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Monte Carlo Modelling of the Transport Phenomena in Magnetic Islands

A. Runow[1], D. Sünder

Max-Planck-Institut für Plasmaphysik, Berlin branch
EURATOM Association
Mohrenstr. 40/41, 101178 Berlin, Germany

[1]Russian Research Centre "Kurchatov Institute"
Moscow, Russian Federation

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Abstract

A Monte Carlo method has been developed that uses two different coordinate systems to model the transport phenomena in the edge plasma of stellarators taking into account the complex magnetic field topology in the stellarator periphery. A self-consistent two-dimensional model is used to investigate the effect of magnetic islands and injected impurities on the transport in the plasma edge. The profiles of the temperature and densities of the plasma, the impurities and the neutral hydrogen particles are calculated for different values of the anomalous perpendicular diffusion coefficient and the impurity source strength and different injection methods. The results show that the existence of magnetic islands significantly influences the transport in the stellarator periphery, and that the injection of impurities into the magnetic island region leads to effective plasma cooling and lowering of the energy flux to the target.

Contents

1 Introduction

It is well known that the most important problem in modelling the stellarator periphery is the complex structure of the magnetic field with magnetic islands, which leads to difficulties in determining the appropriate coordinate system for the finite-volume discretization technique commonly applied. A Monte Carlo method was used to calculate the plasma transport in the edge plasma of W7-AS by a field line tracing code with cross-field diffusion [1],[2]. Another approach to model the transport phenomena in the SOL of tokamaks is to use the B2 multi-fluid code [3],[4]. This code was modified in [5] or W7-X stellarator SOL modelling and island divertor studies. The Chirikov-Taylor map technique [6] for transport modelling in the edge plasma with magnetic islands was used in [7].
Here we propose another approach to the particle and energy transport problem in a plasma with a complicated magnetic field topology: a computational method based on stochastic consideration of the fluid equations [8] which is not very sensitive to the geometry of the plasma boundary and the magnetic field structure. An outline of the proposed method and preliminary results are discussed in [9].
In this paper a two-dimensional model is used to investigate the effect of magnetic islands and impurity injection on transport phenomena in the plasma edge of stellarators with respect to plasma cooling and lowering of the energy flux to the target. The dynamics of the plasma and the injected impurities is described by a self-consistent set of hydrodynamic equations, supplemented by a kinetic equation that describes the dynamics of the neutral particles. Ionization, excitation, charge exchange, neutral particle recycling at the target surface and radiative cooling of the edge plasma by titanium impurity ions are taken into consideration (section 2). The computational method is discussed in section 3. To understand this method, which is based on statistical consideration of the self-consistent set of equations, first the 1D case is investigated (section 3.1). 2D calculations with a magnetic field topology described by the Chirikov-Taylor map technique [7],[9] are analyzed in section 3.2. Numerically obtained results are given in section 4.

2 Model Equations

Let us consider transport phenomena in the periphery of a stellarator with magnetic islands. The set of fluid equations for this problem has the form

$$\frac{\partial n_i}{\partial t} = div(D_\perp \vec{\nabla}_\perp n_i - \vec{V}_{i\|} n_i) \ + S_i, \tag{1}$$

$$\frac{\partial n_z}{\partial t} = div(D_\perp \vec{\nabla}_\perp n_z - \vec{V}_{z\|} n_z) \ + S_z, \tag{2}$$

$$\frac{3}{2}\frac{\partial n_e T}{\partial t} = div(\kappa_\| \vec{\nabla}_\| T + \kappa_\perp \vec{\nabla}_\perp T - \vec{V}_{T\|} T) \ + S_T, \tag{3}$$

where for the vector $\vec{a}$ the following notation is used:

$$\vec{a}_\| = \vec{h}(\vec{a}\cdot\vec{h}), \ \ \vec{a}_\perp = (\vec{h} \text{ x } (\vec{a} \text{ x } \vec{h})), \ \ \vec{h} = \vec{B}/B, \tag{4}$$

n_i, n_z, V_i, V_z are the densities and velocities of the ions and the charge-averaged impurities, respectively, and $V_T = 5/2 \ nV_i$; $D_\perp$ and $\kappa_\perp$ are the corresponding coefficients of the perpendicular anomalous plasma diffusion and heat conduction, $\kappa_\| \sim T^{5/2}$ is the classical heat conduction coefficient, T is the temperature, S_z is a localized homogeneous impurity source, and

$$S_i = n_o n_e < \sigma v >_{ion}, \ \ S_T = -n_e\{n_o L_o(T) + n_o < \sigma v >_{ion} I_{ion} + n_z L_z(T)\}, \tag{5}$$

where n_e is given by $n_e = n_i + n_z < Z(T) >$, and L_o and L_z are the excitation and radiation functions, respectively. In order to complete the particle balance, we use the kinetic equation for the neutrals:

$$\frac{\partial f_o}{\partial t} + \vec{v}\frac{\partial f_o}{\partial \vec{r}} = S_o, \tag{6}$$

where $f_o(\vec{r}, \vec{v})$ is the distribution function of the hydrogen atoms,

$$S_o = -\{< \sigma v >_{cx} n_i + < \sigma v >_{ion} n_e\} f_o + < \sigma v >_{cx} n_o n_i f_{iM}, \tag{7}$$

and f_{iM} is the ion Maxwellian function normalized to 1. In our consideration we use the "closed box" model for the particle balance, $\int (n_o + n_i) d\vec{r} = const., n_o = \int f_o d\vec{v}$, and the following assumptions:

i) $T_i = T_e = T_z = T$,

ii) the magnetic island is described by the Chirikov-Taylor map [6] and

iii) the following atomic physics is taken into account: ionization of the hydrogen atoms by electron impact [10], charge exchange process for hydrogen [11], excitation of hydrogen atoms [12] and radiation of impurities, calculated in the average-ion model [13]. The geometry of the problem and the different impurity sources are shown in Fig. 1, where x and y describe the poloidal and radial distances, respectively. The boundary conditions at the core plasma side are $\Gamma_i = -\Gamma_o$, $q = q_{in}$, $f_o(v_y > 0) = 0$ and at the target $T = T_t \approx 0$, $n_i = 0$, $n_z = 0$, $\Gamma_o = -\Gamma_i$ and $f_o(v_y > 0) = 0, f_o(v_y < 0) = (m_0/2\pi T_t)^{3/2} n_{0,t} exp(-m_0 v^2/2T_t)$, where Γ_i, Γ_o are the corresponding particle fluxes and q is the thermal flux of the plasma. In the poloidal direction periodic conditions for all values are assumed.

3 Computational Method

The idea of the method proposed here is to use 1) the stochastic approach to solve the equations and 2) two different coordinate systems - the "global" and the "local", governed by the boundaries of the problem and the magnetic field geometry, respectively. In the frame of the first system the fluid values (n_i, n_z, T, etc.) are calculated, while the second one is used to calculate the particle jumps [14] along the magnetic field line. The main part of the equations has the usual diffusion-convection-like form. Let us deal here with a "fluid value" N, having no concrete physical meaning.

3.1 1D case

It is well known [8] that the Fokker-Planck equation

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\partial DN}{\partial x} - V_c N\right) \tag{8}$$

can be described by the particle random walk process

$$\Delta x = \sqrt{2D\Delta t}\xi + v_c \Delta t, \tag{9}$$

where ξ is a random number satisfying the conditions $<\xi>=0, <\xi^2>=1$. For computational reasons we use the Gaussian distribution for ξ in these calculations [15]. The diffusion-convection equation for the value N has the form

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial N}{\partial x} - VN\right), \tag{10}$$

which differs from Eq. (8) by the value of the diffusion coefficient gradient. One can redefine the convection as $V_c = V + \partial D/\partial x$ and use the process (9) to obtain the solution of Eq. (10). The outline of the algorithm is as follows. We have a continuous profile $N(t_o)$ at the time t_o and cut it by the preset amount of particles with the initial coordinates $x_j(t_o)$, where the index j is the number of the particle. Using the random process (9), we obtain the new set of the coordinates of the particles $x_j(t_o + \Delta t)$. In order to obtain the profile $N(t_o + \Delta t)$, we use the mesh in the "global" coordinate system (which coincides with the "local" one in the 1D case). The restrictions of the method are as follows: the number of particles in any cell of the mesh must be large, and the time step Δt must be less then ${L_s}^2/2D$, where L_s is the spatial scale length [15]. The latter is important for the nonlinear case (heat conduction equation, where $N \equiv T, D \sim T^{5/2}$). The typical values used in our calculations are about 10^4 particles (total number) and $\Delta t = 10^{-6}$ s. Some additional effects connected with the boundary conditions and sources/sinks are taken into account in terms of the birth and death probabilities of the particles.

3.2 2D case

The 2D transport on the poloidal cross-section consists of two physically separate parts: i) the projection of the transport along the magnetic field line to our fixed cross-section, which describes the particle motion of the magnetic surface on the cross-section in the direction $\vec{h}_p = \vec{B}_p/B_p$,

ii) the isotropic part of the transport (across the magnetic field line). Accordingly, we can rewrite the equation for N in the form

$$\frac{\partial N}{\partial t} = div\{D_p\vec{h}_p(\vec{h}_p\vec{\nabla})N + D_a\vec{\nabla}N - \vec{h}_pV_pN\}, \tag{11}$$

where D_a is the anomalous isotropic part of the diffusion tensor and $V_p = V_{||}\Delta l/L, D_p = D_{||}(\Delta l/L)^2$, Δl is the distance between two neighbouring Chirikov-Taylor points on the cross-section, and L is the distance along the magnetic field line between these points. The corresponding random process can be written as follows:

$$\Delta\vec{r} = \sqrt{2D_a\Delta t}\vec{\xi}_{p\perp} + \sqrt{2D_p\Delta t}\vec{\xi}_{p||} + V_c\Delta t\vec{h}_p, \tag{12}$$

with the same redefined convection velocity V_c, where $V_{||}$ is the local ion sound velocity $c_s(T)$ with the sign determined randomly at the moment of particle birth [1], and with a

set of three random numbers $\xi_{p\mu}(<\xi_{p\mu}>=0, <\xi_{p\mu},\xi_{p\nu}>=\delta_{\mu,\nu})$.
It is easy to show that the process (12) describes the correct fluid equivalent and the cross terms vanish here due to the normalization conditions of the random numbers.
The isotropic part of the transport is quite similar to the 1D case, which is treated in the "global" coordinate system (Cartesian in our case). The parallel part of the transport is analyzed in the "local" curvilinear coordinate system, linked to the magnetic field line for the given particle, and the projection of it to our cross-section is treated.
Let us consider the transport process along the magnetic field lines in the 3D case: $\vec{h}\vec{\nabla}x^i = 0, i = 1,2; x^3 = s$ (magnetic field line length). The Jacobian J of the system in this case has the following property:

$$div\vec{B} = \frac{1}{J}\frac{\partial JB^i}{\partial x^i} = \frac{1}{J}\frac{\partial JB}{\partial s} = 0. \tag{13}$$

The diffusion-convection equation is thus written

$$\frac{\partial N}{\partial t} = \frac{1}{J}\frac{\partial}{\partial s}(J\{D_{\|}\frac{\partial N}{\partial s} - VN\}) = B\frac{\partial}{\partial s}(\frac{1}{B}\{D_{\|}\frac{\partial N}{\partial s} - VN\}) \tag{14}$$

and the corresponding random process for the value N/B is given by

$$\Delta\vec{s} = \sqrt{2D_{\|}\Delta t}\vec{\xi}_{\|} + \{V + B(\frac{\partial}{\partial s}\frac{D_{\|}}{B})\}\Delta t\vec{h}. \tag{15}$$

If we neglect the spatial dependence of B, we obtain the projection of $\vec{\Delta}s$ to our cross-section described in eq. (12).
To estimate the relation of the parallel energy and plasma density transport scale length to the perpendicular one, we can use the expressions

$$\eta_T = 35\frac{\Delta l}{L}\frac{T^{5/4}}{(\alpha n D_a)^{1/2}}, \quad \eta_n = 1\frac{(\Delta l)^{1/2}}{L^{1/2}}\frac{\beta^{1/2}T^{1/4}}{A^{1/4}}, \tag{16}$$

with $T[100\ eV]$, $n[10^{12}/cm^3]$, $D_a[10^4 cm^2/s]$, $\Delta l[cm]$, $L[10^3 cm]$ and $\beta = <V_{\|}>/c_s$.
The dynamics of the neutral hydrogen particles does not depend on the magnetic field topology and is analyzed only in the "global" coordinate system.

4 Numerical Results

The aim of our calculations is to study the efficiency of the plasma cooling in the stellarator periphery by means of impurity injection into magnetic islands. In this case the "benefit" is the reduction of the heat flux onto the divertor target, and the "cost" is the influx (leakage) of the impurities into the core plasma. Of course, a correct statement of the problem requires a self-consistent solution in the frame of a coupled peripheral and central plasma model. A corresponding description is under way, and so here we analyze only the transport in the edge region (Fig. 1), taking into account the described boundary conditions and assuming the following parameters:

a) cross-field transport coefficients: $D_a = (0.1 - 1)\ m^2/s,\ D_a = 0.1\ D_{Bohm},\ \kappa_a = \alpha n D_a$, $\alpha = 3$,

b) impurities: titanium, influx $(0 - 1)10^{20}/m^3 s$,

c) power input: 0.2 MW,

d) energy of the recycled neutrals : 0.1 eV,

e) Chirikov-Taylor map: island perturbation parameter $\epsilon = 0.1$, ι-profil with $\iota_o = 0.9$ and $\iota_1 = 0.2$, L=12 m.

The output values are the 2D profiles of all fluid values (n_i, n_z, n_o, T) for different impurity source strengths S_z, injection localizations (O-, X-point and near the target), and anomalous diffusion coefficients D_a. Furthermore, the heat flux onto the target Γ_t and the loss processes Q_L as a function of the influx of impurities into the main plasma Γ_z are investigated.

Figure 2 shows a strong dependence of the profiles without impurity sources on the structure of the cross-field diffusion coefficient ($a - D_a = 1\ m^2/s,\ b - D_a = 0.1\ D_{Bohm}$). It should be noted that the values D_a outside and especially in the islands are physically not quite justified yet, and this point seems to need further theoretical and experimental investigation taking into account the possibility of neoclassical effects in the magnetic island and that ergodic layers around the islands may exist [7]. The W7-AS periphery plasma modelling makes use of the constant coefficient $D_a = 1\ m^2/s$ [1]. For both values of D_a a plateau in the island region becomes apparent only in the temperature profile (cf. expression η_T (16)). The transition process to the stationary state for the T profile after switching on the island at t = 0 is demonstrated in Fig. 3 in the case $D_a = 1\ m^2/s$ ($t_{st,T} = 8\ ms$). The stationary profiles of the temperature and impurity density for different impurity source strengths S_z are shown in Fig. 4 ($D_a = 1\ m^2/s$) and Fig. 5 ($D_a = 0.1\ D_{Bohm}$), where the source is localized near the O-point (s. Fig. 1). The relaxation after the impurity injection at $t = t_i > t_{st,T}$ to this state for $S_z = 8\ 10^{19}/m^3s$ and $D_a = 1\ m^2/s$ is discussed in Fig. 6. It should be pointed out that in the case of a Bohm-like diffusion coefficient the diffusion of impurities is smaller than that of the main plasma due to the high mean charge state Z(T) [13] for titanium ($D_{Bohm} \sim 1/ < Z >$, $< Z > \sim 10$), and the impurity radiation level becomes very high (s. Fig. 10). For the same source location and $S_z = 2\ 10^{19}/m^3s$ but a smaller constant diffusion coefficient the profiles change and a plateau in the plasma density profile also appears (Fig. 7). The cases of other injection regions are shown in Fig. 8 (X-point) and Fig. 9 (near the target) for $D_a = 1\ m^2/s$. In Fig. 10 the power load deposition on the target surface for different impurity input values is analyzed. The total power loss of the plasma as a function of the impurity influx into the main plasma Γ_z is displayed in Fig. 11. Also shown, in Fig. 12, is the peak power load of the target as a function of Γ_z for different input parameters.

5 Summary

A Monte Carlo method using two different coordinate systems is developed for modelling the transport phenomena in the edge plasma of stellarators taking into account the complex magnetic field topology in the stellarator periphery. In the frame of a self-consistent two-dimensional model the effect of magnetic islands and injected impurities on transport in the plasma edge is studied. The profiles of the temperature and densities of the plasma, the impurities and the neutral hydrogen particles are calculated for different values of the anomalous perpendicular diffusion coefficient and impurity source strengths and different injection methods. The results show that the existence of magnetic islands significantly influences the transport in the stellarator periphery, and that the injection of impurities into the magnetic island region leads to effective plasma cooling and lowering of the energy flux to the target. The efficiency of these processes increases when the anomalous diffusion coefficient for the impurities in the magnetic island decreases, thus increasing the impurity confinement time .

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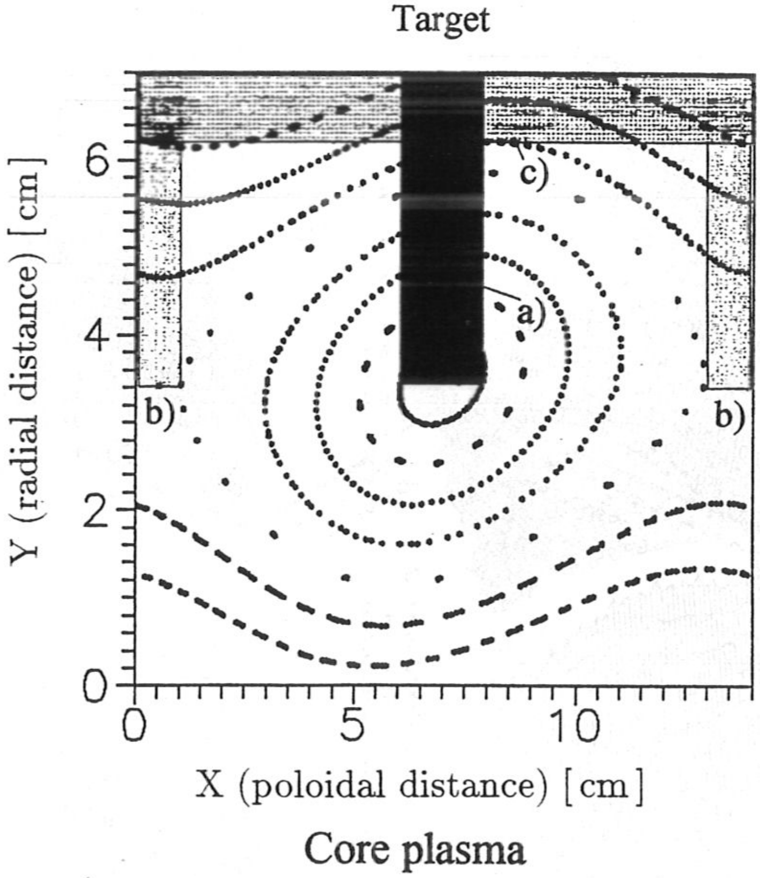

Fig. 1.

Island structure described by the Chirikov-Taylor map technique (perturbation parameter $\epsilon = 0.1$, $\iota_o = 0.9$ and $\iota_1 = 0.2$) and different impurity sources: a) O-point, b) X-point and c) near the target .

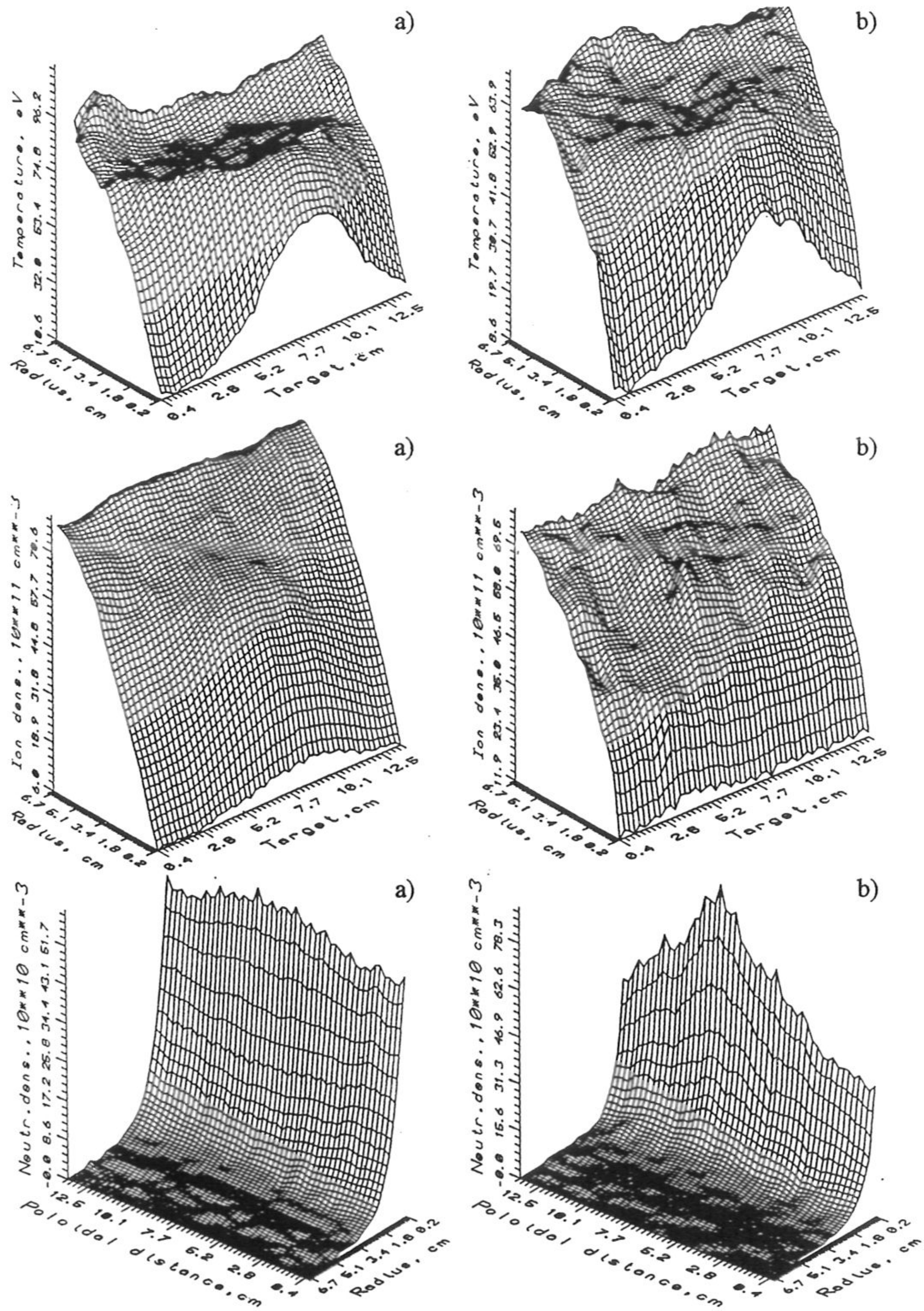

Fig. 2

Profiles of temperature, plasma density and neutral particle density without impurity injection for different cross-field diffusion coefficients $(a - D_a = 1\ m^2/s, b - D_a = 0.1\ D_{Bohm})$.

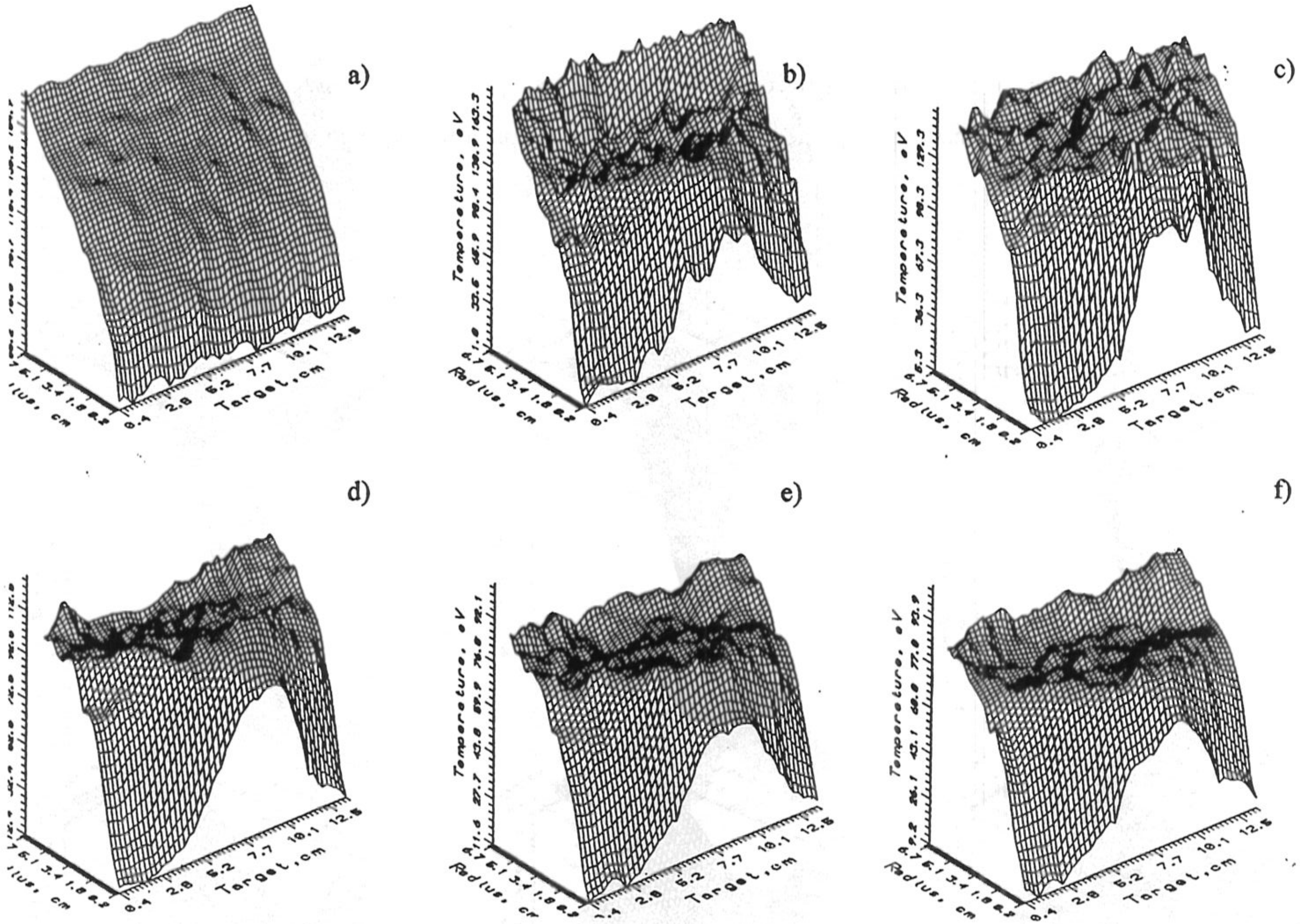

Fig. 3

Time evolution of the temperature profile in the case with $D_a = 1\ m^2/s$ after switching on the island at t = 0 (a), t = 0.01 ms (b), t = 0.1 ms (c), t = 1 ms (d), t = 3 ms (e) to the stationary state $t_{st,T}$= 8 ms (f) .

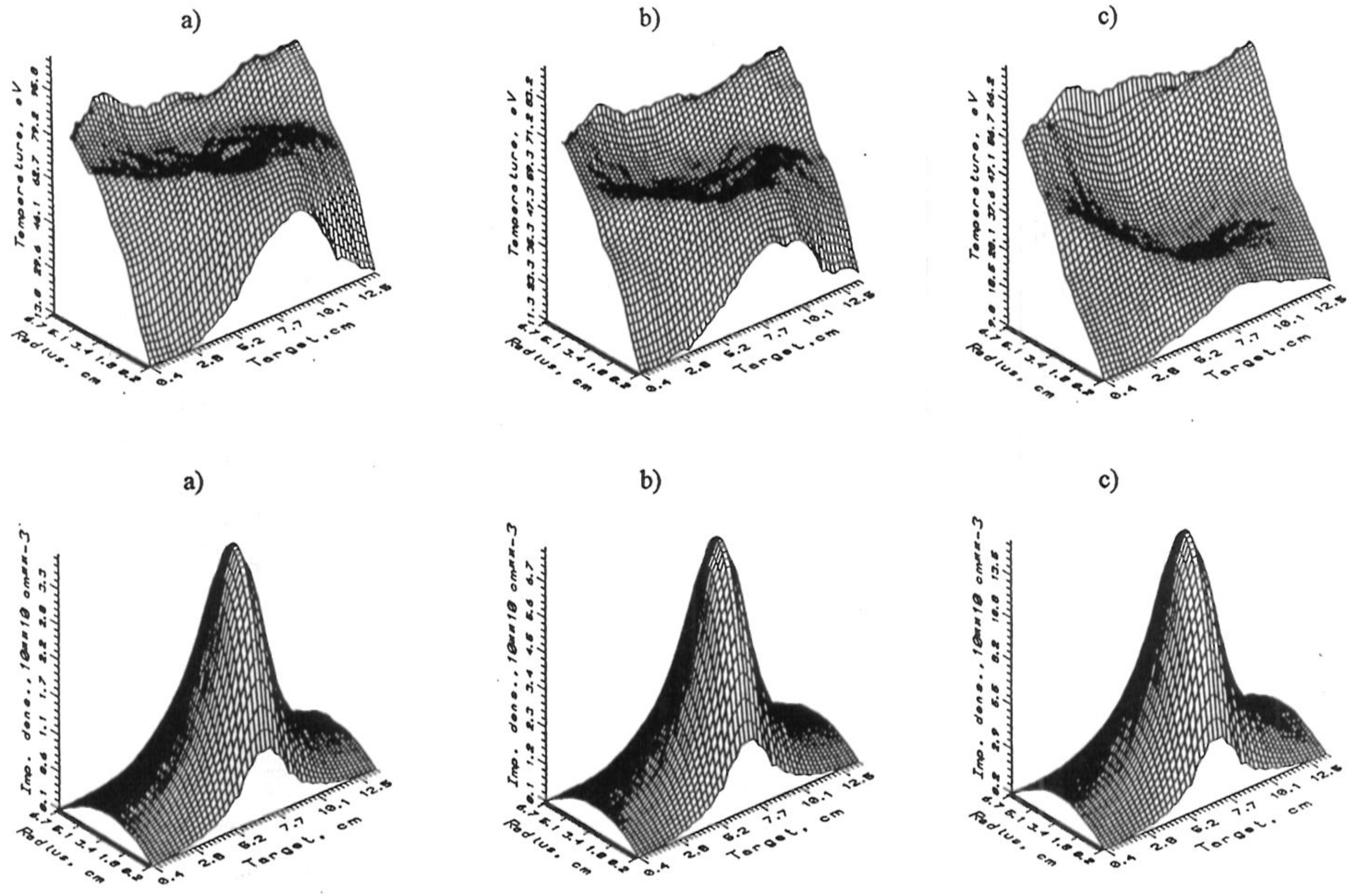

Fig. 4

Stationary profiles of the temperature and impurity density for different impurity source strengths S_z $[10^{19}/m^3s]$ for $D_a = 1\ m^2/s$ and O-point impurity injection : a - 2, b - 4, c - 8 .

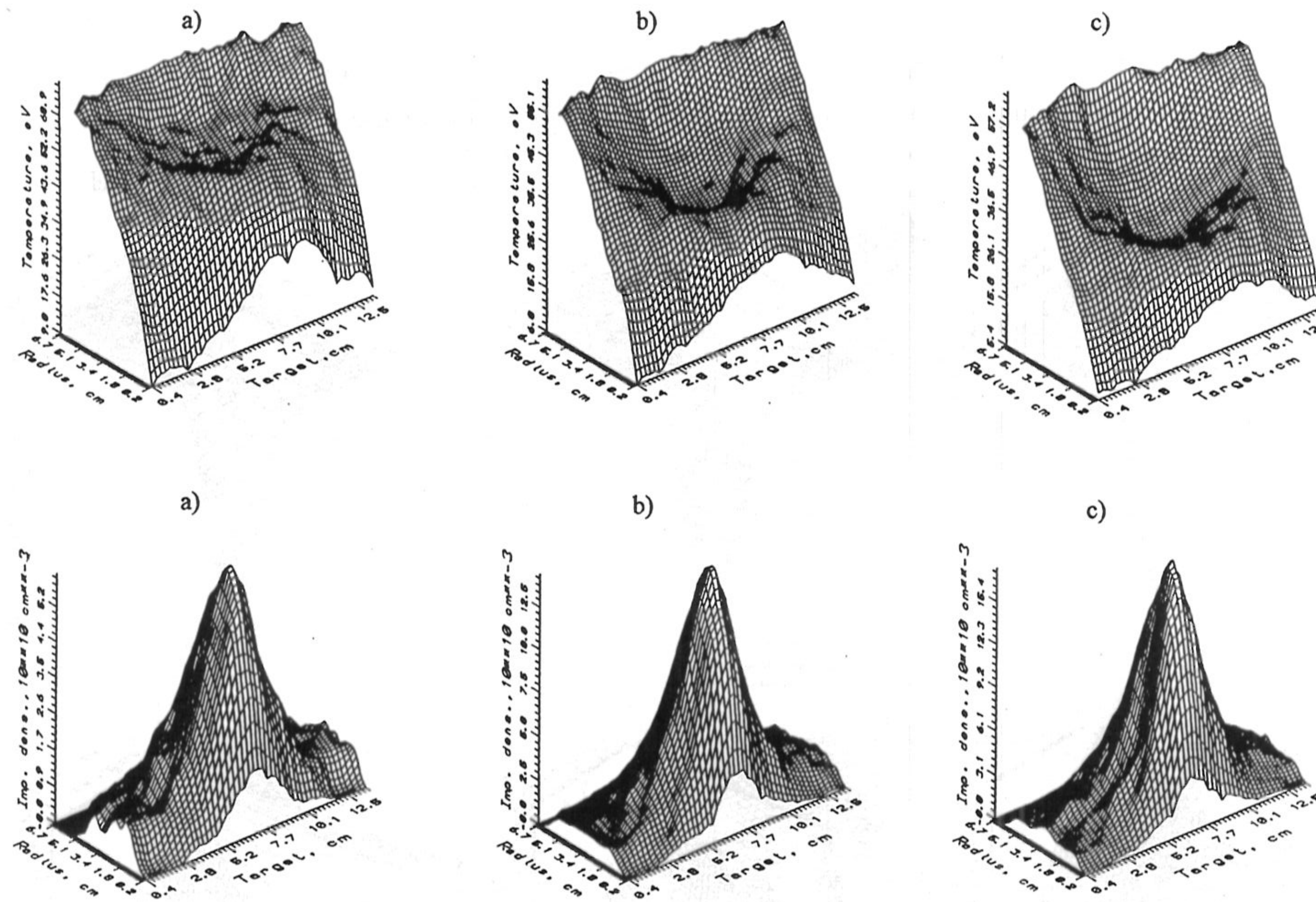

Fig. 5

Stationary profiles of the temperature and impurity density in a plasma with $D_a = 0.1\ D_{Bohm}$ for different impurity source strengths S_z $[10^{19}/m^3s]$ localized at the O-point: a - 0.5, b - 1.0, c - 1.2 .

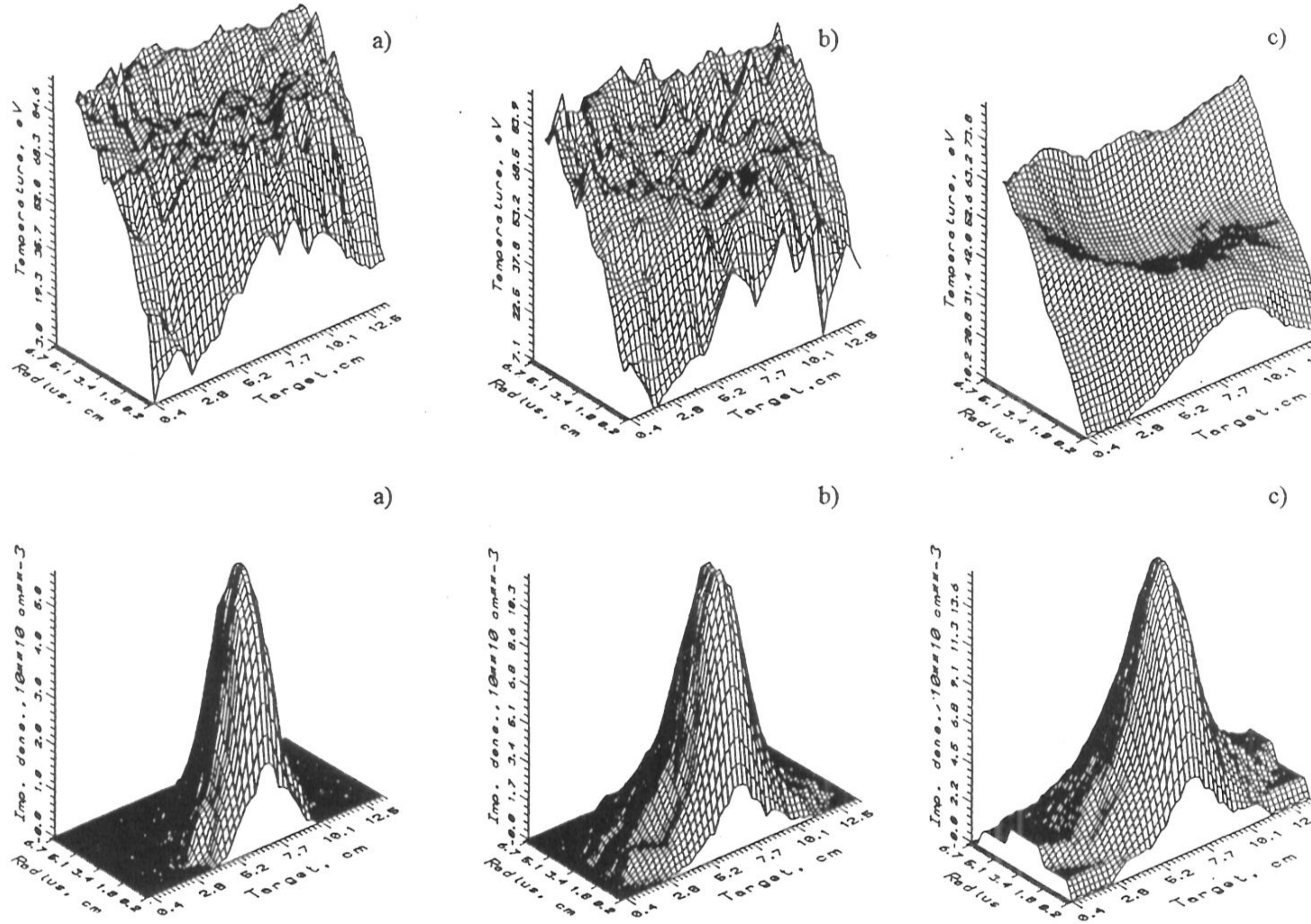

Fig. 6

Relaxation of the temperature and impurity density profiles after the impurity injection at $t = t_z > t_{st,T}$ to the new stationary state for $S_z = 8\ 10^{19}/m^3 s$ and $D_a = 1\ m^2/s$ (τ= 0.1 ms (a), τ= 0.5 ms (b), $\tau_{st,z}$= 9 ms (c), $\tau = t - t_z$).

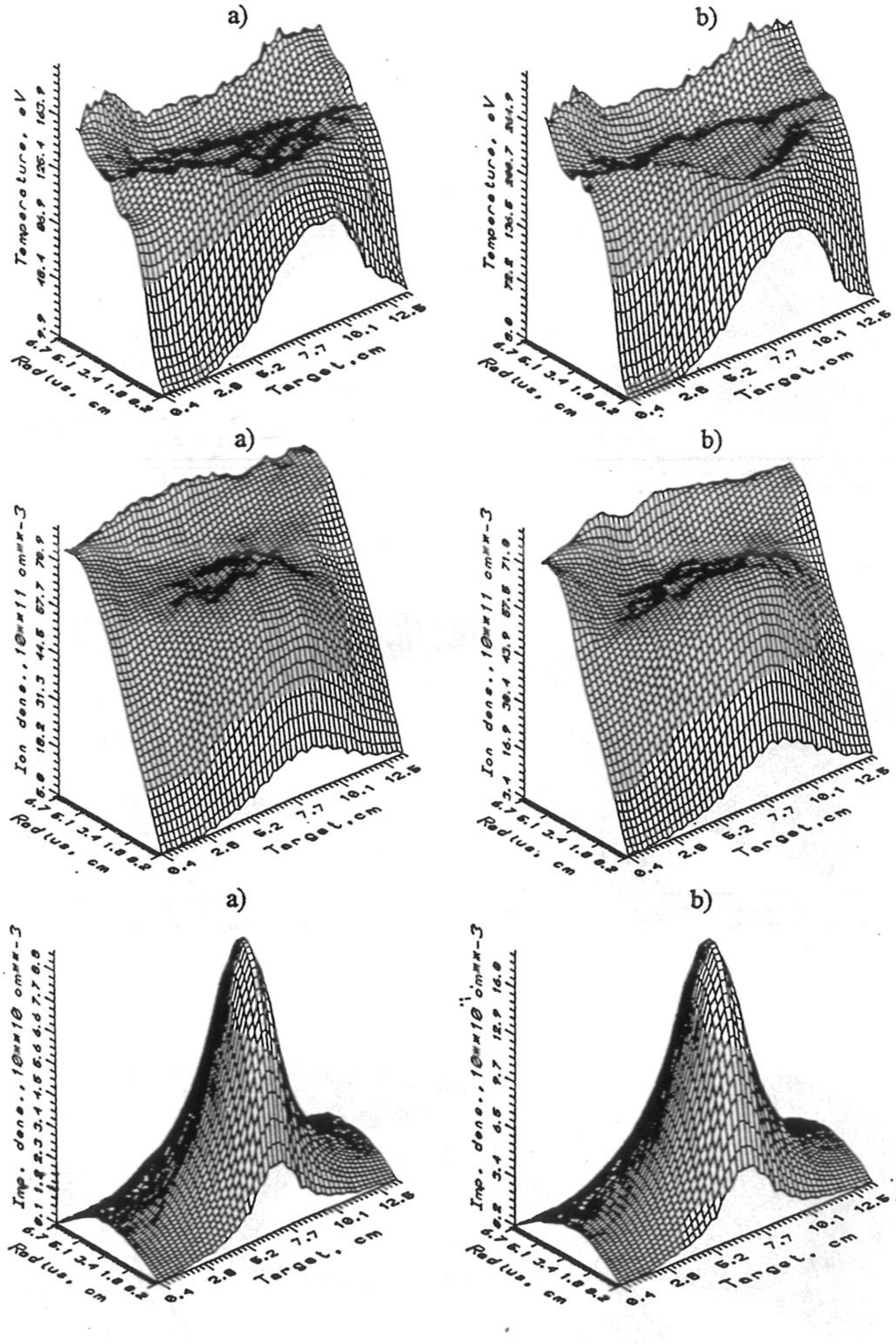

Fig. 7

Stationary profiles of the temperature and impurity and plasma densities in the case of O-point impurity injection, impurity source strength $S_z = 2\ 10^{19}/m^3s$ for $D_a = 0.4\ m^2/s$ (a) and $0.2\ m^2/s$ (b) .

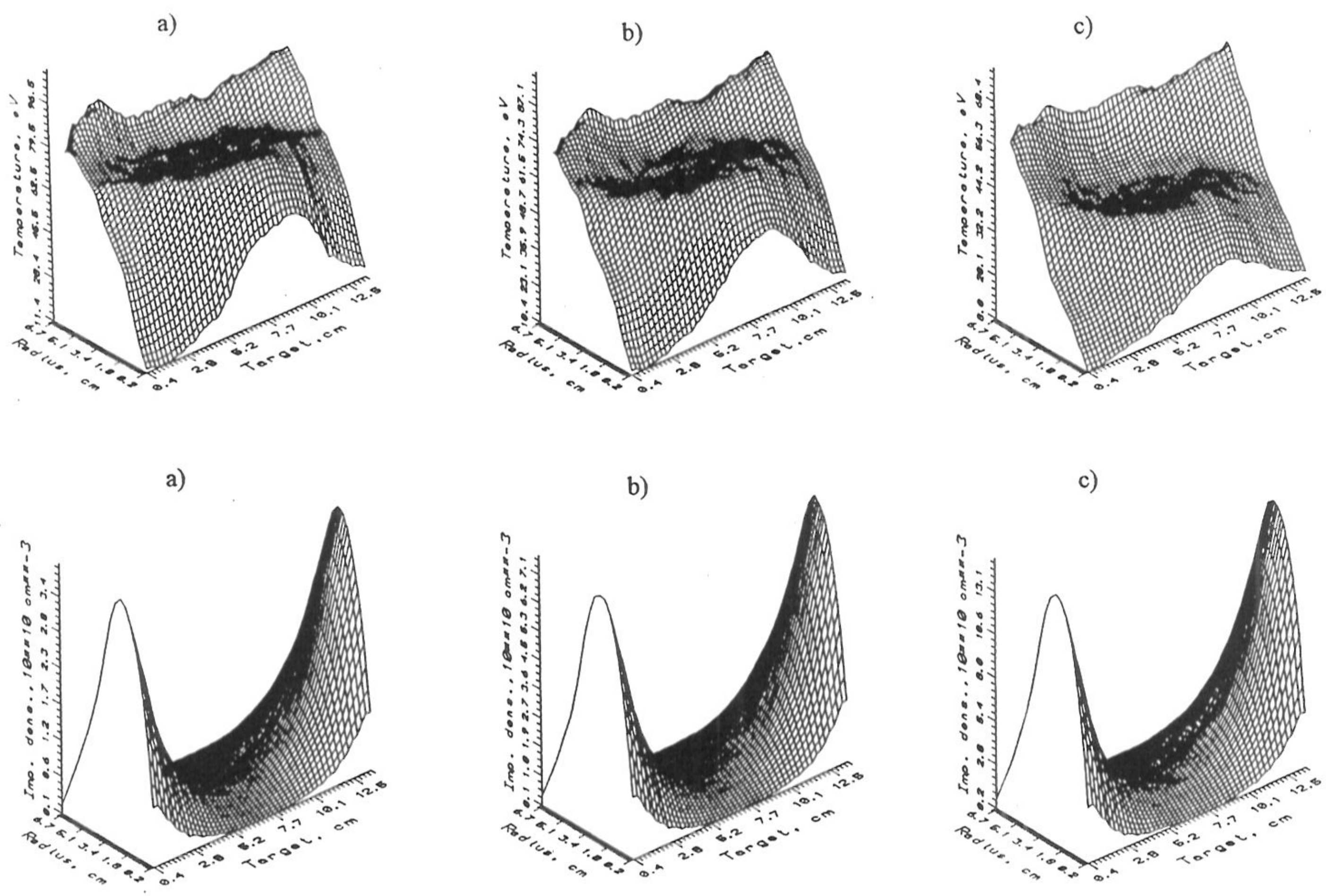

Fig. 8

Stationary profiles of the temperature and impurity density for different impurity source strengths S_z [$10^{19}/m^3s$] in the case of $D_a = 1\ m^2/s$ and X-point source : a - 2, b - 4, c - 8 .

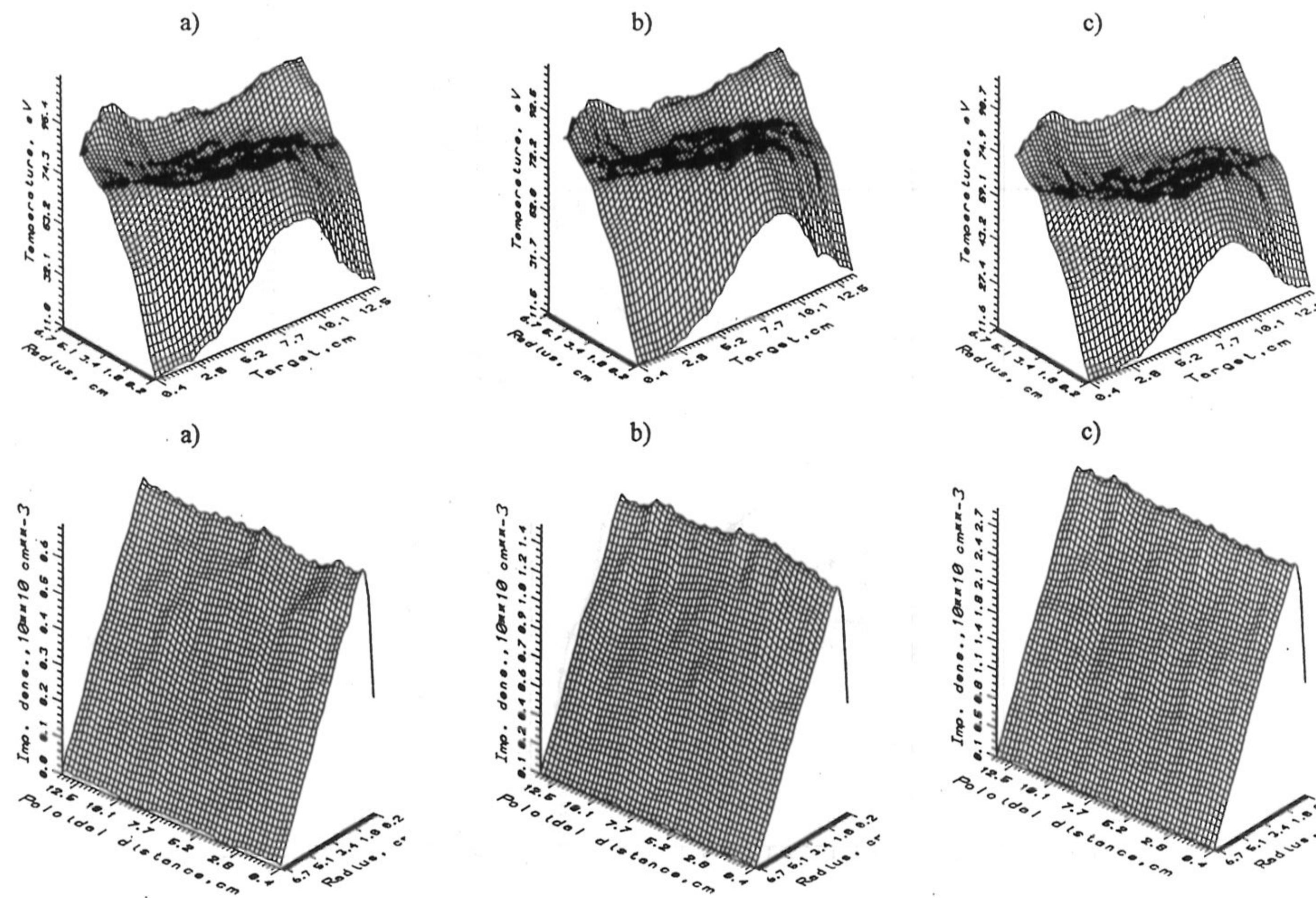

Fig. 9

Stationary profiles of the temperature and impurity density in a plasma with $D_a = 1\ m^2/s$ for the same impurity input values S_z as in Fig. 4 when the injected impurities are localized near the target .

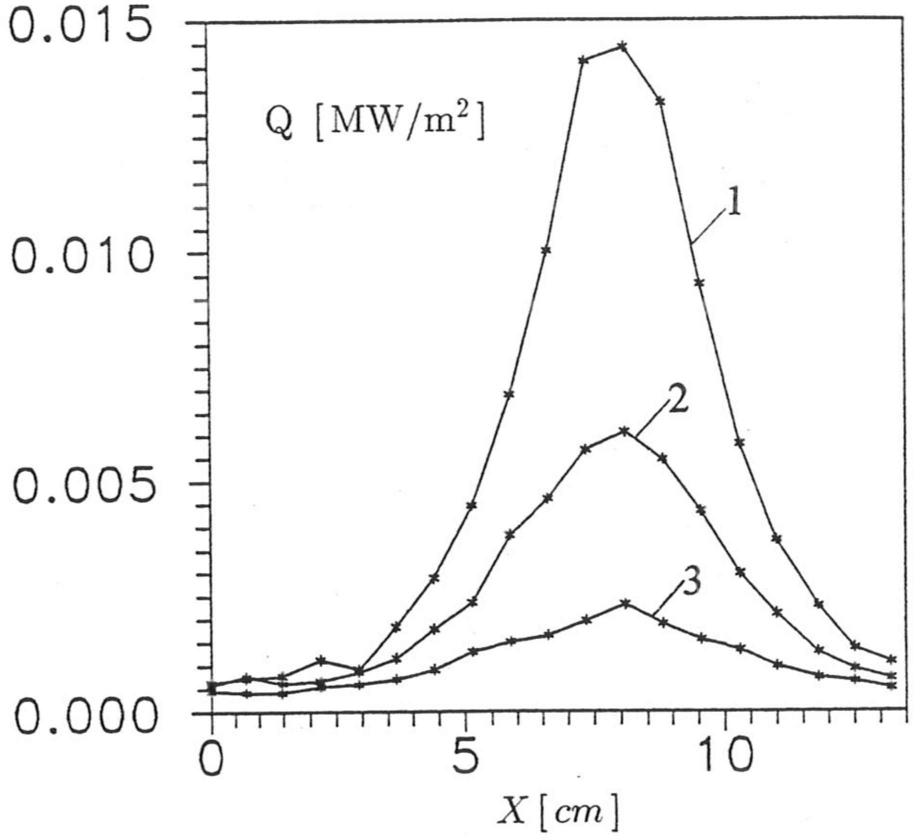

Fig. 10

The power load deposition on the target surface for different impurity input values S_z : 1 - 0, 2 - 6 $10^{19}/m^3s$, 3 - $10^{20}/m^3s$.

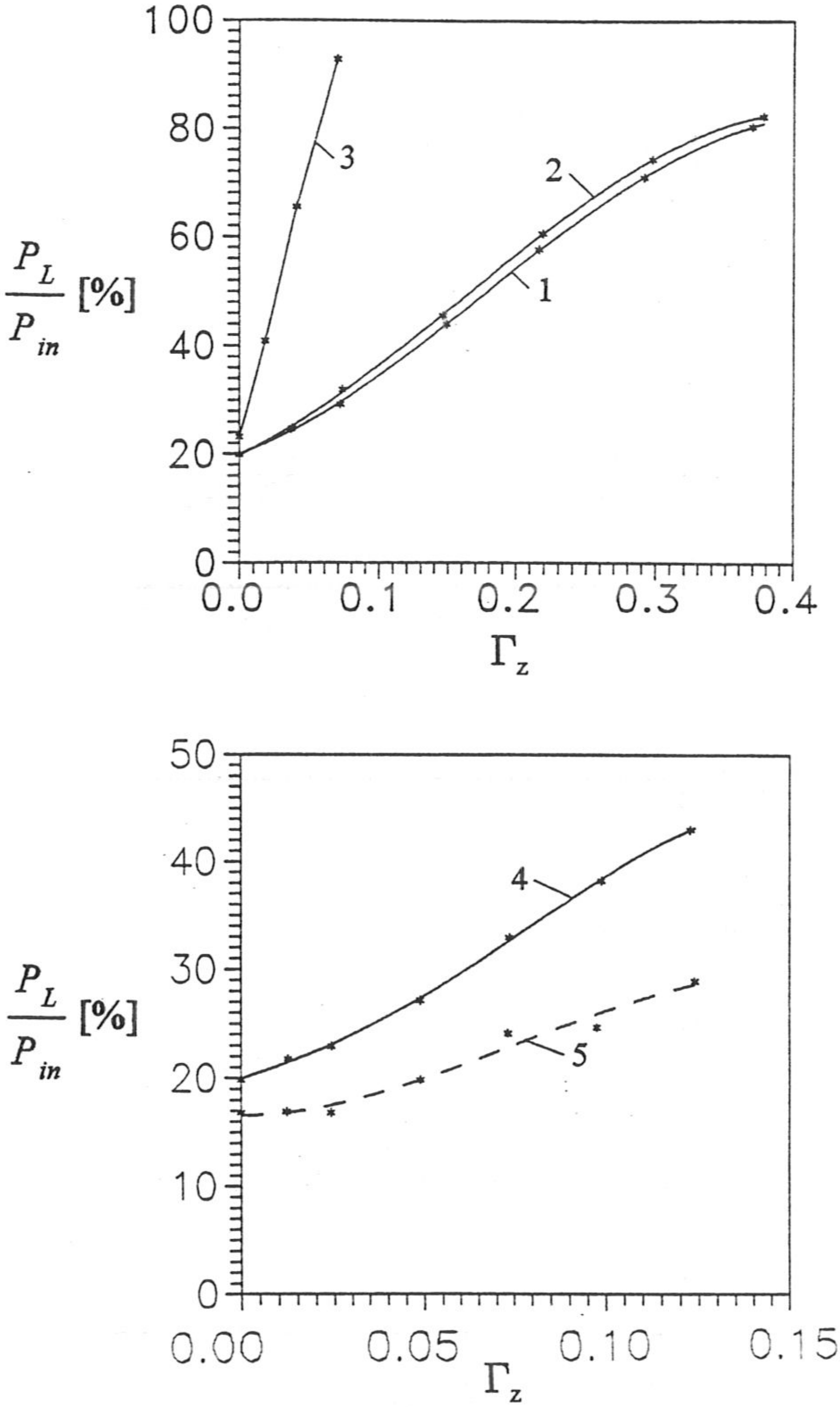

Fig. 11

The relative total power loss as a function of the impurity flux into the main plasma Γ_z $[10^{17}/m^2s]$ for different input parameters: 1 - $D_a = 1\ m^2/s$, X-point source, 2 - $D_a = 1\ m^2/s$, O-point source, 3 - $D_a = 0.1\ D_{Bohm}$, O-point source, 4 - $D_a = 1\ m^2/s$, target source and 5 - $D_a = 1\ m^2/s$, target source without island .

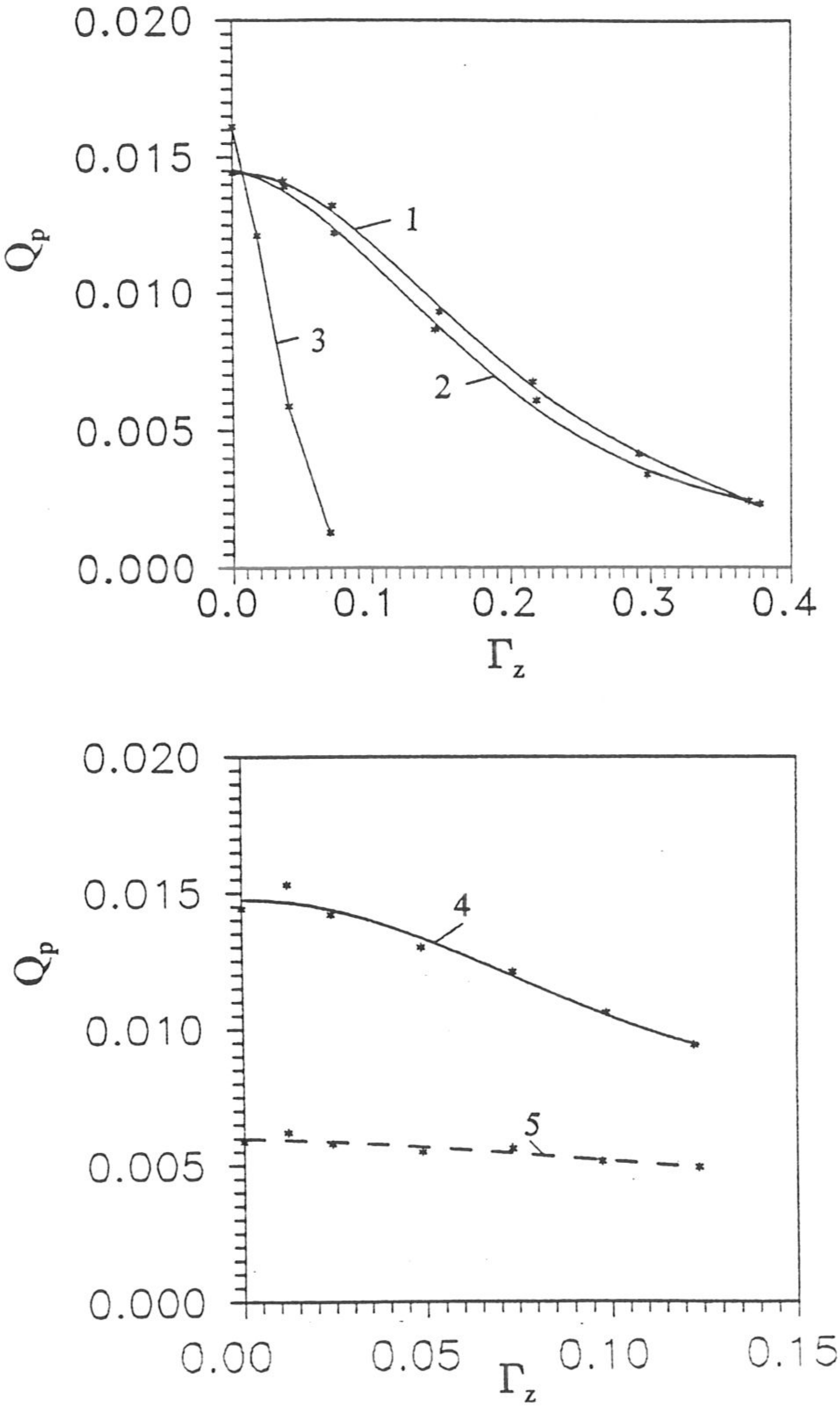

Fig. 12

The peak power load Q_p $[MW/m^2]$ of the target as a function of the impurity flux into the main plasma Γ_z $[10^{17}/m^2s]$ for the same input parameters as in Fig. 11.