MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

Toroidal Plasmas Permeability Tensor and Dissipation of Fast Waves (Methods of Evaluation and Some Results)

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IPP 4/255 July 1992

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Abstract

A method for the analytical treatment of the toroidal plasma electron dielectric permeability tensor is developed.

Simple expressions for some limiting cases are obtained.

Electron Landau and TTMP absorption of the fast waves in tokamaks are discussed in terms of „nonlocality" effects, including the effect of trapped and untrapped particles bounce resonances. Additional dissipation of the fast waves in tokamaks is founded in a comparison with cylindrical model Landau damping.

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

INTRODUCTION.

The fast wave (FW) is widely used for ion-cyclotron resonance heating of large tokamak plasmas (see [1-2]).

Besides, it also be used for current drive in tokamaks [2-4] via absorption of travelling FW due to electron Landau or TTMP damping. In principle, this current can replace the Ohmic current in a tokamak, as also envisaged for the ITER project [5].

For current drive analysis one needs to solve the set of Vlasov- Maxwell equations in toroidal geometry including the specific effect of electron's periodic motion in a tokamak.

Now the computation of this problem is formidable task. As a preliminary step it needs to evaluate the electron part of the permeability tensor.

In the simplest approach that ignores drift motion across the magnetic surfaces and assumes that toroidicity parameter $\varepsilon = r/R \ll 1$, Grishanov and Nekrasov [6] have derived the toroidal plasma dielectric permeability tensor using a boundary value analysis. Puri [7-8] has calculated Landau damping using the Landau pole integration. Some analytic results are obtained earlier [9-11].

Here we prolong analytic treatment of the toroidal plasma dielectric permeability tensor along the lines of Ref.[6] and suggest analytical methods for evaluation of this tensor. These methods are based on transformations with Jacobi functions.

TOROIDAL PLASMAS PERMEABILITY TENSOR EVALUATION

Here we use the simplest axisymmetric tokamak plasma model. The magnetic surfaces are considered circular and concentric. This model is reasonable for tokamaks with low pressure $\beta \ll 1$

and small toroidicity parameter ($\varepsilon = r/R \ll 1$). For this model it is possible to use a quasitoroidal coordinate system with

$$X = (R_o + r\cos\theta)\cos\zeta, \quad Y = -(R_o + r\cos\theta)\sin\zeta, \quad Z = r\sin\theta,$$

and the same type of a stationary magnetic field as in [6]

$$B_r^{(o)} = 0, \quad B_\theta^{(o)} = h_\theta B_o R_o/R, \quad B_\zeta^{(o)} = h_\zeta B_o R_o/R, \quad Q = rh_\zeta/R_o h_\theta = 1/R_o k_o.$$

Further neoclassic conditions for the tokamak will be assumed with $\nu_{ei} \ll \sqrt{\varepsilon/2}\, v_{Te}/RQ$ and small collisions will be taken into account by formal transformation of the wave frequency to $\Omega \to \omega + i\nu_{ef}$ that means τ-approximation for a collision operator $\hat{St}\,\tilde{f} = \nu\,\tilde{f}$.

We begin a finding the solution of linearized Vlasov's kinetic equation for perturbations of an electron distribution function $F_o + \tilde{f}$ by RF fields of the waves $\tilde{f} \sim \tilde{E}_{r,\theta,\zeta}\, exp\; i(n\zeta - \Omega t)$. It will be assumed that frequency of the wave is much less then electron-cyclotron frequency ($\Omega \ll \omega_c$), Larmor radius $\rho_\lambda = v_T/\omega_c$ and thickness of banana trajectories are assumed to be small.

After expansion in Fourier series over angle σ in velocity space $(v_\perp, \sigma, v_\parallel)$ $f = \sum f_l exp\; il\sigma$, Vlasov's equation will be [6]:

$$\partial f_o^{(s)}/\partial\theta + \lambda_o^{(s)} f_o^{(s)} = G_o^{(s)}; \quad \lambda_o^{(s)} = \frac{nQ}{1+\varepsilon\cos\theta} - \frac{s\Omega}{k_o v} \Big/ \sqrt{1-\Lambda/(1+\varepsilon\cos\theta)}; \quad (1)$$

However, unlike Ref. [6] we take into account first order drift corrections to the stationary local Maxwell distribution $F_o = F_M + F_b \sin\sigma$ (where $F_b = \dfrac{v_\perp}{\omega_c}\dfrac{\partial F_M}{\partial r}$), and perturbation function $f = f_o + f_r\cos\sigma + f_b\sin\sigma$ (where $f_r = f_1 + f_{-1}$, $f_b = f_1 - f_{-1}$).

Using Eq.(6) of Ref.[6] we obtain equations for f_o, f_r, f_b after equating coefficients with the same σ-harmonics :

$$-i\Omega f_o + v_\parallel \hat{k}_\parallel f_o - \frac{v_\perp h_\theta}{2\,R}\ \sin\theta\ \hat{L} f_o + \frac{v_\perp}{2}\frac{\partial f_r}{\partial r} + \frac{v_\perp}{2}\hat{k}_b f_b + \frac{v_\parallel}{2}\frac{h_\zeta}{R} n\Big(\sin\theta\ \hat{L} f_b$$

$$- h_\zeta \cos\theta\ \hat{L} f_r\Big) + \frac{h_\zeta}{2v_\perp}\left(\frac{v_\parallel^2}{R}\cos\theta + v_\perp^2 \frac{h_\zeta}{r}\right) f_r - \frac{h_\zeta}{2v_\perp}\frac{v_\parallel^2}{R}\sin\theta\ f_b = -\frac{e}{M}\Big[E_3\frac{\partial F_M}{\partial v_\parallel}$$

$$+ \frac{E_2}{\omega_c}\frac{\partial}{\partial r}\left(F_M + v_\perp \frac{\partial F_M}{\partial v_\perp}\right) + \frac{v_\parallel}{c v_\perp} H_1 F_b\Big]; \qquad (2a)$$

$$\left\{\omega_c + 3\frac{v_\parallel}{2} h_\zeta\left[\frac{h_\theta}{r}\frac{R_o}{R} + \frac{h_\zeta}{3}\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\zeta}\right)\right]\right\} f_b = \left(v_\parallel\ \hat{k}_\parallel - i\ \Omega\right) f_r + v_\perp \frac{\partial f_o}{\partial r}$$

$$-\frac{v_\perp}{2}\frac{h_\theta}{R}\ \sin\theta\ \hat{L} f_r - v_\parallel \frac{h_\zeta^2}{R}\cos\theta\ \hat{L} f_o + \frac{e}{M}\left[E_1\frac{\partial F_M}{\partial v_\perp} + \frac{H_2}{c}\hat{L} F_M - \frac{H_3}{c} F_b\right]; \qquad (2b)$$

$$\left\{\omega_c + 3\frac{v_\parallel}{2} h_\zeta\left[\frac{h_\theta}{r}\frac{R_o}{R} + \frac{h_\zeta}{3}\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\zeta}\right)\right]\right\} f_r = i\ \Omega\ f_b - v_\parallel \hat{k}_\parallel f_b - v_\perp \hat{k}_b f_o +$$

$$\frac{v_\perp}{2}\frac{h_\theta}{R}\ \sin\theta\ \hat{L} f_b - v_\parallel \frac{h_\zeta}{R}\sin\theta\ \hat{L} f_o - \frac{e}{M}\left[E_2\frac{\partial F_M}{\partial v_\perp} + E_3\frac{\partial F_b}{\partial v_\parallel} - \frac{H_1}{c}\hat{L} F_M\right]. \qquad (2c)$$

Where $\hat{k}_b f_{o,b} = \left(\frac{h_\zeta}{r}\frac{\partial f_{o,b}}{\partial\theta} - \frac{in}{R} h_\theta f_{o,b}\right)$; $\hat{k}_\parallel f_{o,b} = \left(\frac{h_\theta}{r}\frac{\partial f_{o,b}}{\partial\theta} + \frac{in}{R} h_\zeta f_{o,b}\right)$

Here indices $1,2,3$ means coordinate system connected with magnetic field lines ($1=r$ - radial, $2=b$ -binormal, $3=\parallel$ -parallel). Substituting Eq.(2b) and (2c) to each other and omitting second order drift and toroidicity terms and exchanging variables as $\theta' = \theta$; $\Lambda = \left(\frac{v_\perp}{v}\right)^2\left(1 + \varepsilon\cos\theta\right)$; $v_\parallel = s\sqrt{1 - \frac{\Lambda}{1+\varepsilon\cos\theta}}$, $s = \pm 1$, we obtain:

$$f_r^{(s)} = -\frac{1}{\omega_c}\Big\{\frac{e}{M}\Big[E_2\frac{\partial F_M}{\partial v}\frac{\Lambda\ v}{1+\varepsilon\cos\theta} + E_3\ s\frac{v}{\omega_c}\sqrt{1 - \frac{\Lambda}{1+\varepsilon\cos\theta}}\frac{\partial F_M}{\partial v\partial r} - \frac{H_1}{c}\hat{L} F_M -$$

$$\left(i\,\Omega - vs\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\theta}}\,\hat{k}_{\parallel}\right)\frac{1}{\omega_c}\,E_1\,\frac{\Lambda v}{1+\varepsilon\cos\theta}\,\frac{\partial F_M}{\partial v}\Bigg] + v\sqrt{\frac{\Lambda}{1+\varepsilon\cos\theta}}\,\hat{k}_b f_o$$

$$- vs\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\theta}}\,\frac{h_\zeta}{R}\,\sin\theta\,\hat{L}f_o \tag{3a}$$

$$f_b^{(s)} = \frac{1}{\omega_c}\left\{\frac{e}{M}\left[E_1\frac{\partial F_M}{\partial v}\frac{\Lambda v}{1+\varepsilon\cos\theta} + \frac{H_2}{c}\hat{L}F_M - \frac{H_3}{c}F_b\right] + \left(v\,s\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\theta}}\,\hat{k}_{\parallel} - \right.\right.$$

$$\left. - i\,\Omega\right)\frac{1}{\omega_c}\left[v\sqrt{\frac{\Lambda}{1+\varepsilon\cos\theta}}\,\frac{\partial f_o}{\partial r} + \frac{e}{M}\,E_2\,\frac{\Lambda v}{1+\varepsilon\cos\theta}\,\frac{\partial F_M}{\partial v}\right]. \tag{3b}$$

After substitution of Eqs.(3b) and (3c) in (2a) and exchanging variable with $\theta' = \theta$ and Λ, one obtains the simplified drift kinetic equation with another right side then in Ref. [6]:

$$\partial f_o^{(s)}/\partial\theta + \lambda_o^{(s)} f_o^{(s)} = \frac{e}{Mk_o}\left[\frac{E_3}{v} - \frac{s\Lambda}{2\omega_c}\,\frac{[\vec{k}\times\vec{E}_\perp]}{\sqrt{1+\varepsilon-\Lambda}}\Big/\sqrt{1-\frac{2\varepsilon\,\sin^2\theta/2}{1+\varepsilon-\Lambda}}\right]\frac{\partial F_M}{\partial v};$$

$$\lambda_o^{(s)} = \frac{nQ}{1+\varepsilon\cos\theta} - \frac{s\Omega}{k_o v}\Big/\sqrt{1-\Lambda/(1+\varepsilon\cos\theta)};\quad [\vec{k}\times\vec{E}_\perp] = \left(\frac{1}{r}\frac{\partial}{\partial r}\left(rE_b\right) - \hat{k}_b E_r\right). \tag{4}$$

Method of solution of this equation is based on using the Jacobi elliptic functions [12,13] with separate treatment [15] for untrapped (u) and trapped particles (t).

1. Untrapped electrons distribution function evaluation.

After exchanging distribution function $\bar{f}_u^s = f_{ou}^{(s)} \exp i\Big[nQ(\theta -$

$\varepsilon \sin\theta) + \varepsilon\,\delta_s^{(u)}\int_o^{\theta/2} d\eta\,\sin^2\eta\,/\sqrt{1-\kappa^2\sin^2\eta}\Big]$, $\quad \delta_s^{(u)} = s\sqrt{2/\varepsilon}\,\frac{\kappa\,\Omega}{v\,k_o}$;

expanding RF field in Fourier series over θ, and introducing the new variable (for the untrapped electrons):

$$w = \int_0^{\theta/2} d\eta/\sqrt{1-\kappa^2\sin^2\eta};\quad \theta/_2 = \text{am}\ w;\quad \frac{d\theta}{dw} = 2\sqrt{1-\kappa^2\sin^2\theta/_2};\quad \kappa^2 = \frac{2\varepsilon}{1+\varepsilon-\Lambda};$$

Eq. (4) reduces to:

$$\frac{\partial \bar{f}^s_u}{\partial w} - i\delta^{(u)}_s(1-\varepsilon/2)\bar{f}^s_u = \frac{2e}{Mk_o}\sum_m\left[\frac{E^m_3}{v}dnw - \frac{\kappa^2(1+\varepsilon)-2\varepsilon}{2\sqrt{2\varepsilon}\ s\ \kappa\ \omega_c}[\vec{k}_m\times\vec{E}^m_\perp]\right]\frac{\partial F_M}{\partial v}\exp\ i\Xi^m_u;$$

$$\Xi^m_u = \left\{2(m+nQ)\ \text{am}\ w - 2\varepsilon\ (\text{sn}\ w)(\text{cn}\ w) + \varepsilon\ \delta^{(u)}_s[w-E(w,\kappa)]/\kappa^2\right\}. \quad (5)$$

Here *sn w*, *cn w*, *am w* and *dn w* are Jacobi functions, and **$E(w,\kappa)$]** is the incomplete elliptic integral of second type [13-14].

An electron distribution function for untrapped electrons is periodic over θ with 2π period then one must be periodic over w with $2K$ period. K is the elliptic integral of first type [12-13]. To find solutions of Eq.(5) that are periodic over w we divide $\text{am}\ w = \tilde{\text{am}}\ w + \frac{\pi w}{2K}$ and $E(w,\kappa) = w\ E/K + \tilde{Z}(w)$ into periodic and nonperiodic parts, then the equation for the periodic part

$$\tilde{f}^s_u = \bar{f}^s_u \exp\left\{-i\left[nQ\ \pi\ \frac{w}{K} + \varepsilon\ \delta^{(u)}_s\ w\left(\frac{K-E}{K\ \kappa^2}\right)\right]\right\}$$

is:

$$\frac{\partial \tilde{f}^s_u}{\partial w} + i\tilde{\Delta}_s\tilde{f}^s_u = \frac{2e}{Mk_o}\sum_m\left[\frac{E^m_3}{v}dn\ w - \frac{\kappa^2-\kappa^2_o}{2s\ \kappa\ \kappa_o\omega_c}[\vec{k}_m\times\vec{E}^m_\perp]\right]\frac{\partial F_M}{\partial v}\exp\ i\tilde{\Xi}^m_u; \quad (6)$$

where $\tilde{\Delta}^s_u = \left\{nQ - s\frac{\kappa\ \Omega}{v\ k}\left[1-\varepsilon\left(\frac{1}{2}-\frac{K-E}{K\kappa^2}\right)\right]/\sqrt{2\varepsilon}\ \right\};\quad \kappa^2_o = 2\varepsilon/(1+\varepsilon).$

After Fourier expansion of the right hand side of Eq.(6) with respect to w are obtained:

$$\frac{\partial \tilde{f}^s_u}{\partial w} + i\tilde{\Delta}^s_u\tilde{f}^s_u = \frac{2e}{k_oM}\sum_{m,l}\left[\frac{E^m_3}{v}A^m_l - \frac{\kappa^2-\kappa^2_o}{2s\ \kappa\ \kappa_o\omega_c}B^m_l\ [\vec{k}_m\times\vec{E}^m_\perp]\right]\frac{\partial F_M}{\partial v}\exp(il\pi\frac{w}{K});$$

Here $A_l^m = \frac{1}{2K}\int_{K}^{K} dw'(dn\, w')\exp\left[i\left(\tilde{\Xi}_u^m - l\frac{\pi w'}{K}\right)\right]$; $B_l^m = \frac{1}{2K}\int_{K}^{K} dw' \exp\left[i\left(\tilde{\Xi}_u^m - l\frac{\pi w'}{K}\right)\right]$; $\tilde{\Xi}_u^m = \left[2(m+nQ)\, a\tilde{m}\, w + m\pi\frac{w}{K} - 2\varepsilon(sn\, w)(cn\, w) - \varepsilon\tilde{Z}\frac{\delta_s^{(u)}}{\kappa^2}\right]$. (7)

The periodic solutions of this equation expanded in Fourier series over w $\left(\tilde{f}_u^s = \sum_l \tilde{f}_{us}^{\,l} \exp i\left(l\,\pi\frac{w}{K}\right)\right)$ are:

$$\tilde{f}_{us}^{\,l} = \frac{2ie}{Mk_o\pi}\sum_{m,l}\frac{K}{1+\Delta_u^s}\left[\frac{E_3^m}{v}A_l^m - \frac{\kappa^2-\kappa_o^2}{2s\,\kappa\,\kappa_o\omega_c}B_l^m\,[\vec{k}_m\times\vec{E}_\perp^m]\right]\frac{\partial F_M}{\partial v}; \qquad (8)$$

Finally, upon back substitution into f_{ou}^s the complete solution of Eq.(4) is:

$$f_{ou}^s = \frac{2ie}{Mk_o\pi}\sum_{m,l}\frac{K}{1+\Delta_u^s}\left[\frac{E_3^m}{v}A_l^m - \frac{\kappa^2-\kappa_o^2}{2s\,\kappa\,\kappa_o\omega_c}B_l^m\,[\vec{k}_m\times\vec{E}_\perp^m]\right]\frac{\partial F_M}{\partial v}\exp i\bar{\Xi}_u; \qquad (9)$$

where $\Delta_u^s = \frac{K}{\pi}\left\{nQ - s\frac{\kappa\,\Omega}{v\,k}\left[1-\varepsilon\left(\frac{1}{2}-\frac{K-E}{K\kappa^2}\right)\right]\Big/\sqrt{2\varepsilon}\right\}$; and phase is given by:

$$\bar{\Xi}_u = \pi(1-m)\frac{w}{K} - (m+2nQ)\,a\tilde{m}\,w + 2\varepsilon\, nQ(sn\, w)(cn\, w)] + \varepsilon\delta_s^{(u)}\frac{Z(w,\kappa)}{\kappa^2}$$

Untrapped electron RF current induced by RF electric fields has the form:

$$(1+\varepsilon\cos\theta)\,\tilde{j}_{r,b}^{(u)} = \frac{\pi e}{2}\int_0^\infty v^3 dv\int_0^{1-\varepsilon} d\Lambda\sum_s f_{r,b,s}^{(u)}\sqrt{\frac{\Lambda}{1-\Lambda+\varepsilon\cos\theta}};$$

$$(1+\varepsilon\cos\theta)\,\tilde{j}_3^u = \pi\, e\int_0^\infty v^3 dv\int_0^{1-\varepsilon} d\Lambda\sum_s s f_{us} \qquad (10)$$

It is convenient to carry out dissipated power analysis for one Fourier harmonic p of oscillatory current $P = Re\sum_{p} \vec{\tilde{j}}^{p}(\vec{E}^{p})^{*}/2$:

$$\left[(1+\varepsilon\cos\theta)\tilde{j}_{3}^{u}\right]^{p} = \pi e\int_{0}^{\infty} v^{3}dv\int_{0}^{1-\varepsilon} d\Lambda \sum_{s} sf_{us}^{p} = 4\varepsilon\pi e\int_{0}^{\infty} v^{3}dv\int_{\kappa_o}^{1} \frac{d\kappa}{\kappa^{3}} \sum_{s} sf_{us}^{p} \quad (10a)$$

where $$f_{us}^{p} = \frac{2ie}{Mk_{o}\pi^{2}} \sum_{m,l} \frac{K^{2}}{1+\Delta_{s}} \bar{A}_{l}^{m}\left[\frac{E_{3}^{m}}{v} A_{l}^{m} - \frac{\kappa^{2}-\kappa_{o}^{2}}{2s\ \kappa\ \kappa_{o}\omega_{c}} B_{l}^{m}\ [\vec{k}_{m}\times\vec{E}_{\perp}^{m}]\right] \frac{\partial F_{M}}{\partial v};$$

and $\bar{A}_{l}^{m} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\ \exp\ i\bar{\Xi}_{u} \equiv A_{l}^{m}$ which is apparent after exchange of variable $\theta = 2\ am\ w$.

For comparison with cylindrical permeability tensor we introduce toroidal tensor-operator:

$$j_{3u}^{p} = \frac{\Omega}{4\pi i}\sum_{m}\left(\varepsilon_{33}^{pm} E_{3}^{m} + \hat{\varepsilon}_{3\perp}^{pm} E_{\perp}^{m}\right); \quad j_{bu}^{p} = \frac{\Omega}{4\pi i}\sum_{m}\left(\hat{\varepsilon}_{b3}^{pm} E_{3}^{m} + \hat{\varepsilon}_{b\perp}^{pm} E_{\perp}^{m}\right); \quad (11)$$

$$\varepsilon_{33}^{pm} = \frac{16\ \sqrt{2}\varepsilon\ \omega_{p}^{2}}{\Omega\ k_{o}v_{T}\ \pi^{3/2}}\int_{0}^{\infty} u^{3}du\ e^{-u^{2}} \sum_{l,s}\int_{\kappa_o}^{1} \frac{d\kappa}{\kappa^{3}} \frac{s\ K^{2}\ A_{l}^{p}}{\pi(1+\Delta_{s})} A_{l}^{m}; \quad (11a)$$

$$\hat{\varepsilon}_{3\perp}^{pm} E_{\perp}^{m} = \frac{8\ \sqrt{\varepsilon}\ \omega_{p}^{2}}{\Omega\ k_{o}v_{T}\pi^{3/2}}\int_{0}^{\infty} u^{4}du\ e^{-u^{2}} \sum_{l,s}\int_{\kappa_o}^{1} \frac{d\kappa}{\pi} \frac{K^{2}A_{l}^{p}}{\omega_{c}\kappa^{4}} \frac{\kappa^{2}-\kappa_{o}^{2}}{(1+\Delta_{s})\kappa_{o}} B_{l}^{m}\ [\vec{k}_{m}\times\vec{E}_{\perp}^{m}].$$

Further we simplify expressions for A_{l}^{m} and B_{l}^{m} using [13-14] q-series of Jacobi functions $\left(q = \alpha+2\alpha^{5}+15\alpha^{9}+..,\ \alpha = \frac{1}{2}\frac{1-(1-\kappa^{2})^{1/4}}{1+(1-\kappa^{2})^{1/4}}\right)$:

$$sn\ w = \frac{2\pi}{\kappa K}\sum_{1}^{\infty} \frac{q^{n-1/2}}{1-q^{2n-1}} \sin\left[(2n-1)\frac{\pi w}{2K}\right]; \quad \tilde{am}\ w = 2\sum_{1}^{\infty} \frac{q^{n}/n}{(1+q^{2n})} \sin\left(\pi n\frac{w}{K}\right);$$

$$cn\, w = \frac{2\pi}{\kappa K}\sum_1^\infty \frac{q^{n-1/2}}{1+q^{2n-1}} \cos\left[(2n-1)\frac{\pi w}{2K}\right]; \quad \tilde{Z}(w) = \frac{2\pi}{K}\sum_1^\infty \frac{q^n}{1-q^{2n}} \sin\left(\pi n \frac{w}{K}\right);$$

$$dn\, w = \frac{\pi}{2K} + 2\frac{\pi}{K}\sum_1^\infty \frac{q^n}{1+q^{2n}} \cos\left(\pi\, n\, \frac{w}{K}\right), \quad \frac{2K}{\pi} = \left(1 + 2\sum_1^\infty q^{n^2}\right)^2; \tag{12}$$

Taking into account only first two terms of this series ($q \ll 1$) after Fourier expansion of exponent factor we obtain:

$$A_1^m = \frac{\pi}{2K^2}\int_{-K}^{K} dw' \left[1 + 4q \cos\left(\pi \frac{w}{K}\right)\right] \sum_g J_g(\beta_m) \exp 2i\left[\pi \frac{w}{K}(m+g-1)\right] =$$

$$\frac{\pi}{2K}\left[J_{1-m}(\beta_m) + 2q\left(J_{1-m-1}(\beta_m) + J_{1-m+1}(\beta_m)\right)\right]; \qquad B_1^m = \frac{\pi}{2K} J_{1-m}(\beta_m);$$

where $$\beta_m = 4q\left[m + nQ\left(1 - \frac{\varepsilon\, \pi^2}{\kappa^2 K^2}\right) - \frac{\varepsilon\, \pi}{2\, K\, \kappa^2}\, \delta_s^{(u)}\right] \tag{13}$$

Simplified form of the coefficients is: $A_1^m \approx B_1^m = \frac{\pi}{2K} J_{1-m}(\beta_m)$.

Using this coefficients, we find components of tensor (11):

$$\varepsilon_{33}^{pm} = \frac{4\sqrt{2}\, \varepsilon\, \omega_p^2}{\Omega\, k_o v_T\, \pi^{1/2}} \int_0^\infty u^3 du\, e^{-u^2} \sum_{1,s} \int_{\kappa_o}^{1} \frac{d\kappa}{\kappa^3} \frac{s}{(1+\Delta_s)} J_{1-p}\, J_{1-m}; \tag{14}$$

$$\hat{\varepsilon}_{3\perp}^{pm} E_\perp^m = -\frac{4\, \varepsilon\, \omega_p^2}{\Omega\, k_o \pi^{1/2}} \int_0^\infty u^4 du\, e^{-u^2} \sum_{1,s} \int_{\kappa_o}^{1} \frac{d\kappa}{\kappa_o} \frac{J_{1-p}}{\omega_c \kappa^4} J_{1-m} \frac{\kappa^2 - \kappa_o^2}{(1+\Delta_s)} [\vec{k}_m \times \vec{E}_\perp^m]$$

It's possible to carry out integrations of Eq.(14) in two cases: a) for small $\beta_m \ll 1$, and b) large $\beta_m \gg 1$.

A. For a small β_m case ($J_{l-p}J_{l-m} \cong \delta_{mp}$).

After exchanging variable $u = \nu\kappa$, the integration over κ gives:

$$\varepsilon^{mm}_{33u} \cong \frac{4\sqrt{2}\ \varepsilon\ \omega_p^2}{\Omega\ k_m v_T\ \pi^{1/2}} \int_{-\infty}^{\infty} \frac{\nu^4 d\nu}{(\nu - z)} \int_{\kappa_o}^{1} e^{-\kappa^2 u^2} \frac{d\kappa^2}{2}\ ;\ z^2 = \Omega^2/2\varepsilon k_m v_T^2; \quad (15)$$

and integration with ν is reduced to Dispersion Function $W(z)$:

$$\varepsilon^{pm}_{33u} = \delta_{pm}\left(\frac{\omega_p}{k_m v_T}\right)^2 \left[\Psi(\kappa_o z) - \kappa_o \Psi(z)\right];\ \Psi(z) - 1 = i\sqrt{\pi} z W(z) = \int_{-\infty}^{\infty} \frac{\nu^2}{\sqrt{\pi}} \frac{e^{-\nu^2} d\nu}{z(\nu - z)};$$

(15a)

$$\varepsilon^{pm}_{3\perp(u)} E^m_\perp = \delta_{pm} \frac{\omega_p^2}{\Omega\ \omega_c\ k_\parallel} \left\{\Psi(\kappa_o z) - \frac{\kappa_o}{2} - \kappa_o\left[1 + (1-\kappa_o^2)\kappa_o^2 z^2\right] \Psi(z)\right\}\left[\vec{k}^m_\perp \vec{E}^m_\perp\right];$$

Calculations for a binormal component of RF untrapped electron current may be carried out in the same manner. Then we obtain:

$$j^{pm}_{2(u)} = \frac{\Omega}{4\pi i}\ \delta_{pm}\left\{ \frac{\omega_p^2}{\Omega\ \omega_c N_e} \frac{\partial}{\partial r}\left[N_e\left(\sqrt{\frac{\varepsilon}{8}}\ T^5_1\ E^m_3 - \frac{v_T}{8}\ T^6_2\ \left[\vec{k}_\perp \vec{E}^m_\perp\right]\right)\right]\right\}; \quad (15b)$$

$$T^\mu_\nu = \frac{v_T}{\sqrt{2\pi}} \int_{\kappa_o^2}^{1} dt \int_{-\infty}^{\infty} \frac{(t-\kappa_o^2)^\nu d\nu\, \nu^\mu}{(k_m v_T \nu - \Omega/\sqrt{2\varepsilon})} \exp\left(-\frac{t\nu^2}{2}\right)$$ - it's Table Integral [13]

Damping of the waves with the phase velocity $v_{ph} = \Omega/k_m \gg 2\sqrt{\varepsilon}\ v_T$ due to the untrapped electrons coincides with that for the cylindrical plasma model. Landau damping for waves with the small phase velocity $v_{ph} \ll \sqrt{2\varepsilon}\ v_T$ in tokamaks is reduced by the factor $z^2 = v_{ph}^2/2\varepsilon v_T^2$, and TTMP damping (due to the binormal components of electrical fields) is reduced more strongly, by the factor $z^4/2$.

For fast waves this fact does not matter in this case because dissipation power is defined by the parallel component of the electric field [11]:

$$E_3 \cong \frac{k_\parallel}{\Omega}\frac{v_T^2}{\omega_c}\frac{dE_b}{dr} \ll E_b, \quad \text{and} \quad P = Re\sum_p \tilde{\vec{j}}^p(\vec{E}^p)^*/2 \cong$$

$$\frac{\Omega}{8\pi}\left[\left(Im\ \hat{\varepsilon}^{mm}_{22} + 2Re\ \hat{\varepsilon}^{mm}_{3\perp}\ Im\ \hat{\varepsilon}^{mm}_{3\perp}/\varepsilon^{mm}_{33}\right) |E^m_\perp|^2 + Im\ \varepsilon^{mm}_{33}|E^m_3|^2\right];$$

$$\left(Im\ \hat{\varepsilon}^{mm}_{22} + 2Re\ \hat{\varepsilon}^{mm}_{3\perp}\ Im\ \hat{\varepsilon}^{mm}_{3\perp}/\varepsilon^{mm}_{33}\right) |E^m_\perp|^2 \cong 0.$$

The same kind attenuation of Landau damping on untrapped electrons for waves with the small phase velocity as analyzed numerically in Ref. [7] and theoretical Eq. (15) are in a good agreement with numerical and theoretical estimations [10-11].

Here we note that for $m + nQ \ll 1$ one needs to take into account q corrections in A^m_l and B^m_l because **the main dissipation** may be on high harmonics $m \pm l$ [14,16].

B. For the large β_m case $(|m + nQ| \geq l \gg 1)$.

In this case it is possible to use Debye asymptotic approximation for Bessel functions of large argument $\left(x^2 = \beta_m^2 - l^2,\ \beta_m \cong \kappa^2(m + nQ)/4\right)$ in Eq.(14):

$$J_l(\beta_m) \sim \sqrt{2/\pi x}\ \cos[x - l\ arc\ tg(x/l) - \pi/4].$$

Exchanging summation with l in Eq.(14) with integration over l (where $x^2 > 0$) and neglecting the fast oscillating terms with x we obtain an estimation of the tensor:

$$\varepsilon_{33}^{mm} \cong \frac{4\sqrt{2}\ \varepsilon\ \omega_p^2}{\Omega\ k_o v_T\ \pi^{3/2}} \int_{\infty}^{\infty} v^4\ dv \int_{-\beta_m}^{\beta_m} \frac{dl}{[(1+\ m\ +\ nQ)v-\ v_\phi]} \int_{\kappa_o}^{1} \frac{d\kappa^2}{x}\ e^{-\kappa^2 v^2} \quad (16)$$

Where $u = v\ \kappa,\ v_\phi^2 = \Omega^2/4\ \varepsilon\ k_o^2\ v_T^2$.

Further we will analyze the situation when Landau damping is going down ($v_{ph} > \sqrt{2}\ v_T$) and take into account additional damping connected with bounce resonances on untrapped electrons when $l^2 \geq \kappa_o^2\ (m+\ nQ)^2/4$. Then omitting factor $exp-(\kappa v)^2$, integrating (16) over κ and by Landau pole method over v we obtain:

$$I_1 = \int_{\kappa_o}^{1} \frac{d\kappa^2}{x}\ e^{-\kappa^2 v^2} = \frac{\sqrt{\pi}}{v\sqrt{\kappa_o^2 + x_1}}\ e^{-\kappa_o^2 v^2} \left[\Phi(v\sqrt{1-x_1}) - \Phi(v\sqrt{\kappa_o^2 - x_1})\right] \text{ for } \kappa_o^2 \geq x_1;$$

$$I_1 = \frac{1}{v}\sqrt{\frac{\pi}{\kappa_o^2 + x_1}}\ e^{-\kappa_o^2 v^2}\ \Phi(v\sqrt{1-x_1});\ \kappa_o^2 \leq x_1;\ \Phi(\bar{x}) \text{ is probability integral.}$$

Integration over $l_o \leq l \leq l_0$ ($l_o = |m\ +\ nQ|/2$) gives us the result:

$$Im\ \varepsilon_{33}^{mm} \cong \frac{2\sqrt{2}\ \varepsilon\ \omega_p^2}{\Omega\ k_m v_T\ \pi^{1/2}}\ \gamma v_\phi^2;\ \text{where } \gamma \cong 1 + \left(2 + \varepsilon v_{ph}^2/v_T^2\right) v_T/v_{ph}, \quad (17)$$

with enhancement in wave dissipation by factor γ.

This enhancement factor will be more if we take into account the β_m dependence of $\varepsilon\ v_T\ \Omega/(\omega_b\ v)$ [16]. When $\Omega \gg k_m\ v_T$, it is possible to integrate (14) by Laplace method over κ and with $\beta_m \cong \sqrt{2}\ \varepsilon\ v_T\ \Omega/(\omega_b v)$, followed by Landau pole method integration:

$$\varepsilon_{33}^{mm} \cong \frac{2\sqrt{2}\ \varepsilon\ \omega_p^2}{\Omega\ k_m v_T\ \pi^{1/2}} \int_{\infty} \sum_{l,s} \frac{v^2 dv}{(v - sz_m)}\ e^{-\kappa_o^2 u^2}\ J_{l-m}^2\left(\beta_m(\kappa_o)\right)\ ;$$

$$Im\ \varepsilon_{33}^{mm} \cong \frac{\sqrt{2\pi}\ \omega_p^2}{\Omega\ k_o\ v_T} \sum_{l} \frac{u_l^2 exp(-u_l^2)}{m + l}\ J_l^2\left[\varepsilon\left(1 + \frac{m + nQ}{2}\right)\right];\ u_l = \frac{\omega}{lk_o v_T \sqrt{2}} \qquad (18)$$

This formula was computed for different $\varepsilon = 0.0 \div 0.3$, $u_l = 2 \div 9$ and results are presented in Fig.1 (v_{ph}/v_T=3.3, ε= 0.25, $\gamma \cong$ 10).

From Eq. (17) and Fig.1 it is possible to conclude that the damping of waves with phase velocities more then thermal ($v_{ph} > \sqrt{2}\ v_T$) due to the untrapped electron bounce oscillations increases with toroidicity factor ε and this damping in tokamaks exceeds the Landau factor $exp[-(v_{ph}/v_T)^2]$ for the cylindrical plasma model if $\varepsilon\Omega/\omega_b > 1$. Width of these resonances $|v/v_{ph} - 1|$ in velocity space increases as $\varepsilon\ nQ/2$. These results are in a good agreement with numerical calculation [8] and greatly differ from those of Refs. [10-11]. Similar conclusions are valid for electron TTMP damping.

2. Trapped electrons distribution function evaluation.

Trapped electrons distribution function analysis will be conducted in the same way as for the untrapped particles.

After expanding $\frac{1}{1+\varepsilon cos\theta}$; $\sqrt{1+\ \varepsilon cos\theta}$ in Taylor series over ε, exchanging distribution function $\bar{f}_t^s = f_{ot}^{(s)} exp\ i\Big[nQ\ (\theta - \varepsilon\ sin\ \theta)$

$$+ \varepsilon\ \delta_s \int_o^{\theta/2} d\eta\ sin^2\eta\ /\sqrt{1-\ \kappa^2 sin^2\eta}\Big],\qquad \delta_s = s\sqrt{2/\varepsilon}\ \frac{\kappa\ \Omega}{v\ k_o};$$

and expanding RF field in Fourier series over θ we introduce the new angle Θ for the trapped electrons:

$$\theta/2 \quad \arcsin\left(\frac{\sin\Theta}{\kappa}\right); \quad \cos\theta/2 \; d\theta/2 = \bar{\kappa} \cos\Theta \; d\Theta; \quad \kappa^2 = \frac{2\varepsilon}{1+\varepsilon-\Lambda} = \frac{1}{\bar{\kappa}} .$$

$$\frac{\partial \bar{f}_t^s}{\partial \Theta} \sqrt{\kappa^2 - \sin^2\Theta} - i\delta_s(1-\frac{\varepsilon}{2})\bar{f}_t^s = \frac{2e}{Mk_o} \sum_m \left[\frac{E_3^m}{v} \cos\Theta - \frac{1-\bar{\kappa}^2\kappa_o^2}{2s\kappa_o\bar{\kappa}\;\omega_c} [\vec{k}_m \times \vec{E}_\perp^m] \right] \frac{\partial F_M}{\partial v} \exp i\bar{\Xi}_t^m;$$

After transformation to Jacobi variables:

(19)

$$w = \int_0^\Theta d\eta / \sqrt{1-\kappa^2 \sin^2\eta}; \quad \Theta = \mathrm{am}\; w; \quad \frac{d\Theta}{dw} = \sqrt{1-\kappa^2\sin^2\Theta}\,; \quad \frac{\theta}{2} = \int^w dw'\, \mathrm{cn}\; w';$$

Eq. (4) becomes:

$$\frac{\partial \bar{f}_t^s}{\partial w} - i\delta_s\bar{\kappa} \; (1-\varepsilon/2)\bar{f}_t^s = \frac{2e\bar{\kappa}}{Mk_o} \sum_m \left[\frac{E_3^m}{v} \mathrm{cn} w - \frac{1-\bar{\kappa}^2\kappa_o^2}{2s\kappa_o\bar{\kappa}\;\omega_c} [\vec{k}_m \times \vec{E}_\perp^m] \right] \frac{\partial F_M}{\partial v} \exp\, i\Xi_t^m;$$

$$\Xi_t^m = \left\{ 2\kappa \left[(m+nQ)\int^w dw'\, \mathrm{cn}\; w' - \varepsilon nQ \; (\mathrm{sn}\; w)(\mathrm{cn}\; w) \right] + \varepsilon \frac{\delta_s}{\kappa} [w - \mathrm{E}(w,\bar{\kappa})] \right\}.$$

We separate out nonperiodical part from this phase and exchange function $\bar{f}_t^s = \tilde{f}_t^s \exp i\left[2\bar{\kappa}\; \varepsilon\; \delta_s w(1-E/K)\right]$.

If an electron distribution function $\tilde{f}_t^s$ is periodic for trapped electrons over Θ with 4π period, it must be periodic over w with $4K$ period. Expanding right hand side of Eq. (19) in Fourier series over w we obtain:

$$\tilde{f}_t^s = \frac{4e\bar{\kappa}\; K}{\pi v k_o M} \sum_{s,m,l} \left[\frac{E_3^m}{v} a_{ls}^m - \frac{1-\bar{\kappa}^2\kappa_o^2}{2s\kappa_o\bar{\kappa}\;\omega_c} b_{ls}^m \; [\vec{k}_m \times \vec{E}_\perp^m] \right] \frac{\partial F_M}{\partial v} \frac{\exp(il\pi w/2K)}{i\;(1-\Delta_t^s)};$$

where $a^{l}_{ms} = \frac{1}{4K}\int_{-4K}^{4K} dw'(cn\, w')\exp\left[i\left(\tilde{\Xi}^{m}_{t} - l\frac{\pi w'}{2K}\right)\right]$; $\kappa^{2}_{o} = 2\varepsilon/(1+\varepsilon)$; (20)

$$b^{m}_{ls} = \frac{1}{4K}\int_{-4K}^{4K} dw'\exp\left[i\left(\tilde{\Xi}^{m}_{t} - l\frac{\pi w'}{2K}\right)\right]; \quad \Delta^{s}_{t} = \sqrt{2/\varepsilon}\,\frac{2sK}{\pi v k_{o}}\left[1 - \varepsilon\left(\frac{1}{2} - \frac{K-E}{K\bar{\kappa}^{2}}\right)\right].$$

After returning to trapped electrons distribution function

$f^{(s)}_{ot} = \tilde{f}^{s}_{t}\exp - i\hat{\Xi}^{m}_{t}$; $\hat{\Xi}^{m}_{t} = \Big[nQ\,(\theta - \varepsilon\sin\theta) - \bar{\kappa}\,\varepsilon\,\delta_{s}w(1 - E/K) +$

$\varepsilon\,\delta_{s}\int_{o}^{\theta/2} d\eta\,\sin^{2}\eta\,/\sqrt{1-\kappa^{2}\sin^{2}\eta}\Big]$ it is possible to express

trapped electron RF current induced by RF electric fields as:

$$(1+\varepsilon\cos\theta)\,\tilde{j}^{(t)}_{r,b} = \frac{\pi e}{2}\int_{0}^{\infty} v^{3}dv \int_{1-\varepsilon}^{1+\varepsilon\cos\theta} d\Lambda \sum_{s} f^{(s)}_{r,b,t}\sqrt{\frac{\Lambda}{1-\Lambda+\varepsilon\cos\theta}};$$

$$(1+\varepsilon\cos\theta)\,\tilde{j}^{(t)}_{3} = \pi e\int_{0}^{\infty} v^{3}dv \int_{1-\varepsilon}^{1+\varepsilon\cos\theta} d\Lambda \sum_{s} s f^{(s)}_{ot}. \qquad (21)$$

Oscillation current for one Fourier harmonic p will be:

$$\left((1+\varepsilon\cos\theta)\tilde{j}_{3t}\right)^{p} = 2\varepsilon e\int_{-\pi}^{\pi} d\theta \int_{o}^{\infty} v^{3}dv\int_{\kappa}^{1}\frac{d\kappa}{\kappa^{3}}\sum_{s} s f^{(s)}_{ot}\exp\left(-ip\theta\right); \qquad (22)$$

Exchanging order of integration over θ and κ in (22) we obtain:

$$j^{p}_{3t} = \frac{\Omega}{4\pi i}\left\{\frac{16\sqrt{2}\,\varepsilon K\omega^{2}_{p}}{\Omega\, k_{o} v_{T}\,\pi^{5/2}}\int_{0}^{\infty} u^{3}du\, e^{-u^{2}}\sum_{s,m,l}\int_{0}^{1}\frac{\bar{\kappa}\, d\bar{\kappa}}{1-\Delta^{s}_{t}}\left[\frac{E^{m}_{3}}{v}\, a^{m}_{ls} - \right.\right.$$

$$\frac{1-\bar{\kappa}^2\kappa_o^2}{2s\kappa_o\bar{\kappa}\ \omega_c}\mathbf{b}_{ls}^m\ [\vec{k}_m\times\vec{E}_\perp^m]\Bigg]\frac{\partial F_M}{\partial v}\int_{-\arcsin\bar{\kappa}}^{\arcsin\bar{\kappa}} d\theta\ \exp\ i\left(l\pi\ \frac{w}{K}-\hat{\Xi}_t^m-p\theta\right)\Bigg\};$$

Finally, exchanging variable θ with w as in Eq.(19) this current will be:

$$j_{3t}^p=\frac{\Omega}{4\pi i}\left\{\frac{32\sqrt{2}\ \varepsilon K^2\omega_p^2}{\Omega\ k_o\ v_T\ \pi^{5/2}}\int_0^\infty u^3du\ e^{-u^2}\sum_{s,m,l}\int_0^1\frac{\bar{\kappa}^3d\bar{\kappa}}{1-\Delta_t^s}\bar{\mathbf{a}}_{ls}^p\left[\frac{E_3^m}{\bar{v}}\mathbf{a}_{ls}^m-\right.\right.$$

$$\left.\left.\frac{1-\bar{\kappa}^2\kappa_o^2}{2s\kappa_o\bar{\kappa}\ \omega_c}\mathbf{b}_{ls}^m\ [\vec{k}_m\times\vec{E}_\perp^m]\ \right]\frac{\partial F_M}{\partial v}\right\}; \tag{23}$$

$$\bar{\mathbf{a}}_{ls}^p=\frac{1}{4K}\int_{-2K}^{2K}dw'(\mathrm{cn}w')\ \exp\ i\left(l\pi\ \frac{W}{K}-\hat{\Xi}_t^m-p\ \mathrm{am}w'\right).$$

Let us simplify formulas (23) taking into account symmetry of coefficients $\mathbf{a}_{-ls}^m=(-1)^{l+1}\mathbf{a}_{ls}^m;\ \mathbf{b}_{-ls}^m=(-1)^l\mathbf{b}_{ls}^m$:

$$j_{3t}^p=\frac{\Omega}{4\pi i}\left\{32\varepsilon\ \frac{K^2\omega_p^2}{\Omega\ \pi^2}\sum_{l=1,m}^\infty\int_o^1\bar{\kappa}^3d\bar{\kappa}\ \mathbf{a}_l^p\left[E_3^m\ \mathbf{a}_l^m\ \mathrm{T}_l^{(4)}-\frac{v_T\ \mathrm{T}_l^{(5)}}{2\kappa_o\bar{\kappa}\ \omega_c}\mathbf{b}_l^m\ [\vec{k}_m\times\vec{E}_\perp^m]\right]\right\};$$

and in the same way: (24a)

$$j_{2t}^p=\frac{\Omega}{4\pi i}\left\{8\ \frac{K^2\omega_p^2}{\Omega\ \pi^2}\frac{\sqrt{2\varepsilon}}{\omega_cN_e}\ \frac{\partial}{\partial r}\int_0^1 N_e\bar{\kappa}^2d\bar{\kappa}\ \left[\sum_m\frac{15\pi v_T}{16K\omega_c}\ \frac{1}{\Omega+i\nu}\mathbf{b}_o^p\mathbf{b}_o^p\ [\vec{k}_m\times\vec{E}_\perp^m]\ +\right.\right.$$

$$\left.\left.\mathbf{b}_l^p\sum_{l=1,m}^\infty\left(E_3^m\ \mathbf{a}_l^m\ \mathrm{T}_l^{(5)}-\frac{v_T\ \mathrm{T}_l^{(6)}}{2\kappa_o\bar{\kappa}\ \omega_c}\mathbf{b}_l^m\ [\vec{k}_m\times\vec{E}_\perp^m]\ \right]\right]\right\}; \tag{24b}$$

$$T_l^{(n)} = \sqrt{\pi/2} \int_{-\infty}^{\infty} \frac{v^n dv \, exp(-v^2/2)}{\pi k_o v_T l v - 2\Omega K\sqrt{2}/\sqrt{\varepsilon}} .$$

It is clear that formulas (24) are more convenient for analysis than the corresponding formulas of Ref. [6].

Coefficients a_l^m, and b_l^m may be simplified using q-series of Jacobi functions (12), and taking into account only first two terms we obtain:

$$a_l^m \cong b_l^m \cong \frac{\pi\sqrt{q}}{\kappa \; K} \left[J_{l-1} + J_{l+1} - \sqrt{\varepsilon} \; \frac{s\pi}{Kv_T} \frac{q\Omega}{uk_o} \left(J_{l-1} - J_{l+1} + J_{l-3} - J_{l+3}\right)\right];$$

$$J_l = J_l(\beta_m^t); \qquad \beta_m^t = \sqrt{q} \left[8(m + nQ) - 2\varepsilon \; nQ \left(\frac{\pi}{K}\right)^2\right]. \tag{25}$$

$$\varepsilon_{33t}^{pm} \cong \frac{\sqrt{2\varepsilon} \; \omega_p^2}{k_o \; v_T} \sum_l \int_o^1 d\bar{\kappa}^2 \left[1 + 2x_l^2\left(1 + i\sqrt{\pi}x_l W(x_l)\right)\right] \frac{J_l(\beta_m^t) \;\; J_l(\beta_p^t)}{(m + nQ)(p + nQ)};$$

where $i\sqrt{\pi} \; x \; W(x) = \int_{-\infty}^{\infty} \frac{v^2}{\sqrt{\pi}} \frac{e^{-v^2} dv}{x(v-x)}; \qquad x_l = \frac{\Omega}{|lk_o| v_T \sqrt{\varepsilon}}$.

The wave damping due to the trapped electrons is essential in those cases, when the damping due to the untrapped particles is small ($v_{ph} < v_T$). Here we write down only imaginary part of the trapped electrons contribution to the transverse components of the permeability tensor, with the real part varying as $\cong \sqrt{2\varepsilon}$:

$$Im\left(\varepsilon_{b\perp}^{pm} \; E_\perp^m + \hat{\varepsilon}_{b3}^{pm} \; E_3^m\right) = \frac{\sqrt{\pi\varepsilon} \; \omega_p^2}{\Omega \; \omega_c N_e} \frac{\partial}{\partial r} \left\{N_e \sum_{l,m} \int_0^1 \bar{\kappa} \; d\bar{\kappa} \; J_l(\beta_p^t) \; J_l(\beta_m^t) \left[\frac{E_3^m}{\bar{K}_m} - \right.\right.$$

$$\left.\left.\frac{\Omega}{k_o^2 \omega_c \varepsilon}\left[\vec{k}_\perp \vec{E}_\perp^m\right]\right] \left(\frac{\Omega}{l\omega_b}\right)^5 \; exp\left(\frac{-\Omega^2}{2l^2\omega_b^2}\right)\right\} \; ; \qquad \omega_b \cong \sqrt{\varepsilon/2} \; k_o v_T \tag{26}$$

The damping due to the trapped electrons is strongest for short waves, when the number of the bounce resonances is of the order of the double parallel wavenumber $l \cong 2(m+nQ)$. In this case the damping on the bounce resonances is more effective than Landau damping on the untrapped particles, and its full value is $\approx 2/\pi$ of Landau damping for the cylindrical model. In these conditions, the absorption, normalized on one trapped particle, is much greater ($\approx 1/\sqrt{2\varepsilon}$), than for the cylindrical plasma model.

If the wave frequency is less than the bounce-frequency $\Omega \leq \omega_b$ (for instance, for global Alfven waves induced by toroidicity) the dissipation of these waves will be very small, and essentially collisionel (see Eq. (24b)).

C O N C L U S I O N

1. Simple expressions for the electron part of toroidal dielectric tensor was obtained. These expressions may be useful for the fast waves RF electron heating and current drive analysis by ray-tracing theory or for the computation of two-dimensional Maxwell equations in tokamaks.

2. The fast waves with phase velocity greater than thermal velocity ($v_{ph} > v_T$), dissipate stronger than Landau damping in cylindrical model because of the untrapped electron bounce-resonance dissipation.

ACKNOWLEDGEMENTS

The authors sincerely thank Prof.R. Wilhelm for support of this work, and Dr. S.Puri for very useful discussions in the course of this study and help in editing the manuscript.

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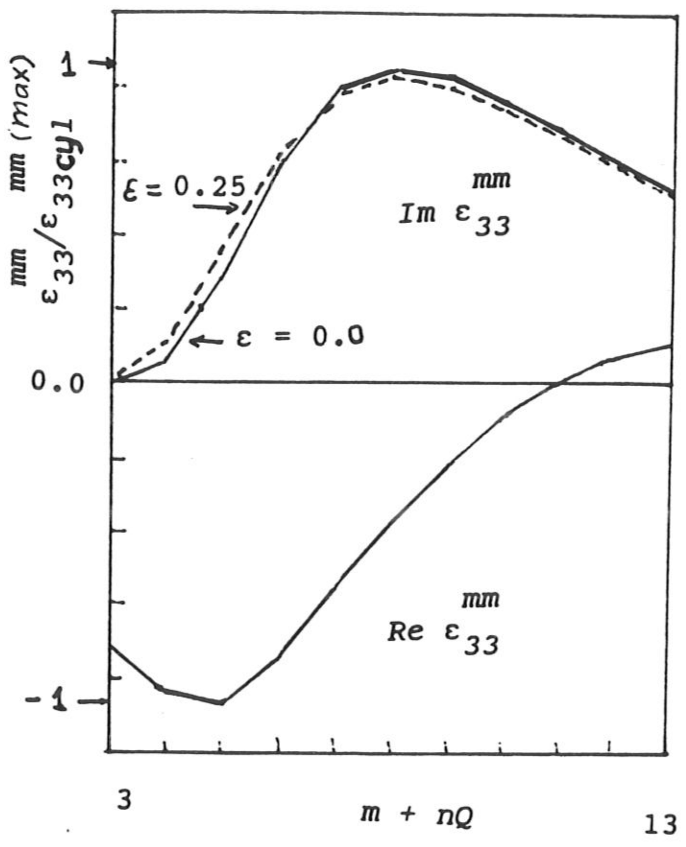

Fig.1 Dependence of parallel component of toroidal dielectric permiability tensor normalised on cylindrical one vrs parallel component of wave vector $k_{\|}RQ = m + nQ$ for $\varepsilon = 0.$ and 0.25; $u_1 = 8.$