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Lower-Hybrid-Current-Drive  
Spectral-Gap Anomaly

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IPP 4/254

April 1992

*Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem  
Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über  
die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.*

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**ABSTRACT**

This paper offers a resolution of the lower-hybrid-current-drive spectral-gap problem via modifications in the Landau damping spectrum for the high-phase-velocity waves in the toroidal geometry. The emergence of parametric resonances produced by the beating of the particles' periodicity with the wave periodicity spreads out Landau damping in velocity space, enhances the net energy absorption, and could lead to the generation of superthermal tails. Also, the existence of the lower-hybrid density limit and alpha-particle heating are explained within the context of linear theory.

PACS numbers: 52.25.Mq;52.35.Hr

Present-day tokamaks rely upon pulsed ohmic-heating transformers to inductively supply the toroidal current needed for sustaining the plasma. Non-inductive current drive necessary for steady-state tokamak operation can be obtained by suitably launched radio-frequency waves in the torus. Lower-hybrid waves have been among the most successful approaches to non-inductive current drive so far.<sup>1-3</sup> Yet the precise nature of their operation is shrouded in mystery. Two of the notably puzzling experimental features of lower-hybrid current drive (LHCD) in a tokamak are: (1) The parallel phase velocity of the waves launched by the antenna typically exceeds the electron thermal speed by a large factor. The small number of resonant electrons corresponding to an initial Maxwellian distribution would be unable to sustain current drive unless a fresh supply was forthcoming, presumably by acceleration of the bulk population. However, it remains unexplained how the electrons in the bulk plasma could interact with the much faster phase velocity of the wave. The anomaly is often referred to as the LHCD spectral-gap problem. (2) Equally strangely, there is circumstantial evidence that some of the tail electrons are accelerated well beyond the wave phase velocity despite the conspicuous absence of an appropriate component of the antenna spectrum.

Critical problems in reconciling the experimental results to the theoretical expectations are outlined in Ref.4. It has long been recognized that only a small amount of power absorbed by the slower electrons in the plasma bulk might account for the observed discrepancy; the absorption presumably caused by an upshift in the parallel-wave-number spectrum.<sup>5</sup> A number of possible explanations for the occurrence of the upshift has been proposed: edge bouncing of the rays<sup>6</sup>, strong edge-density fluctuations<sup>7</sup>, non-linear effects in caustics<sup>8</sup>, effect of Parail-Pogutse instability<sup>9</sup>, ponderomotive effects<sup>10</sup>, diffraction effects<sup>11</sup>, ray stochasticity<sup>12</sup>, wave scattering from toroidal inhomogeneity like the magnetic ripple<sup>13</sup>, and effects of parametric instability<sup>14</sup>.

This paper offers a resolution of the spectral-gap anomaly via modifications in the Landau damping spectrum for high-phase-velocity waves in a toroidal plasma. Also, existence of the lower-hybrid density limit and alpha-particle heating are explained

within the context of linear theory.

In Ref.15, it is shown that the wave absorption in a torus differs significantly from the classical Landau damping result. Instead of the isolated resonance at  $v = \omega/k$ , the parametric beating between the particles' periodicity and the wave periodicity gives rise to an infinite set of discrete resonances. The principal resonance occurs for the group of particles moving on the average at the parallel phase velocity of the wave. Secondary resonances correspond to the case when the particle either gains or loses  $N$  complete wavelengths during one period of its orbit. For  $\varepsilon = r/R_0 \rightarrow 0$  ( $r$  is the radial location of the particle and  $R_0$  is the major torus radius), the secondary resonances contribute very little to the damping process. For larger  $\varepsilon$ , and most particularly for large toroidal wave numbers  $n$ , even a small slippage in phase destroys any semblance of cohesion between the wave and the particle, and Landau damping at the primary resonance would be drastically reduced. However, the secondary resonances begin to gain prominence, both enhancing absorption and enlarging the velocity spread over which Landau damping extends. The enhancement in damping is most pronounced for the large-phase-velocity waves (relative to electron thermal speed) since the secondary resonances occurring at lower velocities involve a much larger population of particles in the acceleration process. At the same time, the secondary resonances on the high-velocity end enhance the production of superthermal electrons beyond that warranted by the runaway mechanism. These effects are documented in Figs.(1-3) of Ref.15.

In cylindrical geometry, the radial absorption length of the lower-hybrid wave may be estimated from the approximate dispersion relation

$$k_{\perp}^2 = \varepsilon_{\parallel} k_{\parallel}^2 \approx \frac{\omega_{pe}^2}{\omega^2} U_p^2 Z'(U_p) k_{\parallel}^2, \quad (1)$$

where  $\varepsilon_{\parallel}$  is the dielectric-tensor component along the magnetic field direction,  $\omega_{pe}$  is the plasma frequency,  $U_p = \omega/\sqrt{2}k_{\parallel}v_{te}$ , and  $Z(U_p)$  is the plasma dispersion function<sup>16</sup>. For  $U_p \gtrsim 2$ , the radial energy absorption length becomes

$$\lambda_{cyl} = (2k_{\perp i})^{-1} \approx \frac{1}{2\sqrt{\pi}n_{\parallel}} \frac{c}{\omega_{pe}} \frac{\exp(U_p^2)}{U_p^3}, \quad (2)$$

where  $k_{\perp i}$  is the imaginary part of  $k_{\perp}$ . Assuming  $n_{\parallel} \sim 2$ , and  $c/\omega_{pe} \sim 10^{-3} m$ , gives  $\lambda_{cyl} \sim 10^{-4} \exp(U_p^2)/U_p^3$ . For  $U_p \gtrsim 3.5$ , the radial damping length exceeds the dimensions of currently operating tokamaks; yet the lower-hybrid wave is completely absorbed in typical experiments with  $U_p \sim 4$ . The anomaly can be resolved by referring to Fig.3 of Ref.15 showing that the toroidal effects enhance Landau damping, particularly for the high-phase-velocity waves. Figure 1 shows the toroidally modified damping length  $\lambda_{tor}$  as a function of  $U_p$  and  $\epsilon$ .

Further broadening of the absorption spectrum occurs due to the finite width of the launched antenna spectrum  $n_{\varphi}$ , as well as from the shift ( $q$  is the safety factor)

$$n_{\parallel} = n_{\varphi} \left( 1 + \frac{m}{nq} \right) \quad (3)$$

in the  $n_{\parallel}$  spectrum due to the finite azimuthal wave numbers. At the high-phase-velocity current drive, even small extensions in the  $n_{\parallel}$  spectrum play a vital role in the wave absorption and contribute towards further reduction of  $\lambda_{tor}$ . Figure.2 shows damping enhancement due to finite  $m$  values, particularly in the plasma interior. Assuming that the lower-hybrid antenna has an azimuthal span of about  $60^{\circ}$  implies that, in comparison with the  $m = 0$  mode, there is approximately 40%, 5% and 2% power in the  $m = 3$ ,  $m = 9$  and  $m = 15$  modes, respectively. The higher  $m$  modes would be the first to be damped in an initially low-temperature plasma, followed by ever decreasing  $m$  modes till the entire spectrum is subject to Landau damping at the elevated values of  $T_e$ . The detailed treatment of the dynamic evolution of the damping process is beyond the scope of this study.

Figure 3 shows the toroidal diffusivity  $D$  deduced from Eq.(10) of Ref.15, the power absorption  $P$ , as well as the power-absorbed per electron  $P/E$  (drawn to relative scales) as a function of  $U_{\parallel}$ . The main absorption occurs near  $U_{\parallel} \sim 3.3$ , although the wave phase velocity corresponds to  $U_p = 5$ . However  $P/E$  has a relatively flat spectrum implying continued acceleration of electrons even at very high phase velocities, greatly in excess of  $U_p$ . This would contribute a mechanism for high-energy tail formation, acting in conjunction with the runaway effect associated with the ohmic heating electric

field.

In the cylindrical geometry, the power is absorbed in a narrow range of  $U_{\parallel}$ ; the resultant diffusivity gives rise to a plateau formation in the Maxwellian tail.<sup>2</sup> However, in a torus, since the diffusivity and power absorption are more evenly distributed, the tail formation does not cause a precipitate flattening of the velocity distribution function. Figures 4 and 5 show diffusivity  $D$ , quasilinear flattening of the distribution function  $-f'/2Uf$ , abundance of tail electrons relative to the Maxwellian  $f/f_m$ , current density  $J$ , power absorption  $P$ , as well as  $P/E$  for three different relative input power levels using the approximate analytical formulae of Ref.2 and employing the diffusivity values obtained from Eq.(10) of Ref.15. Notwithstanding the elevated number of tail electrons, the principal absorption region (as well as the current carrying part of the velocity distribution function) remains essentially that found in the linear results of Fig.3.

During its radial traverse into the plasma interior, the lower-hybrid wave is subject to linear absorption at the ion-cyclotron-harmonic resonances. The wave attenuation due to the  $p$ th harmonic is given by  $\exp(-\Gamma)$  where<sup>17</sup>

$$\Gamma \approx \frac{R_0}{2} \frac{\partial k_{\perp}}{\partial \epsilon_{\perp}} \left( \frac{\omega_{pi}^2}{\omega_{ci}^2} \right)^2 \frac{e^{-\Lambda} I_p(\Lambda)}{\Lambda} \approx \left\{ \frac{R_0}{2} \frac{\omega_{pe}}{c} n_{\parallel} p^2 \right\} \left[ \frac{e^{-\Lambda} I_p(\Lambda)}{\Lambda} \right], \quad (4)$$

$\Lambda = k_{\perp}^2 r_{ci}^2/2$ ,  $r_{ci}$  is the ion gyroradius, and it is assumed that  $\omega \sim \omega_{pi}$ . The expression in the curly brackets typically exceeds the value  $5 \times 10^6$  while that in the square brackets rapidly increases with  $\Lambda$  as the plasma interior is approached, as shown in Fig.6. For  $\Gamma \sim 1$ , the wave would be absorbed by the ions (instead of the electrons), resulting in the cessation of current drive. The density limit for a deuterium plasma assuming  $R_0 \sim 2m$  and  $n_{\parallel} \sim 2$  is shown in Fig.7. The density limit increases for high-frequency operation corresponding to high- $p$  values. For a deuterium plasma, one should be able to operate at densities upto  $3 \times 10^{20} m^{-3}$  for an ion temperature of  $20 keV$ . For the alpha particles in a deuterium plasma<sup>17</sup>

$$\Gamma_{\alpha} \approx \alpha R_0 \frac{\omega_{pe}}{c} n_{\parallel} p_D^2 \frac{e^{-\Lambda_{\alpha}} I_{pD}(\Lambda_{\alpha})}{\Lambda_{\alpha}}, \quad (5)$$

where  $\alpha$  is the fraction of alpha particles while, the subscripts  $D$  and  $\alpha$  refer to deuterium

and alpha particles, respectively. For<sup>18</sup>  $\Lambda_\alpha \sim p_D^2$

$$\Gamma_\alpha^{max} \approx \alpha R_0 \frac{\omega_{pe}}{c} n_{\parallel} \frac{0.242}{p_D}, \quad (6)$$

roughly corresponds to the maximum wave attenuation due to alpha particles. This presents a more formidable hurdle to lower-hybrid wave penetration than the density limit. The actual attenuation due to the alpha particles, after taking their energy distribution into account, might be significantly lower than that given by Eq.(6). In Ref.19, it was shown by comparing linear analytic results with the non-linear computational results that the linear cyclotron-harmonic acceleration remains valid even for extremely large field amplitudes. Alternative derivations of density limit and alpha-particle absorption, using the non-linear stochastic model of Karney<sup>20</sup>, are to be found in Refs.21 and 22.

The derivation of Landau damping and diffusivity tacitly assumes a fixed repetitive step size  $\Delta v$  per period for a given particle. In practice, due to the presence of fluctuations or statistical irregularities, the step sizes  $(\Delta v + \delta)$  are scattered symmetrically around  $\Delta v$ . The actual diffusivity exceeds the calculated value by an amount [ $\langle \delta^2 \rangle / \langle (\Delta v)^2 \rangle$ ]. Thus, the results of this paper represent the lower bound for the linear Landau damping in a torus.

The foremost conclusion of this paper is that the LHCD spectral-gap anomaly can be resolved by properly accounting for Landau damping in toroidal geometry. Also the density limit and the alpha particle acceleration can be explained in terms of ion-cyclotron-harmonic acceleration. Due to the approximate dispersion relation of Eq.(1) and the neglect of the alpha-particle energy distribution, the results obtained in this paper are approximate. Further detailed work is needed to refine the quantitative conclusions, particularly with regard to the alpha-particle acceleration.

We are thankful to Dr. M. Ballico, Dr. F. Leuterer and Dr. J.-G. Wegrowe for their help during the course of this work.

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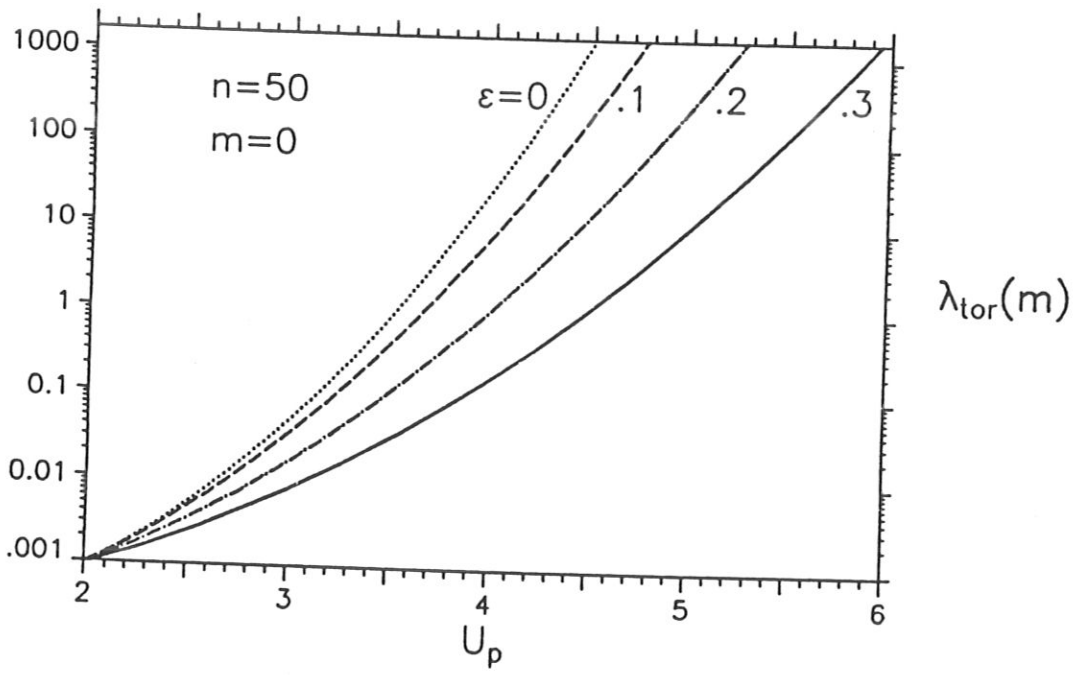


Fig.1 Damping length  $\lambda_{tor}$  in toroidal geometry.

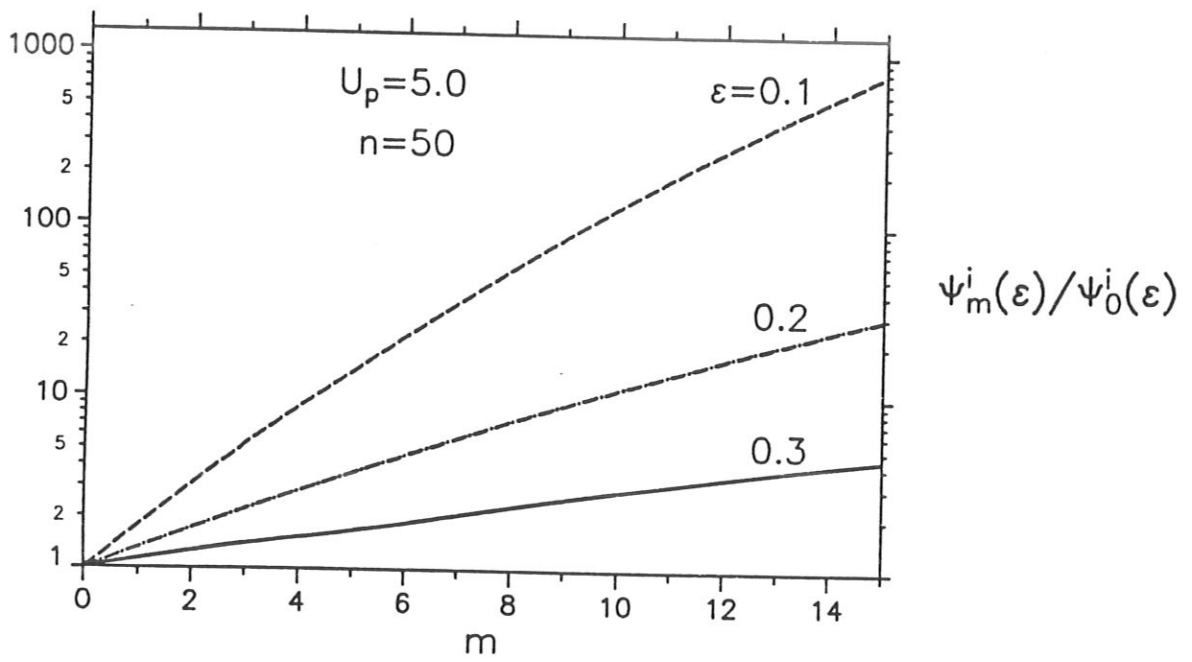


Fig.2 Damping enhancement for finite  $m$ .

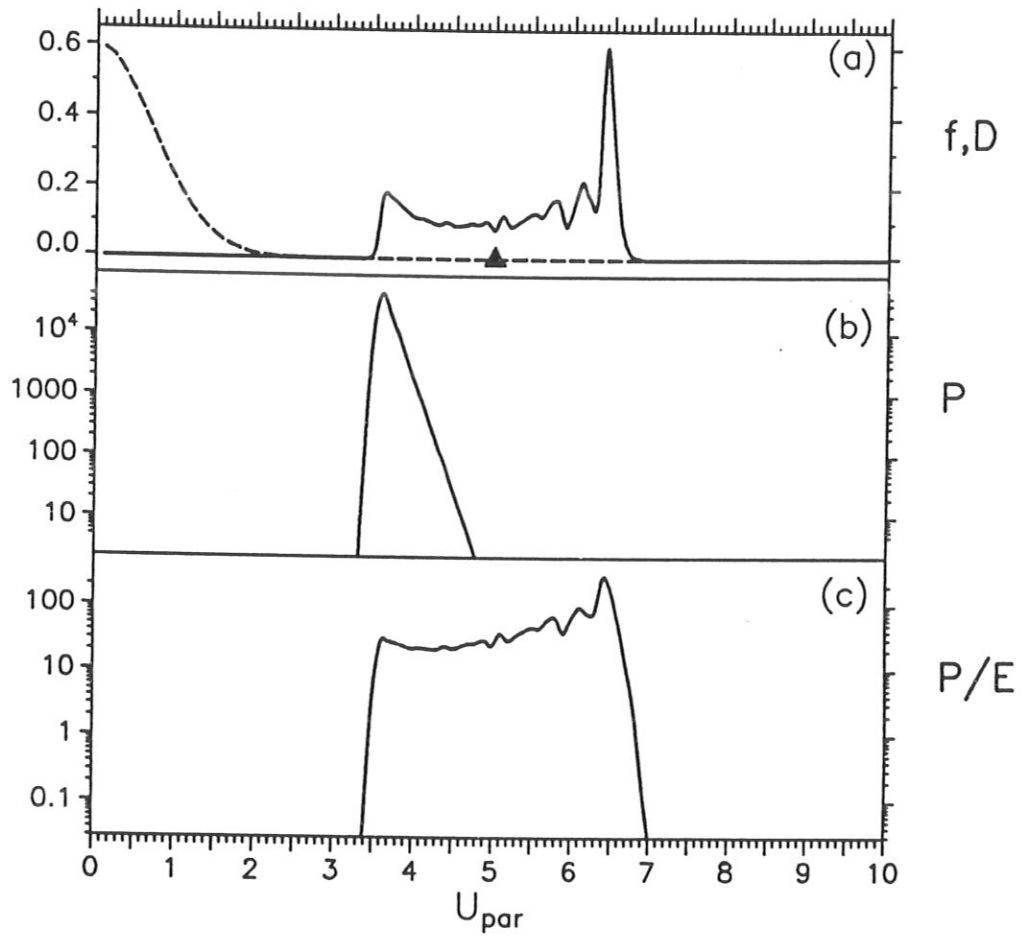


Fig.3  $D$ ,  $P$  and  $P/E$  as a function of  $U_{\parallel}$ .

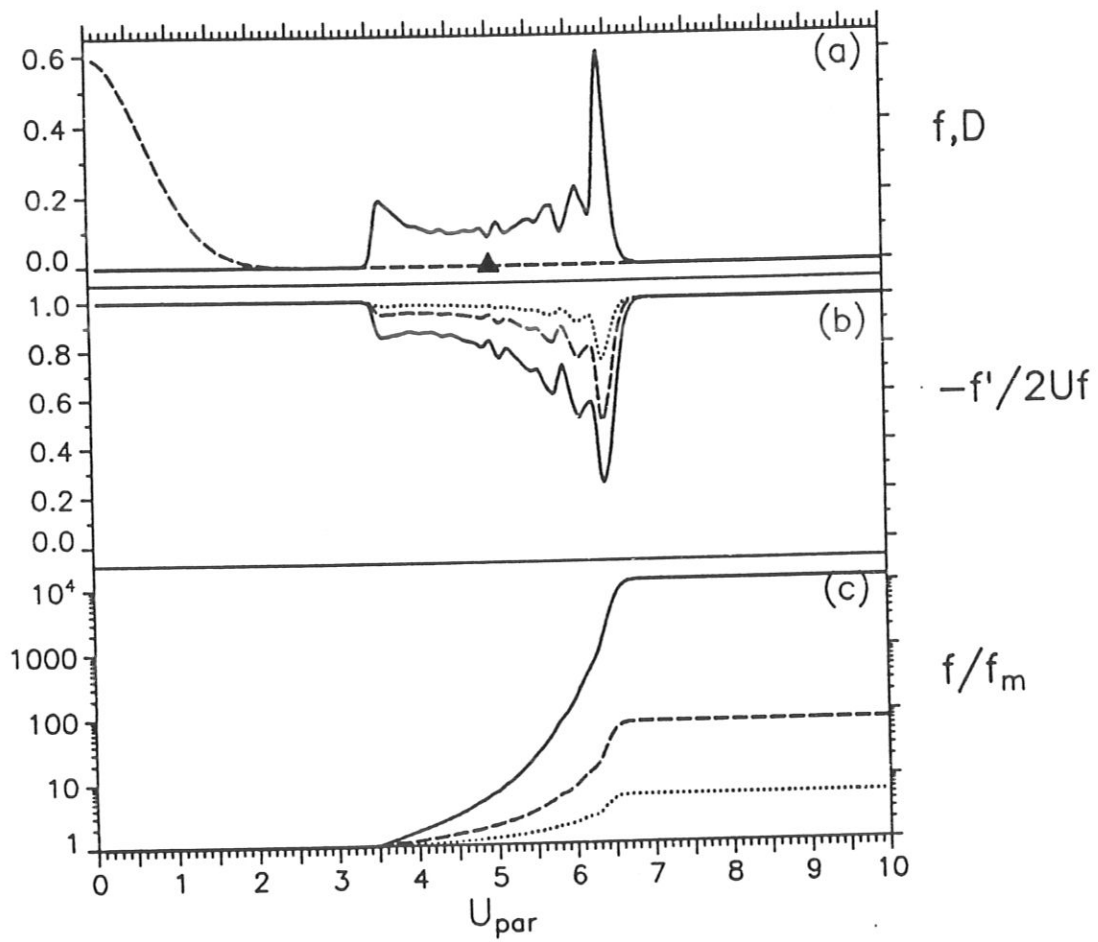


Fig.4 *D*, quasilinear  $-f'/2Uf$  and  $f/f_m$  versus  $U_{\parallel}$ .

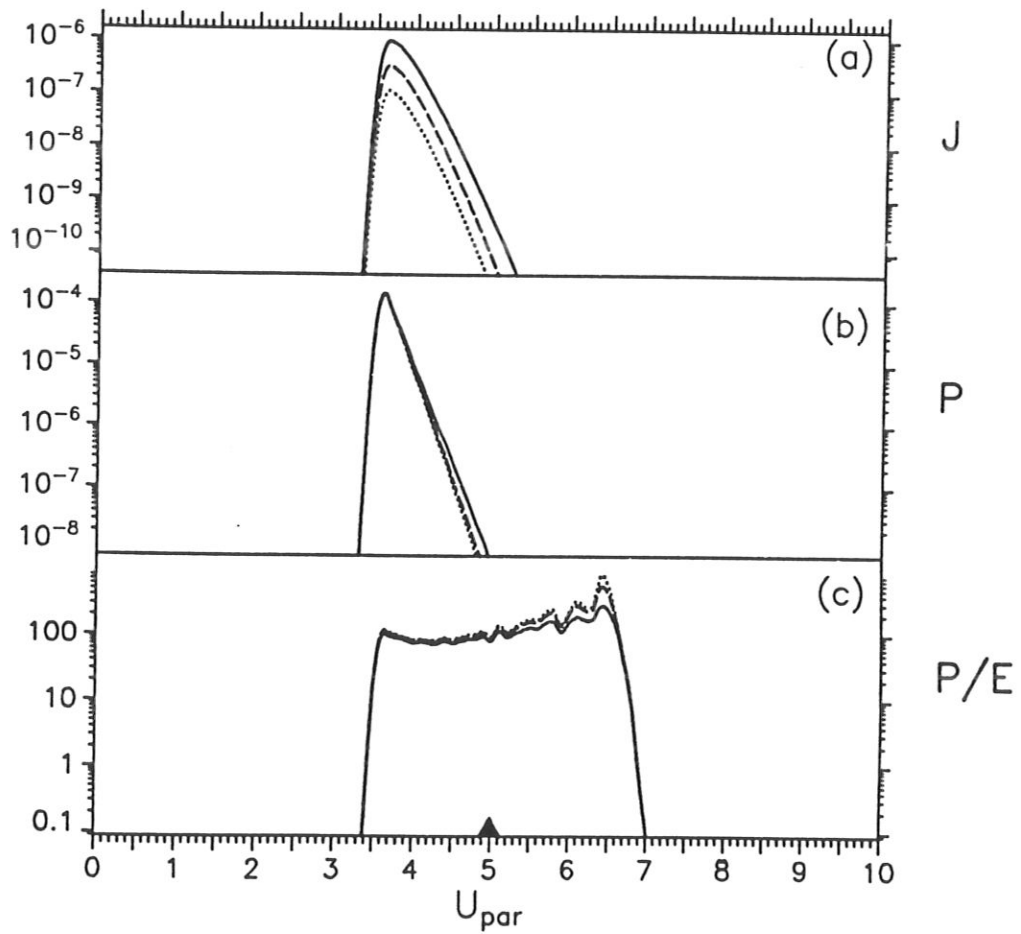


Fig.5 Quasilinear  $J$ ,  $P$  and  $P/E$  versus  $U_{\parallel}$ .

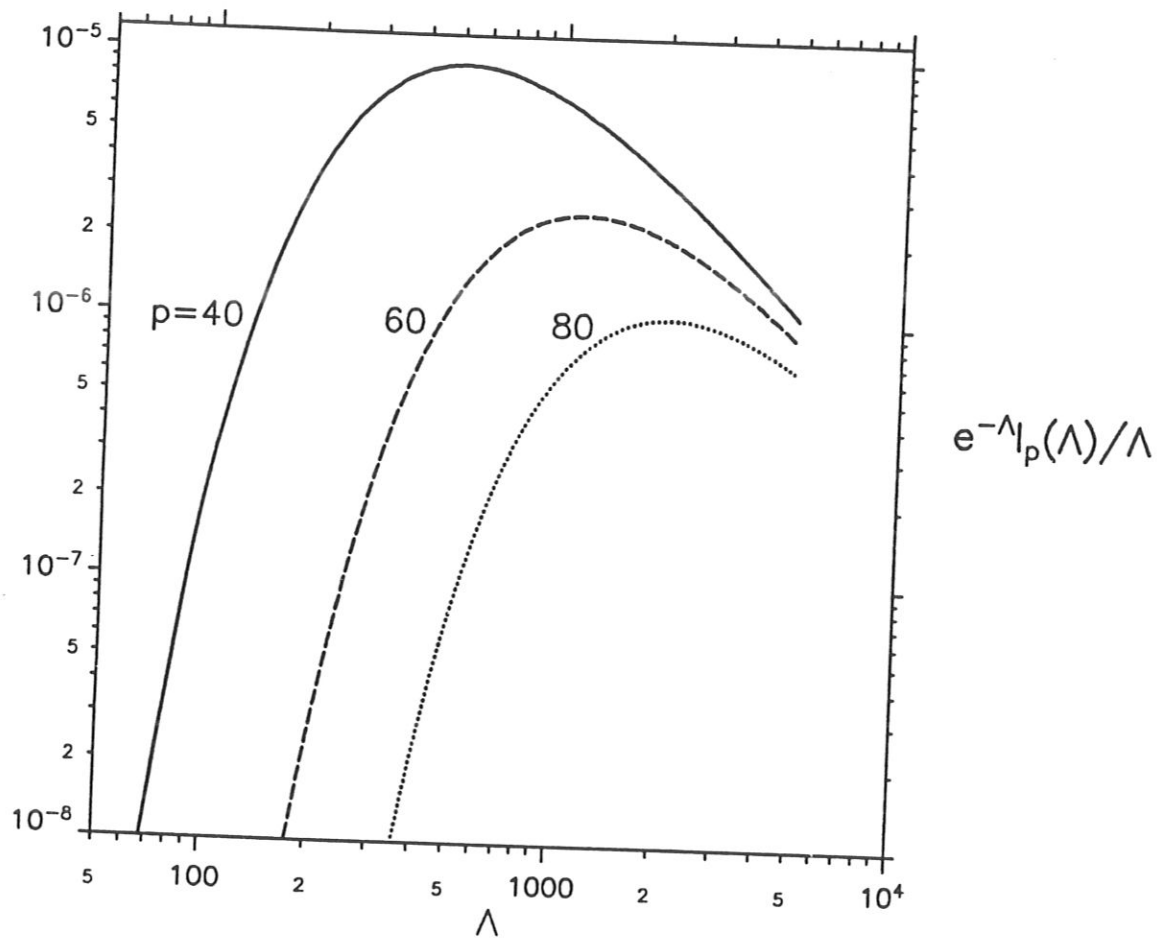


Fig.6  $e^{-\Lambda} I_p(\Lambda) / \Lambda$  versus  $\Lambda$ .

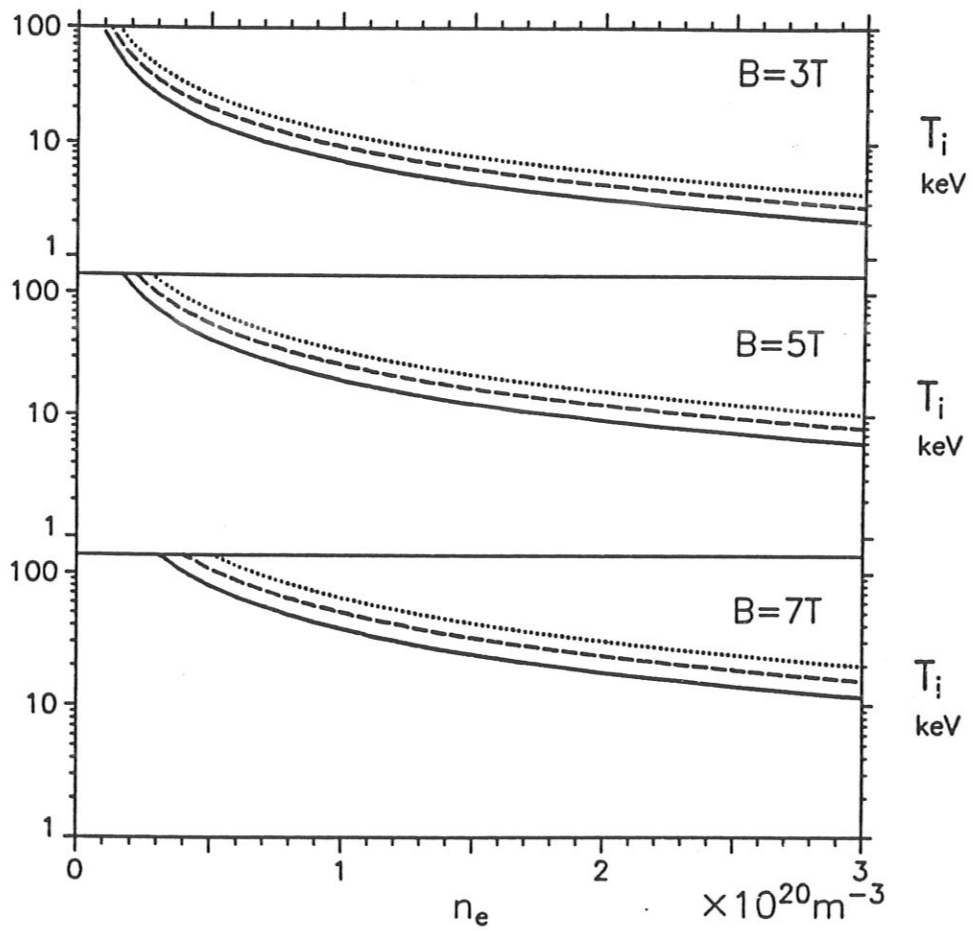


Fig.7 Density limit for a deuterium plasma for  $p = 70$  (solid curve),  $p = 80$  (dashed curve) and  $p = 90$  (dotted curve).