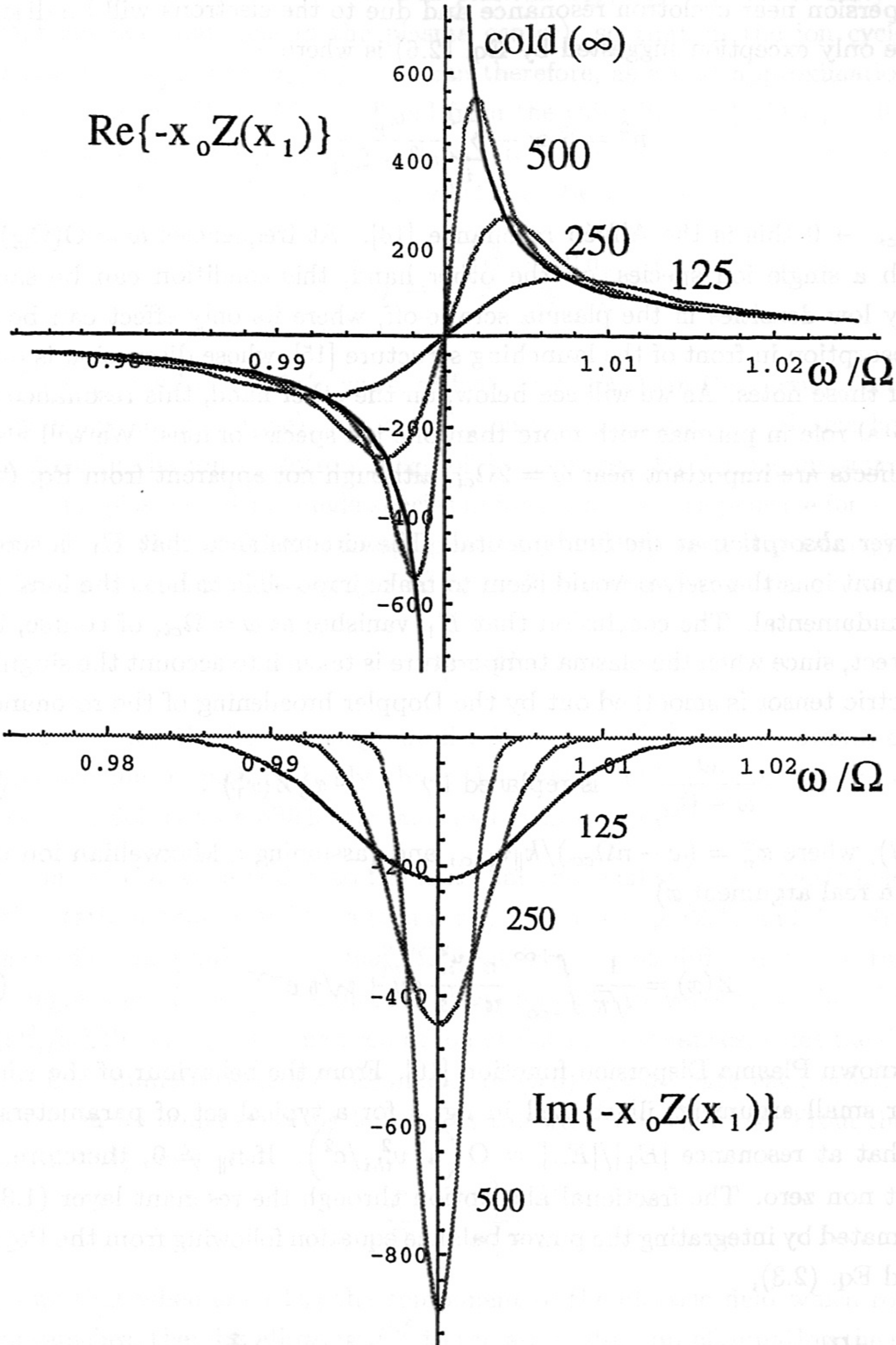

Fig. 4 - The function $-x_oZ(x_1)$ with $x_1 = (\omega-\Omega/\omega)*x_o$, and $x_o = \omega/k_{//}v_{th}$.
The curves shown are for x_o^{-1}= 125, 250, 500, and, for the real part, ∞ (cold limit or perpendicular propagation).

Here X and Y are horizontal and vertical coordinates in the poloidal plane, and we have for simplicity assumed that the wave propagates horizontally, i.e. $k_\perp = k_X$. Only the argument x_1^i of the exponential factor in $\mathrm{Im}(L)$ needs to be regarded as space dependent, while all other quantities can be taken at $\Omega_{ci} = \omega$. Using (2.8) and (2.11) one easily obtains

$$\frac{\Delta P_X}{P_X} \simeq \frac{\omega}{c} R_{tor} \left(\frac{\omega_{pi}^2}{\Omega_{ci}^2} \right)^{1/2} n_\parallel^2 \frac{v_{thi}^2}{c^2} \qquad (2.14)$$

Thus ion heating at the cyclotron frequency does occur, but with a very low efficiency. As we will see below, first harmonic heating of a single species plasma, although a FLR effect, is much more efficient.

2.4 – *Ion-ion resonances and minority heating.* The best efficiency, however, is obtained with IC fundamental heating of a minority species sufficiently diluted to be unable to screen E_+. We have then a situation in which the wave polarization is determined by the majority, which is not resonant, and the absorption by the resonant minority, which can then be heated with great efficiency [17]. The most popular minority heating scheme in medium size tokamaks has been fundamental IC heating of a small fraction of H^+ ions in a deuterium plasma. Other minority heating scenarios, e.g. He_3^{++} in D^+, however, have also been successfully attempted, and might be more interesting under reactor conditions.

In a multi-species plasma L and S have a zero between each pair of cyclotron frequencies. The quantitative investigation of minority heating must therefore take into account the existence of a perpendicular resonance (ion-ion or Buchsbaum resonance [18])

$$n_\perp \to \infty \qquad \text{where} \qquad n_\parallel^2 = S \qquad (2.15)$$

and an associated cutoff

$$n_\perp \to 0 \qquad \text{where} \qquad n_\parallel^2 = L \qquad (2.16)$$

separated by a layer of evanescence. As long as $n_\parallel^2 \ll \omega_{pi}^2/\Omega_{ci}^2 = O(m_i/m_e)$ these conditions can be approximated by $S = 0$ and $L = 0$, respectively, and depend essentially only on the relative concentrations $\nu_i = n_i/n_e$ (with $\sum Z_i \nu_i = 1$). Denoting the majority and minority species with indexes M and m, from (2.5), (2.9) and taking into account the horizontal variation of B one finds

$$\begin{aligned} \frac{X_S}{R_o} &\simeq 1 + \frac{1}{2} \nu_m Z_m \left\{ \frac{Z_M/A_M}{Z_m/A_m} - \frac{Z_m/A_m}{Z_M/A_M} \right\} \\ \frac{X_L}{R_o} &= 1 + \nu_m Z_m \left\{ \frac{Z_M/A_M}{Z_m/A_m} - 1 \right\} \end{aligned} \qquad (2.17)$$

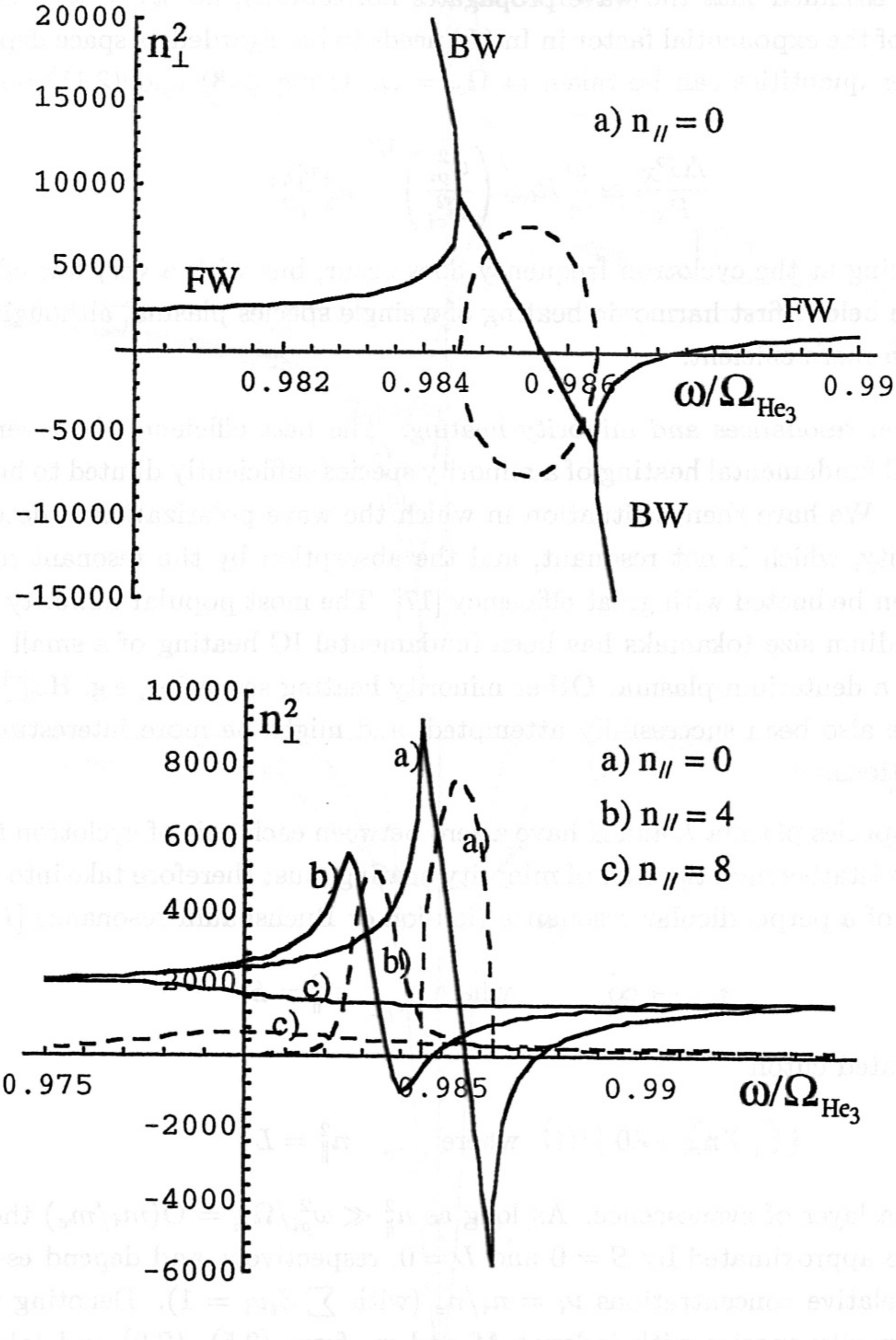

Fig. 5 - He_3^{++} minority in D^+: dispersion relation in the vicinity of the ion-ion hybrid resonance. $n_{He}/n_e = 0.025$, $\omega_{pe}^2/\Omega_{ce}^2 = 0.5$, $\beta_i = \beta_e = 0.01$ (T = 5.1 keV). Full lines: real part; dashed lines: imaginary part.

where X_S, X_L, are the horizontal distances of the ion-ion resonance and cutoff, respectively, from the cyclotron resonance of the minority ions located at the toroidal radius R_o. The wave singularities approach the latter point when the minority concentration tends to zero. Both lie on the high-field side of this layer if the minority is the "lighter" species (i.e. the one with the largest charge to mass ratio), and viceversa. Taking into account that the ratios Z/A are never larger than unity, moreover, one can show that the wave resonance is always to the high-field side of the cut–off. An example of perpendicular index in the equatorial plane of a tokamak near an ion-ion resonance (2.5% He_3^{++} in D^+) is shown in fig. 5.

2.5 – *The Budden model.* When the compressional wave is launched from the outside towards an ion-ion resonance it will be partly reflected from the cutoff and partly transmitted through the evanescence layer. In a plane stratified model in which only the horizontal variation of B is taken into account, the cold-plasma wave equation which describes propagation through the ion-ion layer can be cast into the form [19]

$$\frac{d^2}{dX^2}\left[\left(1-\frac{X_S}{X}\right)E_+\right]+n_{\perp F}^2\left(1-\frac{X_L}{X}\right)E_+=0 \tag{2.18}$$

where $n_{\perp F}^2$ can be taken at $X=0$ (minority cyclotron resonance). A simple change of variable transforms this into a Whittaker equation (a particular case of the confluent hypergeometric). The solutions satisfying the appropriate conditions at large distance ('outward radiation conditions' on the side opposite to incidence) yield the coefficients of transmission and reflection. They can be written

$$\begin{aligned} &R_+=(1-e^{-2\eta_1})^2 \qquad T_+=e^{-2\eta_1} \qquad \text{incidence from the cutoff side}\\ &R_-=0 \qquad\qquad\qquad\; T_+=e^{-2\eta_1} \qquad \text{incidence from the resonance side}\end{aligned} \tag{2.19}$$

where

$$\eta_1=\frac{\pi}{2}n_{\perp F}\frac{\omega}{c}|X_s-X_L| \tag{2.20}$$

is the 'optical thickness' of the evanescence layer. For typical parameters η_1 is of order unity, so that both reflection and transmission depend sensitively of the precise experimental conditions. Note that $R_\pm$ and $T_\pm$ do not add up to unity; the missing power is interpreted as absorbed, with absorption coefficient

$$A_+=e^{-2\eta_1}\left(1-e^{-2\eta_1}\right) \qquad\qquad A_-=1-e^{-2\eta_1} \tag{2.21}$$

Within the present model, absorption at ion-ion resonances (which should not be confused with cyclotron damping) is explained by the fact that the group velocity of the wave tends to zero as X_S is approached; its true origin, namely mode conversion to a

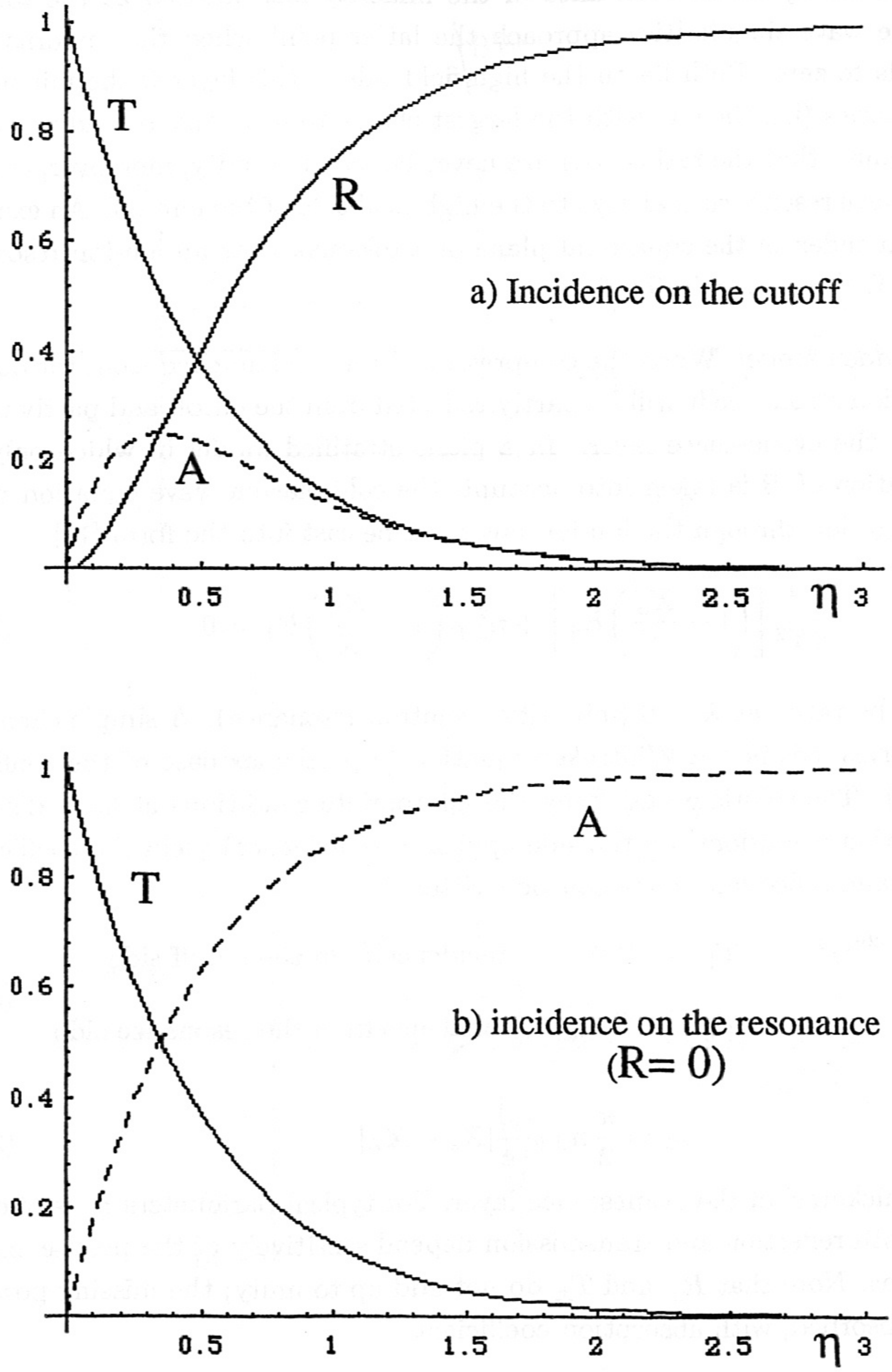

Fig. 6 - Budden reflection, transmission and absorption coefficients from an ion-ion resonance-cutoff layer (perpendicular propagation).

short wavelength electrostatic wave, can be clarified only when finite temperature effect are taken into account. For a wave incident first on the resonance (high-field-side launching) absorption is nearly complete when $\eta_1 \gg 1$; for a wave incident first on the cutoff (low-field launching) it cannot exceed 1/4, the optimum being reached when $\eta_1 = 1/2$ (fig. 6). In applying these results, however, one should be aware that the launched wave can be reflected further from the low-density cutoff $n_\parallel^2 = R$ or from the vacuum vessel, so that a partially standing wave can exist in some region of the plasma; this obviously alters the total power absorbed at the ion-ion resonance. Equations (2.21) give only the so-called 'first transit' absorption.

2.6 – *Minority heating and mode conversion regimes.* In writing Eq. (2.18) we have neglected cyclotron damping by the minority ions. It can be estimated iteratively from Eq. (2.12), including, however, only the minority ions contribution to Im(L), while the solution of the wave equation is used to relate $|E_+|^2$ to the incident flux. In this way it is found that

$$\frac{\Delta P_X}{P_X} \simeq \eta_{cycl} = \frac{1}{1+\epsilon_m}\left(\frac{\omega}{c}R_o\right)\left(\frac{\omega_{pM}^2}{\Omega_{cM}^2}\right)^{1/2}\nu_m \qquad (2.22)$$

with

$$\epsilon_m = \alpha\left(n_\parallel^2\frac{v_{thm}^2}{c^2}\right)^{-1}\nu_m^2 \qquad (2.23)$$

where $\alpha = O(1)$ depends on Z/A of the two ion species. $\Delta P_X/P_X$ is proportional to ν_m at very low concentrations, $\epsilon_m \ll 1$, and inversely proportional to it in the opposite limit, $\epsilon_m \gg 1$. The best cyclotron absorption occurs when ϵ_m is about unity (the peak value of η_{cycl} as evaluated in this way can be comparable or even larger than unity; in this case a better approximation would be $\Delta P_X/P_X \simeq 1 - e^{-\eta_{cycl}}$).

The existence of two regimes can be easily understood by looking again at the dispersion relation and the polarization of the compressional wave in the vicinity of the cyclotron resonance of the minority. If $\epsilon_m \lesssim 1$ the thermal Doppler broadening ΔX_{cycl} of the cyclotron layer is of the same order or larger than the distance to the ion-ion resonance, so that the singularity associated with the latter is smeared out and mode conversion suppressed (cfr. again fig. 5 with $n_\parallel = 8$). It is easily seen, moreover, that under these conditions the minority ions are not sufficient in number to screen E_+: this is the minority heating regime proper. At larger concentrations (or smaller values of $n_\parallel$) the ion-ion resonance is well separated from the layer where the imaginary part of $n_\perp)_F$ is appreciably different from zero, while screening of E_+ by the minority ions begins to reduce cyclotron absorption, so that mode conversion dominates.

Since the values of $n_\parallel^2$ which can be launched are limited by the conditions imposed by the R-cutoff, the optimum ν_m is quite small, typically a few percent. It is not always easy to control the plasma composition to such a precision, since differential absorption and recycling by the walls often play a larger role in determining the relative concentrations than direct gas feeding. One should also be aware that antennas launching the fast wave are always much shorter than the vacuum wavelength, and therefore excite a rather broad spectrum of $n_\parallel$ values (cfr. fig. 8). The condition $\epsilon_m \lesssim 1$, therefore, should strictly speaking be interpreted as determining which portion of the spectrum is absorbed predominantly by cyclotron damping. Even averaged over the power spectrum of real antennas, however, first transit absorption by the minority shows the characteristic dependence on ν_m predicted by Eq. (2.23), and the distinction between the two regimes remains valid.

Finally, we may note that since antennas can be located in practice only on the outer side of the torus, minority heating will be more efficient in the case of 'lighter' minority ions. In the case of an heavier minority the evanescence layer screens the cyclotron resonance from the antenna, so that in Eq. (2.23) P_X must be interpreted as the transmitted flux only. In this configuration, absorption by the minority decreases exponentially with ν_m when the concentration exceeds the optimum value. For lighter minority it decreases only as ν_m^{-1}, so that control of the plasma composition is much less critical. The case of H^+ in D^+ belong to the latter category, but is peculiar, because the minority fundamental coincides with the first harmonic of the majority; to fully understand this situation it is necessary to take into account FLR effects. This will be done in section 2.9 below.

2.7 – *Launching the compressional wave.* The dispersion relation also gives useful insight on the design and performances of IC antennas. Since the oscillating magnetic field of the compressional wave is parallel to the static magnetic field, it must be excited by h.f. currents flowing in the poloidal direction. Thus antennas for IC heating consist of one or more poloidally oriented conductors. In the simplest configuration (fig. 7 a) each conductor has two feeders in push-pull at the two ends, and a central short to ensure symmetry. A more complicated design used in ASDEX Upgrade in Garching is shown in fig. 7 b. The purpose of such complex configurations is to present to the plasma a poloidal current distribution $J_\theta(\theta)$ as uniform as possible, and in any case in the same direction throughout, over a sufficiently large frequency range. If J_θ changes sign, as it does when the electrical length of the antenna exceeds half of the wavelength, its poloidal spectrum is rich in components with large poloidal wavenumber, which excite waves evanescent in vacuum and therefore poorly coupled to the plasma. Non-conventional launchers, in particular using ridged or folded waveguides, have also been proposed [20]–[21]. They have advantages, but since they work only above a minimum size, they can hardly be tested in small tokamaks, and are therefore still in the development phase. Finally we may mention that most IC antennas today have lateral protecting limiters and a Faraday screen. The latter consists of a more or less densely packed array of toroidally oriented metallic stubs, shorted on the lateral limiter or to the wall, whose purpose is to optically isolate the main conductors from the plasma, and to short stray fields parallel to the static magnetic field, which could lead to parasitic absorption in the low density edge plasma where they are still not completely screened by the electrons. At the same time they are essentially transparent to the radial and poloidal components of the electric field, which are those which excite the compressional wave in the plasma. Cooling the Faraday screen adequately in a reactor, however, would be exceedingly difficult; IC antennas without screen have been recently successfully tested.

Antennas with two or more conductors which can be excited with a controllable phase shift relative to each other allow shaping the launched power spectrum (repartition of the power between partial waves with different $n_\parallel$) in order, for example, to optimize minority heating, or to have a sufficient directivity for current drive. It has become customary, although strictly speaking incorrect, to call a configuration exciting a symmetric spectrum peaked at $n_\parallel = 0$ a 'monopole' antenna, one exciting an antisymmetric spectrum with no power in the $n_\parallel = 0$ component a 'dipole' antenna (fig. 8).

Spectral shaping is also useful to reduce the excitation of Fourier components with $n_\parallel^2 \lesssim 1$. These components can excite surface modes guided between the wall of the vacuum vessel and the plasma surface. Because of the long parallel wavelength of these modes, the associated parallel electric field, although weak, can correspond to potential drops of hundreds of Volts along open field lines terminating on the wall or on metallic limiters at some distance from the antenna. This is made particularly dangerous for the production of impurities by a nonlinear mechanism known as h.f. sheath rectification [22]. Electrons in the scrape-off plasma screen E_z along the open field line, but cannot short out the inductively excited potential. Most of the potential drop, therefore, occurs in a non-neutral sheath near the metallic obstacle, whose thickness is of the order of the Debye length, or, if the field line is incident at grazing angle, of the ion Larmor radius. Since the transit time of the ions in this sheath is shorter than the wave period, this mechanism allows the acceleration of escaping ions through the full potential drop. The energy of ions hitting the wall is then sufficient to extract impurities from the wall with high efficiency [23]-[24].

The power spectrum of an antenna can be qualitatively predicted by an equation of the form

$$P_X(n_\parallel) = P_o \, |J_a(n_\parallel)|^2 \exp\left(-2 \int_a^{co} |n_\perp)_F| \, \mathrm{d}X\right) \tag{2.24}$$

The toroidal Fourier spectrum $J_a(n_\parallel)$ of the antenna current, in turn, is the product of a factor describing each conductor, and a factor which takes into account the number and relative phase of conductors. The former is always very broad because the width d of

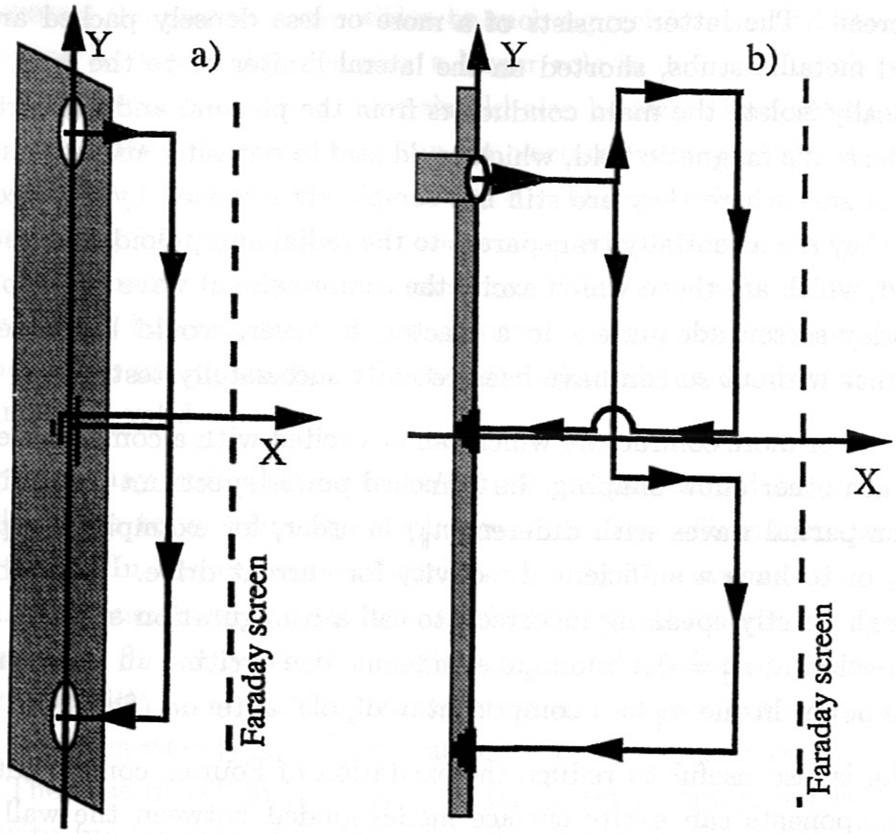

Fig. 7 - Schematic view of two IC antennas: a) push-pull antenna; b) the antenna of the ASDEX-Upgrade IC experiment.

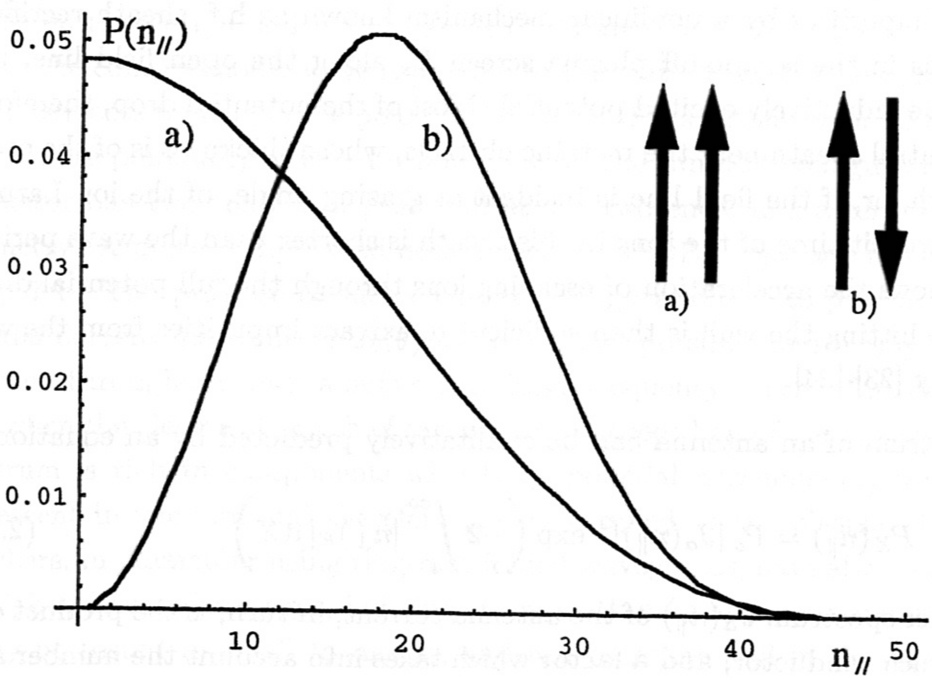

Fig. 8 - Typical nominal antenna power spectra for the symmetric (a) and antisymmetric (b) configurations.

each conductor is a small fraction of the vacuum wavelength; the second modulates the spectrum into separate 'lines' whose width is inversely proportional to the number of conductors, and whose position depends on the phases. The exponential factor in (2.24) takes into account the evanescence from the antenna to the R-cutoff. As already remarked, its presence puts an upper limit on the values of $n_\parallel^2$ which can be efficiently launched, in order of magnitude $n_\parallel^2 \lesssim$ a fraction of $(\omega_{pi}^2/\Omega_{ci}^2) = O(m_i/m_e)$. This restricts the flexibility of spectral shaping. A more severe limitation is the fact that $n_\parallel$ does not remain constant during propagation in toroidal geometry (section (2.15)).

It should also be remarked that Eq. (2.24) is a good approximation only if absorption is sufficiently strong to rule out the existence of high-quality cavity eigenmodes of the plasma-filled vessel. If this is not the case, the values of $k_\parallel$ which are closer to such eigenmodes are preferentially excited, and the actual power spectrum will consist of well separated 'lines', and thus differ appreciably from (2.24). The position of the eigenmodes can be easily estimated in simplified geometry (slab or cylindrical models of the plasma). Their 'exact' position in the spectrum and the amount of broadening due to damping, however, can be predicted accurately only by solving the wave equation in realistic geometry. It is clear, in any case, that global eigenmodes play a more important role in small than in large device: both their separation and their quality factor (sharpness) are likely to decrease as the plasma dimensions and performances increase.

As an element of the h.f. system, the antenna is fully characterized by its complex imput impedance Z_a. The imaginary, or reactive part of Z_a can be estimated by considering the conductor as a piece of transmission line, with appropriate boundary conditions at feeders and shorts. The resistive part of Z_a, on the other hand, can be evaluated accurately only by solving the wave equations in the plasma. Because of time limitations, we cannot discuss these calculations; details and further references can be found in [25]. It will suffice here to mention that, due to the fact that most of the power is radiated in Fourier components with $n_\parallel^2 > 1$ and must tunnel through the evanescence layer between the antenna and the R-cutoff, the load is rather sensitive to the density profile near the plasma edge. To get a radiation resistance of the order of at least a few Ohms, as required for a stable matching of the whole system, the main conductors cannot be located too far from the plasma: this is perhaps the most difficult constraint to be satisfied to implement ICR heating or FW current drive in the reactor. Antisymmetric or current-drive antennas, moreover, are more demanding than symmetric ones in this respect. Abrupt changes in the edge profile, such as those which occur at the L-to-H confinement transition, or even during short periodic bursts of losess (ELM's) in the H-regime, can also influence the loading resistance to the extent that feedback matching might be required to protect the amplifiers from excessive reflection.

2.8 – *The FLR dispersion relation.* To complete our understanding of the dispersion relation in the IC range of frequencies it is important to exhamine a few effects which were previously omitted, namely ion FLR effects, finite electron inertia, and finite electron temperature. Inclusion of finite electron inertia makes $H(\omega, \vec{k})$ a second order polynomial in $n_\perp^2$. Ion FLR effects introduce perpendicular dispersion, i.e. make ϵ_{ij} depend on $k_\perp^2$. To the lowest significant order in the Larmor radius, the dispersion becomes then a third order polynomial in $n_\perp^2$. In other words, each effect adds a new wave: electron inertia brings in the shear Alfvén wave, FLR effects a pressure-driven wave.

From the form of the vacuum term in Eq. (2.3) it is not difficult to see that the FLR contribution responsible for the pressure-driven wave is the one of the ions to ϵ_{xx}. Writing

$$\epsilon_{xx} = S - \sigma_2 n_\perp^2 \qquad \sigma_2 = \frac{1}{2}(\lambda_2 + \rho_2) \tag{2.25}$$

where λ_2 and ρ_2 are the FLR corrections to L and R, respectively, we obtain the FLR dispersion relation in the form

$$\begin{aligned} 0 = &-\sigma_2 n_\perp^6 + (S + P\sigma_2) n_\perp^4 \\ &+P\left\{ \left[(n_\parallel^2 - S) + (n_\parallel^2 - R)\lambda_2 + (n_\parallel^2 - L)\rho_2 \right] n_\perp^2 + (n_\parallel^2 - R)(n_\parallel^2 - L) \right\} \end{aligned} \tag{2.26}$$

(FLR terms must be retained also in the coefficients of $n_\perp^4$ and $n_\perp^2$ because P is so large). The coefficient σ_2 is easily obtained from the hot plasma dielectric tensor by expanding in the Larmor radius, and contains in principle a term resonant at the fundamental and one resonant at the first harmonic. The former is always a small correction to the zero Larmor radius part S, which also resonant at the fundamental. We therefore keep only the latter,

$$\lambda_2 = \frac{1}{2}\sum_\alpha \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2}\frac{v_{th\alpha}^2}{c^2}\left(-x_o^\alpha\, Z(x_2^\alpha)\right) \qquad \rho_2 = \frac{1}{2}\sum_\alpha \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2}\frac{v_{th\alpha}^2}{c^2}\frac{\omega}{\omega + 2\Omega_{c\alpha}} \tag{2.27}$$

which is very large near $\omega = 2\Omega_{ci}$ where S is regular. It turns out that this is not only a convenient simplification, but actually gives a much better agreement of Eq. (2.26) with the full hot plasma dispersion relation near the fundamental cyclotron resonance (the failure of the complete FLR expansion near $\omega = \Omega_{ci}$ is due to the fact all roots of the disperion relation except the FW have a wavelength much shorter than the ion Larmor radius in this frequency domain).

As long as the roots of (2.26) are well separated, the smallest one is just the FW (2.6), with a small correction due to the finite pressure. Near the Alfvén resonance (2.9), however, this root is no more divergent, but has a confluence with a short-wavelength wave. The nature of the new root depends on the value of the ratio $|P\sigma_2/S| = \beta_i \cdot (m_e/m_i) \cdot f(\omega/\Omega_{ci})$, where f is a numerical factor of order unity (but much larger than unity near the first cyclotron harmonics), and $\beta_i = (\omega_{pi}^2/\Omega_{ci}^2)(v_{thi}^2/c^2)$ the normalized ion pressure.

a) If the plasma is very tenuous and cold,

$$\beta_i \ll m_e/m_i \tag{2.28}$$

the confluence occurs with a wave whose dispersion relation can be written

$$n_\perp^2 = n_\perp^2)_S = -(n_\parallel^2 - S)\frac{P}{S} \tag{2.29}$$

If $\omega/k_\parallel \gg v_{the}$ this is the cold-plasma shear Alfvén wave (SAW), and is propagative on the low magnetic field side of the confluence; in the opposite limit it is called 'kinetic Alfvén wave' (KAW), and is propagative on the high magnetic field side. The third root does not satisfy the condition $k_\perp^2 v_{thi}^2/\Omega_{ci}^2 \ll 1$, and must be discarded.

b) In a typical fusion plasma, however, the opposite condition usually prevails, namely

$$\beta_i \gtrsim m_e/m_i \tag{2.30}$$

In this case, the confluence occurs with the pressure-driven root, whose dispersion relation is

$$n_\perp^2 = n_\perp^2)_B \simeq -\frac{n_\parallel^2 - S}{\sigma_2} \tag{2.31}$$

This wave is the first member of the family of electrostatic waves which exist near harmonics of the ion cyclotron frequency, known as Ion Bernstein waves (IBW) [26]; it is always propagative on the high magnetic field side of the resonance. In this case SAW or KAW still exist as solutions of the hot plasma dispersion relation, but do not play any role near the Alfvén resonance. In other words, condition (2.30) justifies the zero electron inertia approximation, $|P| \to \infty$, which simplifies (2.26) to

$$0 = \sigma_2 n_\perp^4 + \left[(n_\parallel^2 - S) + (n_\parallel^2 - R)\lambda_2 + (n_\parallel^2 - L)\rho_2)\right] n_\perp^2 + (n_\parallel^2 - R)(n_\parallel^2 - L) \tag{2.32}$$

In the following we will consider mainly situations where this approximation is valid.

2.9 – *First harmonic heating in a single species plasma.* In the limit of perpendicular propagation

$$\lambda_2 \to \frac{\beta_i}{2}\frac{\omega}{\omega - 2\Omega_{ci}} = \frac{\hat{\lambda}_2}{X} \qquad \hat{\lambda}_2 = \frac{\beta_i}{2}\frac{\omega}{c}R_{tor} \tag{2.33}$$

diverges at $\omega = 2\Omega_{ci}$. Hence ion FLR effects are always important near the first cyclotron harmonic, although this could not have been predicted from (2.6) alone. The dispersion curves (fig. 9) show a double confluence between the FW and the IBW, separated by a 'frequency gap' in which the two roots of (2.32) are complex conjugate, and the two waves evanescent. The confluences are easily located analytically by searching for the roots of the discriminant of (2.32):

$$\left.\frac{2\Omega_{ci} - \omega}{\omega}\right)_{conf} = \beta_i \frac{h_F^2}{1 \pm \sqrt{h_F^2}} \qquad h_F^2 = -\frac{n_\parallel^2 - R}{n_\parallel^2 - L} > 0 \tag{2.34}$$

($h_F^2 \simeq 1/3$ in a deuterium plasma). They both lie on the high-field side of the resonance; the optical thickness of the evanescence layer can be estimated as

$$\eta_2 = \frac{\pi}{8}\beta_i\, n_\perp)_F \frac{\omega}{c}R_{tor} \tag{2.35}$$

with $n_\perp)_F$ taken at the resonance in the cold limit.

To evaluate reflection and transmission through the resonance layer the wave equation must be solved. Because the FLR terms in the dispersion relation depend on the wavevector, however, in a nonuniform configuration they become second order differential operators, whose form is not uniquely determined by the dispersion relation alone. It turns out [27] that the correct FLR wave equation in this case is

$$0 = \left(\frac{1}{2}\frac{d^2}{dX^2} + R\right)\left[\frac{d}{dX}\left(\frac{\hat{\lambda}_2}{X}\frac{dE_+}{dX}\right)\right] + \frac{d^2}{dX^2}(S\,E_+) - R\,L\,E_+ \tag{2.36}$$

where $R = -S = (2/3)(\omega_{pi}^2/\Omega_{ci}^2)$, $L = -2(\omega_{pi}^2/\Omega_{ci}^2)$, and $\hat{\lambda}_2$ can be regarded as constants. The solutions of this self-adjoint equation can be written in the form of Laplace integrals [28], and from their asymptotic behaviour for large X and the radiation conditions the reflection and transmission coefficients can be determined. The result is identical with that of the Budden model, Eq. (2.19), with, however, η_2 replacing η_1. The 'absorbed' power in this case appears as power transported away from the resonance region by the Berstein wave, a phenomenon known as (linear) mode conversion. The IBW can be absorbed by electron Landau damping if its parallel phase velocity is sufficiently small; alternatively, it can be absorbed by stochastic acceleration of the ions, which can occur when the wavelength becomes shorter than the ion Larmor radius [29].

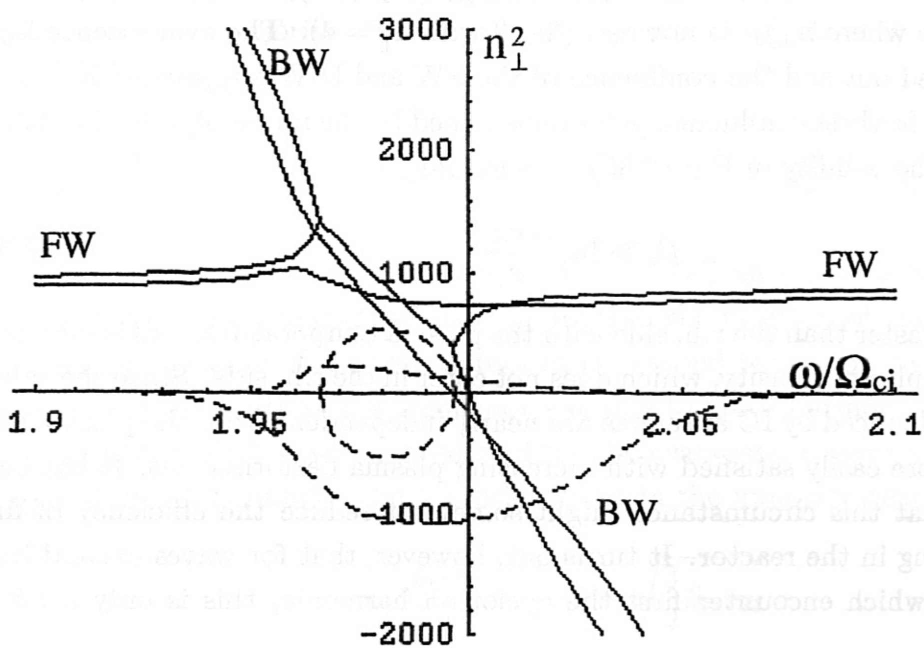

Fig. 9 - Dispersion curves near the first ion cyclotron harmonic for perpendicular ($n_{//} = 0$) and oblique ($n_{//} = 4$) propagation. $\omega_{pe}^2 / \Omega_{ce}^2 = 0.5$, $\beta = 0.02$ ($T_e = T_i = 5.11$ keV), hydrogen plasma.

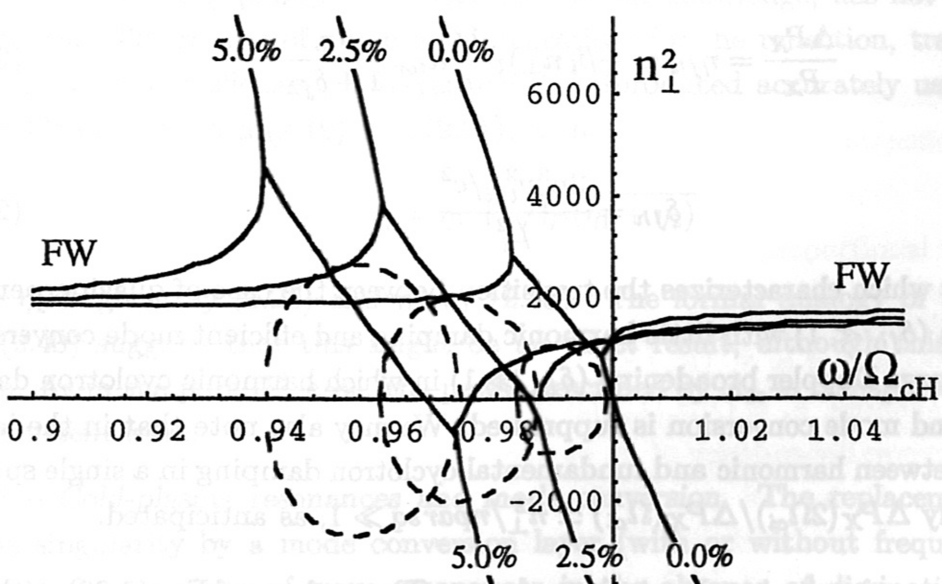

Fig.10 - Hydrogen minority in Deuterium plasma, $\beta = 0.02$, $\omega_{pe}^2/\Omega_{ce}^2 = 0.5$, $T_e = T_i = 5.11$ keV. Hot-plasma dispersion curves near the ion-ion resonance, perpendicular propagation.

If $n_\parallel$ is not zero, there is a layer of witdth ΔX_{cycl} (Eq. (2.3)) around the cyclotron harmonic resonance where $n_\perp)_F$ is not real (fig. 9 with $n_\parallel = 4$). The evanescence layer is completely washed out and the confluence of the FW and IBW suppressed if ΔX_{cycl} extends beyond the farthest confluence point determined by the minus sign in Eq. (2.34). The condition for the validity of Eq. (2.36) is therefore

$$\beta_i \gg |n_\parallel| \frac{v_{thi}}{c} \tag{2.37}$$

The l.h. side grows faster than the r.h. side with the plasma temperature, and is moreover proportional to the plasma density, which does not enter in the r.h. side. Since the values of $n_\parallel$ which can be launced by IC antennas are nearly independent from the plasma size, this condition is more easily satisfied with increasing plasma performances. It has been pointed out [30] that this circumstance might somewhat reduce the efficiency of first harmonic ion heating in the reactor. It turns out, however, that for waves excited from the low field side, which encounter first the cyclotron harmonic, this is only a minor effect.

The amount of harmonic cyclotron damping suffered by the FW wave itself while transiting through the first harmonic resonance can be easily estimated in the two limits of negligible and very large Doppler broadening, using once more an equation similar to (2.12), but with $\mathrm{Im}(\lambda_2)$ on the r.h. side. An equation which interpolates well between these two extremes is [31]

$$\frac{\Delta P_X}{P_X} = \eta_{fh} \simeq \frac{\pi}{4} \beta_i \, n_\perp)_F \frac{\omega}{c} R_{tor} \frac{\delta_{fh}}{1+\delta_{fh}} \tag{2.38}$$

where

$$\delta_{fh} = \frac{2 n_\parallel^2 v_{thi}^2 / c^2}{\beta_i^2} \tag{2.39}$$

is the parameter which characterizes the transition between the case of quasiperpendicular propagation ($\delta_{fh} \ll 1$) with little harmonic damping and efficient mode conversion, and the case of large Doppler broadening ($\delta_{fh} \gg 1$) in which harmonic cyclotron damping dominates and mode conversion is suppressed. We may also note that in the latter limit the ratio between harmonic and fundamental cyclotron damping in a single species plasma is roughly $\Delta P_X(2\Omega_{ci})/\Delta P_X(\Omega_{ci}) \simeq n_\perp^2/nparsq \gg 1$, as anticipated.

2.10 – *Hydrogen minority in a Deuterium plasma.* The analogy of Eq. (2.38) with the two heating regimes in a two-species plasma will have been noted. Indeed, as illustrated by the dispersion curves of fig. 10, minority heating of hydrogen in deuterium goes over continously into the pure D^+ case as the H^+ concentration tends to zero. The dispersion relation for this scenario for nearly perpendicular propagation can be discussed as in the previous case, taking into account the singularities of both L due to the minority, and σ_2 due to the majority. The two confluences between the FW and the IBW are located where

$$\left.\frac{\Omega_{cH}-\omega}{\omega}\right)_{conf} = -\frac{3}{2}\nu_H - \beta_D \pm \sqrt{\frac{3}{4}\beta_D\left(\beta_D + 2\nu_H\right)} \simeq -\frac{3}{2}\nu_H \pm \sqrt{\frac{3}{2}\beta_D\nu_H} \qquad (2.40)$$

the last approximation being valid if ν_H, although small, satisfies $\nu_H \gg \beta_D$. In this limit the farthest confluence asymptotically approaches the position of the cold-plasma ion-ion resonance. Because of the square root in the second term, however, at low minority concentrations the width of the frequency gap due to finite Larmor radius effects easily exceeds substantially the width predicted for the evanescence layer by the cold plasma approximation. Accordingly, the condition to be in the minority heating regime can be written

$$|n_\parallel|\frac{v_{thH}}{c} \gtrsim \frac{3}{2}\nu_H + \sqrt{\frac{3}{2}\beta_D\nu_H} \qquad (2.41)$$

We can add that in this regime the minority is preferentially heated as soon as $\nu_H \gtrsim \beta_D$, and second harmonic heating of deuterium correspondingly reduced compared to the pure D^+ case.

The wave equation in the mode-conversion regime is a fourth-order differential equation (since it must describe both the FW and the IBW) which combines the singularities of Eq. (2.18) and Eq. (2.36). This equation, to our knowledge, has not been solved in closed form. The results of numerical integrations for the reflection, transmission and mode-conversion coefficients, however, can be reproduced accurately using once again the Budden expressions (2.19) and (2.21), with

$$\eta = \eta_1 + \sqrt{\eta_2(\eta_1 + \eta_2)} \qquad (2.42)$$

where η_1 is given by (2.20) and η_2 by (2.35). The formal analogy of this expression with (2.40) suggests that this might be an exact result, although this has not been proven. As in the pure D^+ case, the power described by $A_\pm$ is transported away by the ion Bernstein wave.

2.11 – *Cold-plasma resonances and mode conversion.* The replacement of a cold-plasma singularity by a mode conversion layer (with or without frequency gap) is a universal characteristic of wave resonances in the absence of dissipation when finite pressure effects are taken into account. Thus for nearly perpendicular propagation the confluence of the FW with the pressure-driven wave (or with the shear Alfvén wave in very low β plasmas) occurs also in the scenarios where the minority resonance does not coincide with the first harmonic of the majority. In this case, however, σ_2 is much smaller and the wavelength of the IBW correspondingly shorter, so that the confluence layer is very narrow. In this case any residual damping (ion cyclotron damping of the minority in the tail of the Doppler broadened cyclotron resonance, electron Landau damping, or even collisional absorption) is sufficient to eliminate the confluence; the cold-plasma description of the wave resonance is then perfectly adequate, as the dispersion curves in fig. 5 for oblique propagation clearly illustrate.

2.12 – *Electron heating.* In hot tokamak plasmas damping of the FW on the electrons always competes with ion heating. One can also purportedly avoid damping on the ions when the FW is used for current drive. To estimate the importance of electron damping one must in the first place take into account parallel dispersion in ϵ_{zz}, rewriting the last line of Eq. (2.5) as

$$\epsilon_{zz} \simeq -\frac{\omega_{pe}^2}{\omega^2}\, x_e^2\, Z'(x_e) \qquad\qquad x_e = \frac{\omega}{k_\parallel v_{the}} \tag{2.43}$$

In addition, as pointed out by Stix [2], it is necessary to take into account the FLR corrections contributed by the electrons to $\epsilon_{yy} \simeq S - n_\perp^2(\sigma_2 + 2\tau_e)$ and $\epsilon_{yz} = -\epsilon_{zy} \simeq i n_\perp n_\parallel \xi_e$. Expanding the hot plasma dielectric tensor to second order in the electron Larmor radius, and keeping only the leading terms in ω/Ω_{ce}, one finds

$$\tau_e = \frac{1}{2}\frac{\omega_{pe}^2}{\Omega_{ce}^2}\frac{v_{the}^2}{c^2}\{-x_e\, Z(x_e)\} \qquad\qquad \xi_e = \frac{1}{2}\frac{\omega_{pe}^2}{\omega\Omega_{ce}}\frac{v_{the}^2}{c^2}\, x_e^2\, Z'(x_e) \tag{2.44}$$

Although not contributing significantly to the FLR dispersion relation, τ_e and ξ_e are required to determine correctly the parallel component of the FW electric field and P_{abs}^e. From the third line of (2.1) one gets

$$E_z = \frac{n_\perp n_\parallel}{n_\perp^2 - P}\{E_x - i\xi_e E_y\} \tag{2.45}$$

The refractive index of the fast wave satisfies always $n_\perp^2 \ll |P|$; moreover, except near the Alfvén resonance $n_\parallel^2 = S$, $|E_x/E_y| = O(\omega/\Omega_{ci})$ is of order unity or smaller. If $\beta_e \ll m_e/m_i$, therefore, $\xi_e = O(\beta_e\Omega_{ce}/\omega)$ is negligible, and this equation predicts the characteristic inverse dependency of E_z on the density due to screening by the electrons in a cold plasma. Under the more typical condition $\beta_e \gtrsim m_e/m_i$, on the other hand, the first term within the bracket in Eq. (2.45) is negligible, and one obtains

$$E_z =\simeq i n_\perp n_\parallel \frac{\omega}{\Omega_{ce}}\frac{v_{the}^2}{c^2} E_y \tag{2.46}$$

In this regime E_z is independent from the plasma density, and proportional instead to the plasma temperature. This residual parallel field is required to maintain charge neutrality. Note that ξ_e/P is a real quantity independent from the parallel phase velocity of the wave; Eq. (2.46), therefore, is valid even if $|x_{oe}| \lesssim 1$, i.e. when damping by the electron is very strong.

According to Eq. (2.3), the density of power absorption by the electrons can be written

$$P^e_{abs} = \frac{\omega}{8\pi}\Big\{ -2n_\perp^2 \,\mathrm{Im}\,(\tau_e)\,|E_y|^2 + 2n_\perp n_\parallel \,\mathrm{Im}\,(\xi_e)\,\mathrm{Im}\,(E_y^* E_z) + \mathrm{Im}\,(P)\,|E_z|^2 \Big\} \qquad (2.47)$$

The last term is the well-known electron Landau damping (ELD), due to the secular acceleration of the electrons along the static magnetic field by the parallel component of the wave electric field when the Cerenkof resonance condition $\omega = k_\parallel v_\parallel$ is satisfied. The first term is known as Transit Time Magnetic Pumping (TTMP, a name with only historical justification); it is similar in nature to Landau damping, but is driven by the $\vec{\mu}\cdot\vec{\nabla} B$ force due to the compressional magnetic field of the wave, where $\vec{\mu}$ is the magnetic moment of the gyrating electron. TTMP is therefore a finite Larmor radius effect, as the form of τ_e immediately makes clear. The second is known as 'mixed' term. In the high-beta regime the three terms are of comparable magnitude, i.e. Landau damping and Transit Time cannot be considered separately. Inserting (2.46) into (2.47) one obtains

$$P^e_{abs} = \frac{\omega}{8\pi} n_\perp^2 \,\beta_e\, \sqrt{\pi}\, x_e e^{-x_e^2} |E_y|^2 = \frac{\omega}{8\pi} \sqrt{\pi}\, x_e^3\, e^{-x_e^2} |E_z|^2 \qquad (2.48)$$

The first expression is half magnetic pumping alone (i.e. evaluated as if E_z where exactly zero); the second is just half the Landau damping which would correspond to the electric field E_z alone.

Damping of the compressional wave on the electrons is proportional to $n_e T_e$. Thus, although not 'localized' in the same sense as cyclotron damping, it is strongest in the plasma core. It also increases with increasing plasma performances. In medium-size tokamaks it is rather inefficient until the central temperature reaches a few keV. In the reactor, however, easily half or more of the launched power will be absorbed by the electrons, even when the conditions for IC damping are optimized.

We may finally notice that since the electrons satisfying the Cerenkoff resonance condition are accelerated along the static magnetic field, ELD and TTMP are not very sensitive to toroidal trapping. Neither $k_\parallel$ nor $v_\parallel$, however, are constant when toroidicity is taken into account. It follows that an electron can satisfy the Cerenkoff condition only over a finite segment of its orbits. The conseqences of this when only one wave is present and collisions are neglected have been explored in [32]. The effects of the broad

$k_\parallel$-spectrum typical of FW propagation in tokamaks and of collisions or perturbations by low-frequency fluctuations are however likely to dominate. The electron distribution function is then determined by a quasilinear equation similar (except for the inclusions of FLR effects in TTMP and in the mixed term) to the equation used to describe h.f. current drive in the Lower Hybrid frequency domain [33].

2.13 – *Ion Bernstein Wave heating.* A review of IC heating would not be complete without mentioning that Ion Bernstein waves have also been proposed for auxiliary heating of tokamak plasmas. For details and references, we must refer the reader to a dedicated review [34]. Here it will suffice to note that since IBWs are electrostatically polarized and have a very large perpendicular index, their main electric field is along the direction of propagation. Such a polarization can hardly be excited directly from outside the plasma. Instead, an antenna with current oriented parallel to the static magnetic field can be used to launch first the slow cold-plasma wave (2.29), which then converts into a IBW a few centimeters inside the plasma. Good loading of the antenna is possible only if the density near the antenna is sufficiently low to allow propagation of the SW in the plasma scrape-off, and the first cyclotron harmonic resonance of the ions is located just outside the plasma, so that the perpendicular index of the IBW in the conversion layer is not excessively large [35].

Since the excitation of the SW occurs mainly through E_z, which in turn is proportional to $n_\parallel$ (cfr. Eq.(2.45)), launching with an antisymmetrically phased array of waveguides mounted flush on the vacuum vessel with their long side in the poloidal direction is likely to be preferable [36]. Waveguide arrays can be used also to launch higher-order IBWs. The principle of such arrays is identical to that of the 'grill' which is used to launch the slow wave in the Lower Hybrid domain [37]; the main restriction to their use in the IC range of frequencies is access, since the lower frequency implies larger guide dimensions.

Once inside the plasma, IBWs can be absorbed by electron Landau damping if their parallel phase velocity is slow enough, or by cyclotron harmonic damping as they cross the next harmonic resonance in the poloidal cross-section. Nonlinear heating mechanisms are also possible [29].

The use of IBWs for auxiliary heating in large tokamaks, however, is likely to pose serious problems. Being electrostatic waves, their perpendicular group velocity is very slow, in fact comparable to the ion thermal velocity. This implies that to transport a given power flux across magnetic surfaces the electric field amplitude must be much larger than in the case of the compressional wave, whose perpendicular group velocity is of the order of the Alfvén speed. The power flux, moreover, is not associated with the Poynting vector (which vanishes for an electrostatic wave), but with a flow of coherent oscillation energy of the ions. Finally, due to the very short wavelength, the optical path from the edge to the absorption region is very large. As a consequence of all this, any linear or nonlinear parasitic absorption mechanism, even quite weak, might easily compromise the penetration of IBWs into large plasmas.

2.14 – *FLR wave equations for IC waves in tokamaks.* The elementary considerations presented to this point give a sufficiently complete coverage of the physics of IC heating, so that it has been possible to write a simple subroutine based on the equations of this overview and allowing a rapid evaluation of ICR heating scenarios [38]. It is clear, however, that quantitative results on antenna loading, power deposition profiles, etc., require the solution of appropriate wave equations. The limited space available does not allow to present details on how such equations are derived and solved; we will limit ourselves to mention the main principles, referring to the literature for further information.

The constitutive relation (i.e. the relation between the h.f. electric field and the h.f. current) of a hot, inhomogeneous plasma can be obtained by integrating the linearized Vlasov equation along the 'unperturbed' orbits of charged particles in the static magnetic field. This leads to a non-local relation, since particles contributing to $\vec{J}_{hf}$ at a given time and point in space, but having different unperturbed velocities there, have seen different h.f. fields along their previous trajectories (this property of hot plasmas is known as 'spatial dispersion'; in the uniform limit it implies that the dielectric tensor $\underline{\underline{\epsilon}}$ depends on the wavevector $\vec{k}$). Once $\vec{J}_{hf}$ is known, the wave equations are easily obtained by writing Maxwell's equations

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \frac{\omega^2}{c^2} \left\{ \vec{E} + \frac{4\pi i}{\omega} \vec{J}_{hf} \right\} \tag{2.49}$$

in a coordinate system appropriate to the geometry of the confined plasma. Because of space dispersion, these equations in a hot plasma are strictly speaking integro-differential equations [39].

In the direction perpendicular to $\vec{B}_o$, however, one can take advantage of the fact that in the IC frequency domain the wavelength of the FW, and in the most interesting regions that of the IBW as well, is large compared to the ion Larmor radius. A Taylor expansion around the ion guiding centers then puts the wave equations in differential form in this direction. This way of deriving the FLR wave equations has been followed first in [40] and [27] assuming for simplicity a plane-stratified geometry; for details and further references cfr. [41]. In the approximation corresponding to the FLR dispersion relation (2.32) it is sufficient to evaluate in this way the ion FLR current $\vec{J}_i$ in response to the perpendicular field component $\vec{E}_\perp$. Separating zero and first order part in the Larmor radius, it can be written in rotating components

$$\vec{J}_i = \vec{J}_i^{(0)} + \vec{J}_i^{(2)} \tag{2.50}$$

The zero Larmor radius part is

$$\frac{4\pi i}{\omega}\vec{J}_i^{(0)} = \hat{L}\,E_+\vec{u}_+ + R\,E_-\vec{u}_- \tag{2.51}$$

where we have introduced the integral operator

$$\hat{L}E_+ = -\sum_i \int_{-\infty}^{+\infty} du\,\frac{e^{-u^2}}{\sqrt{\pi}}\left(-i\omega\int_{-\infty}^{t} dt'\,e^{i\int_{t'}^{t}(\omega-\Omega_g')\,d\tau}E_+'\right) \tag{2.51}$$

(R, being non-resonant, can be approximated by the local limit (2.5)). The FLR part is found to be

$$\begin{aligned}\frac{4\pi i}{\omega}\vec{J}_i^{(2)} = \frac{c^2}{\omega^2}\underline{\underline{R}}\cdot\Bigg\{&\vec{\nabla}_\perp\Big[\sigma^{(2)}\,\vec{\nabla}_\perp\cdot(\underline{\underline{R}}\cdot\vec{E}_\perp) - i\delta^{(2)}\,\vec{\nabla}_\perp\cdot\left(\vec{u}_\zeta\times\underline{\underline{R}}\cdot\vec{E}_\perp\right)\Big]\\ &+\left(\vec{u}_\zeta\times\vec{\nabla}_\perp\right)\Big[\sigma^{(2)}\,\vec{\nabla}_\perp\cdot\left(\vec{u}_\zeta\times\underline{\underline{R}}\cdot\vec{E}_\perp\right) + i\delta^{(2)}\,\vec{\nabla}_\perp\cdot(\underline{\underline{R}}\cdot\vec{E}_\perp)\Big]\Bigg\}\end{aligned} \tag{2.52}$$

where

$$\hat{\sigma}_2 = \frac{\rho_2+\hat{\lambda}_2}{2} \qquad\qquad \hat{\delta}_2 = \frac{\rho_2-\hat{\lambda}_2}{2} \tag{2.53}$$

with

$$\hat{\lambda}^{(2)}\,E_+ = \frac{1}{2}\sum_i \frac{\omega_{pi}^2}{\Omega_{ci}^2}\frac{v_{thi}^2}{c^2}\int_{-\infty}^{+\infty} du\,\frac{e^{-u^2}}{\sqrt{\pi}}\left(-i\omega\int_{-\infty}^{t} dt'\,e^{i\int_{t'}^{t}(\omega-2\Omega_g')\,d\tau}E_+'\right) \tag{2.54}$$

and ρ_2 given by (2.27). In (2.52) $\vec{u}_\zeta$ is a unit vector along the static magnetic field, and the matrix $\underline{\underline{R}} = \underline{\underline{R}}^{-1}$ is the reflection matrix with respect to the plane containing $\vec{B}_o$ and $\vec{\nabla}B_o$:

$$\underline{\underline{R}}\cdot\vec{E}_\perp = \vec{E}_\perp - 2\,\vec{u}_g\times\left(\vec{E}_\perp\times\vec{u}_g\right) \tag{2.55}$$

where $\vec{u}_g$ is a unit vector in the direction of the perpendicular part of the gradient of the static magnetic field.

If toroidal curvature is neglected, i.e. if the tokamak is assimilated to a plasma 'slab', the h.f. field can be assumed to be a plane wave with a constant $k_\parallel$, and the integral over the parallel motion and over the distribution of parallel velocities can be performed, leading to the Plasma Dispersion function as in the homogeneous case; the FLR wave equations are then purely differential [41]. This approximation is sufficient, in particular, to develop quantitative models of the antenna coupling [25]. The only new feature with respect to the cold limit is that the presence of a pressure-driven wave in the plasma requires additional boundary conditions to be imposed at the vacuum-plasma interface; the nature and physical meaning of these conditions are discussed in [42].

2.15 – *Toroidal effects on wave propagation.* Meaningful power deposition profiles, on the other hand, can be obtained only by solving the wave equations in fully toroidal geometry. Being in vector form, Eqs. (2.51)–(2.55) can be easily written explicitly also in this case (for an alternative derivation directly in toroidal geometry which, however, ignores parallel dispersion, cfr. [43]). Due to parallel dispersion, they remain, however, an integral, non-local relation along magnetic field lines.

A discussion of the time and parallel velocity integrals in Eqs. (2.52) and (2.53) which takes into account the effects of toroidicity on the ion motion discussed in section 1.9 can be found in [44]. For most purposes a simpler approximation, equivalent to the one made to obtain the quasilinear diffusion coefficient (1.12), is sufficient. It is convenient to expand the wave field into toroidal and poloidal Fourier components,

$$\vec{E} = e^{in\varphi} \sum_{m=-\infty}^{+\infty} \vec{E}^m(n;r)\, e^{im\vartheta} \qquad (2.56)$$

Because of axisymmetry there is no coupling between different n components, but due to the poloidal variation of the coefficients of the wave equation a large number of m modes are strongly coupled to each other. To each value of m, however, corresponds locally the parallel wavenumber

$$k_{\parallel}(n,m) = \frac{n}{R_{tor}} \cos\Theta + \frac{m}{r} \sin\Theta \qquad (2.57)$$

where $\tan\Theta = B_{pol}/B_{tor}$ is the ratio of the poloidal to the toroidal component of the static magnetic field. If $\Omega_{ci}(r,\vartheta)$ where taken at the observation point instead than at the particle position, the integrals would reduce again to Plasma Dispersion functions, with a different value of $k_{\parallel}$ for each Fourier component. The correct integrals obviously have the same asymptotic behaviour far from cyclotron resonances; close to such resonances they can be evaluated exploiting the stationarity of the phase [45]. An interpolation between these two limits, which is uniformly very accurate and easily implemented numerically, has been proposed in [46]. It consists in using the Plasma Dispersion function, with, however, an effective value of the parallel wave number

$$k_{\parallel})_{eff} = k_{\parallel}(n,m)\frac{\sqrt{1+4\gamma}-1}{\gamma} \qquad \gamma = \frac{\omega}{2k_{\parallel}^2 R_{tor} v_{thi}} |\sin\theta \sin\Theta| \qquad (2.58)$$

The γ–dependent factor adds to the thermal Doppler broadening of the cyclotron layer a correction which takes into account the finite length of the resonances of individual ions (neglecting those with turning point close to the resonance itself). Thus $k_{\|})_{eff}$ does not vanish even in the limit $k_{\|} \to 0$, a result which is easily seen to be physically correct.

The Ansatz (2.56) tranforms the integro-differential wave equations in tokamak geometry into a (large) set of coupled differential equations for the radial variation of the amplitudes of the poloial Fourier components. Methods for their solution and examples of results can be found in [47]–[48]; a review of other approaches for the solution of the wave equations in toroidal geometry and further references can be found in [49]. Here we mention only that the second term in Eq. (2.57) is responsible for considerable broadening and even scrambling of the $k_{\|}$-spectrum excited by the antenna, particularly near the magnetic axis $r \to 0$.

2.16 – ***Quasilinear effects in the propagation of IC waves.*** The coefficients of the constitutive relation (and those of the local dispersion relation (2.32)) have been written assuming Maxwellian ion distribution functions. They can be easily modified, however, to take into account the deviations of the ion distribution functions from thermal equilibrium discussed in section (1). This task is greatly facilitated by the fact that in the FLR approximation the distribution of perpendicular velocities enters in these coefficients only through its two lowest momenta, namely the density and the perpendicular pressure. The quasilinear distribution of parallel velocities, on the other hand, must replace the Maxwellian in the definition of the Plasma Dispersion function.

Clearly, the increase in perpendicular pressure due to heating will influence in particular the efficiency of mode conversion near first harmonic resonances. The presence of suprathermal ions with excess of energy in the parallel direction, on the other hand, broadens the cyclotron resonance layers, and enhances ion cyclotron damping. The results of section 1.8 suggest simple analytic approximations by means of which the modified Plasma dispersion function can be easily evaluated and the importance of these quasilinear effects estimated [50].

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