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AN ABLATION RATE MODEL FOR TIME-DEPENDENT PELLET ABLATION STUDIES AT MAX-PLANCK INSTITUTE

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Abstract

The ablation cloud of a pellet injected into a plasma consists of two fairly distinct regions: (1) a cold, dense, weakly ionized core region, whose expansion is spherically symmetric and (2) a warm plasma outer region which forms an elongated channel parallel to $\vec{B}$. The core region is transonic; The pellet mass ablation rate depends only on the attenuated incident-electron-energy-flux falling on the sonic surface. The attenuation is dictated by the total $\int ndl$ in the outer region. The quasi-steady "fast" core region provides a particle source for the more slowly evolving outer region, described using a time-dependent model under development at IPP-Garching. By joining the two regions a self-consistent ablation model can be constructed for refuelling studies on fusion grade plasmas.

1. Introduction

Oscillations in the light intensity [1,2] of the luminous portion of ablation clouds surrounding pellets injected into plasmas seem to indicate that the dynamics of pellet ablation may be inherently non-steady.

One of the primary purposes for developing a time-dependent pellet ablation code is to uncover intrinsically time-dependent phenomena occuring in the pellet's ablation cloud, if it exists.

From the pellet's point of view, the background plasma appears to be streaming past at the local $E \times B$ drift velocity at a steady rate. Here E is the electric field seen by the pellet: the sum of the motionally induced (poloidal) electric field, $E_p = V_p B$, and the radial electric field, E_R, naturally occuring inside tokamak plasmas. Since the pellet velocity V_p is constant, and the background plasma may be considered uniform (except for possible small-scale rational surface induced electron-heat-flux irregularities, which have been ruled out for smooth q-profiles as a cause for non-steady ablation rate [3]), one might wonder how pellet ablation could be non-steady.

Recent theoretical models [4–6] have shown that the charge separation inside the ablation plasma effectively screens out the E-field near the pellet. This makes the cloud velocity across the B-field nearly the same as the pellet velocity. Now the ablation cloud in the immediate vicinity of the pellet is a cold, dense, high-pressure weakly-ionized plasma that undergoes an almost spherically symmetric expansion because: (1) the deposition of the incident electron energy is nearly spherically symmetric (see appendix for details) and (2) the magnetic $J \times B$ force compared to the hydrodynamic force driving the cloud expansion is usually well below 1%. However, the very nature of a radial expansion is that both the ram pressure and the kinetic pressure decay rapidly with distance. So eventually the magnetic force decelerates the transverse flow, gradually collimating the flow pattern from radial flow near the pellet to 1-D channel flow along the magnetic field lines.

As the plasma mass builds up along the direction of the magnetic field, while being carried along with the pellet, it could eventually cut down the heat flux to the spherically symmetric core region and reduce the ablation rate. The remaining ablation mass is eventually heated up and swept "behind" the pellet – the E-field screening is not perfect – and the whole process repeats itself. This is only mere conjecture, but it may prove to be the reason for the 10 μs oscillations seen universally in both hydrogen and carbon pellet experiments [1,2]. There may be some indirect experimental evidence supporting this view. It has been noted that the fluctuations, normally present in ohmic plasmas, are markedly reduced in LH or ECRH heated plasmas [2], where supra-thermal electrons are produced. Such electrons clearly dominate the ablation rate because pellet penetration is much shallower in these discharges. However, compared to thermals these supra-thermals would be much less shielded by a long plasma channel, resulting in a lower fluctuation level.

This elongated plasma mass offers the possibility of additional attenuation of the incident electron-heat-flux [7,8]. Hence the shape and density of the entire ablation cloud must be obtained self-consistently with the feedback mechanism connecting the attenuation to the mass ablation rate.

In this report we present a simple method of calculating the pellet mass ablation rate as a function of the attenuated heat-flux falling on the spherically symmetric region. For definiteness, the boundary of this region is the sonic surface of the flow. It can be shown rigorously that a sonic surface exists even in the presence of the magnetic field, provided the magnetic field's influence is weak. In that case the flow parameters at the sonic radius in terms of the heat-flux there deviate only slightly from the flow parameters already obtained from the transonic flow model of Parks [9,10] for the case of no magnetic field. Although the transonic flow model is steady-state, the part of the model which deals with the sub-sonic region, $r_p < r < r_*$, is still applicable here simply because of the short time to establish the flow in this region; r_*/v_*, is only $0.1\mu s$. This is much less than the $10\mu s$ oscillation time-scale which presumably would be the time-scale for modulation of the heat-flux into the fast core region due to the slower evolution of the outer long channel flow region.

2. Existence of a Sonic Surface in a Magnetic field.

The transonic flow method [9,10] can be extended to include the presence of a magnetic field using the following anzatz. In the radial momentum equation the radial component of the magnetic force has the form

$$F_r = \sigma v_r B^2 sin^2\theta \tag{1}$$

where $\sigma(T,\rho)$ is the electrical conductivity. The angular dependence makes the force weaker as the polar angle θ gets smaller.

Let us now imagine a ficticious magnetic force which is "spherically symmetric". By dropping the angular dependence this becomes a friction like force:

$$F_r = \sigma v_r B^2 \tag{2}$$

Using this anzatz, the spherical flow equations can be analyzed to show that a sonic surface indeed exists for B-fields below a certain value. We therefore expect a sonic surface to exist in the real case since the magnetic force given by Eq. (1) is less restrictive. Furthermore, if the dimensionless parameter characterizing the ratio of magnetic force to the inertial force at $r = r_*$ given by

$$N_* = \frac{\sigma_* r_* B^2}{\rho_* v_*} \tag{3}$$

is small, then the new sonic flow quantities deviate from their zero B-field counterparts by an amount of order N_*. In section 4. we show that N_* is, indeed, small for most cases of interest to pellet fuelling.

3. Mass Ablation Rate – the Feed Back Mechanism

To evaluate the mass ablation rate, and eventually N_*, we need the sonic flow quantities: T_*, ρ_*, and v_*. The expressions for these spring from the theory of transonic flow [10]:

$$T_* = \frac{2^{-2/3}}{\gamma} m_H^{1/3} [Q(\gamma - 1) q_* r_* \Lambda_*]^{2/3} \tag{4}$$

$$\rho_* = \frac{\lambda_* m_H}{\Lambda_* r_*} \tag{5}$$

$$v_* = \left[\frac{Q(\gamma - 1) q_* r_* \Lambda_*}{2 m_H} \right]^{1/3} \tag{6}$$

In these formulas γ is the ratio of specific heats, m_H is the mass of a hydrogenic atom, Λ_* is the energy-flux-cross-section for mono-energetic electrons arriving with energy E_* at $r = r_*$ and q_* is the energy flux at $r = r_*$. The set of gas-dynamic equations coupled to the stopping process of the incident electrons were posed in dimensionless form, containing a single eigenvalue λ_*. The existence of transonic-flow solutions depends upon finding a real value for λ_* which allows the solutions to satisfy certain boundary conditions at the pellet surface. As the equations are highly non-linear only a machine solution can determine λ_* numerically. It was found that $\lambda_* = 0.77$ and $r_* = 1.72r_p$ for $\gamma = 5/3$. The symbol Q appearing in Eqs. 4 and 6 takes care of reductions in the ablation temperature, and ablation rate due to dissociation and ionization processes, i.e., $Q < 1$. Its exact value is tricky to estimate, but one can use the tabulated results from Ref. [11] to obtain some estimate of Q.

These are how the sonic flow quantities appeared before they were "prettied-up" by connecting q_* and E_* to their asymptotic values at infinity, q_∞ and E_∞. Of course, this connection was based on a spherically symmetric supersonic region extending to infinity as the medium providing the attenuation of the heat flux. Recall the "attenuation coefficients" were given such that $q_* = 0.678q_\infty$ and $E_* = 0.902E\infty$ for $\gamma = 5/3$. But because of the magnetic field we know that these relations are no good because for $r > r_*$ the flow pattern is no longer spherically symmetric nor is is it necessarily steady-state, (or quasi-steady state). In addition, thermal heat conduction within the outer region can become appreciable [12]. All of these effects taken together can alter the $\int ndl$ through the outer region and change the attenuation. Therefore we must write in general

$$q_* = F_q q_\infty \tag{7}$$

and

$$E_* = F_e E_\infty \tag{8}$$

where F_q and F_e are the new attenuation coefficients which the time-dependent code is supposed to update at each point in time. In terms of these coefficients the desired feed back on the mass ablation rate can be written as

$$G = 4\pi r_*^2 v_* \rho_* \tag{9}$$

which for $\gamma = 5/3$ (dissociated to atoms) becomes

$$G = 7.4 \times 10^{-17} Q^{1/3} n_e^{1/3} T_e^{1/2} F_q^{1/3} r_p^{4/3} M^{2/3} / \Lambda_*^{2/3}(E_*) \tag{10}$$

where cgs units are used except T_e and E are in eV and M is the atom mass in AMU. It will be shown that under fusion plasma conditions $T_* \sim 1eV$ and the fractional ionization is negligible. However the gas is completely dissociated in the subsonic region. Dissociation processes absorb heat causing a slower ablation rate by about 80%. To reflect the dissociation effect we take $Q^{1/3} = 0.8$ or $Q = .512$. The temperature is accordingly reduced by $Q^{2/3}$ so that

$$T_{*,Diss.} \cong 0.64 T_* \tag{11}$$

where T_* is given in Eq. (4) for no dissociation. The expression for $\Lambda(E)$ is given in the appendix for mono-energetic incident electrons. In order to take into account the fact that the actual Maxwellian electron distribution is more penetrating than the equivalent mono-energetic distribution, we have to reduce Λ by a factor of 3 to account for this [13,14] i.e.,

$$\Lambda(E) \rightarrow \frac{1}{3}\Lambda(E) \tag{12}$$

4. Evaluation of N_*

Using the results of the previous section we can now evaluate N_* in Eq. (3) together with the ablation rate G given in Eq. (10) for different F_q and F_e and background plasma conditions. Since F_q and F_e are tied together by Eq. (A10) in the appendix, we take

$$F_q = F_e^2$$

The conductivity is calculated from

$$\sigma = \frac{\sigma_{spitzer}}{(1 + \nu_{en}/\nu_{ei})}$$

where the electron neutral collision rate ν_{en} is given in Ref. [15]. The ratio ν_{en}/ν_{ei} depends on the ionization fraction, fi, which is calculated using the Saha equation.

The following table shows how G and N_* vary under different shielding conditions represented by the parameter F_e. Corresponding to a given F_e is an outer cloud thickness $\int ndl$ which is tabulated in arbitrary units. The first row ($F_e = 0.9$) corresponds to the weak shielding found in the spherically symmetric supersonic flow model [9,10]. (The ablation rate in this case is in close agreement with similar models described in Refs. 13a and 13b and lies between the two when $Q = 1$.) In all cases N_* is a small parameter, which was to be justified.

Table I

Ablation rate, G, and N_* under different outer cloud shielding conditions represented by parameter $F_e = E_*/E_\infty$. The background plasma parameters are $n_{e\infty} = 2 \times 10^{14} cm^{-3}, T_{e\infty} = 10keV, B = 4T$. The pellet is DT with a radius $r_p = 0.25cm$, and $Q = 0.512$.

F_e	$T_*(ev)$	f_{i*}	$G(g/s)$	$\int ndl$ (Arb. Units)	N_*
0.9	.96	.002	4287	1	.0011
0.7	.94	.002	2650	2.766	.002
0.5	.92	.0023	1393	4.20	.004
0.3	.88	.0025	526	5.27	.011

Conclusion

The key result of this paper is the pellet mass loss rate expression given in Eq. 10. It depends upon the attenuation coefficients, F_q and F_e, which connect the heat flux, and average incident electron energy at the sonic surface to their asymptotic values at infinity; thus providing a feed back to a time-dependent numerical study of the ablation cloud in the outer region beyond the sonic surface. On account of the small values of N_* and T_*, neither the magnetic field nor thermal heat conduction will significantly alter the basic spherically symmetric transonic behavior of the flow near the pellet.

Appendix

Part 1: Amended Electron heat-flux transport model for mono-energetic electrons

In the theory of Parks [9,10] it was supposed that the degradation of the hot incident electrons streaming through the ablation cloud was caused by two processes. The influence of inelastic collisions, or energy loss, is eqivalent to a retarding force $nL(E)$ in the Fokker-Planck equation. The influence of 90^o scattering from the forward hemisphere to the backwards hemisphere was described by a mean cross-section $\sigma_t(E)$. If the magnetic field lies in the $\hat{z}$ direction the electron-energy-flux transported along the field lines has the form

$$\vec{q} = q\hat{z} \tag{A3}$$

and the attenuation of the energy-flux, q, due to the combined effect of the above processes is, in the mono-energetic approximation, given by

$$\frac{\partial q}{\partial z} = \frac{n}{< \cos\theta >}[L(E)/E + \sigma_T]q \tag{A2}$$

where n is the density of atom nuclei and $< cos\theta > \approx 1/2$. In a cylindrical coordinate system with coordinates p, z the partial derivative in (A2) is taken holding p fixed, i.e., staying on a given field line.

Parks, in his original theory, over-estimated σ_T by a factor of 3 by errorneously including single back-scattering cross-section events which really have no physical significance [16]. However, the 90^o scattering caused by cumulative small-angle collisions still remains. This corrected result makes the contributions from energy loss and scattering about equal in Eq. A2. Our amended equation governing q can be written as

$$\frac{\partial q}{\partial z} = \frac{\rho}{m_H}\Lambda(E)q \tag{A3}$$

where $\Lambda(E)$ is the effective energy-flux cross section, to be used in the text were it appears, viz.,

$$\Lambda(E) = \frac{1}{< cos\theta >}[L(E)/E + \sigma_{eff}] \tag{A4}$$

where σ_{eff} is the scattering cross-section in hydrogen plasma with fractonal ionization fi:

$$\sigma_{eff} = (1 - fi)\frac{1.3\times10^{-13} ln\Lambda_0}{E^2} \qquad + \qquad fi\frac{2.6\times10^{-13} ln\Lambda_e}{E^2}$$

(Neutral contribution) (Plasma (e+i) contribution)

and the neutral logarithmic factor is given by [14]

$$\Lambda_0 = \frac{E^{1/2}}{3} \tag{A6}$$

and the plasma one is given by [17]

$$\Lambda_e = \frac{\lambda_{Debye}}{\lambda_{Debroglie}} = 3.8\times10^8 E^{1/2} T^{1/2}/n_e^{1/2} \tag{A7}$$

where T and n_e refer to the ablation temperature and electron density respectively. All units are cgs except for E and T which are in eV. Since the energy loss functions for plasma and neutral atoms is essentially the same for high speed electrons we use, for convenience, the familiar Bethe-Block equation for electrons on hydrogen atoms:

$$L(E) = \frac{1.3\times10^{-13}}{E} ln(E/5.5) \tag{A8}$$

so

$$\frac{\partial E}{\partial z} = \frac{1}{< cos\theta >}\frac{\rho}{m_H} L(E) \tag{A9}$$

A general relationship between q and E can be obtained by dividing Eq. (A3) by Eq. (A9) and integrating. The relation holds at any point in the cloud, viz.,

$$\frac{q}{q_\infty} = \left(\frac{E}{E_\infty}\right)^\alpha \tag{A10}$$

where α is bounded: it is 1.5 for $fi = 0$ and 2 for $fi = 1$.

Part 2 Justification for a spherically symmetric heat source

The ablation flow close to the pellet was assumed spherically symmetric, but how symmetric is the heat source owing to the deposition of the incident electron energy? The volumetric heat source is given by

$$S = -\nabla \cdot \vec{q} \tag{A11}$$

We want to show that at a constant radius, r, the θ variation of S is relatively weak. We can compute $\nabla \cdot \vec{q}$ in the r, θ spherical coordinate system, but it is of course the same in the cylindrical p, z coordinate system (Fig. 1). From Eq. A3 this is

$$S = \left.\frac{\partial q}{\partial z}\right|_p = \frac{q\Lambda\rho}{m_H} \tag{A12}$$

All we have to do now is to show that the product, $q\Lambda$, is nearly uniform over a spherical surface. For definiteness let the surface in question be the sonic surface $r = r_*$, and for $r > r_*$ assume the density falls off like $n(r) = n_*(r_*/r)^2$ for simplicity. By integrating Eq. (A9) along z from ∞ to $z = (r_*^2 - p^2)^{1/2}$ we obtain E_* as a function of $sin\theta = p/r_*$:

$$E_*(\theta) = \{E_\infty^{1.72} - 2.3 \times 10^{-13} n_* r_* \Gamma(\theta)\}^{(\frac{1}{1.72})} \tag{A13}$$

where

$$\Gamma(\theta) = \frac{1}{sin\theta}\{\frac{\pi}{2} - cos^{-1}(sin\theta)\} \tag{A14}$$

and for convenience we approximated Eq. (A8) by taking $L(E) \cong 6.66\times10^{-14}/E^{0.72} eV-cm^2$. From Eq. (A10), $q_* \propto E_*^2$, and $\Lambda(E_*) \propto L(E_*)/E_* \propto E_*^{-1.72}$ so that we have finally the angular variation of S at $r = r_*$:

$$S(r_*, \theta) \propto E_*^{0.28}(\theta) \tag{A15}$$

This is tabulated below

θ *deg*	$S(r_*,\theta)/S(r_*,0)$
0	1
30	0.994
60	0.975
90	0.926

The reason for the good symmetry is obvious now: E_* and q_* decrease with increasing θ but Λ_* increases. Hence, the product $q_*\Lambda_*$ remains fairly constant. Since the θ-variation is weak we can approximate the source term in Eq. (A12) as

$$S = \frac{\partial q}{\partial z}\Big|_{p=0} \qquad (A16)$$

This is equivalent to the result normally seen in the spherically symmetric models, namely

$$S = \frac{dq}{dr} \qquad (A17)$$

This completes our proof. It should be noted that the flux in Eq. (A3) is the one-sided flux; the pellet blocks the flux coming from the opposite direction. However, on field lines not threading the pellet one must take into account the flux from both directions. In fact at the point $r = r_*, \theta = \pi/2$ the value of S contributed by both fluxes is the same so really the value of $S(r_*,\pi/2)/S(r_*,\theta)$ should be $2 \times 0.926 = 1.85$. It appears therefore that the heating of the cloud is maximum at $\theta = 90^o$.

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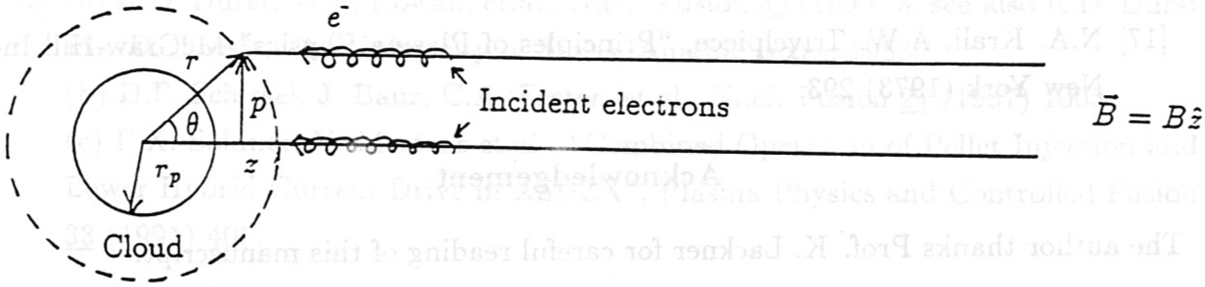

Fig. 1
The r, θ and p, z coordinate systems used to describe heat deposition in the ablation cloud by incident electrons streaming parallel to $\vec{B}$.