How Ripples of the Steady Magnetic Field in Tokamaks Influence Propagation, Conversion and Absorption of Alfvén and Fast Magnetosonic Waves

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ABSTRACT

Results of analytical research into the effect of steady magnetic field ripples on Alfven waves (AW) and fast magnetosonic waves (FMSW) propagation, conversion and absorption are overviewed. Numerical estimations of the considered effects for the parameters typical for ASDEX-Upgrade are carried out. Possible applications of the obtained results to the experiments in ASDEX-U are discussed.

1 Introduction

The magnetic field of the confinement systems $\vec{B}_0 = \vec{e_r}B_{0r} + \vec{e_z}B_{0z}$ (in cylindrical coordinates) is often rippled:

$$B_{0z} = B_0(1 + \varepsilon(r)cos(k_b z)), \qquad B_{0r} = B_0\frac{\varepsilon'}{k_b}sin(k_b z). \qquad (1)$$

(Here $\varepsilon' = \frac{\partial\varepsilon}{\partial r}$, $k_b = \frac{2\pi}{L_b}$, and L_b is the ripple period). This is true for adiabatic systems with a ripple magnetic field, namely, for tokamaks, where the ripple is related to discreet toroidal magnetic field coils and also for the toroidal systems with the ripple magnetic field of the ELMO bumpy torus type. The ripple parameter is usually small ($\mid \varepsilon \mid \ll 1$). Dependence of the ripple parameter doubled 2ε versus the radius in ASDEX-U is shown in the Fig. 1, which is kindly given by Mr. P.Martin (IPP). The assumption of the small ripple allows us to use a perturbation technique to solve the problem.

We consider AW and FMSW because they are actively used for additional plasma heating in fusion devices.

The plasma pressure is assumed to be negligibly small in comparison with the magnetic field pressure. In this plasma, the equilibrium density $n(r,z)$ can be introduced as a function of a single variable that is the magnetic surface $n(r,z) = n(r_0)$, where

$$r_0 = r + \frac{cosk_b z}{r}\left(1 - \frac{\varepsilon}{2}cosk_b z\right)\int_0^r r\varepsilon dr + O(\varepsilon^3). \qquad (2)$$

We can now write down the expressions for the components $\varepsilon_{1,2}$ of the cold plasma dielectric permittivity tensor in the form of expansion in small parameter powers with keeping the terms of the second order in ε:

$$\varepsilon_{1,2}(r,z) = \varepsilon_{1,2}^{(0)}(r) + \varepsilon_{1,2}^{(1)}(r)cosk_b z + \varepsilon_{1,2}^{(2)}, \qquad (3)$$

here $|\varepsilon_{1,2}^{(i+1)}| \sim |\varepsilon\varepsilon_{1,2}^{(i)}|$.

When solving the set of Maxwell equations, we neglect collisions, electron inertia, and also the effect of the finite ion Larmor radius $\rho_{Li} = v_{Ti}/\omega_{ci}$ ($v_{Ti} = \sqrt{T_i/m_i}$ and T_i are the ion thermal velocity and temperature, respectively) everywhere except the region of fundamental Alfven resonance (AR) and except the regions of satellite ARs (SARs).

Distribution of axial component of RF magnetic field $B_z^{\sim}$ is determined using the perturbation technique with taking into account small summand $B^{(2)}(r)$ of the second order for the amplitude of the fundamental harmonic $\propto exp(ik_z z)$ and of the first order - for satellite harmonics $\propto exp[i(k_z \pm k_b)z]$,

$$\begin{aligned} B_z^{\sim} &= exp[i(k_z z + m\vartheta - \omega t)][B^{\sim}(r) + \\ &+ B^{(+)}(r)exp(ik_b z) + B^{(-)}(r)exp(-ik_b z)], \end{aligned} \tag{4}$$

here $B^{\sim}(r) = B^{(0)}(r) + B^{(2)}(r)$, $|B^{(2)}| \sim |\varepsilon^2 B^{(0)}|$ and $|B^{(\pm)}| \sim |\varepsilon B^{(0)}|$. Distribution $B^{(0)}(r)$ is supposed to be known from the zeroth approximation.

2 Effect of $\vec{B}_0$ ripples on eigen modes and eigen frequencies. Nonresonant case ($k_z \neq \dfrac{\pi}{L_b}$)

Dispersion relation for the MHD waves

$$D^{(0)} + D^{(2)} = 0, \tag{5}$$

has been obtained [1]. The shift $\Delta\omega$ of the MHD waves eigen frequency ω caused by the inhomogeneity of $\vec{B}_0$ has been shown to be small value of the second order,

$$\Delta\omega = -D^{(2)}(\partial D^{(0)}/\partial\omega)^{-1} \mid_{\omega=\omega_0}, \tag{6}$$

$\Delta\omega \sim \varepsilon^2\omega_0$, here ω_0 is eigen frequency determined in the zeroth approximation ($D^{(0)}(\omega_0) = 0$).

If the period of ripples is small ($k_b d \gg 1$, here d is the radius of coils of the toroidal magnetic field), as it takes place in ASDEX-U, then the value ε decreases approximately exponentially with going away from the plasma periphery: $\varepsilon(r) = \varepsilon(d)exp[-k_b(d-r)]$. In this case correction to the frequency

$$\omega_0 = \omega_{ci}^{(0)}(1 - k_A^2/k_z^2), \tag{7}$$

of short wavelenght AW ($k_A^2 \ll k_z^2$)is the following

$$\frac{\Delta\omega}{\omega_0} = \frac{k_b k_z^2 b}{j_{1,s}^2 k_A^2}\varepsilon^2(d)exp[-2k_b(d-b)], \tag{8}$$

here $j_{m,k}$ is the value of k-th root of the Bessel function of m-th order, b is the radius of the metal chamber, $k_A = \omega\omega_{pi}/(c\omega_{ci})$ is Alfven wave number. If AW propagates in ASDEX-U with the frequency $\omega_0 = (2/3)\omega_{ci}$ then the ratio (8) is equal by the order to 10 % .

Both eigen frequency ω_0 determined in the zeroth approximation ($\varepsilon = 0$) and the shift $\Delta\omega$ are even functions of axial wavenumber k_z for nonresonant perturbations (the longitudinal wavelength is not close to the double ripple period), $\omega_0(k_z) = \omega_0(-k_z)$ and $\Delta\omega(k_z) = \Delta\omega(-k_z)$.

3 Splitting the eigen frequency with rippled magnetic field

It has been shown in [2], that the splitting of eigen frequency,

$$\omega = \omega_0 \pm \delta\omega, \tag{9}$$

takes place for the resonant perturbation,

$$\frac{2\pi}{k_z} = 2L_b. \tag{10}$$

The following dispersion relation which is similar to a secular equation in the conventional perturbation theory,

$$D^{(0)^2} - D^{(1)^2} = 0, \tag{11}$$

is obtained (compare with (5)). The frequency shift $\delta\omega$,

$$\delta\omega = D^{(1)}(\partial D^{(0)}/\partial\omega)^{-1} \mid_{\omega=\omega^{(0)}}, \tag{12}$$

caused by the weak inhomogeneity of $\vec{B}_0$, is shown to be the small value of the first order, $\delta\omega \sim \varepsilon\omega_0$. Splitting of the FMSW eigen frequency caused by the ripples is expected to be in ASDEX-U more significant than that caused by the steady poloidal magnetic field for FMSW with the smallest values of wave numbers $\mid m \mid < 3$.

MHD waves with axial period $(2L_b)$ exist in plasma cylinder with bumpy magnetic field (1) in the form of two standing waves,

$$\begin{aligned} B_z^{\sim} = exp[i(m\vartheta - \omega t)][(C_0^{(+)}\psi_1^{(0)}(r) + C_1^{(+)}\psi_1^{(+)})e^{ik_z z} + \\ +(C_0^{(-)}\psi_1^{(0)}(r) + C_1^{(-)}\psi_1^{(-)})e^{-ik_z z} + \\ +C_3^{(+)}\psi_3^{(+)}e^{3ik_z z} + C_3^{(-)}\psi_3^{(-)}e^{-3ik_z z}], \end{aligned} \tag{13}$$

here $C_0^{(+)} = \pm C_0^{(-)}$, with close frequencies (9).

Crests (or nodal points) of the standing wave with the higher (or lower) frequency are situated just in the middle of the coils of the longitudinal magnetic field. This fact is explained as follows. Eigen frequencies of FMSW and AW increase with increasing external magnetic field, and the standing wave with the crests (or nodal points) in the middle of the coils exists in those areas where the magnetic field is higher (or lower).

Beat waves arise as a result of superposition of these standing waves. Measurements of the frequency, $\delta\omega$ (12), of these beat waves (which is expected to be equal in ASDEX-U approximately to 500 Hz if the frequency of AW $\omega_0 = (2/3)\omega_{ci}$) can be used for diagnostics of plasma density. In tokamaks with sixteen coils of the toroidal magnetic field these beat waves have the axial wave number $k_z = 8/R$, here R is the large radius of a tokamak. For ASDEX-U, this axial wave number $k_z \approx 0.05 cm^{-1}$.

Simple analytical expressions are obtained for the shift $\delta\omega$ of eigen frequency in the case of MHD oscillations in the thin ($\kappa a \ll 1$, $k_\perp a \ll 1$) plasma cylinder with the radius a with the uniform density which is separated from metal chamber by the wide ($\kappa b \gg 1$) vacuum layer, then the effect of $\vec{B}_0$ inhomogeneity is sufficient in the narrow layer $\Delta r \leq (2k_z)^{-1}$ near the magnetic coils. In this case shift $\delta\omega$ is exponentially small. For example, for AW

$$\omega_0 = \omega_{ci}^{(0)} \cdot \left(1 - \left(k_A^2/(2\kappa^2)\right)\right), \qquad (14)$$

$$\delta\omega = -\varepsilon(b) \cdot exp\Big(-2 \cdot |k_z| \cdot (b-a)\Big) \cdot \frac{N_z^2 - 1}{2 \cdot |m| + 1} \cdot \frac{\omega_{ci}^{(0)3} \cdot a}{\omega_{pi}^2 \cdot c} \cdot \omega_0. \qquad (15)$$

Here $\kappa^2 = k_z^2 - \omega^2/c^2$, $k_\perp$ is radial wave number,

$$k_\perp^2 = (\omega/c)^2\left(\varepsilon_1^{(0)} - N_z^2\right)(1-\mu^2), \mu = \varepsilon_2^{(0)}/\left(\varepsilon_1^{(0)} - N_z^2\right). \tag{16}$$

4 Plasma heating within satellite Alfven resonances (SARs)

In this chapter, we show that, in the confinement systems whith rippled magnetic field (1) along with the fundamental AR $r = r_A^{(0)}$ within which

$$\varepsilon_1^{(0)}(r_A^{(0)}) = N_z^2, \tag{17}$$

two extra resonances $r = r_A^{(\pm)}$ within which

$$\varepsilon_1^{(0)}(r_A^{(\pm)}) = (N_z \pm N_b)^2, \tag{18}$$

can occur [3]. Here, $N_z = \dfrac{ck_z}{\omega}$ is the longitudinal refractive index, and $N_b = \dfrac{ck_b}{\omega}$. It is natural to refer to these resonances as the satellite Alfven resonances (SAR). Note that, in spite of the fact the RF power absorbed in the vicinity of SAR is the value of the second order in ε, in some cases, the considered effect can be impotrant for plasma heating. This power is related to the work of the waves fields on both the radial microwave currents, $j_r \propto exp[i(k_z z + m\vartheta - \omega t)]$,

$$P_r^{(\pm)} = \pi r_A^{(\pm)} Re \int\limits_{r_A^{(\pm)}-x}^{r_A^{(\pm)}+x} j^* E_r^{(\pm)} dr = \frac{\pi\omega}{4}\left(\left|\frac{\partial \varepsilon_1^{(0)}}{\partial r}\right|^{-1} r\varepsilon_2^{(0)2}\right)_{r=r_A^{(\pm)}} \left|F_1^{(\pm)}\right|^2, \tag{19}$$

and the axial ones

$$P_z^{(\pm)} = \pi r_A^{(\pm)} Re \int_{r_A^{(\pm)}-x}^{r_A^{(\pm)}+x} j^* E_z^{(\pm)} dr = \int_{-\infty}^{+\infty} \left|\frac{\partial u_0}{\partial \zeta}\right|^2 d\zeta \times$$

$$\times \left(\frac{r\omega}{4} \left| \frac{\partial \varepsilon_1^{(0)}}{\partial r} \right|^{-1} \frac{\varepsilon_2^{(0)2} Im \varepsilon_3^* \varepsilon_1^{(0)}}{\mid \varepsilon_T \varepsilon_3^2 + \varepsilon_1 \varepsilon_3 \mid} \right)_{r=r_A^{(\pm)}} \left| F_1^{(\pm)} \right|^2. \quad (20)$$

Here

$$F_1^{(\pm)} = \left\{ \frac{-1}{2\varepsilon_2^{(0)}} \left[\varepsilon_1^{(1)} E_r^{(0)} + i\varepsilon_2^{(1)} E_\vartheta - \frac{c}{\omega}(N_b \pm N_z) \times \right.\right. \quad (21)$$

$$\left. \times \frac{\partial}{\partial r} \left(\frac{\varepsilon'}{k_b} E_r^{(0)} \right) \mp \frac{c\varepsilon' N_z}{\omega r k_b} \frac{\partial}{\partial r} (r E_r^{(0)}) \mp \frac{im\varepsilon' c}{\omega r k_b} N_z E_\vartheta^{(0)} \right] -$$

$$\left. \left. - \frac{cm}{r\varepsilon_2^{(0)}\omega} B^{(\pm)} - iE_\vartheta^{(\pm)} \right\} \right|_{r_A^{(\pm)}} \sim \varepsilon E_\vartheta^{(0)}.$$

For the driven MHD oscillations, the distribution of the field $B^{(0)}(r)$, and consequently, the value of $F_1^{(\pm)}$ (see (21)) are defined by an antenna current. Note, although the dependence of $B^{(\pm)}(r)$ in cold approach has a singulatity at the point $r_A^{(\pm)}$, the sum of the last two terms

$$\left(\frac{cm}{\omega r \varepsilon_2^{(0)}} B^{(\pm)} - iE_\vartheta^{(\pm)} \right)_{r=r_A^{(\pm)}}, \quad (22)$$

contained in the expression (21) varies slowly in the vicinity of SARs.

Behavior of fields within SAR regions was studied incorporating the electron inertia, the effect of the finite ion Larmor radius, and the collisions.

Plasma heating within SARs can be significant in the following cases.

(a) If the value of k_z is rather small, then the resonance $r = r_A^{(0)}$ is localized near the plasma boundary. At the same time, for sufficiently large values of

k_b, the SARs can lie deep inside the plasma. According to expression (19), the ratio of $P_r^{(\pm)}$ to the RF power $P_r^{(0)}$ absorbed within the fundamental AR can be estimated as

$$\frac{P_r^{(\pm)}}{P_r^{(0)}} \propto \frac{(k_z \pm k_b)^2}{k_z^2} \left| \frac{F_1^{(\pm)}}{F_1^{(0)}} \right|^2 . \tag{23}$$

Therefore, the value of $P_r^{(\pm)}$ can be larger than that of $P_r^{(0)}$ for sufficiently large values of k_b. In this case, as a result of small k_z, the field $B^{(0)}$ can penetrate deep into the plasma and even into the regions of rather high plasma density, $\left(\omega_{pi}^2/(\omega_{ci}(\omega+\omega_{ci})) > N_z^2\right)$, where it corresponds to an FMSW. In the ripple magnetic field, this FMSW interacts with the Alfven wave with the axial wave number $k_z \pm k_b$ and as a result is resonantly absorbed at the points $r_A^{(\pm)}$. The wave $B^{(0)}$ can be a slow-damping eigenmode of the considered plasma waveguide (or plasma resonator). In the case of a plasma torus, this resonator is a plasma cylinder with identical end-sides. In this case, the satellite harmonics $B^{(\pm)}$ grow resonantly and consequently, the power absorbed at the points $r_A^{(0)}$ and $r_A^{(\pm)}$ increases.

If the MHDW, with the longitudinal wave number $k_z \pm k_b$ is the eigen wave of the waveguide, then the absorption at the satellite hatmonics in the points $r_A^{(\pm)}$ can also be enhanced. In this case, the last two terms in expression (21), which are shown in (22), increase resonantly.

(b) In the situation corresponding to the low-density plasma, the condition $N_z^2 > \varepsilon_1^{(0)}$ holds everywhere, and there is no fundamental resonance. In this case, the SAR $r_A^{(-)}$ is possible, which can provide the absorption of the pump field. In this resonance, the absorption can also be enhanced because the satellite wave $B^{(-)}$ is an eigenmode of the waveguide.

5 Effect of $\vec{B}_0$ ripples on waves conversion within the fundamental Alfven resonance regions

Amplitudes of satellite harmonics are small as compared with that of the fundamental harmonic everywhere except the region of the fundamental Alfven resonance and except the regions of satellite ARs. Behavior of these amplitudes within the resonance regions is similar to that of associated oscillators. Width k_1^{-1} of FAR depends strongly upon the value of the small ripple parameter ε along with the collisions, finite ion Larmor radius and electron inertia, here

$$k_1 = \Big(-\frac{\omega^2}{c^2}\frac{\partial\varepsilon_1}{\partial r}\frac{\varepsilon_3}{\varepsilon_1+\varepsilon_T\varepsilon_3+\varepsilon_b\varepsilon_3}\Big)^{\frac{1}{3}} \sim (\rho_{Li}^2 a)^{-\frac{1}{3}}, \tag{24}$$

$$\varepsilon_b = \frac{\varepsilon'^2}{4k_b}\left[\frac{k_b+2k_z}{k_-^2}+\frac{k_b-2k_z}{k_-^2}\right]. \tag{25}$$

Contribution of the $\vec{B}_o$ inhomogeneity is comparable with that of finite ion Larmor radius if $\varepsilon^2 \succeq 10^{-5}$, with that of electron inertia if $\varepsilon^2 \succeq 10^{-4}$, with that of collisions if $\varepsilon^2 \succeq 4\cdot 10^{-6}$.

FAR region moves away from the plasma axis at the small distance

$$\delta r = -\varepsilon_1^{(2)}\left(\frac{\partial\varepsilon_1^{(0)}}{\partial r}\right)^{-1}_{r_A^{(0)}} \sim \varepsilon^2 a, \tag{26}$$

due to $\vec{B}_o$ ripples.

6 Discussion

Ripples of toroidal magnetic field are small in ASDEX-U (see Fig. 1). Small parameter ε of ripples does not exceed the value of 3 % . This circumstance proves applicability of the perturbation technique for studying the effect of ripples on the waves propagation, conversion and absorption. In spite of the smallness of the ripple taking this type of $\vec{B}_o$ inhomogeneity into account leads to new effects. Information about these effects can be applied in experiments which are carried out at IPP.

Firstly, as far as current drive is produced by travelling waves and waves with axial wave number (10) exist in plasma with rippled magnetic field in the form of standing waves (13), then it is preferable to avoid excitation of these waves in experiments on current drive. This note is expected to be true also for the other types of waves.

Secondly, unfortunately, it is impossible to use the obtained results for the diagnostics of the density profile of the separate ion specie. Such diagnostics need some mechanism which is sensitive to the presence of one separate specie only. But if somebody proposes such diagnostics then we can propose the mechanism for checking it up. Our check up consists in comparison of the eigen density shift δn caused by the ripples and which is to be obtained from our calculations (if profiles given by some diagnostics are substituted) with the direct RF measurements of δn. Here δn is the shift between the values of the electron density at the axis which are eigen for the waves with opposite signes of axial wave number.

Thirdly, contribution of satellite Alfven ersonances to plasma heating in

ASDEX-U is not expected to be significant. But in Wendelstein 7AS, it can be so due to large ripples, the ratio (23) of the power absorbed within SAR to that absorbed in the fundamental AR is expected to exceed 30 % if the axial wave number of the fundamental harmonic is small ($| k_z | \ll k_b$).

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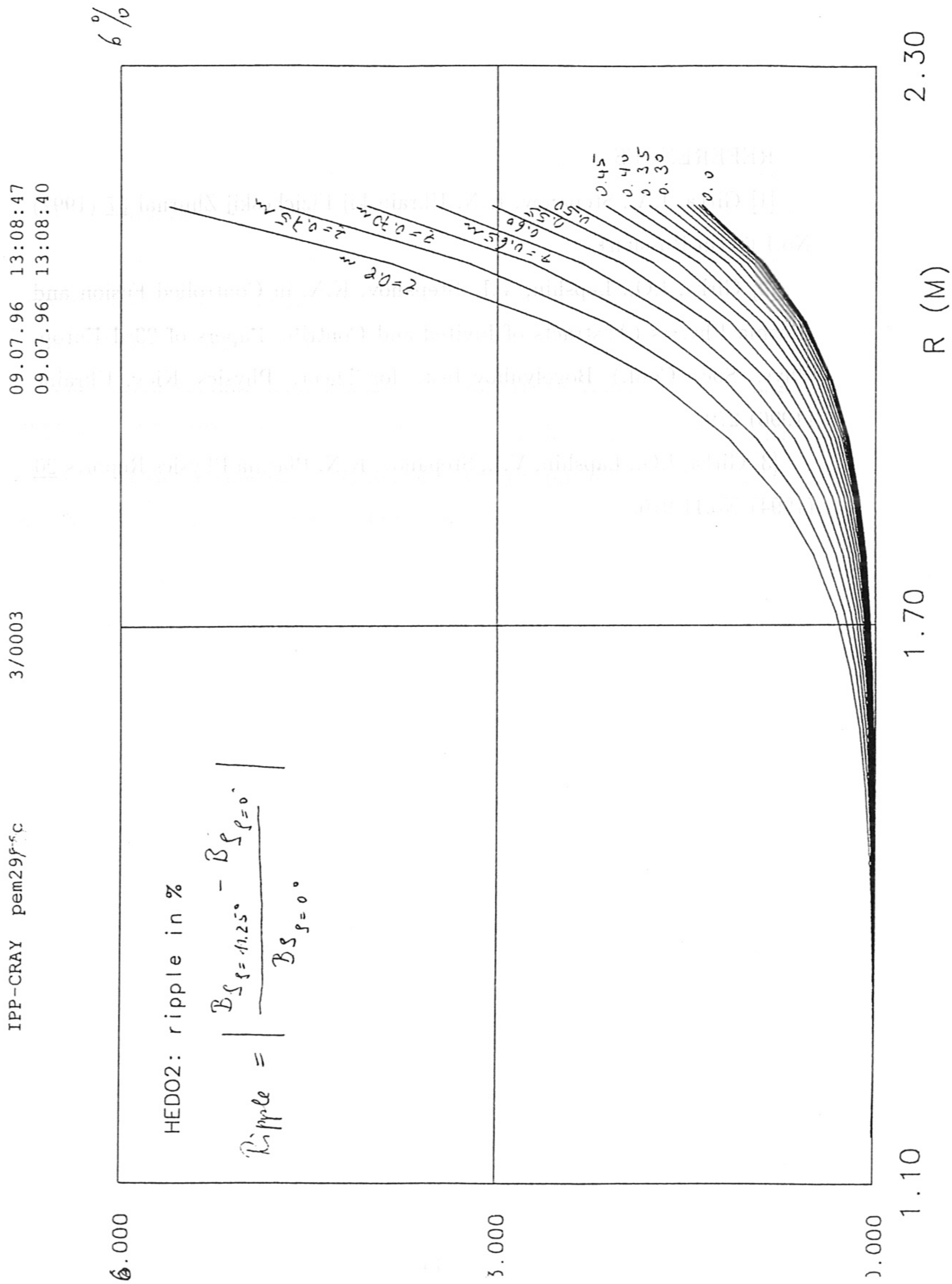

Fig. 1. Dependence of ripple parameter doubled versus the radius in ASDEX-U.