Scaling Investigations and Statistical Profile Analysis

by

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3.1 L-mode scalings for energy confinement

The ITER L-mode global confinement database, containing contributions from a large range of Tokamaks, and collected by Kaye et al., has been analysed with special attention to the derivation of global power-law scalings, and estimates of their statistical uncertainty. In cooperation with collegues from each of the 4 'nations' involved in the ITER project, this led to a joint publication in Nuclear Fusion (Yushmanov et al.). The major results of this article are an ITER89-P (Power-law) and an ITER89-OL (offset-linear) scaling, which both give a prediction of about 2 seconds for the average ITER L-mode operation, with a roughly estimated uncertainty of 20%. The geometrical scaling of ITER89-P is $\tau \sim R^{1.2}a^{0.3}\kappa^{0.5}$. Moreover, two directions of small data variation were identified, one corresponding to q_{cyl} and the other to $(R/a)a^{0.25}k^{0.5}$. More data in those two directions would increase the precision and reliability of ITER L-mode predictions. Moreover, a better distinction between diamagnetic and MHD measurements of the confinement time, and preparations to a future merge with the ITER H-mode database, would be further steps to improve these, already valuable, results.

3.2 Compilation of the ASDEX global H-mode dataset

This dataset was compiled as part of the ITER global H-mode database. It consists of a cross-section of all ASDEX H-mode discharges from 1982-1986 with NBI as the only additional heating method. Seven timepoints ('observations') per discharge are provided (3 ohmic points, 1 L-mode point, and 3 H-mode points), the last H-mode point corresponding to $\dot{W} = 0$. The variable list for each observation consists of 76 global plasma parameters. Reliable determination of $\dot{W}_{dia}$, and $\dot{W}_{mhd}$ for the first two H-mode points, and the ELM classification (no ELM's, small ELM's, large ELM's) required time-intensive manual intervention. In the so-called 'standard dataset' (which excludes observations with low q_{cyl}, high fast particle contributions, high radiation, or transient behaviour), there are 160 ASDEX observations with large ELM's, 80 with small ELM's, and 16 without ELM's.

3.3 H-mode scalings for energy confinement

In cooperation with collegues from DIII-D, JET, JFT-2M, PDX and PBX/M, the ITER H-mode Database ('ITER HM-DB-90') has been assembled, and preliminary analyses of the confinement time scaling have been performed. This resulted in an IAEA contribution (Christiansen et al.), while a more extensive report is in preparation. The work at ASDEX concentrated on analysis of the condition of the dataset and performing global power-law type regression analyses. As the logarithms of $B_t, I_p, P_{abs}, \overline{n}_e, R, a$ and b (the symbols having their usual meaning) are used as regressors for the confinement time, the data are geometrically represented by a large number of points in 7 dimensional space. The condition of the dataset can now be looked at from two points of view, (a) the ratio between the data variation and the measurement errors (b) the ratio between the data variation and the distance from the database-average to ITER. Both ratios are considered along the principal directions, which point from the center of the database along the principal axis of the ellipse fitting the data-cloud. The first point of view indicates how reliable the individual regression coefficients can be estimated from ordinary least squares regression. From

the second point of view, one can see how much the various principal directions contribute to the statistical uncertainty in the prediction for ITER, provided the condition is 'good' from the first point of view. If the condition is 'moderately bad' to 'just good' from the first point of view (ratios between 1 and 4, say), then improved estimates can be obtained by errors-in-variable regression, and some other 'advanced' techniques. The condition of both the ELMy and ELM-free datasets appears to be 'just good' with respect to the above 7 variables from both points of view, provided that all Tokamaks are used in the regression. This implies, however, that no allowance is made for triangularity, 'bean shapedness' (PBX/M), plasma-wall distance, and type of divertor (open/closed). In fact, the open divertor Tokamaks tend to have high kappa values, such that the effects of divertor type and kappa on confinement cannot be separated from the present dataset. Also, the isotope dependence can hardly be extracted from the present dataset. Finally, it appears that some Tokamaks are heavily represented by ELMy, and others heavily by ELM-free discharges.
A sensible choice seems to use diamagnetic confinement times for the Tokamaks with (roughly) parallel beam injection (ASDEX, JFT-2M) and mhd confinement times for the Tokamaks with perpendicular beam injection (DIII-D, PDX, PBX/M). For PDX and PBX/M, only τ_{mhd} is available. For JET, we consider both cases. Taking τ_{dia} for JET, one gets for the confinement time in seconds:

$$\tau_E = 0.022 I_p^{0.85} B_T^{0.65} (P_{abs} - \dot{W})^{-0.5} (A/Z)^{0.5} R^{1.9} \kappa^{0.65} (a/R)^{-0.2} \overline{n}_e^{0.1}$$

with a rmse of 13.7% for the ELM-free discharges, and

$$\tau_E = 0.028 I_p^{0.55} B_T^{0.50} (P_{abs} - \dot{W})^{-0.5} (A/Z)^{0.5} R^{2.5} \kappa^{0.5} (a/R)^{0.4} \overline{n}_e^{0.25}$$

with a rmse of 13.0% for the ELMy discharges. The units are MA, T, MW, m, and $10^{19}/m^3$. The result is obtained by standard least squares regression of the so-called standard dataset (see above, in addition the shots from PDX where the ratio between the divertor and midplane H_α signal is less then 4 are excluded), and imposing the exponent of A/Z. The exponents are slightly rounded, and satisfy, within the famlily of power-law scalings, to good approximation the constraint imposed by the quasi-neutral, collisional, high-beta Fokker-Plank model, which states that τ_E equals the Bohm time times a function of 7 dimensionless parameters. The high coefficient of B_t is persistent under various variations of the regression set-up, but not in accordance with single parameter scans in some of the Tokamaks. Hence, this is a topic of further investigation. The above scaling gives confinement predictions of 7.0 (1 ± 0.2) for the ELMy and 8.5 (1 ± 0.15) for the ELM-free confinement time for ITER with $I_p = 22$ MA, $B_t = 5$ T, $\overline{n}_e = 1.25 \times 10^{20}$ m^{-3}, $P_{abs} = 160$ MW, $A/Z = 2.5$, $R = 6$, $a = 2.15$, $b = 4.75$, where R, a, and b refer to the *plasma* dimensions. In brackets an estimate of the uncertainty (3 std. dev. from OLS, roughly 2 std. dev. if the errors in the regression variables are taken into account) of the *average* ITER performance, under the assumption that the regression model is correct. However, if τ_{mhd} instead of τ_{dia} is used for JET, then one gets $5.3(1 \pm 0.2)$ s for ELMy, and 7.8 (1 ± 0.15) s for ELM-free ITER discharges. This is due to the fact that τ_{dia} is notably higher than τ_{mhd} for the ELMy shots at JET (on average about 25%). Further activities are planned for more refined analyses, especially of the thermal energy confinement time.

3.4 ELMy H-mode scalings based on JET and ASDEX data

The dataset consists of the ELMy H-mode dataset discussed in the previous annual report (which is, except for the JET shots from 1989, also included in 'ITER HM-DB90'), extended with 52 boronised ASDEX shots from 1989 (20 of which D into D^+). In this analysis, only 1 point per shot is analysed (where W_{dia} is maximal and, in any case $\dot{W}_{dia} < 0.3 P_{tot}$). The condensed results of the analysis have been presented on an EPS poster, which is also available as ITER-1L-PH-4-0-E-10. We discuss some particular aspects: (All results are based on simultaneous regression analysis, using q_{cyl}, $B_t I_p$, $P_{tot} - \dot{W}$, n_e, $L \sim (Rab)^{1/3}$, A/Z and indicators for DN/SN, and wall conditioning (C, Bo, Be), as regression variables. All uncertainties, unless stated otherwise, indicate 1 std. dev.) $\tau_E \sim (A/Z)^{0.45-0.55}$ in the joint regression, and $\tau_E \sim (A/Z)^{0.55-0.65}$ in the regression of the ASDEX data alone. (The lower values correspond to τ_{dia}, and the upper values to τ_{mhd}; the JET data consisted exclusively of D into D^+ discharges.) Each of the two datasets exhibited a negative curvature in the dependence of $\log \tau$ with respect to $\log P_{tot}$ ($\log \tau_E \sim (-0.5 \pm 0.15)(\log P)^2$) not present in the joint regression. This suggests an interaction between the heating power and the size of the machine on confinement time. Both carbonised and boronised ASDEX ELMy H-mode shots have about 15 to 20% ($\pm$ 3%) smaller confinement time than the standard stainless steel discharges in the old (closed, DV I) divertor configuration. It should be mentioned that the boronised ASDEX shots had partly carbonised walls and were all with the new (closed, DV II) divertor type. The DN shots at ASDEX had about 10-15% ($\pm$ 3%) better confinement than the SN shots, whereas at JET no significant difference was observed. At ASDEX there seems to be a higher B_t and a smaller I_p dependence for $q_{cyl} < 3$ than for $q_{cyl} > 3$. This effect is less strong than the one presented in the previous annual report, possibly related to the simplifying assumption, now made, that the ratio of the confinement times of the boronised, DV II shots and the stainless steel, DV I shots is independent of the plasma parameters. For JET, the diamagnetic and mhd confinement times give conflicting results (for mhd as in ASDEX, for dia there was no difference between the two regions). In both cases, the drawing of any firm conclusions on this aspect of the scaling is hampered by the relatively small variation in q_{cyl}. The size scaling is $\tau_{dia} \sim L^{2.2}$, and $\tau_{mhd} \sim L^{1.8}$, with $L \sim (Rab)^{1/3}$. Given this somewhat primitive model for the scaling with R, a and b, the predictions for ITER at $q_{cyl} = 1.8$ are: $4.2(1 \pm 0.3)$ sec for τ_{mhd} and $6.9(1 \pm 0.3)$ sec for τ_{dia}. (In brackets roughly 2 std. dev. from OLS). The difference between these two results can be understood from the fact that τ_{mhd} at ASDEX is about 25% larger than τ_{dia}, largely due to the parallel beam injection, whereas for the ELMy shots at JET (with perpendicular beam injection) about the reverse is true. Hence, from this JET-ASDEX comparison, one expects the ITER ELMy H-mode confinement time to be somewhere between 4 and 7 s.

3.5 Statistical profile analysis

The work on statistical profile parameterisation has come to a temporarily conclusion by two IPP reports (5/34 and 5/35). The theoretical report gives a precise account of the statistical models and techniques to analyse the influence of plasma parameters on e.g. density and temperature profiles as measured by YAG-laser Thomsen scattering experiments. Some continuous profile models are discussed with emphasis on (general) spline representations with fixed knot positions, and applied to the logarithm of the profile measurements. Those representations also describe the influence of the plasma variables on the profiles. Special attention is given to the modelling of the propagation of errors. An account is given of relevant estimation methods, and of statistical tests to decide whether various sub-models are in accordance with the data (with profile invariance, and the hypothesis of gaussian profile shapes as interesting special cases). A derivation and explicit formulas are given for local and global confidence bands for the underlying profile (for any specified values of the plasma parameters). Similarly, a discussion is given for prediction bands of new single profile measurements, and asymptotic confidence intervals are derived for volume averaged values of density, temperature and pressure, obtained by (numeric) integration of the measured n_e and T_e profiles. The addition of a running example of practical analyses is planned the make the report more accessible to a larger audience.

The practical report describes the analysis of 2 extensive ohmic datasets in the Saturated Ohmic Confinement regime (105 temporally compressed profiles from 'high density', 'low q_a', Ti-gettered discharges, and 38 from 'medium density', 'medium q_a', non-gettered discharges). An extended version of this report is submitted for publication. Global confinement scalings are given of all major plasma parameters,

$$W_{dia}, W_{mhd}, W_{th}, V_{loop}, Z_{eff}, < T_e >,$$

confinement times, etc. A sensitivity analysis was made to investigate the influence of various assumptions on the scaling of the energy estimates. One aspect of these analyses is that, in contrast to the results some previous single scan experiments, in general no confinement degradation (only a saturation) at high values of n_e is observed. The scaling of the temperature peaking factor is rigorously shown to be in good agreement with the ansatz of classical resistive equilibium, which implies that

$$\frac{< T^{3/2} >_{r=0}}{< T^{3/2} >_{r=a}} = \frac{q_{r=a}}{q_{r=0}}.$$

A detailed profile analysis has been performed, with a careful analysis of the influence of outliers and of the propagation of the various error components. In addition to the regression formulae, the results are summarised by convenient graphical representations, from which one can quantitatively judge both the hypothesis of profile invariance and of gaussian profile shapes. It appears that q_{cyl} influences the profile shapes of T_e, and n_e (and hence p_e) noticeably in the region $0.2 < r/a < 0.6$, and that the hypothesis of gaussian profiles is severely rejected.

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