MAX-PLANCK-INSTITUT FŪR PLASMAPHYSIK

GARCHING BEI MŪNCHEN

Ion Resistivity Matrix of a tokamak plasma

Riccardo Croci

IPP 6/331 June 1995

Die nachstehende Arbeit wurde im Rahmen des Vetrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt

Abstract

The relation between the Fourier transform (in the poloidal and toroidal coordinates) of the ion current density and the electric field of a Vlasov plasma in a tokamak is a matrix – the resistivity matrix – for which no approximated form appropriate for analytical and numerical problems yet exists. Here an approximated form is deduced by assuming that a Fourier component of the ion current is given, and then solving for the electric field the first moment of the Vlasov equation. The matrix elements thus obtained have a simple form suitable for the applications.

Introduction

The relation between the ion current density and the electric field of a Vlasov plasma in a tokamak, although first derived long ago (Cattanei & Croci, 1976), and then rederived many times, has not yet received the attention necessary to give it an approximated form suitable for analytical and numerical problems. Usually, the Fourier transform in the poloidal and toroidal coordinates of the fields are used; in this case the relation (sometimes called constitutive relation) is a matrix — the ion resistivity matrix — that becomes tridiagonal when the ion thermal velocity v_t is zero. The elements of the resistivity matrix are given in Cattanei & Murphy, 1991, as series of Bessel functions whose indices and arguments depend on the parameters. The authors use it only when a symmetry condition (the resonance surface $\omega = \Omega$, where Ω is the local ion cyclotron frequency, goes through the plasma centre) allows a drastic approximation. Otherwise the complexity of the representation makes it not very useful in practice, also for numerical problems (cited paper and private communication). More usual is a local approximation (see, for example, Brambilla & Krücken, 1988), which avoids the difficulties for the price of restricted validity.
Here an approximated form of the ion resistivity matrix is deduced by assuming that a Fourier component of the ion current is given, and then solving for the electric field the first moment of the Vlasov equation. In this procedure a function, G in the text, plays an important role; partial results on the approximated form of G can be found in the literature (Faulconer, 1987); however, they are not used to derive the elements of the resistivity matrix. This paper generalizes the previous results by a different method; the approximations, which cover all the range of the poloidal and toroidal wave numbers, are then used to derive the resistivity matrix. Some insight into the range of validity of the approximations and into the form of the current is reached by considering how particles that are before or beyond the value of θ for which their resonance condition is fulfilled contribute to the current. The approximated matrix elements thus obtained have a simple form suitable for both analytical and numerical problems.

1. Resistivity due to an elementary ion current

For our purposes only the contribution of the ions to f_1 need be considered since the relation between electron current and electric field is well known; thus, in the expression

$$f_1 = f_M \frac{e}{mv_t^2} \int_{-\infty}^{0} \left[v_x(t') E_x(\theta(t'), t+t') + v_y(t') E_y(\theta(t'), t+t') \right] dt' \tag{1}$$

m is the ion mass; f_M is the Maxwellian velocity equilibrium distribution function. The quantities in the integral of (1) are the projections of the vectors on the axes of an orthogonal system. The toroidal plasma is approximated by a straight cylinder; it is postulated that all quantities are periodic in z, with period $2\pi R$. The toroidal effects considered are those due to a magnetic field described by $\Omega(\theta) \equiv \omega + \Omega_o r/R(\cos\theta - \cos\theta_r)$, with $\omega = \Omega_o(1 - (r/R)\cos\theta_r)$. The value of the magnetic field encountered by an ion at a point (r, θ) is approximated by the value at the gyrocentres: this should be sufficient for the derivation of approximations of the resistivity. Moreover, when $r \ll R$ trapped or quasi–trapped particles can be neglected. Then for the gyrocentres $\theta(t') \equiv \theta + (v_z/qR)t'$. Along the characteristics one has

$$v_x(t') = v_x \cos N(t') - v_y \sin N(t'), \quad v_y(t') = v_x \sin N(t') + v_y \cos N(t'), \tag{3}$$

where $N(t') \equiv \int_0^{t'} \Omega(t')\, dt'$. With $s \equiv v_t t'/qR$, N is evaluated by expanding $\theta(s)$ for $s \ll 1$. (a procedure equivalent to the assumption that the resistivity, although not local, is independent of the value of the fields at a poloidal distance comparable to r; since for $t' = 1/\Omega$ one has $s = \rho/qR \ll 1$, in the time interval considered the ions gyrate many times). The result is (with $\alpha \equiv qr/\rho$)

$$N(s) = (qR/v_t)\big(\Omega(\theta)s + (v_z/2v_t)\Omega'(\theta)s^2\big)$$

$$= \omega(R/\Omega_o r)\alpha s + (\cos\theta - \cos\theta_r)\alpha s - \sin\theta\, (v_z/v_t)\alpha s^2/2\,. \tag{2}$$

Suppose that the elementary ion current

$$j_{px} \sin(\omega t - p\theta - k_z z), \quad j_{py} \cos(\omega t - p\theta - k_z z), \tag{4}$$

is given; our problem is then to deduce from (1) an approximated form of the electric field. Let us try with the ansatz

$$E_x = \big(\mathcal{E}^+(\theta) + \mathcal{E}^-(\theta)\big)\cos(\cdot), \quad E_y = \big(\mathcal{E}^+(\theta) - \mathcal{E}^-(\theta)\big)\sin(\cdot), \tag{5}$$

where $\mathcal{E}^+$ is the coefficient of the electric field component that rotates as the ions. The results for the complex representation of the electric field are given in the next section.

With (3), (4) and (5) the integrand of (1) becomes (with the notations $n_\parallel \equiv p + k_z qR$ and $P \equiv \omega(R/\Omega_o r)\alpha s - (v_z/v_t)n_\parallel s$) :

$$\Big[(\mathcal{E}^+ + \mathcal{E}^-)(v_x \cos N - v_y \sin N)\cos P + (\mathcal{E}^+ - \mathcal{E}^-)(v_x \sin N + v_y \cos N)\sin P\Big]\cos(\cdot) +$$

$$+ \Big[(\mathcal{E}^+ + \mathcal{E}^-)(-v_x \cos N + v_y \sin N)\sin P + (\mathcal{E}^+ - \mathcal{E}^-)(v_x \sin N + v_y \cos N)\cos P\Big]\sin(\cdot)\,. \tag{6}$$

Hence the coefficients of $\cos(\omega t - p\theta)$ and of the sinus of the same argument are

$$v_x\big(\mathcal{E}^+\cos(N-P) + \mathcal{E}^-\cos(N+P)\big) - v_y\big(\mathcal{E}^+\sin(N-P) + \mathcal{E}^-\sin(N+P)\big)\,,$$

$$v_x\big(\mathcal{E}^+\sin(N-P) - \mathcal{E}^-\sin(N+P)\big) + v_y\big(\mathcal{E}^+\cos(N-P) - \mathcal{E}^-\cos(N+P)\big)\,. \tag{7}$$

With $v \equiv v_z/v_t$ and $A \equiv \cos\theta - \cos\theta_r$ one has

$$N \pm P = \Big[A + (1 \pm 1)\omega R/\Omega_o r + (A's/2 \mp (p + k_z qR)/\alpha)v\Big]\,\alpha s\,. \tag{8}$$

After the obvious integrations over v_x and v_y the components of the ion current that follow from (1) are (the remaining integration over v is not indicated)

$$j_{px}\sin(\cdot) = \frac{e^2 nqR\, e^{-v^2}}{4\pi^{1/2} m v_t}\int_{-\infty}^{0}\Big[\mathcal{E}^+\big(\theta + (v_z/v_t)s\big)\big(\cos(N-P)\cos(\cdot) + \sin(N-P)\sin(\cdot)\big) +$$

$$+\mathcal{E}^-\big(\theta + (v_z/v_t)s\big)\big(\cos(N+P)\cos(\cdot) - \sin(N+P)\sin(\cdot)\big)\Big]\,ds\,,$$

$$j_{py}\cos(\cdot) = \frac{e^2 nqR\, e^{-v^2}}{4\pi^{1/2} m v_t}\int_{-\infty}^{0}\Big[\mathcal{E}^+\big(\theta + (v_z/v_t)s\big)\big(-\sin(N-P)\cos(\cdot) + \cos(N-P)\sin(\cdot)\big) -$$

$$-\mathcal{E}^-\big(\theta + (v_z/v_t)s\big)\big(\sin(N+P)\cos(\cdot) + \cos(N+P)\sin(\cdot)\big)\Big]\,ds\,. \tag{9}$$

These equations are equivalent to (again without the integration signs):

$$e^{-v^2}\big[-\mathcal{E}^-\sin(N+P) + \mathcal{E}^+\sin(N-P)\big] = v_t C j_{px}\,,$$

$$e^{-v^2}\big[\mathcal{E}^-\cos(N+P) + \mathcal{E}^+\cos(N-P)\big] \ll v_t C|j_{px}|\,,$$

$$e^{-v^2}\big[-\mathcal{E}^-\sin(N+P) - \mathcal{E}^+\sin(N-P)\big] = v_t C j_{py}\,,$$

$$e^{-v^2}\big[\mathcal{E}^-\cos(N+P) + \mathcal{E}^+\cos(N-P)\big] \ll v_t C|j_{py}|\,, \tag{10}$$

where $C \equiv 16\pi^{3/2}/qR\,\omega_{pi}^2$. Equations (10) can be written in the simpler form

$$e^{-v^2}\mathcal{E}^\pm(\theta + vs)\sin(N \mp P) = (\pm j_{px} - j_{py})v_t C\,,$$

$$e^{-v^2}|\mathcal{E}^{\pm}(\theta+vs)\cos(N\mp P)| \ll |(j_{px},j_{py})|v_t C\,. \tag{11}$$

An approximation for $\mathcal{E}$ is obtained (in the spirit of the initial expansion in powers of s) by writing $\mathcal{E}(\theta+vs)\approx\mathcal{E}(\theta)+vs\,\mathcal{E}'(\theta)$. It is now appropriate to define the following functions:

$$F \equiv \int_{-\infty}^{0} \exp\left(-ias - i\alpha A' vs^2/2\right) ds\,, \tag{12}$$

where a is the part proportional to s in $N\pm P$;

$$\int_{-\infty}^{0} vs\,\sin(as + A'vs^2/2)\,ds = -\frac{\partial F_R}{\partial n_\parallel}\,; \tag{13}$$

and finally $G \equiv \int F\,dv$. The first of (11) then yields

$$G_I\,\mathcal{E} \pm \frac{\partial G_R}{\partial n_\parallel}\,\mathcal{E}' = (\pm j_{px} - j_{py})v_t C\,. \tag{14}$$

A form of G useful in the following is obtained by first integrating over v, leaving an integration over s (upper sign for $\mathcal{E}^+$) :

$$G = \pi^{1/2}\int_{-\infty}^{0} e^{-ibs}\exp\left(-(\pm n_\parallel + \alpha A' s/2)^2 s^2/4\right) ds\,, \tag{15}$$

where $b \equiv (A + 2\omega R/\Omega_o r)\alpha$ for $\mathcal{E}^-$, or $b\equiv \alpha A$ for $\mathcal{E}^+$.
The function G is particularly simple for $\mathcal{E}^-$ because it is (almost) independent of θ for all $n_\parallel$; then (14) yields the result (already known to the lowest order in v_t) :

$$G_I\,\mathcal{E}^- = -(j_{px} + j_{py})v_t C\,. \tag{16}$$

Approximations for G_I are easily obtained from (15) by partial integrations (b is the dominant parameter).
The function G for $\mathcal{E}^+$ has the symmetry properties $G_I(-A) = -G_I(A)$ and $G_R(-A) = G_R(A)$. Since far from θ_r one has $|G_I| \gg |\partial G_R/\partial n_\parallel|$, but $G_I(\theta_r)=0$, one immediately obtains for $\mathcal{E}^+$ the approximation far from θ_r

$$\mathcal{E}^+ \approx \frac{(j_{px}-j_{py})v_t C}{G_I}\,, \tag{17}$$

and in the neighbourhood of θ_r

$$\mathcal{E}^+ \approx \frac{(j_{px}-j_{py})v_t C}{\partial G_R/\partial n_\parallel}\,(\theta-\theta_R)\,. \tag{18}$$

The discussion that follows is mainly based on known properties of the function F that are now summarized. First, F is given by

$$F = \frac{-i\zeta}{a} Z(\zeta), \tag{19}$$

where Z is the plasma dispersion function and $\zeta \equiv a/2(i\alpha A'v/2)^{1/2}$.
When $a^2 \gg 2\alpha|A'v|$ one has

for $\zeta_I > 0$: $F \approx i/a - \alpha A'v/a^3$;

for $\zeta_I < 0$: $F \approx i/a - \alpha A'v/a^3 + (2\pi^{1/2}\zeta)/a \exp(-\zeta^2)$;

when $a^2 \ll 2\alpha|A'v|$ one has

$$F \approx a/\alpha A'v + (\pi^{1/2}\zeta/a)\exp(-\zeta^2). \tag{20}$$

The easiest way to determine the sign of ζ_I is to note that an exponential term appears in the asymptotic form of F when $\zeta_I < 0$. This term arises when the integrand of (12) has a point of stationary phase; since such a point is given by the condition $s = -a/\alpha A'v$, and since $s < 0$, one finds that $\zeta_I < 0$ when $a/\alpha A'v > 0$. Equivalent to this is the remark that $a = 0$ determines the value of θ for which a particle is at resonance with the field (velocity and other parameters being given), and that $a/A'v > 0$ characterizes the region after the resonance. By taking these results into account, ζ for $\mathcal{E}^+$ can be conveniently written as

$$\zeta \equiv \left(1 - i(A'v/|A'v|)\right) \frac{n_\parallel v + \alpha A}{2|A'v|^{1/2}}. \tag{21}$$

We are now in a position to discuss the contribution of a group of ions to the current, without explicitly evaluating G. Consider the group of particles that meet the condition $|\zeta| < 1$ (resonant particles), for definiteness with $v > 0$ and $n_\parallel < 0$; when the interval (v_1, v_2) which defines this group is such that $v_1 \ll 1$ and $(v_2 - v_1) \gtrsim 1$, the resonant particles determine the current due to $v > 0$. For the ions moving toward θ_r (i.e. those with $vA > 0$), the equation $(n_\parallel v + \alpha A)^2 = 2\alpha|A'|v$ shows under what conditions the resonant particles determine the ion current in the corresponding direction:

for $n_\parallel = 0$: $v_1 = \alpha A^2/2|A'|$ and $v_2 \to \infty$; hence only where $\alpha A^2 < |A'|$;

$v_2 = 1$ for $n_\parallel = n_1 \equiv \alpha A - (2\alpha|A'|)^{1/2}$; then $v_1 = \alpha^2 A^2/n_1^2$, and therefore only where $\alpha A^2 < |A'|$.

for $|n_\parallel A| \gg |A'|$: $v_1 \approx -\alpha A/n_\parallel$; $v_2 - v_1 \approx 2\alpha(2A|A'|)^{1/2}/|n_\parallel|^{3/2}$; this is never possible.

Hence, the resonant particles moving toward θ_r determine the ion current in the corresponding direction where $\alpha A^2 < |A'|$ (the resonance zone), if $0 \leq |n_\parallel| \leq n_1$. The equation for the particles with $|\zeta| < 1$ coming from θ_r $(vA < 0)$ is $(n_\parallel v + \alpha A)^2 = 2\alpha|A'|v$;

$|\zeta|$ reaches its minimum $2n_{\|}A/|A'|$ for $v = \alpha A/n_{\|}$. Hence it must be $|n_{\|}| < |A'/2A|$. A discussion similar to the previous one yields the same result, namely that the particles with $|\zeta| < 1$ determine the ion current only in the resonance zone $\alpha A^2 < |A'|$ and if, moreover, $|n_{\|}| \lesssim (2\alpha|A'|)^{1/2}$. The ion current is determined by the non-resonant ions $(|\zeta| > 1)$ either outside the resonance zone or, for $|n_{\|}| \gtrsim (2\alpha|A'|)^{1/2}$, in the resonant zone.

The expression to be evaluated is

$$G = -i \int_{-\infty}^{\infty} e^{-v^2} \frac{\zeta Z(\zeta)}{\alpha A + n_{\|} v} \, dv \,, \tag{22}$$

with ζ given by (21). The previous discussion allows us to use the approximation for $|\zeta| < 1$, where $\alpha A^2 < 1$, if $|n_{\|}| < (2\alpha A')^{1/2}$. Then (22) becomes

$$\begin{aligned} G &\approx \pi^{1/2} \int e^{-v^2} \frac{1 - iA'v/|A'v|}{2|\alpha A'v|^{1/2}} \, dv + \int e^{-v^2} \frac{\alpha A + n_{\|} v}{\alpha A' v} \\ &\to \frac{\pi^{1/2}}{\alpha|A'|} \int_0^{\infty} e^{-v^2} \frac{dv}{v^{1/2}} + \frac{n_{\|}}{\alpha A'} \int e^{-v^2} \, dv \end{aligned} \tag{23}$$

or

$$G \approx \frac{\pi^{1/2}\Gamma(1/4)}{4(\alpha|A'|)^{1/2}} + \frac{\pi^{1/2} n_{\|}}{\alpha A'} \,. \tag{24}$$

Thus the quantity needed in (18) is

$$\left(\frac{\partial G_R}{\partial n_{\|}}\right)_{A=0} = \frac{\pi^{1/2}}{\alpha A'} \,, \tag{25}$$

which is proportional to v_t and independent of $n_{\|}$.

The approximation for G in the resonance zone for $n_{\|} > (2\alpha A')^{1/2}$, and outside the resonance zone for every $n_{\|}$, is obtained with the asymptotic approximation for the function Z, integrated over the appropriate interval of v, defined as follows: let (for definiteness) $A' < 0$; then $\zeta_I < 0$ in the intervals

when $n_{\|} > 0$: outside $(-\alpha A/n_{\|}, 0)$, or $(0, -\alpha A/n_{\|})$;

when $n_{\|} < 0$: inside $(-\alpha A/n_{\|}, 0)$, or $(0, -\alpha A/n_{\|})$.

Since the term $-1/\zeta$ in the expansion of Z is present for both signs of ζ_I, the approximation is (with obvious symbols)

$$G \approx \frac{\pi^{1/2}}{2(\alpha|A'|)^{1/2}} \not\int \frac{e^{-v^2}}{v^{1/2}} (1 - iv/|v|) \exp(-\zeta^2) \, dv - i \int_{-\infty}^{\infty} \frac{e^{-v^2}}{n_{\|} v - \alpha A} \, dv \,. \tag{26}$$

Clearly the last term in (26) is $-i(\pi^{1/2}/n_\parallel)Z(\alpha A/n_\parallel)$. Outside the resonance zone the first term (which is due to the ions that have already passed the resonance) is exponentially small. Where $\alpha A^2 < 1$ the integration interval for $n_\parallel < 0$ disappears, whereas for $n_\parallel > 0$ all particles contribute. Then the first term becomes

$$\frac{(1+i)}{(2\alpha A')^{1/2}} \int_0^\infty \frac{e^{-v^2}}{v^{1/2}} \exp\left(i{n_\parallel}^2 v/2\alpha A'\right) dv \; + c.c. \; \approx 2\pi/n_\parallel \, . \tag{27}$$

The quantity needed in (18) is now

$$\left(\frac{\partial G_R}{\partial n_\parallel}\right)_{A=0} = -\frac{3\pi}{{n_\parallel}^2} \qquad (-\pi/{n_\parallel}^2 \; if \;\; n_\parallel A' < 0)\, , \tag{28}$$

which is independent of v_t.
The results deduced up to now generalize those obtainable from (15) and partly given in Faulconer, 1987; the expression

$$\left(\frac{\partial G_R}{\partial n_\parallel}\right)_{A=0} = \frac{\pi^{1/2}}{\alpha A'}\, , \tag{25}$$

immediately follows when $|n_\parallel| \ll 1$, whereas the corresponding result (24) was deduced under more general conditions. An approximation valid when $|n_\parallel|$ is sufficiently large is obtained by taking into account that the second exponential function in (15) has two maxima: in $s = 0$ and in $s = -2n_\parallel/\alpha A'$ (the last one only if $0 < 2n_\parallel/\alpha A' \ll 1$ because of the initial hypothesis on s). They result from the equality of the reciprocal phases of ions and electric field at the times $t' = t$ and $t' = t - 2n_\parallel/\alpha A'$. Since the minimum of the exponential function between the two maxima (when they exist) is $\exp\left({n_\parallel}^4/16\alpha^2 A'^2\right)$, the two maxima are well separated if ${n_\parallel}^2 \gtrsim 4\alpha|A'|$. The integral can then be evaluated as follows (the second integral appearing only if $0 < 2n_\parallel/\alpha A' \ll 1$) :

$$G \approx \pi^{1/2} \int_{-\infty}^{0} e^{-i\alpha A s} \exp\left(-{n_\parallel}^2 s^2/4\right) ds + \pi^{1/2} \int_{-\infty}^{\infty} e^{-i\alpha A s} \exp\left(-{n_\parallel}^2 (s - 2n_\parallel/\alpha A')^2/4\right) ds$$
$$\rightarrow -\frac{i\pi^{1/2}}{n_\parallel} Z(\alpha A/n_\parallel) + \frac{2\pi}{n_\parallel} e^{-2in_\parallel A/A'} e^{-\alpha^2 A^2/{n_\parallel}^2} \, . \tag{30}$$

The first term in (30) can be obtained directly from (15) for $A' = 0$, i.e. if the resistivity is considered as a local quantity. Again, (30) is in accord with (26) and (27), which were derived under more general conditions.

E R R A T A

p. 7: *instead of the equation number* (25) *read* (29)

p. 8: *equation (33) should read*

$$\frac{2isv_t\, e^{is\theta_r}}{3\alpha^{3/2}\left(\partial G_R/\partial n_{\parallel}\right)_{\theta_r}} .$$

p. 8: *the second line of equation (34) should read*

$$+\frac{2isv_t\, e^{is\theta_r}}{3\alpha^{5/2}\left(\partial G_R/\partial n_{\parallel}\right)_{\theta_r}}+\frac{2isv_t\, e^{-is\theta_r}}{3\alpha^{5/2}\left(\partial G_R/\partial n_{\parallel}\right)_{-\theta_r}}\Bigg] \qquad (34)$$

2. Fourier components

The complex representation of the field is obtained by noting that the addition of $\pi/2$ to the phase of the circular functions in (4) does not change $\mathcal{E}_p^{\pm}(\theta)$. An appropriate linear combination of the solutions obtained in this way yields the following form of the electric field due to the ion current: $j^+(\theta) = \sum_p j_p^+ \exp\left(i(\omega t - p\theta - k_z z)\right)$ (with the substitution ij_{px} for j_{px} in order to have a neater representation):

$$E_x^{\pm} = 2\mathcal{E}^{\pm} \exp\left(i(\omega t - p\theta - k_z z)\right).$$

Here $\mathcal{E}^{\pm}$ is given by the equations obtained before with the substitutions ij_p^+ for $(j_{px} - j_{py})$, and ij_p^- for $-(j_{px} + j_{py})$.

The Fourier components of $\mathcal{E}^+$ can be evaluated as the sum of two parts:

1) The contribution of (17); for $|n_\parallel| < \alpha|A|$ from (26) one has

$$\frac{\mathcal{E}_{ps}^+}{(j_{px} - j_{py})C} \approx \frac{v_t\alpha}{\pi^{1/2}} ⨍ e^{is\theta} A\, d\theta, \tag{31}$$

where the integration interval does not include the resonance zones $\theta \approx \pm\theta_r$. When the integral is extended to $(0, 2\pi)$ (as required when $v_t = 0$), the result of the integration is temperature independent (as it must be) because it is proportional to αv_t; it contains the terms $s = 0, \pm 1$ only. The correction to be subtracted is (when $s \ll \alpha^{1/2}$ and for $\theta \approx \theta_r$)

$$\frac{\alpha v_t}{2} \int_{\theta_r - \epsilon}^{\theta_r + \epsilon} e^{is\theta}(\cos\theta - \cos\theta_r)\, d\theta \to -\frac{\alpha v_t}{4}\epsilon^2 \sin\theta_r\, e^{is\theta_r} \to \frac{v_t}{4} e^{is\theta_r}. \tag{32}$$

For $|s| > \alpha^{1/2}$ it decreases as $\alpha v_t / s$.

2) The contribution due to the effective value of the electric field in the resonance zone, given by (18); that is, for $|s| \ll \alpha^{1/2}$ and for $\theta \approx \theta_r$,

$$\frac{v_t\, e^{is\theta_r}}{\alpha\left(\partial G_R/\partial n_\parallel\right)_{\theta_r}}. \tag{33}$$

This contribution is proportional to v_t when ${n_\parallel}^2 < 2\alpha|A'|$; otherwise, with (28) it is proportional to v_t^2 and larger than the correction deduced in 1).

These results together with the contribution of $-\theta_r$ yield the required ion resistivity matrix

$$\begin{aligned}\mathcal{E}_{ps}^+ &\approx (j_{px} - j_{py})\frac{2\pi^{1/2} r}{\sigma^- R}\Bigg[-\cos\theta_r\,\delta_s + \frac{1}{2}\delta_{s-1} + \frac{1}{2}\delta_{s+1} + \\ &\quad + \frac{v_t\, e^{is\theta_r}}{\alpha\left(\partial G_R/\partial n_\parallel\right)_{\theta_r}} + \frac{v_t\, e^{-is\theta_r}}{\alpha\left(\partial G_R/\partial n_\parallel\right)_{-\theta_r}}\Bigg] \\ &\equiv (j_{px} - j_{py})\rho_{ps},\end{aligned} \tag{34}$$

where $\sigma^- \equiv \omega_{pi}^2/4\pi(\omega+\Omega_o) \approx \omega_{pi}^2/8\pi\omega$. It is important to note that the matrix ρ has an imaginary part because the value of the derivative of G_R with respect to $n_\parallel$ depends on the sign of θ_r, through A'. In conclusion, the Fourier components of $E^+(\theta)$ (divided by $\exp(i\omega t - ik_z z)$) are

$$E_s^+ = \sum_p \mathcal{E}_{p,s-p}^+ = i\sum_p j_p^+ \rho_{p,s-p} \,. \tag{35}$$

REFERENCES

Brambilla, M. & Krücken, T., 1988 *Nucl. Fusion* **28**, 1813

Cattanei, G. & Croci R., 1977 *Nucl. Fusion* **17**, 239

Cattanei, G. & Murphy, A.B., 1991 *Nucl. Fusion* **31**, 219

Faulconer, D. W., 1987 *Plasma Phys. & Controlled Fusion* **29**, 433