

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK
GARCHING BEI MÜNCHEN

Calculation of Metric Coefficients
for a Divertor Tokamak Configuration

C.V. Atanasiu¹⁾, A. Moraru²⁾, A.A. Subbotin³⁾

IPP 5/63

August 1995

- ¹⁾ Permanent address: *Institute of Atomic Physics, IFTAR L22, R-76900 Bucharest-Magurele, Romania*
- ²⁾ University of Bucharest, Polytechnical Institute, R-77206 Bucharest, Romania
- ³⁾ Russian Research Center, I.V. Kurchatov Institute, 123182 Moscow, Russia

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Abstract

Most toroidal flux coordinate systems tacitly assume a nested flux surface structure. However, in a diverted torus the presence of a separatrix breaks this structure and the usual toroidal flux coordinates can not be used directly. In this paper we present a method to calculate the metric coefficients, necessary for MHD equilibrium and stability computations, in an axisymmetric divertor tokamak configuration. A "classical" flux coordinate system, amended by a "cast function", has been used. Thus, the unknown moments - the solution of the equilibrium equation - are determined by the difference between the real flux surfaces and those described by the cast function only. With this procedure, the necessary number of moments to describe the flux surfaces in a quite complicated separatrix configuration is small enough to make computations time-efficient. As an example of our approach, the separatrix of a particular equilibrium configuration of the ASDEX-Upgrade tokamak has been considered and, for a given surface dependence of the toroidal plasma current density, some metric coefficients and the rotational transform have been computed considering the cast function only.

KEY WORDS MHD equilibrium and stability; tokamak; magnetic divertor; flux coordinates

I. INTRODUCTION

X points occur in a wide variety of plasma confinement devices. In order to control impurities, diverted toroidal configurations - the object of our investigation - use solid conductors to draw off field lines from the outside of the plasma, leading them to a neutralizer divertor plate. A typical configuration with a separatrix and X points, presented in Fig. 1, results from two parallel current distributions, the first given by the current density J_1 distributed up to the separatrix and the second by the current I_2 . The separatrix surface "separates" the surfaces which enclose one current distribution from those which enclose both. The X line, resulting from the statement that the transverse components of B vanish, represents the meeting place of the two branches of the separatrix.

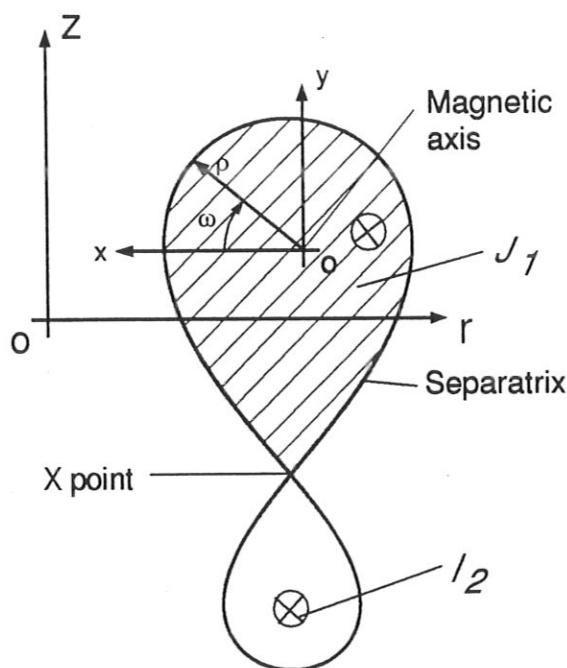


Figure 1: Parallel current distribution producing a magnetic separatrix.

Most toroidal flux coordinate systems¹⁻² implicitly assume a nested flux surface structure. However, in a diverted torus the presence of a separatrix breaks this structure and the usual toroidal flux coordinates can not be used directly.

Different papers have considered the separatrix problem. In Ref. 3 a method for determining the region of ergodic field lines about the separatrix magnetic surface is given. A generalization of this method, with a two-wire divertor model, is presented in Refs. 4-5. More recently, accurate and efficient calculations of the axisymmetric equilibrium up to the separatrix have been performed in Ref. 6, using a combination of the boundary layer expansion near the separatrix and a spectral method in the core with matching at a virtual boundary, while in Ref. 7 a coordinate system that is well behaved in the X point region is presented.

Our approach consists of introducing a "cast function" which describes the separatrix

exactly (i.e. with a prescribed accuracy) while the internal flux surface contours are represented approximately by this cast function. The moments to be determined, in order to describe the MHD plasma equilibrium, are related now to the difference between the real flux surface contours and the contours described by the cast function only.

The adopted system of flux coordinates, the momentum equilibrium differential equation and the method of using cast functions for the calculation of the metric coefficients are presented in Sec. II. A separatrix configuration of a particular equilibrium at the ASDEX-Upgrade tokamak, with the relevant data (given in Appendix) obtained by the equilibrium interpretation code DIVA⁸ has been considered. In Sec. III, as an exemplification of our approach, some metric coefficients, necessary for equilibrium and stability calculations, are computed for a given distribution of the toroidal current density $J(a)$ using the cast function only.

The next two Sections present in some detail two models of cast functions to be used. Thus, in Sec. IV the cast function f_A has been deduced starting from physical considerations related to a plasma current density distribution with solid divertor conductors. This function leading to the solving of a transcendental equation (minimization algorithms are necessary), the identification of its parameters in order to describe a real separatrix curve can not be realized directly by using a classical least-squares method. With this in view, in Sec. V, a more flexible cast function f_B , leading to a non-transcendental equation, has been drawn from geometrical considerations only.

Finally, a brief discussion of the method is given in Sec. VI.

The relevant parameters of the separatrix obtained at the 5000th shot of the ASDEX-Upgrade tokamak can be found in Appendix.

II. THE CAST FUNCTIONS METHOD

Considering now the system of coordinates (a, ω, ζ) with a an index of the magnetic surfaces, $\omega \in [0, 2\pi]$ a poloidal anglelike coordinate and $\zeta \in [0, 2\pi]$ a toroidal anglelike coordinate and considering also the local coordinate system (Fig. 1), it is possible to represent the coordinate transformation through Fourier series in ω :

$$x = \rho(a, \omega) \cos \omega \quad (1)$$

$$y = \rho(a, \omega) \sin \omega$$

$$\rho^2(a, \omega) = a^2 + \text{Real} \left[\sum_{m=-\infty, m \neq 0}^{\infty} \delta_m e^{im\omega} \right] \quad (2)$$

where $\delta_m(a)$ are the complex moments ($\delta_m = \delta_{-m}^*$, the star designating the complex conjugate moments) and, by definition,

$$a^2 = \frac{\Phi(a)}{\pi B_0} \quad (3)$$

where Φ is the toroidal flux, while B_0 is the toroidal magnetic field at the magnetic axis ($a = 0$); therefore, a has the signification of an equivalent radius. As the moments are complex variables, no assumption that the flux surfaces possess up-down symmetry has to be made.

Starting from the pressure equilibrium equation and the Maxwell equations, written for an axisymmetric configuration, and using this representation of magnetic surfaces we obtain the following system of complex differential momentum equations⁹

$$Y_m'' + \left(3 + 2\frac{\mu' a}{\mu}\right) \frac{Y_m'}{a} - (m^2 - 1) \frac{Y_m}{a^2} = -2 \frac{W_m}{\mu a^2} \quad (4)$$

where $Y_m = \delta_m/a$, $\mu = 1/q$ is the rotational transform, $W_m = W_m(a, \mu(a), g_{ik}(a, \omega), p'(a), B_0, R, \delta_{l,l \neq m})$ is a nonlinear functional, p is the scalar plasma pressure and g_{ik} are the metric coefficients; prime indicates derivation with respect to a .

By solving equation (4) with the boundary conditions given at the magnetic axis ($a = 0$) and at the plasma boundary ($a = 1$)

$$\delta_m(0) = 0 \quad (5)$$

$$\delta_{m,m \neq 1}(1) = \text{given};$$

one obtains the $\delta_m(a)$ dependence over the full plasma region and thus the full equilibrium description of the considered plasma.

It is known that for a separatrix-like function, the expansion in series, based on any set of orthogonal functions, converges slowly near the X point, so it is difficult to separate the high order terms. The $\rho(a = 1, \omega)$ function, owing a discontinuity of the derivation, the Fourier coefficients are decreasing proportionally with m^{-2} . An accurate description of the separatrix corner would require a large number of harmonics. Thus, for the separatrix obtained from the shot no. 5000 at 1.550 seconds of ASDEX-Upgrade⁸ and described by 12 complex moments, one obtains a maximum relative error in ρ , at the X point, of 7.3 %, while with 24 moments and 60 moments respectively, one obtains an error of 3.2 % and 0.8 % respectively (Fig. 2). For more than 40 complex moments, a classical least-square method for $\delta_m(1)$ determination would not work as the system to be solved is ill-conditioned and special procedures have to be used.

Even if the relative error could be made acceptable by identifying a sufficient number of moments, the running time, necessary to solve the system of m complex differential equations (4), would be prohibitive for practical calculations.

Introducing now a cast function f which describes with a desired accuracy the given separatrix curve, we may write :

$$f^2 = f_0^2 + \tilde{f}^2 = f_0^2 + \text{Real} \left[\sum_m \hat{\delta}_m e^{im\omega} \right] \quad (6)$$

where $f_0^2 = \langle f^2 \rangle_\omega$ is the averaged part of f^2 , while $\tilde{f}^2 = f^2 - f_0^2$ is the periodic part of f^2 .

With these notations, we can write equation (2) in the form :

$$\rho^2(a, \omega) = a^2 + \text{Real} \left[\sum_m \bar{\delta}_m e^{im\omega} \right] + \tilde{f}^2 \quad (7)$$

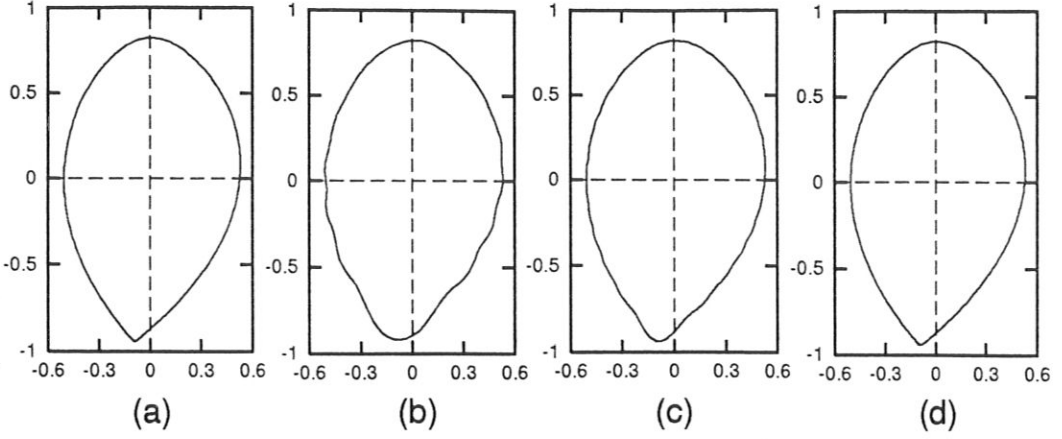


Figure 2: Moment description of an ASDEX-Upgrade separatrix configuration. Real configuration (a), modelling with 12 complex moments (b), with 24 complex moments (c) and with 60 complex moments (d), respectively.

where $\bar{\delta}_m = \delta_m - \hat{\delta}_m$. Thus, the $\bar{\delta}_m$ moments describe a function of class C^1 or higher. It is easy to prove that the necessary and sufficient continuity condition for the difference function $\rho^2 - f^2$ at the X point is that both angles generated by the functions f and ρ at this point are equal (i.e. in the vicinity of the X point, it is not necessary for both functions to be “identical”)

In Sec. IV and Sec. V respectively, two different cast functions will be given.

III. CALCULATION OF METRIC COEFFICIENTS

Making the following notations

$$\bar{\Delta} = \text{Real} \left[\sum_m \bar{\delta}_m e^{im\omega} \right] \quad (8)$$

$$\alpha = \bar{\Delta} + \tilde{f}^2 \quad (9)$$

$$\gamma = a^2 + \alpha \quad (10)$$

the metric coefficients, used in our equilibrium and stability calculations, become

$$g_{22} = \gamma + \frac{\alpha_\omega^2}{4\gamma} \quad (11)$$

$$g_{12} = \frac{\alpha_\omega}{2a} + \frac{(a\alpha' - 2\alpha)\alpha_\omega}{4a\gamma} \quad (12)$$

and the Jacobian

$$D = a + \frac{\alpha'}{2} \quad (13)$$

where, prime indicates, as before, derivation with respect to a , while the subscript ω indicates derivation with respect to ω .

Considering now a surface distribution of the toroidal plasma current density of the form :

$$J(a) \propto a^2 - \frac{a^4}{2} \quad (14)$$

and taking f_B as cast function (given in Sec. V and related to the shot 5000 of the ASDEX-Upgrade tokamak), the averaged values of the metric coefficients $\langle \rho(a, \omega) \rangle_\omega$, $\langle g_{22}(a, \omega) \rangle_\omega$, $\langle D(a, \omega) \rangle_\omega$ and of some coefficients involved in our equilibrium code like $\langle g_{12}(R - \rho \cos \omega)/D \rangle_\omega$ and $\langle g_{22}/(R - \rho \cos \omega)/D \rangle_\omega$ have been computed; their surface dependence is presented in Figs. 3a - 3e, with R the radius of the magnetic axis.

For the current density distribution given by equation (14), the surface dependence of the rotational transform

$$\mu(a) = \frac{J(a)}{aR \left\langle \frac{g_{22}(a, \omega)}{(R - \rho(a, \omega) \cos \omega)D} \right\rangle_\omega} \quad (15)$$

has been calculated (Fig. 3f).

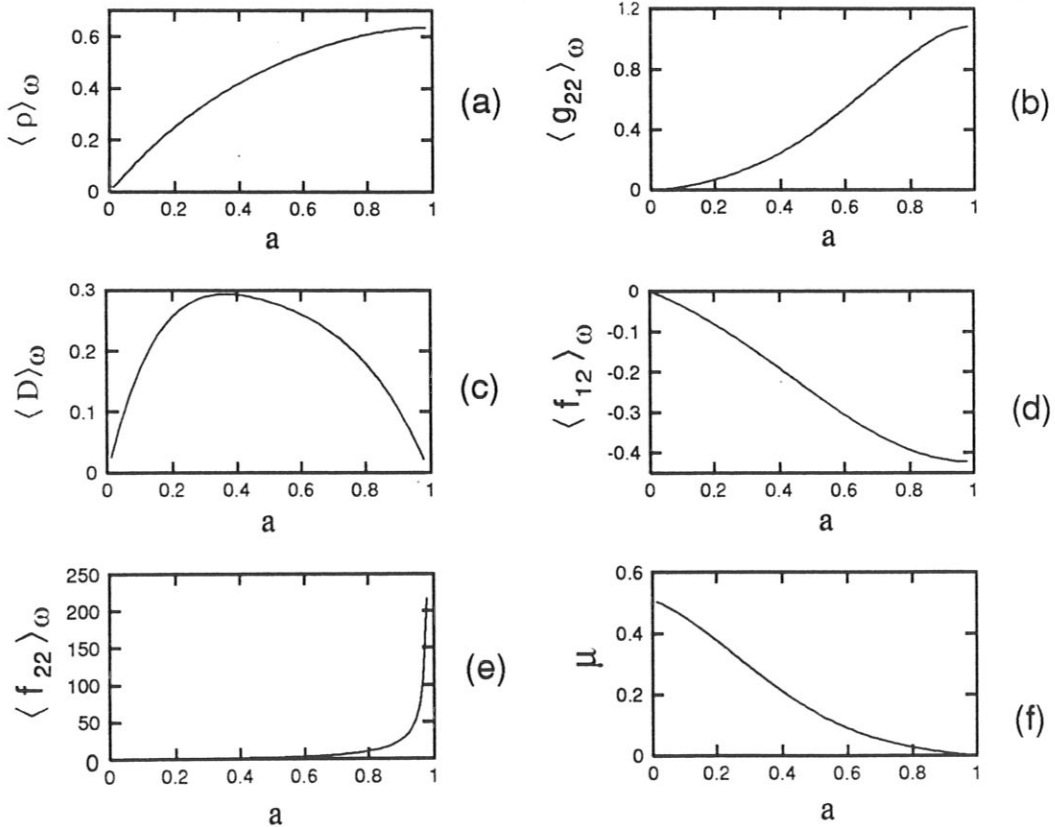


Figure 3: Various profiles of metric coefficients (a) - (e) and of the rotational transform (f) calculated using the cast function f_B only; the current density distribution is given by equation (14).

IV. 1st CAST FUNCTIONS MODEL

A first model for a cast function has been naturally suggested by the currents distribution given in Fig. 1; in place of concentrated currents, a current density distribution J_1 has been considered to avoid the singularity of the magnetic field at the magnetic axis. For simplification and without loss of generality, in the following, a cylindrical current distribution will be considered.

a) Symmetrical cast functions

We are considering now a Cartesian coordinate system (x, y) with the origin at the axis of the J_1 current density domain and the x axis passing through the axis of the I_2 current (Fig. 4a). For the sake of simplicity, at the beginning, only symmetrical cast functions with respect to the y axis ($f_A(x, y) = f_A(x, -y)$) will be considered. At the point (x, y) , the magnetic field and the magnetic vector potential respectively, due to a constant current density distribution are given by the relations

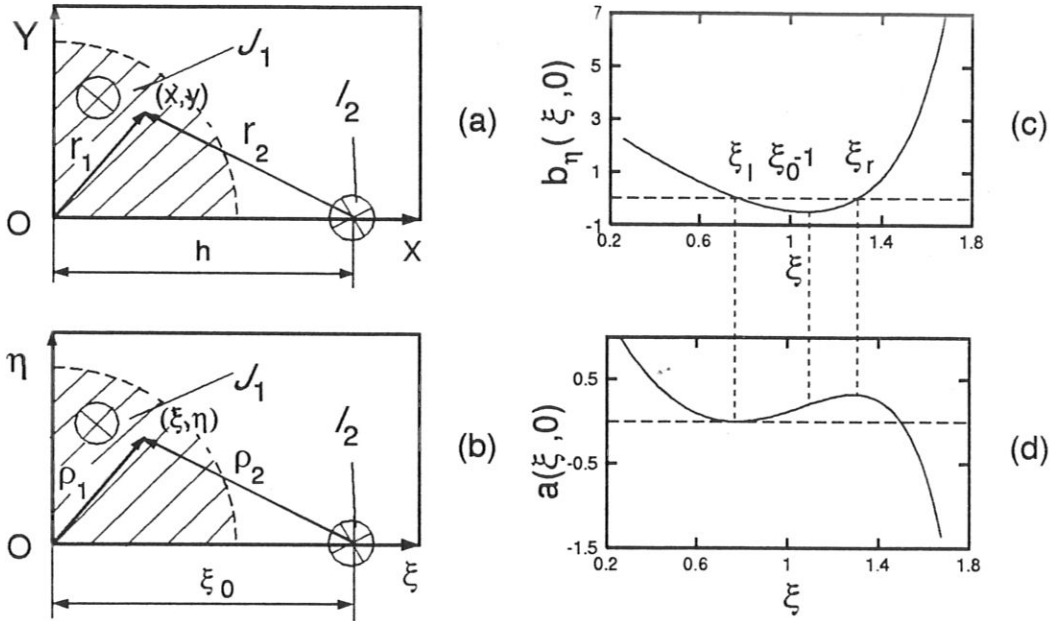


Figure 4: Model for the f_A cast function calculation: coordinates (a) - (b), magnetic field distribution (c) and magnetic flux distribution (d).

$$B_1(r_1) = \frac{\mu_0}{2} J_1 r_1; \quad A_1(r_1) = \frac{\mu_0}{4} J_1 r_1^2 \quad (16)$$

Similarly, for the current I_2 one has

$$B_2(r_2) = \frac{\mu_0 I_2}{2\pi r_2}; \quad A(r_2) = \frac{\mu_0}{4\pi} I_2 \ln r_2^2 + C \quad (17)$$

where, $r_1 = (x^2 + y^2)^{1/2}$, $r_2 = ((x - h)^2 + y^2)^{1/2}$, C is an integration constant, while μ_0 is the magnetic permeability of free space.

Introducing the non-dimensional coordinates (Fig. 4b)

$$\xi = \frac{x}{r_0}, \quad \eta = \frac{y}{r_0}, \quad \xi_0 = \frac{h}{r_0}, \quad \rho_1 = \frac{r_1}{r_0}, \quad \rho_2 = \frac{r_2}{r_0} \quad (18)$$

and the non-dimensional magnetic field and magnetic vector potential

$$b_\xi = \frac{B_x}{B_0}, \quad b_\eta = \frac{B_y}{B_0}, \quad a = \frac{A}{\Phi_0} \quad (19)$$

where

$$r_0 = \left(\frac{I_2}{\pi J_1} \right)^{1/2}, \quad B_0 = \frac{\mu_0}{2} J_1 r_0, \quad \Phi_0 = B_0 r_0 \quad (20)$$

one obtains the following expressions which depend on the ξ_0 parameter only

$$\rho_1 = (\xi^2 + \eta^2)^{1/2}, \quad \rho_2 = ((\xi - \xi_0)^2 + \eta^2)^{1/2} \quad (21)$$

$$a = \frac{1}{2} \rho_1^2 + \ln \rho_2 + C \quad (22)$$

$$b_\xi = \eta \left(1 + \frac{1}{\rho_2^2} \right), \quad b_\eta = (\xi - \xi_0) \left(1 + \frac{1}{\rho_2^2} \right) - \xi \quad (23)$$

On the axis $\eta = 0$, the b_η component has the expression

$$b_\eta(\xi, 0) = \frac{1}{\xi_0 - \xi} - \xi \quad (24)$$

with a maximum

$$b_\eta^{max}(\xi, 0) = 2 - \xi_0 \quad (25)$$

at the point $\xi = \xi_0 - 1$; $b_\eta(\xi, 0)$ vanishes at the points (at the same points, the magnetic flux has two extrema, Fig. 4c)

$$\xi_{r,l} = \frac{1}{2} \xi_0 \pm \left(\frac{1}{4} \xi_0^2 - 1 \right)^{1/2} \quad (26)$$

It is obvious that real solutions exist only if $\xi_0 \geq 2$.

For this configuration the $(\xi_r, 0)$ point represents the X point of the separatrix, where the flux has a maximum, while the $(\xi_l, 0)$ point, with a minimum for the flux, represents the magnetic axis. It is convenient to choose an integration constant C such that the flux vanishes at the magnetic axis; thus, the flux expression becomes (Fig. 4d)

$$a = \frac{1}{2} (\rho_1^2 - \xi_l^2) - \ln \frac{\xi_r}{\rho_2} \quad (27)$$

and the maximum flux value, with which the flux has to be normalized, is

$$a_r = \frac{1}{2} (\xi_r^2 - \xi_l^2) - \ln \frac{\xi_r}{\xi_l} \quad (28)$$

The slopes of the constant flux surface contours are

$$\eta' = \frac{d\eta}{d\xi} = \frac{-\xi(\rho_2^2 + 1) + \xi_0}{\eta(\rho_2^2 + 1)} \quad (29)$$

On the axis $\eta = 0$ all the constant flux surface contours have an infinite slope, with the exception of the separatrix curve whose slope is given by

$$\eta' = \pm \left(\frac{1 - \rho_2^2}{1 + \rho_2^2} \right)^{1/2} \quad (30)$$

At the X point $(\xi_r, 0)$ the slope takes the values

$$\eta'_r = \pm \left(\frac{1 - \xi_l^2}{1 + \xi_l^2} \right)^{1/2} \quad (31)$$

It is interesting to remark that at the magnetic axis $(\xi_l, 0)$, the zero measure constant flux surface presents a "corner" too, with the slope equal to η'_l .

If the slope $\eta'_r = p$ is given as input data, then the necessary values of ξ_l^p and ξ_0^p are given by

$$\xi_l^p = \left(\frac{1 - p^2}{1 + p^2} \right)^{1/2} \quad (32)$$

and

$$\xi_0^p = \xi_l^p + \frac{1}{\xi_l^p} \quad (33)$$

Obviously, $|p| \leq 1$, corresponding to a half-angle at the X point $\theta_p/2 \leq \pi/4$; the real case of an angle greater than $\pi/2$ will be treated later. In Fig. 5, constant flux surface contours obtained by a symmetrical cast function with $\xi_0 = 2.2$ and a scaling factor $\kappa_\eta = 0.6$, defined later in equation (49), are presented.

b) Non-symmetrical cast functions

The real separatrix is not symmetric with respect to the η axis, thus, to the previous symmetrical flux function a non-symmetrical, odd with respect to η , additional flux function a_a has been added

$$a = \frac{1}{2} (\rho_1^2 - \xi_l^2) - \ln \frac{\xi_r}{\rho_2} + \frac{1}{2} a_a \quad (34)$$

The simplest form that a_a could take is

$$a_a = \eta \psi_a \quad (35)$$

where ψ_a is an even function with respect to η . Knowing that on a constant flux surface contour $da = 0$, one obtains

$$\xi + \eta \eta' + \frac{\xi - \xi_0 + \eta \eta'}{\rho_2^2} + \frac{1}{2} \left[\left(\psi_a + \eta \frac{\partial \psi_a}{\partial \eta} \right) \eta' + \eta \psi'_a \right] = 0 \quad (36)$$

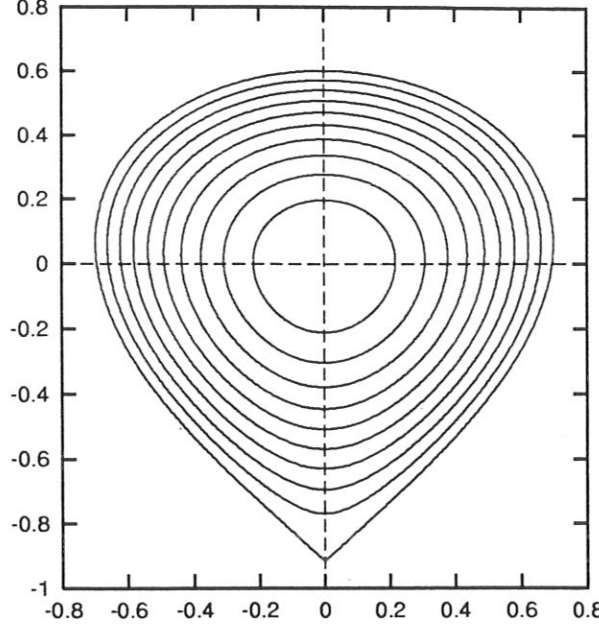


Figure 5: Constant flux surface contours given by the symmetrical f_A cast function with $\xi_0 = 2.2$ and $\kappa_\eta = 0.6$; the angle at the X point is equal to the real one.

To maintain the position of the X point at $\xi = \xi_r$ and $\eta = 0$, ψ_a must vanish at this point, i.e. ψ_a has to be of the form

$$\psi_a = (\xi - \xi_r)(\alpha + \beta(\xi - \xi_r)) + \gamma\eta^2 \quad (37)$$

α , β , and γ being arbitrary “asymmetrisation factors”.

The slope of this new curve is given by

$$\eta' = -\frac{\xi - \xi_0 + \rho_2^2 \left[\xi + \eta \left(\frac{1}{2}\alpha + \beta(\xi - \xi_r) \right) \right]}{\eta + \rho_2^2 \left[\eta + \frac{1}{2}((\xi - \xi_r)(\alpha + \beta(\xi - \xi_r)) + 3\gamma\eta^2) \right]} \quad (38)$$

The two different slopes of the function at the point $(\xi = \xi_r, \eta = 0)$ are given by

$$\eta'_{1,2} = \frac{(\alpha^2 + 4\xi_r^2 - 4)^{1/2} - \alpha}{2(1 + \xi_r^2)} \quad (39)$$

The flux center (ξ_c, η_c) - the point where the flux function reaches its minimum - can no more be determined analytically; a minimization algorithm for non-linear functions has to be used.

c) Elongation of the constant flux surface contours

From Fig. 5 we can see that, at the opposite side of the X point, the constant flux surface contours present a too flat structure in comparison to the real separatrix. To improve in this sense the form of the theoretical curve, on the same $\eta = 0$ axis one introduces two currents of the same sign. Noting with d the distance from point $(\xi_r, 0)$ to these two new

currents, located symmetrically at the left and the right of the X point and with δI_2 the intensity of these currents, the additional flux is given by the relation

$$a_d = \delta I_2 \ln(\rho_3 \rho_4) \quad (40)$$

where

$$\rho_3^2 = (\xi - \xi_r + d)^2 + \eta^2, \quad \rho_4^2 = (\xi - \xi_r - d)^2 + \eta^2 \quad (41)$$

Thus, the flux function becomes

$$a = \frac{1}{2} \rho_1^2 + \ln \rho_2^2 + a_a + a_d \quad (42)$$

One might prove that, in this new configuration, the point $(\xi_r, 0)$ remains an X point. At the X point, the slopes of the cast function are given by the relation

$$\eta'_{1,2} = \frac{\pm \left(u^4 - 1 + \frac{\alpha^2}{4} \right)^{1/2} - \frac{\alpha}{2}}{u^2 + 1} \quad (43)$$

where

$$u^2 = \xi_d^2 + \frac{2\delta}{d^2} \quad (44)$$

d) Coordinate space deformation

Investigating the cast functions, we have observed that for $\xi_0 \geq 2.2$ the curve becomes concave near the X point, while the real separatrix is convex there. But for $\xi_0 \simeq 2$, the angle of the corner is less than $\pi/3$, while the same angle of the considered separatrix is greater than $\pi/2$; thus, a space deformation along the η coordinate was necessary.

Let p_{1e} , p_{2e} and θ_e , respectively represent the two slopes and the angle at the X point of the real separatrix

$$\theta_e = \arctan p_{1e} - \arctan p_{2e} \quad (45)$$

From equation (43), for the calculated $p_{1,2}$ slopes one has the relation

$$p_1 p_2 = -\frac{u^2 - 1}{u^2 + 1} \quad (46)$$

Scaling now the η coordinate with the factor κ_η , to obtain the real angle θ_e , the following relation has to be satisfied

$$\theta_e = \arctan(\kappa_\eta p_1) - \arctan(\kappa_\eta p_2) \quad (47)$$

or

$$\frac{\kappa_\eta(p_1 - p_2)}{1 + \kappa_\eta^2 p_1 p_2} = \tan \theta_e \quad (48)$$

Thus, for the scaling factor κ_η one obtains the two solutions

$$\kappa_\eta = \frac{-\frac{\left(u^4 - 1 + \frac{\alpha^2}{4}\right)^{1/2}}{\tan \theta_\epsilon} \pm \left(\frac{u^4 - 1 + \frac{\alpha^2}{4}}{\tan^2 \theta_\epsilon} + u^4 - 1\right)^{1/2}}{u^2 - 1} \quad (49)$$

retaining only the positive one.

e) Determination of the cast function f_A

From equation (7), it is possible to see that the calculation of our metric coefficients is easier if the cast function has the explicit form

$$f_A = \rho(a, \omega) \quad (50)$$

where ω is some polar angle representing the angular gap between the vector radius ρ (characterizing the distance between the "flux center" - the magnetic axis - located at (ξ_c, η_c) and the current point on a constant flux surface contour) and the axis of the considered currents distribution. In this model, the f_A function can be put in the form of an implicit (with respect to the independent variables a and ω) transcendental equation only.

Making the following notations

$$\xi = \xi_c + f_A^{1/2} \cos \omega, \quad \eta = \eta_c + f_A^{1/2} \sin \omega \quad (51)$$

$$\rho_1^2 = \xi^2 + \eta^2, \quad \rho_2^2 = (\xi - \xi_0)^2 + \eta^2 \quad (52)$$

$$\rho_3^2 = (\xi - \xi_d + d)^2 + \eta^2, \quad \rho_4^2 = (\xi - \xi_d - d)^2 + \eta^2 \quad (53)$$

$$a_a = \eta((\xi - \xi_d)(\alpha + \beta(\xi - \xi_d) + \gamma\eta^2)) \quad (54)$$

the flux function may be written as

$$a(f_A, \omega) = \frac{1}{2} (\rho_1^2 + \ln \rho_2^2 + \delta \ln(\rho_3^2 \rho_4^2) + a_a) \quad (55)$$

For a given flux a_i , at constant angle ω , the cast function f_A is obtained by solving the following transcendental equation

$$F(f_A) = a(f_A, \omega) - a(0, \omega) - a_i = 0 \quad (56)$$

To correlate now these results with a real separatrix curve, given by (r, z) points, the following algorithm has to be followed :

- from the input data one determines the θ_e angle at the X point;
- one chooses an initial slope $|p| \leq 0.2$ for a symmetrical configuration;
- one gives such initial values to the parameters α, β, γ, d and κ_η as to describe a curve in the system of coordinates $(\xi, \kappa_\eta \eta)$ with the same angle θ_e at the X point and having the flux center located at $(\xi_c, \kappa_\eta \eta_c)$;
- one scales the abscissa of the cast function in order to obtain the same abscissae of the distance flux center- X point for both curves;
- modifying now the parameters, one realizes the superposition of the ordinates of the flux centers.

V. 2nd CAST FUNCTIONS MODEL

In the following, a cast function f_B , easier to use for the metric coefficients calculation, has been obtained just through geometrical considerations.

Let us consider the ASDEX-Upgrade separatrix configuration given in Fig. 6, with the origin of the coordinates system at the magnetic axis. For the beginning, we will write the equation of a separatrix characterized by the following parameters only : a_s the distance from the magnetic axis to the X point, a_c the distance from the magnetic axis to the intersection of the OX line with the separatrix, $\pi - \phi_0 - \phi_1$ the angle at the X point and θ_0 the angle between the vector radius connecting the magnetic axis and the X point with the z axis

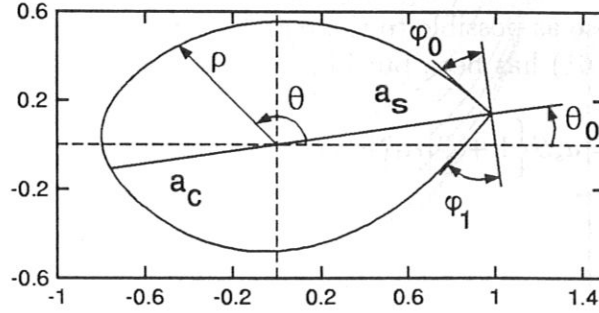


Figure 6: Geometrical parameters for the f_B cast function calculation.

$$\rho(a, \theta) = a_s \left[1 + \alpha_1(1 - \cos t) + \beta_1 \sin t - \alpha_0(1 - \cos t)^{1/2} \right] \quad (57)$$

with $t = \theta - \theta_0$ and $\omega = \theta - \pi/2$.

It is easy to find the relations connecting the above-mentioned parameters with the coefficients α_0, α_1 , and β_1

$$\alpha_0 = \frac{\tan \phi_0 + \tan \phi_1}{\sqrt{2}} \quad (58)$$

$$\beta_1 = \frac{-\tan \phi_0 + \tan \phi_1}{2} \quad (59)$$

$$\alpha_1 = \frac{\alpha_0}{\sqrt{2}} - \frac{a_s - a_c}{2a_s} \quad (60)$$

To give some degrees of freedom in order to identify a curve close to the real separatrix, but satisfying the values of the above-mentioned four parameters, equation (57) can be put in the form

$$\rho(a_s, \theta) = a_s \left[1 + \alpha_1(1 - \cos t) + \beta_1 \sin t + \beta \left(\sin t - \frac{1}{2} \sin 2t \right) - \alpha_0(1 - \cos t)^{1/2} + \sin^2 t (x_0 + x_1 \cos t + x_2 \cos 2t + \dots + y_1 \sin t + y_2 \sin 2t + \dots) \right] \quad (61)$$

with the coefficients $\beta, x_0, x_2, \dots, y_1, y_2, \dots$ to be determined by a least squares method in order to be "close" with our curve to the real separatrix.

An alternative form that we have investigate is given by the relation

$$\rho^2(a_s, \theta) = a_s^2 \left[1 + \alpha_1(1 - \cos t) + \beta_1 \sin t + \beta \left(\sin t - \frac{1}{2} \sin 2t \right) - \alpha_0(1 - \cos t)^{1/2} + \sin^2 t (x_0 + x_1 \cos t + x_2 \cos 2t + \dots + y_1 \sin t + y_2 \sin 2t + \dots) \right] \quad (62)$$

Using equations (61) or (62), the real separatrix curve can be reproduced with an accuracy depending on the number of determined coefficients. To represent now the internal constant flux surface contours, using the same cast functions, in such a way that these curves have to be as close as possible to the real constant flux distribution of the considered equilibrium, equation (61) has been put in a very flexible form

$$\rho(a, \theta) = a_s S \left\{ 1 + S \left[\alpha_1(1 - \cos t) + \beta_1 \sin t + \beta \left(\sin t - \frac{1}{2} \sin 2t \right) - \alpha_0 \left(\gamma^2 \frac{(a - a_s)^2}{a_s^2} + 1 - \cos t \right)^{1/2} + \gamma^2 \frac{(a - a_s)^2}{a_s^2} \sin^2 t (x_0 + x_1 \cos t + x_2 \cos 2t + \dots + y_1 \sin t + y_2 \sin 2t + \dots) \right] \right\} \quad (63)$$

where, S is a function of a and may have one of the following kinds of dependence

$$S_1(a) = \frac{3}{2} \frac{a}{a_s} - \frac{1}{2} \left(\frac{a}{a_s} \right)^3 \quad (64)$$

$$S_2(a) = 2 \left(\frac{a}{a_s} \right)^2 - \left(\frac{a}{a_s} \right)^4 \quad (65)$$

$$S_2(a) = \frac{3}{2} \left(\frac{a}{a_s} \right)^3 - \frac{1}{2} \left(\frac{a}{a_s} \right)^9 \quad (66)$$

and γ is a free parameter.

Accordingly, equation (62) has been put in the form

$$\begin{aligned} \rho^2(a, \theta) = a^2 & \left[1 + \alpha_1(1 - \cos t) + \beta_1 \sin t + \beta(\sin t - \frac{1}{2} \sin 2t) - \right. \\ & \left. \alpha_0 \left(\gamma^2 \frac{(a - a_s)^2}{a_s^2} + 1 - \cos t \right)^{1/2} \right. \\ & \left. + \sin^2 t (x_0 + x_1 \cos t + x_2 \cos 2t + \dots + y_1 \sin t + y_2 \sin 2t + \dots) \right] \end{aligned} \quad (67)$$

In Fig. 7, constant flux surface contours using only the cast function (63) are presented for a $S(a) = S_1(a)$ dependence and $\gamma = 0.5$.

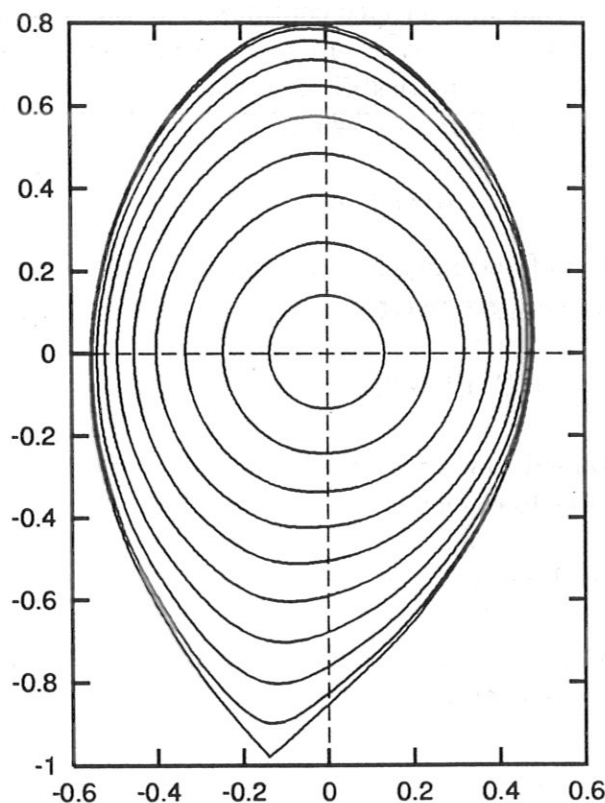


Figure 7: Constant flux surface contours given by the f_B cast function for the ASDEX-Upgrade separatrix configuration given in Appendix; $S(a) = S_1(a)$ and $\gamma = 0.5$.

IV. DISCUSSION

In the present paper, a method for the calculation of the metric coefficients in diverted toroidal configurations is presented. The metric coefficients, to be used in stability calculations (where the potential energy presents itself with the simplest form if it is expressed in a flux coordinate system), have to be calculated by equilibrium codes, taking into

account the real plasma configuration and parameters. Due to the presence of the separatrix, direct use of a flux coordinate system can not be made. To overcome this, the cast functions method has been introduced. The cast functions describe the separatrix with a prescribed accuracy and approximately the internal constant flux surface contours. Using a moment equilibrium solver, the moments to be determined are related to the difference between the real flux surface contours and the surface contours described by the cast function only. At the separatrix, these moments take the boundary values corresponding to the difference between the curve described by the cast function and the separatrix itself; these boundary values are, in general, zero. Such an approach permits to use a reasonable number of moments to describe accurately a quite complicated separatrix configuration and to ensure the time efficiency of our computations.

Two models of cast functions have been considered : if for the first model, deduced from physical considerations, the constant flux surface contours near the magnetic axis are closer to the real a^2 surface dependence, for the second model, a better modelling of the separatrix and of the constant flux surface contours near the separatrix is obtained.

A generalization of the cast functions form, especially for the first model, could be possible. Thus, other current densities distributions could be considered, for example, a parabolic radius dependence as in real plasmas

$$J_1(r) = J_1^0(1 + 4\varepsilon\rho^2) \quad (68)$$

where the same definition of the reference radius r_0 , given in equation (20), has been considered, while ε is a negative parameter. With this current density distribution one obtains

$$B_1(r) = B_0\rho(1 + 2\varepsilon\rho^2); \quad A_1 = \frac{1}{2}\Phi_0\rho^2(1 + \varepsilon\rho^2) \quad (69)$$

The relations to be obtained now are more complicated than in the previous case ($\varepsilon = 0$) but the supplementary freedom degrees could better approximate the constant flux surface contours.

For the second model, an optimization of the $S(a)$ functions given by equations (64) - (66) has to be considered.

As an example of application of our approach, a particular equilibrium configuration of the ASDEX-Upgrade has been considered.

APPENDIX : RELEVANT DATA FOR THE CONSIDERED SHOT

Shot no. : 5000 at time : 1.550 s

Coordinates of the magnetic axis point : $r = 1.685$ m; $z = 0.132$ m

Coordinates of the X point : $r = 1.546$ m; $z = -0.848$ m

Separatrix curve described by the following 155 (r,z) points (in meters) :

1.5455	-0.8481	1.5323	-0.8331	1.4976	-0.7924	1.4671	-0.7548	1.4386	-0.7188
1.4121	-0.6838	1.3874	-0.6497	1.3644	-0.6165	1.3429	-0.5841	1.3230	-0.5523
1.3044	-0.5212	1.2872	-0.4907	1.2712	-0.4608	1.2564	-0.4314	1.2427	-0.4025
1.2300	-0.3741	1.2183	-0.3462	1.2075	-0.3188	1.1976	-0.2918	1.1886	-0.2652
1.1802	-0.2390	1.1727	-0.2132	1.1658	-0.1877	1.1597	-0.1625	1.1541	-0.1377
1.1492	-0.1131	1.1449	-0.0889	1.1412	-0.0648	1.1380	-0.0410	1.1353	-0.0174
1.1332	0.0060	1.1315	0.0293	1.1304	0.0524	1.1297	0.0754	1.1295	0.0982
1.1297	0.1210	1.1304	0.1437	1.1316	0.1664	1.1333	0.1890	1.1354	0.2116
1.1380	0.2343	1.1411	0.2569	1.1446	0.2796	1.1487	0.3023	1.1533	0.3252
1.1583	0.3481	1.1639	0.3711	1.1701	0.3943	1.1769	0.4176	1.1843	0.4410
1.1924	0.4646	1.2011	0.4884	1.2106	0.5123	1.2208	0.5363	1.2319	0.5605
1.2438	0.5848	1.2567	0.6092	1.2704	0.6337	1.2852	0.6582	1.3011	0.6827
1.3180	0.7071	1.3362	0.7314	1.3555	0.7555	1.3762	0.7791	1.3982	0.8022
1.4216	0.8245	1.4465	0.8457	1.4728	0.8656	1.5005	0.8836	1.5296	0.8994
1.5600	0.9122	1.5914	0.9215	1.6236	0.9269	1.6562	0.9280	1.6887	0.9247
1.7208	0.9173	1.7521	0.9062	1.7823	0.8920	1.8113	0.8753	1.8388	0.8566
1.8649	0.8363	1.8895	0.8149	1.9127	0.7927	1.9345	0.7700	1.9549	0.7469
1.9741	0.7237	1.9921	0.7003	2.0089	0.6769	2.0247	0.6536	2.0393	0.6303
2.0530	0.6071	2.0657	0.5841	2.0775	0.5612	2.0884	0.5384	2.0985	0.5158
2.1078	0.4934	2.1162	0.4711	2.1238	0.4489	2.1307	0.4270	2.1370	0.4052
2.1425	0.3836	2.1473	0.3622	2.1516	0.3409	2.1552	0.3199	2.1583	0.2990
2.1608	0.2783	2.1628	0.2578	2.1643	0.2374	2.1653	0.2172	2.1658	0.1971
2.1659	0.1772	2.1655	0.1574	2.1647	0.1377	2.1635	0.1181	2.1618	0.0986
2.1597	0.0792	2.1572	0.0598	2.1543	0.0404	2.1510	0.0211	2.1472	0.0018
2.1431	-0.0175	2.1385	-0.0368	2.1335	-0.0562	2.1281	-0.0757	2.1222	-0.0952
2.1159	-0.1148	2.1091	-0.1345	2.1018	-0.1544	2.0939	-0.1744	2.0856	-0.1946
2.0767	-0.2149	2.0672	-0.2355	2.0571	-0.2564	2.0463	-0.2774	2.0349	-0.2988
2.0227	-0.3205	2.0098	-0.3425	1.9960	-0.3649	1.9814	-0.3877	1.9658	-0.4109
1.9492	-0.4345	1.9316	-0.4586	1.9127	-0.4832	1.8927	-0.5084	1.8712	-0.5341
1.8484	-0.5605	1.8239	-0.5875	1.7978	-0.6153	1.7698	-0.6439	1.7398	-0.6735
1.7075	-0.7042	1.6729	-0.7363	1.6354	-0.7700	1.5948	-0.8059	1.5608	-0.8352

These data have been obtained by the equilibrium interpretation code DIVA⁸

REFERENCES

- ¹ L.M.Degtyarev, V.V.Drozdo, M.I.Mikhailov, V.D.Pustovitov, and V.D.Shafranov, *Fiz. Plazmy*, **9**, 22 (1985) [*Sov. J. Plasma Phys.* **11**, 39 (1985)].
- ² W.D.D'haeseleer, W.N.G.Hitchon, J.D.Callen, and J.L.Shohet, *Flux Coordinates and Magnetic Field Structure, A Guide to a Fundamental Tool of Plasma Theory* (Springer-Verlag, Berlin, 1991).
- ³ A.B.Rechester and T.H.Stix, *Phys. Rev. Lett.* **36**, 587 (1972).
- ⁴ A.H.Boozer and A.B.Rechester, *Phys. Fluids* **21**, 682 (1978).
- ⁵ S.P.Auerbach and A.L.Boozer, *Phys. Fluids* **23**, 2396 (1980).
- ⁶ L.E.Zakharov, A.H.Glasser, 15th International Conference on the Numerical Simulation of Plasmas, (1994)
- ⁷ N.Mattor, *Phys. Plasmas* **2**, 594 (1995).
- ⁸ H.P.Zehrfeld, private communication.
- ⁹ C.V.Atanasiu, A.A.Subbotin (to be published).