Uncertainties in the confinement time prediction for ITER

by

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What is a 95% interval estimate for the (average) confinement time of a plasma in ITER operating at certain plasma parameters, and how has this estimate to be interpreted?

According to the classical frequentist definition, a 95% confidence interval for τ_E is a random interval which, with 95% probability, covers the 'true' (average) confinement time (of say 1000 discharges, all made at the same operating point) in ITER, given that a specific model, e.g. a simple power law scaling, is correct. Such an interval is routinely calculated by regression procedures in e.g. SAS and S-PLUS. Basically, the error propagation of the estimated regression coefficients is calculated, taking into account the distance (in a suitable norm) of the operating point of ITER to the center of gravity of the database. A specific formula using principal components was derived and used in [1]. The scale factor in the error propagation formula is the root mean squared error (rmse) of the fit divided by the square root of the number of 'effective' observations, N_{eff}, which is the number of observations, N, divided by some factor to account roughly for (a) the correlations between the observational errors (several time slices per shot have been taken) and (b) the fact that measurement errors in the regression variables have been neglected. However, there are more definitions.

During the ITER CDA phase, T. Takizuka once remarked, while discussing a draft version of the L-mode confinement paper [2], 'a 95% confidence interval means that the confinement time in ITER must be situated within that interval, otherwise I lose my job'. This is a second definition.

During one of the ITER CDA Meetings in Garching, K. Riedel gave a third definition, by stating as a *Gedankenexperiment*, that if one would build 1000 ITER machines, and perform in each of them one discharge, all of them at the same operating point, then 95% of those 1000 discharges should have a confinement time within the 95% prediction interval. Replacing 'one discharge' in the above definition by 'the average of a large number of discharges', all performed at the same operating point, we have a definition of a 95% interval estimate of the true confinement time at that operating point. In order to estimate such an interval before all those machines are built, one has to assume that building a new device constitutes in a certain sense a 'gamble' with respect to the confinement time. The standard deviation of the distribution due to this gambling has to be estimated from the presently built devices [3].

A fourth definition is, to consider the interval obtained by interchanging for each tokamak the two or three measurements of the thermal energy (Wdia/Wmhd, both corrected for fast particles, and Wkin) that are available in the database. This was, in a simplified version, suggested by O. Kardaun [4].

A fifth definition, which was essentially suggested by B. Dorland and M. Kotschenreuther at the third ITER expert Meeting in Naka, is to consider the interval that contains 95% of all 'admissible' non-linear fits (on logarithmic scale) to the data. Admissible means that the rmse decreases significantly with respect to the best fitting log-linear model (simple power law), taking into account the degrees of freedom due to the increased flexibility of the non-linear model, and that the model selection has been based on more or less plausible physical considerations or simple model extensions, rather than on automatic selection from large classes of flexible models, or on 'devious' mathematical construction.

Related to the fifth definition is the aspect of 'hidden variables' that influence the confinement time and are that are neither included in the regression equation nor accounted for by the selection criteria of the so-called 'standard datasets' (on which the standard 'ITER scalings' are based). It is useful in this context to make the distinction between hidden variables of an 'engineering character', which can be experimentally controlled, and those of a 'physical character', which are interesting from a physical point of view, but not (yet) under experimental control. It is difficult, and will presumably remain always rather speculative, to construct an interval estimate that accounts for the uncertainty due to (unknown) hidden variables. The most effective way to proceed is to disclose the influence of these variables by experimental investigation, and take them into account either in the regression equation(s), or by restricting the definition of the standard dataset, or by performing randomised experiments over these variables.

Each of these definitions covers only a partial aspect of the complex real situation, and each of them has to be accounted for to ascertain the prediction margin of an important device like ITER.

It is remarked that the probabilistic interpretative framework of these intervals according to these various definitions ranges from objectivistic statistics (probability interpreted as relative frequency of repeated measurements under 'nearly identical' situations), Bayesian statistics ('probability' as a personal strength-of-belief, to a lesser or to a stronger extent influenced by the data and by other expert opinions), and, a fruitful synthesis of these, distributional inference ('credence' corresponding to the weight of scientific evidence based on a loss function approach), developed by Schaafsma et al. [5-7].

Let us call a standard deviation of the 'credence distribution' of the confinement time, based on the considerations described above, 'a technical standard deviation'. The technical standard deviation has to be assessed on the basis of statistical data analysis as well as on additional information, obtained during intensive discussions between specialists that have been investigating the confinement time prediction problem from various sides.

In the light of the available data and based on the discussions at the third ITER Database and Modelling Working Group Workshop in Naka, the technical standard deviation for ITER is estimated to be 20 to 25%. Since the point prediction of the ITER-EDA confinement time (at the 'standard design parameters'), according to both the ITERH92-P ELMy and 0.85 times the ITERH93-P ELM-free scaling, is about 6.0 sec, this gives a 95% confidence interval, to be interpreted in the sense described above, of some 3.5 to 9 sec.

It must be remarked that the inferential distribution corresponding to this interval covers a considerable fine structure, which to unravel is an important area of future research in plasma physics. This requires input from the experimental side, from plasma theory as well as from data analysis.

It has been reported that once, in a scientific discussion, a medical doctor mentioned that he wanted to express the results of his research into just one number and that he was reluctant to give any confidence interval, because in his view 'uncertainty had a negative utility' and 'he wanted his voice to be heard'. This last type of argument seems, fortunately, alien to plasma physicists. However, it remains true that uncertainty has a negative utility, in that it is associated with increased construction costs. On the long run this may turn out to be a serious obstacle to downsize the successors of ITER to commercially viable reactors.

— *References* —

1 Christiansen J.P., et al., Nuclear Fusion **32** (1992) 291.

2 Yushmanov P.N., et al., Nuclear Fusion **30** (1990) 1999.

3 Riedel, K., Nucl. Fusion **30** (1990) 755.

4 IPP Annual Report 1990, p. 78-79.

5 Kroese, A.H., Distributional Inference: A Loss Function Approach, Thesis Groningen, 1994.

6 Van der Meulen, E.A., Assessing Weights of Evidence for Discussing Classical Statistical Hypotheses, Thesis Groningen, 1992.

7 Kardaun, O., Confidence bands and the Relation with Decision Analysis, in: Handbook of Statistics, Vol. 8, Elsevier, 1991.

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