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**Profile constraining mechanism for $T_e$ profile invariance in tokamaks**

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ABSTRACT. A new model for the electron temperature profile resilience in the outer half of tokamak plasmas is proposed and investigated. It is shown that introducing a negative feedback term into the scaling of the electron heat diffusivity $\chi_e(r) = \chi_{ec}(r)\left(1 - \alpha\frac{a^2}{T_{eo}}\frac{d^2T_e}{dr^2}\right)$ with $\alpha \gtrsim 3$ results in the observed $T_e$ profile stiffness against variations of the density and heating profiles. In addition, the feedback by itself yields the measured approximately linear $T_e(r)$ shape for $r/a \gtrsim 0.5$ in contrast to previous profile constraining models.

The study of electron temperature profiles in ohmically heated (OH) and L-mode plasmas on ASDEX [1] and TFTR [2] and in H-mode [3] and OH plasmas [4] on ASDEX yielded a type of $T_e$ profile invariance which is different in several respects from the old principle of profile consistency [5]. It was found that the shape of the $T_e$ profile is conserved for $r/a \gtrsim 0.5$ and that it does not vary with the density profile, heat deposition profile and $q$ value. In addition, a canonical temperature profile shape was observed during current ramps and lower hybrid current drive, i.e. for highly different current density profiles and with $j(r)$ and $T_e(r)$ decoupled. The mechanism constraining the temperature profile shape in the range $0.5 \lesssim r/a \lesssim 1.0$ has not yet been identified. All the same, $T_e$ profile invariance was shown to be capable of describing essential properties of bulk $L$ and $H$ confinement [3] and transport with peaked density profiles [6].
The electron temperature profiles in the $OH, L$ and $H$ regimes of ASDEX were studied by statistical analysis [7]. It was shown that the $T_e(r)$ shape is conserved in the outer half of the plasma in these regimes and that it is invariant to the heating power, heating profile, density, density scale length, $q$ value and ion mass. For $r/a \gtrsim 0.5$, the canonical electron temperature profile can be approximated by a straight line

$$\frac{T_e(r)}{T_{eo}} = 1 - \frac{r}{a_T} \tag{1}$$

so that the scale length $r_{Te} = -T_e/(dT_e/dr)$ reads

$$r_{Te}(r) = a_T - r \tag{2}$$

For ASDEX, the plasma radius $a$ is $40cm$ and $a_T$ is $43.2cm$. Constancy of the $T_e$ profile shape means that $r_{Te}(r)$ is conserved and that $a_T$ is constant. $T_e$ profile invariance associated with an almost linear shape is also observed in many other tokamaks (TFTR, JET, DIII-D, JT-60, T-10, TORE SUPRA, FT). It is important that this behaviour is found in many confinement regimes (OH, L, H, PEP, supershot, hot-ion mode), with various heating schemes (OH, NBI, ICRH, ECRH, LHH), with LHCD and with

different fuelling methods (gas puffing, pellet injection). In the central region, however, large variations of the temperature profile are observed. We conclude that in the outer half of tokamaks $T_e$ profile invariance is a ubiquitous phenomenon and that a profile constraining mechanism is present under a large variety of conditions, which yields an approximately linear $T_e(r)$ shape.
The existence of such a process was also inferred from sensitivity studies in which the density profiles $n_e(r)$ and electron heating profiles $q_h(r)$ defined by

$$q_h(r) = (4\pi^2 R_o)^{-1} \ \frac{P_e(r)}{r} \tag{3}$$

were varied [8]. Here, $R_o$ is the major plasma radius and $P_e(r)$ is the integrated net electron heating power within the radius $r$. It was shown that local transport laws of the form

$$\chi_e(r) = s(r) r_{Te}^{-\alpha}(r) \tag{4}$$

with $\alpha \gtrsim 4$ and

$$\chi_e(r) = s(r)\Big(\frac{dT_e}{dr}\Big)^{\alpha} \tag{5}$$

with even $\alpha$ values ($\alpha \gtrsim 2$) yield the experimental stiffness of the temperature profile. These transport laws provide, in principle, an efficient profile constraining mechanism but the high $\alpha$ values required conflict with empirical $\chi_e$ scalings. Moreover, to obtain the linear $T_e$ profile with Eqs. (4) and (5), special functions $s(r)$ have to be prescribed. An approximately linear $T_e(r)$ is only reached when $q_h/(n_e s)$ is about constant. In the case of a wrong choice of $s(r)$ resulting in a marked $r$ dependence, other temperature profile shapes are obtained.
For the zone $0.5 \lesssim r/a \lesssim 1.0$, we propose the following scaling with negative feedback:

$$\chi_e(r) = \chi_{ec}(r)\Big(1 - \alpha\frac{a^2}{T_{eo}}\frac{d^2T_e}{dr^2}\Big) \tag{6}$$

where $\alpha$ is a positive constant. The feedback term is proportional to the deviation of the curvature of $T_e(r)$ from the vanishing curvature of the linear (reference) profile. This scheme by itself generates a shape close to a straight line. The new factor acting on the confinement regime dependent scaling $\chi_{ec}(r)$ takes into account that $T_e$ profile invariance is observed in all confinement regimes. The response of the temperature profile to changes in the $n_e(r)$ and $q_h(r)$ shapes will now be explored.
Under steady-state conditions, the conductive electron heat flux density is equal to $q_h(r)$:

$$q_e = -n_e \chi_e \frac{dT_e}{dr} = q_h \tag{7}$$

Substituting Eq. (6) into Eq. (7), one obtains the differential equation

$$-n_e(r)\chi_{ec}(r)\Big(1-\alpha\frac{a^2}{T_{eo}}\frac{d^2T_e}{dr^2}\Big)\frac{dT_e}{dr}=q_h(r) \tag{8}$$

It is obvious that for sensitivity studies only the variation of $q_h/(n_e\chi_{ec})$ with the radius is important, the $r$ dependences of $q_h$, $n_e$ and $\chi_{ec}$ entering with the same weight. We use the linear $T_e(r)$ given by Eq. (1) as reference profile since the sensitivity to variations of $n_e(r)$ has to be determined there. Transport simulations of neutral-beam-heated ASDEX plasmas have resulted in a roughly constant $q_h$ profile in the confinement zone. Moreover, the change in heat deposition caused by the variation of $n_e(r)$ is compensated by the modification of the convective power loss, so that $q_h(r)$ remains almost unchanged [9]. Setting in Eq. (8) $q_h = const/_r$ and $\kappa_{ec} = n_e(r)\chi_{ec}(r) = const/_r$, i.e.

$$\chi_{ec}(r)\propto\frac{n_{eo}}{n_e(r)} \tag{9}$$

yields the reference profile with the feedback term vanishing. The density profile is varied between an almost parabolic shape defining $\chi_{ec}(r)$ and a rectangular form $n_e = n_{eo}$. During the sensitivity test $\chi_{ec}(r)$ is kept fixed. By integration of Eq. (8) with

$$\frac{n_e(r)}{n_{eo}}=\Big[1-0.9\Big(\frac{r}{a}\Big)^2\Big]\Big[1-0.3\frac{a_T}{a}\frac{1}{\alpha}\Big(\frac{r}{a}\Big)^3\Big] \tag{10}$$

one obtains the exact solution

$$\frac{T_e(r)}{T_{eo}}=1-\frac{r}{a_T}+0.075\frac{1}{\alpha}\Big(\frac{r}{a}\Big)^4 \tag{11}$$

As expected, the deviation from the canonical profile becomes smaller with increasing parameter $\alpha$. The density and temperature profiles for $\alpha = 1$ and 3 are plotted in Figs. 1 and 2. It is obvious that for $\alpha = 3$ the $T_e(r)$ shape is invariant within the measurement error in spite of the large change in density profile. For $\alpha = 1$, however, the temperature profile is found to be not sufficiently stiff.
The effect of the negative feedback term in Eq. (6) on the $\chi_e$ profile is shown for $\alpha = 3$ in Fig. 3. According to Eqs. (9) and (10), the $r$ dependence of $\chi_{ec}$ is almost totally determined by the factor $[1-0.9(r/a)^2]^{-1}$. Substituting the solution into Eq. (6) it follows that $\chi_e(r) = \chi_{ec}(r)[1-0.9(r/a)^2]$, i.e. the strong variation of $\chi_{ec}$ with the radius is offset by the feedback term despite the rather small deviation of $T_e(r)$ from a straight line (see Fig. 2).
The influence of peaked and peripheral heating on the temperature profile shape is investigated, with $\chi_{ec}(r)$ and $n_e(r)$ being kept fixed and $\kappa_{ec} = n_e(r)\chi_{ec}(r) = const/_r$. By setting $q_h = const/_r$ in Eq. (8), the linear reference $T_e$ profile is recovered (see Figs. 2 and 4). Solving Eq. (8) with a peaked heating profile

$$\frac{q_h(r)}{q_h(a)}=C\Big[1-0.9\Big(\frac{r}{a}\Big)^2\Big]\Big[1-0.3\frac{a_T}{a}\frac{1}{\alpha}\Big(\frac{r}{a}\Big)^3\Big] \tag{12}$$

with

$$C = 10\left(1 - 0.3\frac{a_T}{a}\frac{1}{\alpha}\right)^{-1} \tag{13}$$

again yields the solution given by Eq. (11). The change in heating profile for $\alpha = 3$ shown in Fig. 4 thus corresponds to the small modification of $T_e(r)$ for $\alpha = 3$ in Fig. 2. The response is identical to that obtained for the density profile variation.
To study the effect of peripheral heating, we again start with $q_h = const/_r$ (see Fig. 5), yielding the reference temperature profile in Fig. 6. By integration of Eq. (8) with

$$\frac{q_h(r)}{q_h(a)} = C\left[1 + 0.9\left(\frac{r}{a}\right)^2\right]\left[1 + 0.3\frac{a_T}{a}\frac{1}{\alpha}\left(\frac{r}{a}\right)^3\right] \tag{14}$$

and

$$C = 0.53\left(1 + 0.3\frac{a_T}{a}\frac{1}{\alpha}\right)^{-1} \tag{15}$$

it follows that

$$\frac{T_e(r)}{T_{eo}} = 1 - \frac{r}{a_T} - 0.075\frac{1}{\alpha}\left(\frac{r}{a}\right)^4 \tag{16}$$

For $\alpha = 3$, the heating profile and resulting temperature profile are plotted in Figs. 5 and 6, respectively. It is evident that the $T_e$ profile is invariant within the measurement error.

In conclusion, a $T_e$ profile constraining factor acting on the $\chi_e$ scalings in all confinement regimes was introduced and its effect on the $T_e(r)$ shape studied. It was shown that the negative feedback term with $\alpha \gtrsim 3$ in Eq. (8) provides an efficient profile constraining mechanism which can cause temperature profile resilience upon large variations of density and heating profiles. This mechanism should also work with significant changes in the $\chi_{ec}(r)$ shape. The feedback by itself yields the approximately linear $T_e(r)$ for $r/a \gtrsim 0.5$. It is thus superior to the profile constraining models given in Eqs. (4) and (5), where special functions $s(r)$ have to be prescribed.
It is difficult to find the origin of the $d^2T_e/dr^2$ term. As $T_e$ profile invariance is also observed in situations with the temperature and current profiles decoupled, a connection with $dj/dr$ and tearing instabilities can be excluded. By contrast, strong turbulence induced by drift instabilities or pressure driven modes is a possible cause of profile resilience and the proposed scaling. The nonlinear polarization drift term $-\vec{v}\cdot\nabla\nabla^2\tilde{\phi}$ plays an important role in nonlinear drift wave turbulence and, owing to $e\tilde{\phi} \simeq T_e\tilde{n}_e/n_e$, might be responsible for the feedback term.

## REFERENCES


[1] Murmann, H., Wagner, F., Becker, G., et al., in Controlled Fusion and Plasma Heating (Proc. 13th Europ. Conf. Schliersee, 1986), Vol. 10C, Part I (1986) 216.

[2] Fredrickson, E.D., McGuire, K.M., Goldston, R.J., et al., Nucl. Fusion 27 (1987) 1897.

[3] Becker, G., Nucl. Fusion 31 (1991) 663.

[4] McCarthy, P.J., Riedel, K.S., Kardaun, O.J.W.F., et al., Nucl. Fusion 31 (1991) 1595.

[5] Coppi, B., Comments Plasma Phys. Controll. Fusion 5 (1980) 261.

[6] Becker, G., Nucl. Fusion 31 (1991) 1243.

[7] Becker, G., Nucl. Fusion 32 (1992) 81.

[8] Becker, G., Nucl. Fusion 32 (1992) 1064.

[9] Becker, G., ASDEX Team, Neutral Injection Team, Nucl. Fusion 28 (1988) 139.


## FIGURE CAPTIONS

Fig. 1 Variation of density profile between essentially parabolic and rectangular shapes.

Fig. 2 Response of electron temperature profile with $\chi_{ec}(r)$ kept fixed. The straight line is the reference profile.

Fig. 3 Influence of the feedback term on the $\chi_e(r)$ shape.

Fig. 4 Variation of heating profile between constant $q_h$ and peaked shape.

Fig. 5 Variation of heating profile between constant $q_h$ and peripheral shape.

Fig. 6 Response of electron temperature profile with $n_e(r)$ and $\chi_{ec}(r)$ kept fixed.

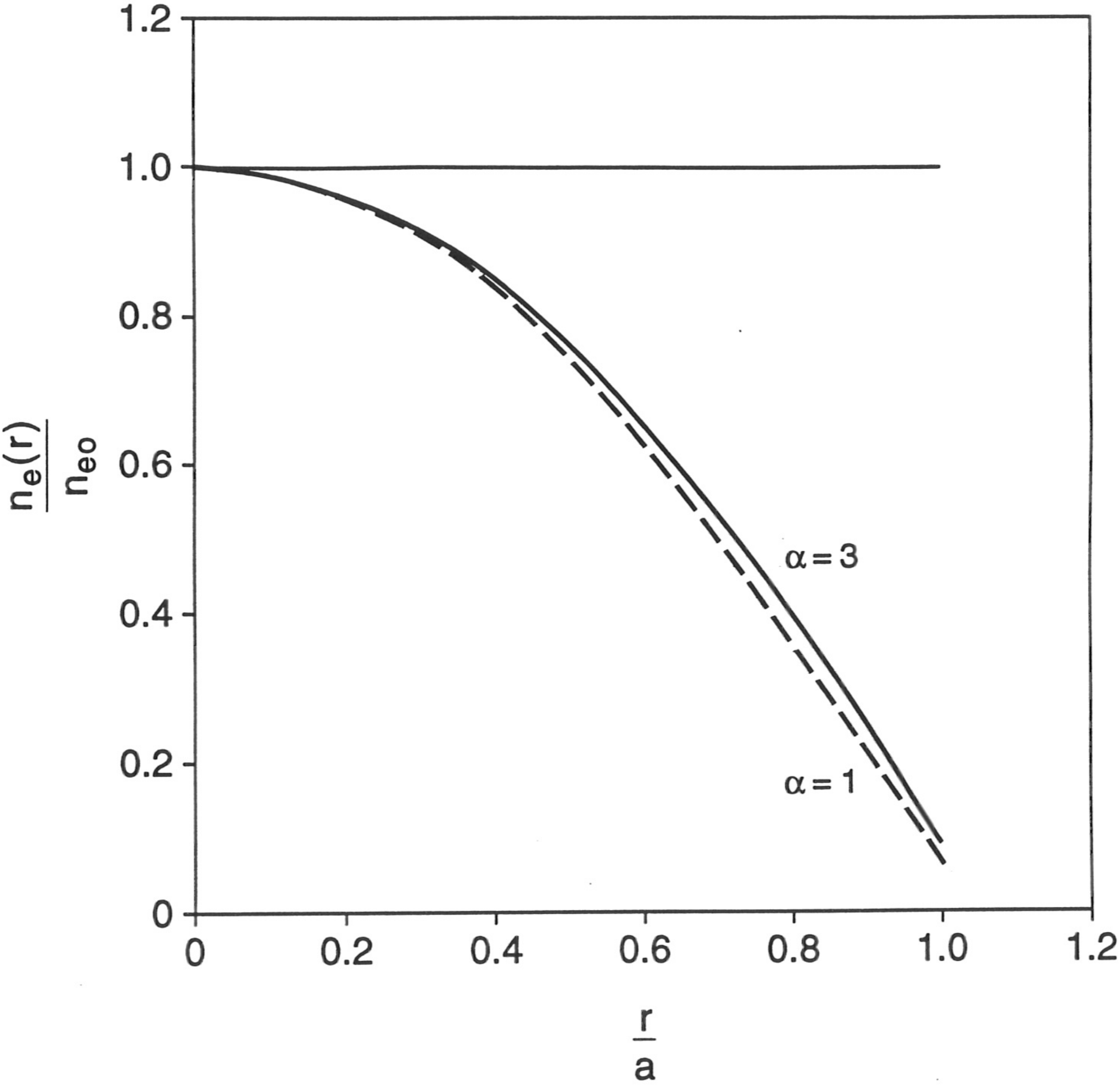



Fig. 1

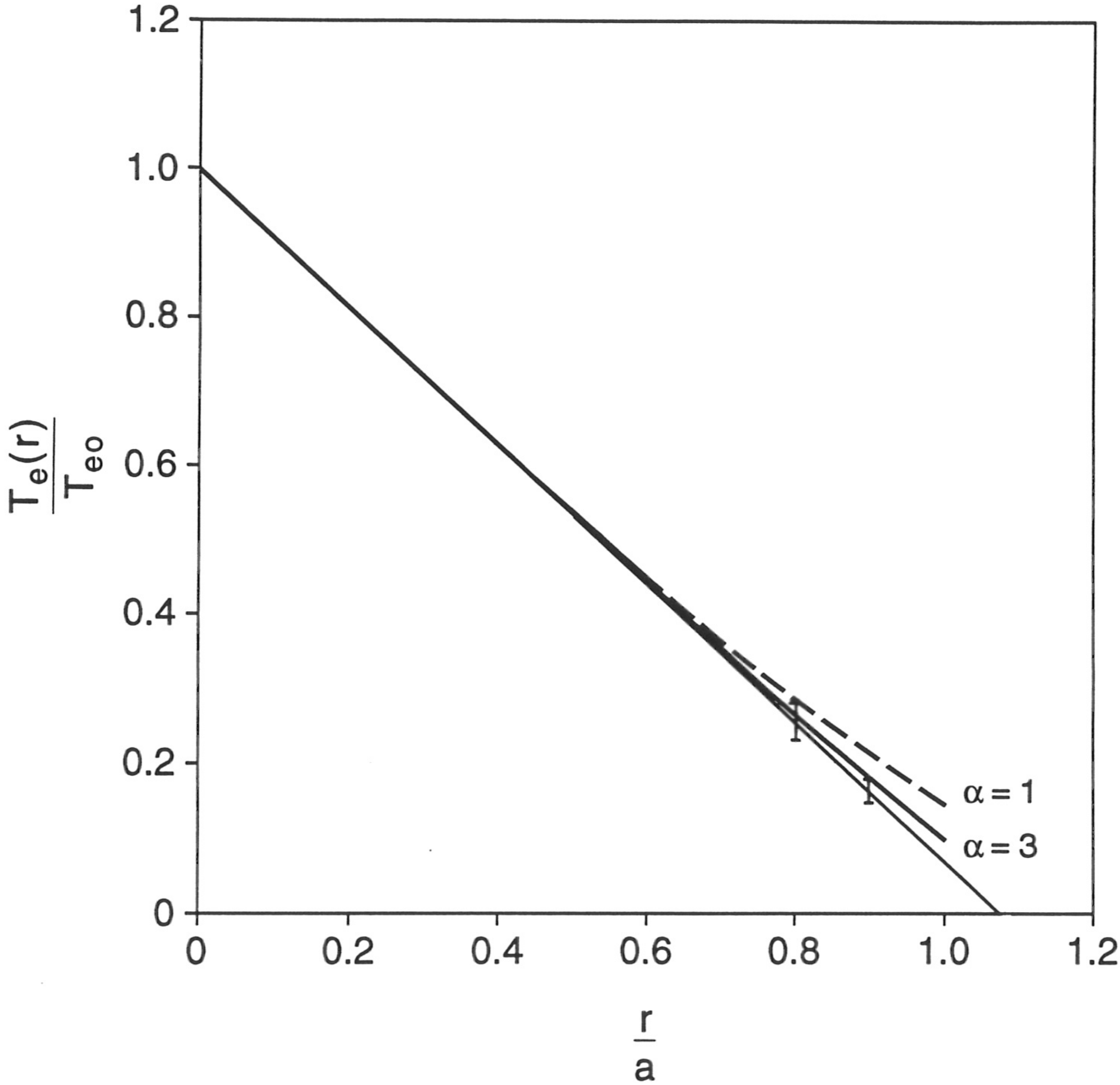
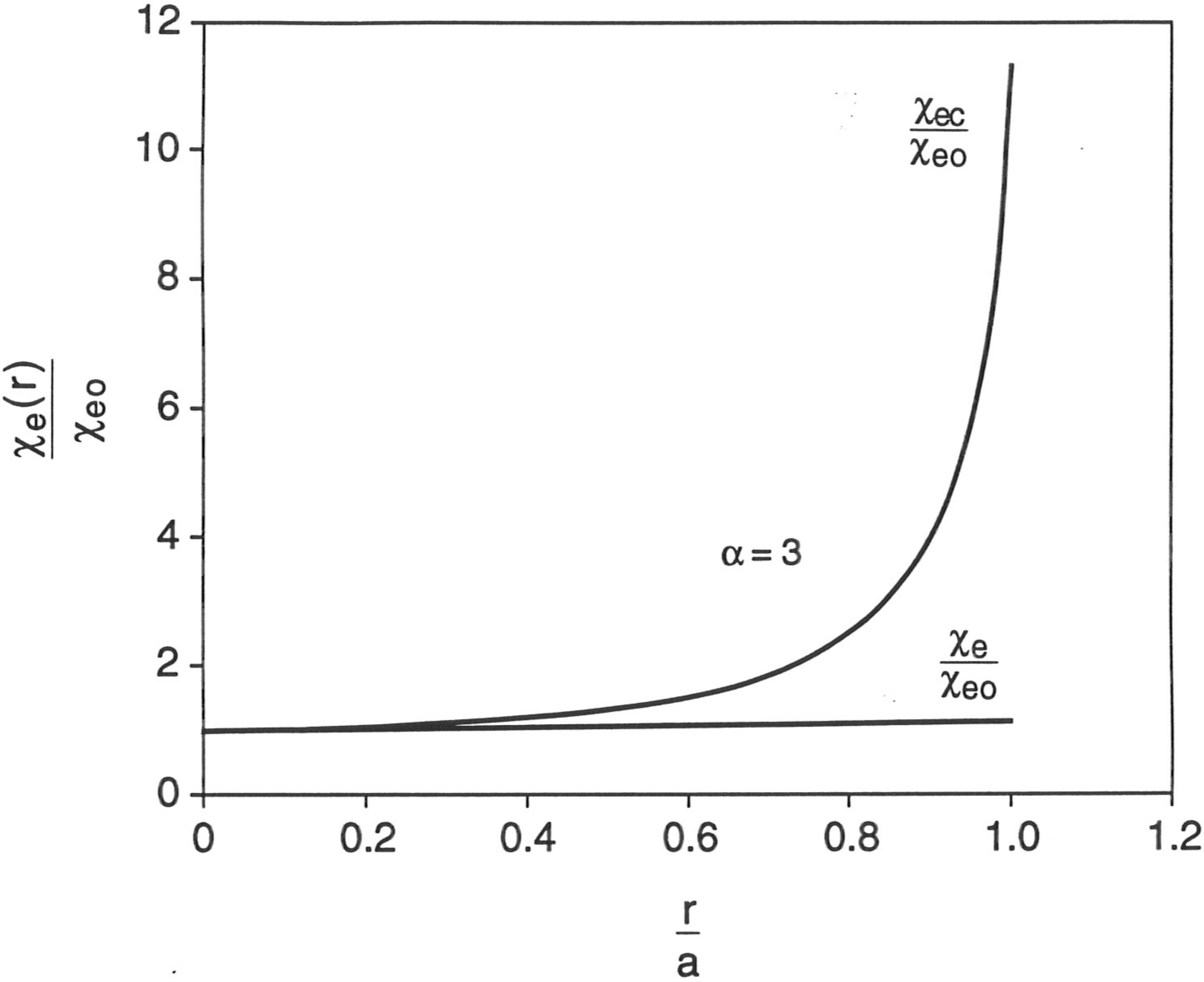



Fig. 2



12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1.0
1.2
$\frac{\chi_e(r)}{\chi_{eo}}$
$\frac{r}{a}$
$\frac{\chi_{ec}}{\chi_{eo}}$
$\alpha = 3$
$\frac{\chi_e}{\chi_{eo}}$


Fig. 3

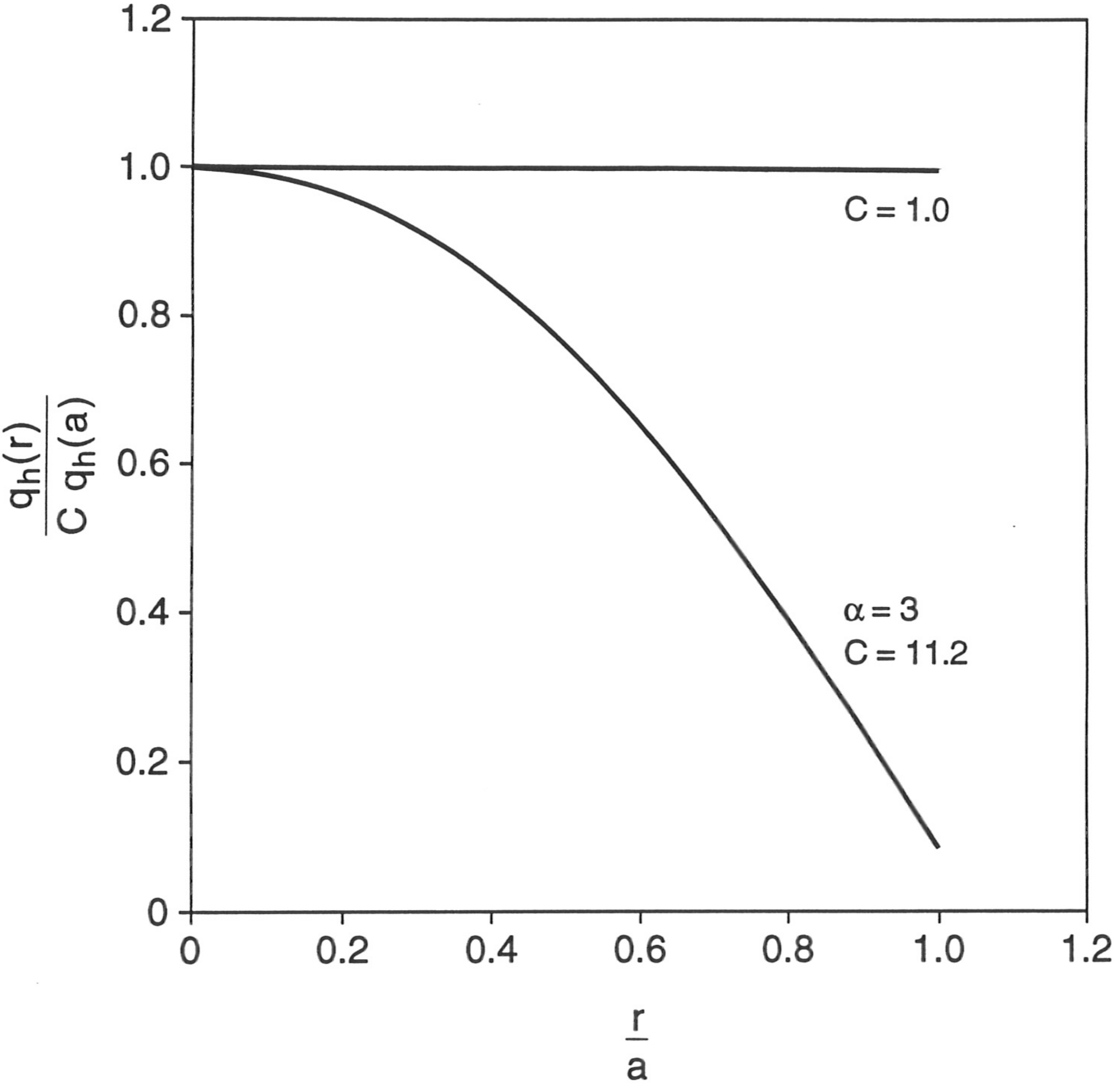

1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{q_h(r)}{C\, q_h(a)}$
C = 1.0
α = 3
C = 11.2
0
0.2
0.4
0.6
0.8
1.0
1.2
$\frac{r}{a}$

Fig. 4

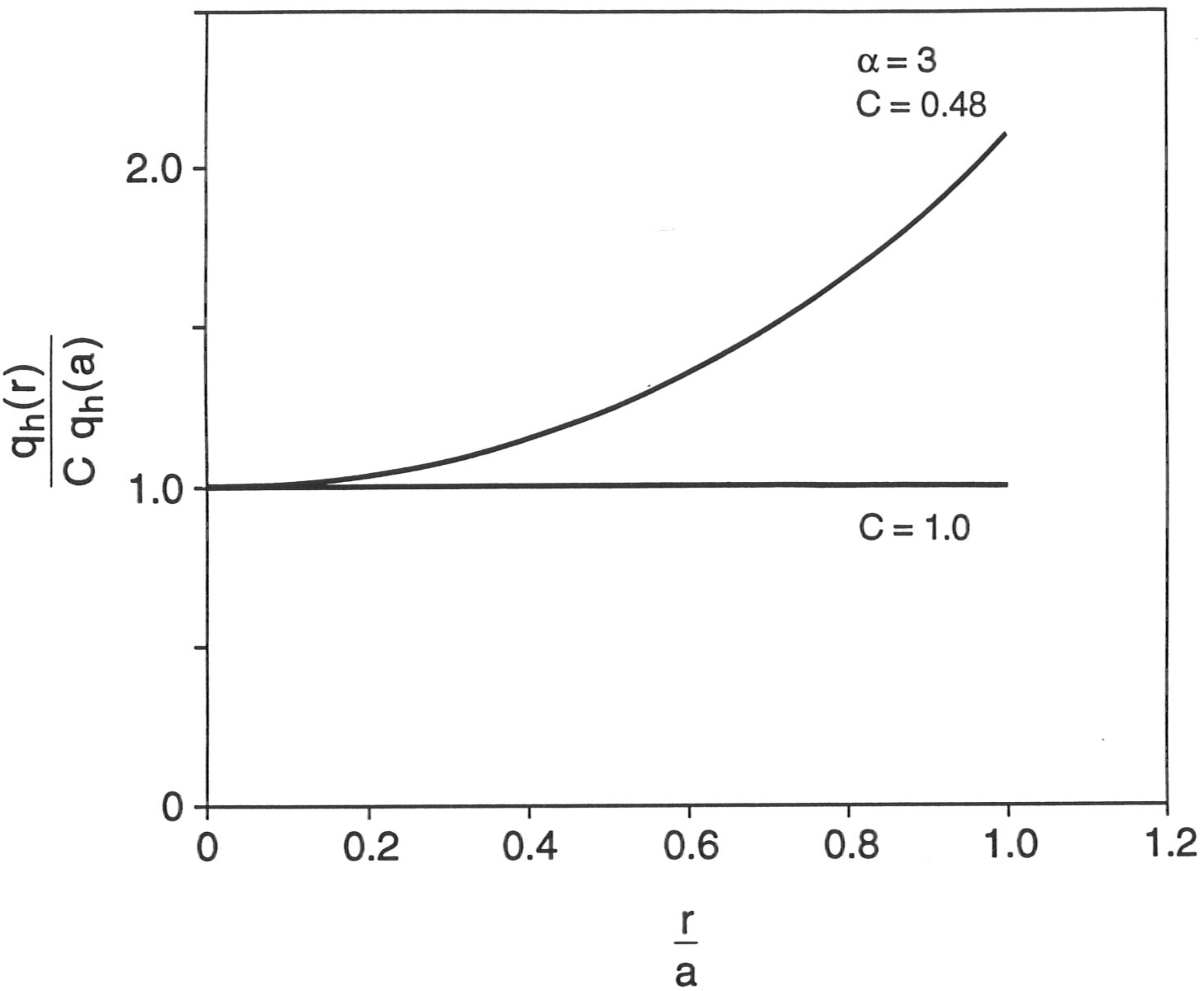



Fig. 5

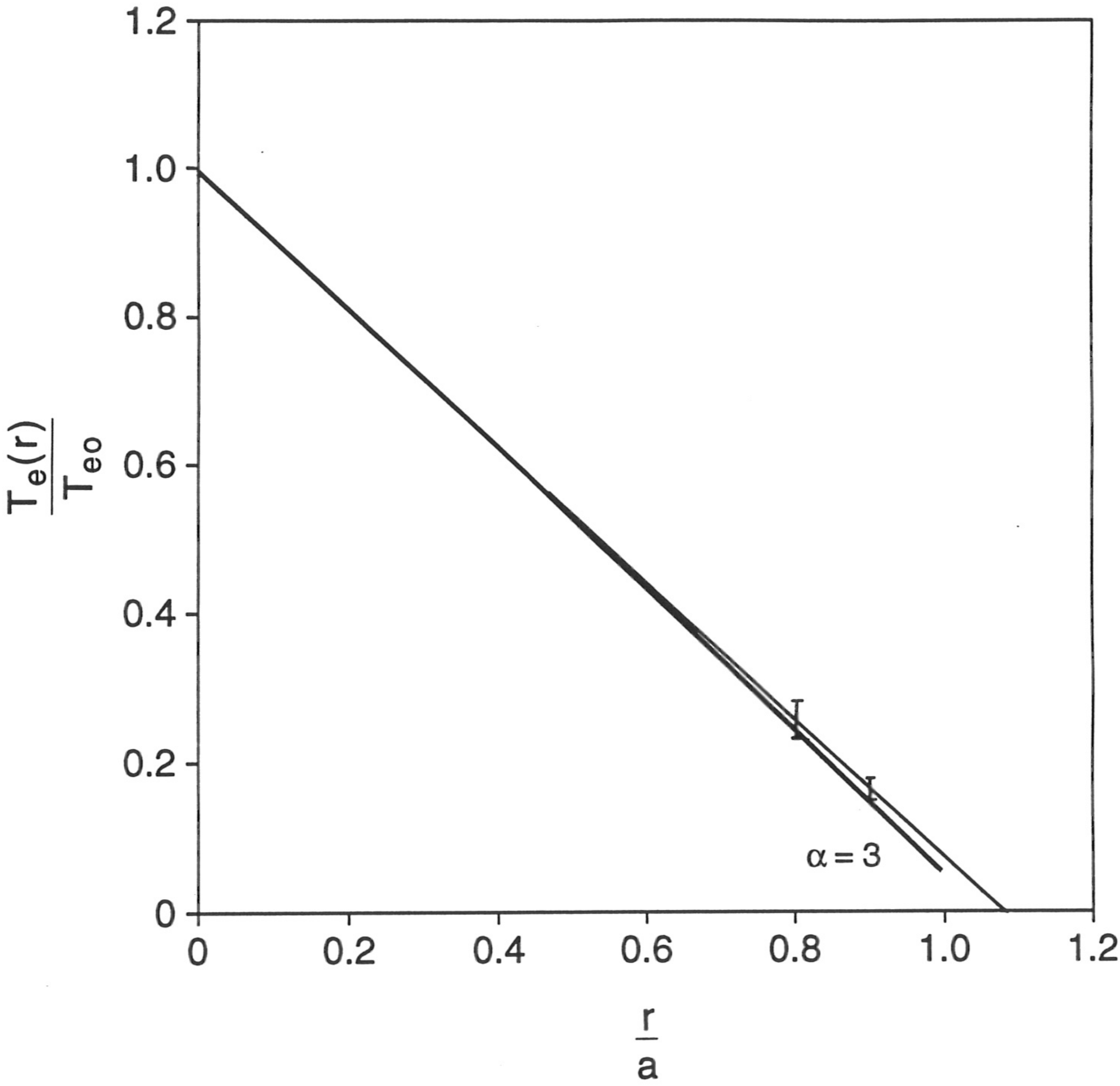



Fig. 6