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IPP 6/311 January 1993

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Negative Energy Waves in a Magnetically Confined Guiding Center Plasma

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Abstract

The general expression for the second-order wave energy of a Maxwell-drift kinetic system derived by Pfirsch and Morrison is evaluated for the case of a magnetically confined plasma for which the equilibrium quantities depend on a Cartesian coordinate y. The conditions for the existence of negative-energy waves for electrostatic initial perturbations are also obtained. If the equilibrium guiding center distribution function $f_{g\nu}^{(0)}$ of any species ν has locally the property $q_4 \dfrac{\partial f_{g\nu}^{(0)}}{\partial q_4} > 0$, where q_4 is essentially the velocity parallel to the magnetic field, and if this holds in a frame of reference in which the center-of-mass velocity parallel to $B^{(0)}$ vanishes, negative-energy waves exist with no restriction on either the orientation or the magnitude of the wave vector, while if $q_4 \dfrac{\partial f_{g\nu}^{(0)}}{\partial q_4} < 0$ the possible negative-energy waves are nearly perpendicular. The condition for purely perpendicular negative-energy waves reads $\dfrac{dP^{(0)}}{dy}\dfrac{\partial f_{g\nu}^{(0)}}{\partial y} < 0$, where $P^{(0)}$ is the plasma pressure. For the cases of tokamak-like and shearless stellarator-like equilibria, which are derived on the basis of, respectively, a slightly modified Maxwellian and a Maxwellian distribution function, the existence of perpendicular negative-energy waves is related to the threshold value 2/3 of the quantity $\eta_\nu \equiv \partial \ln T_\nu / \partial \ln N_\nu$, where T_ν is the temperature and N_ν the density of some species. This is lower than the critical η_ν-value for the trigger of *linear* temperature-gradient driven modes.

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The conditions for the existence of negative-energy waves for electrostatic initial perturbations (which could be nonlinearly unstable and cause anomalous transport) are investigated for an equilibrium with space dependent, sheared magnetic field. The method of investigation consists in evaluating the general expression for the second-order wave energy derived by Pfirsch and Morrison [1] for the case of the Maxwell-drift kinetic theory, based on the Lagrangian formulation of the quiding center theory given by Littlejohn [2] and later regularized by Correa-Restrepo and Wimmel [3]. In Cartesian coordinates, the equilibrium magnetic field is given by $\boldsymbol{B}^{(0)} = B_x^{(0)}(y)\boldsymbol{e}_x + B_z^{(0)}(y)\boldsymbol{e}_z$, and the mean Lorentz force, which balances the force due to the pressure gradient, acts along the y-axis. The guiding center drift velocity $\boldsymbol{v}_{g\nu}^{(0)}$ of any species ν possesses, in addition to the component $v_{g\nu\|}^{(0)} = q_4/g'_\nu$ parallel to the magnetic field, a perpendicular component due to the grad-B drift (q_4 is a velocity variable, the function $g_\nu(z)$ regularizes the theory, and the rest of the symbols are illustrated in Table 1). The equilibrium guiding center distribution function $f_{g\nu}^{(0)} = f_{g\nu}^{(0)}(y, q_4, \mu)$ (with μ the magnetic moment), which must be spatially dependent for at least one species, is weakly constrained because of the equilibrium condition $P^{(0)} + (B^{(0)})^2/8\pi =$ constant. That is, it belongs to a specific class of functions such that its potential spatial dependence through the magnetic field involves the magnetic field modulus only (the function $f_{g\nu}^{(0)}$ is free to depend on y either explicitly or implicitly through any other quantity not related to $\boldsymbol{B}^{(0)}$).

After a lengthy calculation, the second-order wave energy for the equilibria considered and for purely electrostatic initial perturbations can eventually be cast in the neat form

$$F^{(2)} = -S\sum_\nu \int dq_4 d\mu dy\, |G_\nu^{(1)}|^2 \frac{B_\nu^{\star(0)}}{m_\nu} \left(\boldsymbol{k}_{xz}\cdot \boldsymbol{v}_{g\nu}^{(0)}\right) \left(k_\| \frac{\partial f_{g\nu}^{(0)}}{\partial q_4} - k_\perp \frac{g'_\nu}{\omega_\nu^{\star(0)}} \frac{\partial f_{g\nu}^{(0)}}{\partial y}\right),$$

where $G_\nu^{(1)}(y, q_4, \mu)$ is a first order function which is connected with the perturbations around the equilibrium state, and $\boldsymbol{k}_{xz}$, $k_\|$ and $k_\perp$ are, respectively, the wave vector lying on the x-z plane and its components parallel and perpendicular to $\boldsymbol{B}^{(0)}$. The following conditions for the existence of negative-energy waves, which need only be satisfied locally in y, q_4 and μ, obtain if the reference frame is one of minimum energy, i.e. if in it the center-of-mass velocity parallel to $\boldsymbol{B}^{(0)}$ vanishes. The conditions for oblique propagation ($k_\| \neq 0$) are substantially different from those for perpendicular propagation ($k_\| = 0$). The former case is thus discussed separately from the latter as follows.

1. Oblique waves If any $f_{g\nu}^{(0)}(y, q_4, \mu)$ has the property $\dfrac{q_4}{g'_\nu}\dfrac{\partial f_{g\nu}^{(0)}}{\partial q_4} > 0$, negative-energy waves exist with no restriction on either the orientation or the magnitude of the wave vector $\boldsymbol{k}_{xz}$ (other than $k_\| \neq 0$). This condition also obtains for an

Symbol	*Meaning or Definition*
m_ν	mass of particles of species ν
e_ν	charge of particles of species ν
c	speed of light
S	normalization surface
κ	Boltzmann's constant
v_0	constant velocity
z	q_4/v_0
g'_ν	dg_ν/dz
$B^{(0)}$	magnetic field modulus
$\boldsymbol{b}^{(0)}$	$\boldsymbol{B}^{(0)}/B^{(0)}$
$\boldsymbol{B}_\nu^{\star(0)}$	$\boldsymbol{B}_\nu^{(0)} + (m_\nu c/e_\nu)v_0 g(z)\, curl\, \boldsymbol{b}^{(0)}$
$B_\nu^{\star(0)}$	modulus of $\boldsymbol{B}_\nu^{\star(0)}$
$\omega_\nu^{\star(0)}$	$e_\nu B_\nu^{\star(0)}/cm_\nu$

Table 1: Notation

inhomogeneous force-free equilibrium with sheared magnetic field of constant modulus, which can be described by taking $B_x^{(0)} = B^{(0)}\sin(\alpha x)$, $B_z^{(0)} = B^{(0)}\cos(\alpha x)$ [α^{-1} is the shear length, and $f_{g\nu}^{(0)} = f_{g\nu}^{(0)}(q_4, \mu)$]. The last result agrees with that obtained by Correa-Restrepo and Pfirsch [4], in the context of Maxwell-Vlasov theory. In addition, the same condition is valid for a homogeneous magnetized plasma ($\boldsymbol{B}^{(0)}$ = constant), a result which was first derived by Pfirsch and Morrison [1].

If $\frac{q_4}{g'_\nu}\frac{\partial f_{g\nu}^{(0)}}{\partial q_4} < 0$, a condition which is more frequently satisfied, only waves for which the quantity $\frac{k_\parallel}{k_\perp}$, ($k_\perp \neq 0$), is restricted within the interval [$min(\Lambda_\nu, M_\nu)$, $max(\Lambda_\nu\ M_\nu)$]($\Lambda_\nu \equiv -\frac{4\pi g'_\nu \mu\, dP^{(0)}/dy}{m_\nu q_4 B^{(0)}\omega_\nu^{\star(0)}}$ and $M_\nu \equiv \frac{g'_\nu \partial f_{g\nu}^{(0)}/\partial y}{\omega_\nu^{\star(0)}\partial f_{g\nu}^{(0)}/\partial q_4}$) can possess negative energy. For particles with thermal velocities this condition implies that the ratio $\frac{k_\parallel}{k_\perp}$ must take values of the order of the quantity $\frac{(r_{L\nu})_{th}}{L} \ll 1$, with $(r_{L\nu})_{th}$ the Larmor radius at thermal velocities and L the macroscopic scale length. The possible waves are therefore nearly perpendicular.

2. Perpendicular waves If $\frac{dP^{(0)}}{dy}\frac{\partial f_{g\nu}^{(0)}}{\partial y} < 0$, negative-energy waves exist for any wave number $k_\perp$, irrespective of the sign of the quantity $\frac{q_4}{g'_\nu}\frac{\partial f_{g\nu}^{(0)}}{\partial q_4}$. Since the most important negative-energy perturbations concern perpendicular waves, the last condition is further examined for tokamak-like and stellarator-like equilibria.

To simplify the notation, the superscript (0) will be suppressed in the following, on the understanding that all the quantities pertain to equilibrium.

2.1 Tokamak-like equilibria To describe an equilibrium of this kind, the space-dependent, shifted Maxwellian distribution function

$$f_{g\nu} \propto \exp\left[-\frac{\mu B(y) + 1/2m_\nu(v_{g\nu\|} - V_\nu(y))^2}{\kappa T_\nu(y)}\right]$$

is used. The small parallel shift velocity $V_\nu(y)$ $[\frac{V_\nu}{(v_\nu)_{th}} \approx \frac{(r_{L\nu})_{th}}{L} \ll 1$, where $(v_\nu)_{th}$ stands for a thermal velocity] produces a net "toroidal current" (the coordinates x and z represent the poloidal and toroidal directions, respectively). In the case $\eta_\nu \geq 0$ for all ν, if $\eta_\nu > 2/3$ for at least one ν, negative-energy waves exist for any perpendicular wave number $k_\perp$, except $k_\perp = 0$. The existence of negative-energy waves is therefore related to the threshold value 2/3 of the quantity η_ν, a quantity which usually expresses the trigger for the temperature-gradient driven modes. This is lower than the critical value η_ν^c, which is determined in the framework of a linear stability analysis (for example, performing a linear kinetic stability analysis of the ion temperature-gradient mode, Dominguez and Rosenbluth [5] obtained a critical value $\eta_i^c \geq 1$). Accordingly, the value $\eta_\nu^{sc} = 2/3$ is subcritical and the possible existence of negative-energy waves below the instability threshold implies that self-sustained turbulence may be present in a linearly stable tokamak regime. This result agrees with numerical results obtained by Scott [6] as well as by Nordman, Pavlenko and Weiland [7] within the framework of a nonlinear collisional and a nonlinear collisionless fluid model, respectively.

Furthermore, the part of the phase space and, consequently, the fraction of the particles which are associated with negative-energy waves are determined. (Henceforth particles of this kind will be called active particles.) This is accomplished by using analytic tokamak-like solutions of the drift kinetic equilibrium equation for cold ions, which have the following characteristics: constant toroidal magnetic field, poloidal magnetic field of the form $B_x \propto \tanh\rho$ ($\rho = y/L$) and peaked pressure as well as current density profiles ($P, j_z \propto 1/\cosh^2\rho$). With the electron shift velocity profile appropriately chosen, the following three equilibria with typical η_e-values, compared with the subcritical value, are considered:

- $\underline{\eta_e = 1 \text{ for any } \rho.}$ In this case both the density profile and the temperature profile are peaked (N_e, $T_e \propto 1/\cosh\rho$). Although the value of η_e is equal to the critical value for linear stability or probably a little lower, nearly one-third of the electrons that possess thermal velocities are active.

- $\underline{\eta_e \to \infty \text{ for any } \rho}$. In this case the density profile is flat and the temperature profile is peaked ($T_e \propto 1/\cosh^2\rho$). All the electrons are now active, as expected, because η_e approaches an extremely large value.

- $\eta_e = 0$ for any ρ. Conversely, this equilibrium exhibits a flat temperature and a peaked density profile ($N_e \propto 1/\cosh^2\rho$). In this case it is shown that the plasma has no negative-energy waves, as again expected, because η_e takes its lowest non-negative value well below the subcritical one.

Two equilibria with negative values of η_e, for which the above criterion does not obtain, are also examined. In the one, the density profile is peaked ($N_e \propto 1/B\cosh^2\rho$) and the temperature profile is hollow ($T_e \propto B$), while the other possesses inverse characteristics ($N_e \propto B$, $T_e \propto 1/B\cosh^2\rho$). For both equilibria negative-energy waves exist for any ρ without restriction on the fraction of the active thermal electrons.
For the majority of the equilibria considered (either $\eta_e > 0$ or $\eta_e < 0$) the fraction of the active particles increases as one proceeds from the center ($\rho = 0$) to the edge ($\rho = 1$) This indicates that self-sustained turbulence exists to a higher degree in the edge region.

2.2 Stellarator-like shearless equilibria The distinguishing feature of these equilibria in comparison with the tokamak-like ones is that the net plasma current vanishes. The single toroidal component of the magnetic field exhibits a hollow profile ($B_z = (B_c^2 + B_s^2\tanh^2\rho)^{1/2}$, where B_c and B_s are constants) and the associated poloidal current density is an odd function of ρ ($j_x \propto \tanh\rho/\cosh^2\rho$). Thus, the current in the one half-space ($\rho > 0$) flows in the opposite direction to that in the other ($\rho < 0$). To derive equilibria of this kind, an appropriate distribution function is a y-dependent Maxwellian ($V_\nu \equiv 0$). By treating this distribution function, the condition $\frac{dP}{dy}\frac{\partial f_{g\nu}}{\partial y} < 0$ furnishes the same subcritical value $\eta_\nu^{sc} = 2/3$. Equilibria which exhibit electron density and temperature profiles identical to those of the tokamak-like ones discussed above are also examined. It is shown that the results of the tokamak-like equilibria that concern the fraction of the active particles are also valid in the stellarator-like regime. It therefore turns out that, as far as the existence of negative-energy waves is concerned, within the approximation considered in the present work the two confinement systems are equivalent.

ACKNOWLEDGMENTS

We would like to thank D. Correa-Restrepo and P.J. Morrison for useful discussions.
One of the authors (G.N.T.) acknowledges the support by Commission of the European Communities, Fusion Program, Contract No. B/FUS*-913006.

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