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Satish Puri

Max-Planck Institut für Plasmaphysik, EURATOM Association,

Garching bei München, Germany

ABSTRACT

Toroidal transport due to the anomalous slowing down of low-parallel-velocity electrons (with gyroradius less than the Debye length) by the enhanced electron-ion collisions is investigated. Expressions for the particle, energy and canonical-angular-momentum diffusivities are obtained. Large diffusivities are found to occur at the cold, low-density edge and may be eliminated by a nominal increase of ion temperature at the edge, resembling the L-H mode transition in Tokamak plasmas.

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Electron-ion momentum-transfer collisions parallel to the magnetic field direction are anomalously enhanced if the electron gyroradius, r_{ce} is much less than the Debye length, λ_D; the collision frequency may exceed the corresponding Spitzer value by a factor of one thousand for electrons with parallel velocity of the order of ion-thermal speed.[1,2] Trapped electrons in a torus are subject to this anomaly (henceforth referred to as the Matsuda anomaly) near their turning points, leading to the conjecture that the Matsuda anomaly might provide an additional channel for neoclassical diffusion.[1,2]

In this Letter the diffusivity contribution of the Matsuda anomaly is investigated and the underlying transport mechanism is identified. Particle (D_e), energy (χ_e) and canonical-angular-momentum (Ψ_e) electron diffusivities in a large-aspect-ratio torus of concentric, circularly nested flux surfaces are determined. The canonical-angular-momentum loss pertains to the possibly deleterious effect on the recovery of stored momentum via the inverse Ware[3] pinch in subthermal-current-drive schemes[4,5]. Resemblence of the observed enhanced diffusivity near the plasma edge with the L-H mode transition[6] in a Tokamak is pointed out.

The transport mechanism may be understood by referring to Fig.1 depicting a trapped particle orbit subject to the Matsuda anomaly. The thousandfold enhancement in the collision frequency could effectively capture the electrons near their turning points, where the parallel velocity is maintained at the ion thermal speed through continual collisions. These *stagnant* electrons escape upon acquiring (i) either a large enough parallel velocity through anomalous collisions with ions, or (ii) by an increase in the gyroradius through energy exchange collisions with ions. The freed electrons resume their banana orbits and are prone to another capture incidence at the subsequent turning point. *The gradient drift occuring during the stagnation interval constitutes the mechanism for enhanced diffusivity.* Under conditions of extremely large collisionality (for low ion temperature at the plasma edge), the escape probability may become small enough to preclude escape altogether, leading to a *stagnation catastrophe* signaling the onset of convective transport, and the diffusion model ceases to be valid.

Electrons and ions are assumed to possess isotropic Maxwellian velocity distributions (at $\theta = 0$) with temperatures T_e and T_i, respectively. As the electrons slow down near their turning points, a fraction α_{cap} is captured through the anomalous collisions. The capture probability per unit length is given by

$$P_{cap} = \frac{1}{\lambda_{mfp}} \exp\left[-\int^{\lambda} \frac{d\lambda}{\lambda_{mfp}} \right] ,$$

where the integration extends over the particle's trajectory, $d\lambda = qR_0\, d\theta$, $\lambda_{mfp} = v_{\|}/\nu_a$, ν_a is the anomalously enhanced electron-ion collision frequency which for $\lambda_D = \sqrt{T_e/m}\sqrt{T_i/(T_i + ZT_e)}/\omega_{pe} > r_{ce}$ (or $\eta = (\omega_{ce}/\omega_{pe})\sqrt{.5T_i/(T_i + ZT_e)} > u_{\perp/e}$) is given by[2]

$$\nu_a \approx \frac{\gamma Z}{4\pi} \frac{v_{ti}^2}{v_{\|}^2} \frac{\omega_{pe}^4}{n_e v_{\|}^3} \ln\left(\frac{\eta}{u_{\perp/e}} \right) , \tag{1}$$

$$\gamma = \frac{\hat{n}_i}{n_i} \frac{< v_i^2 >}{v_{ti}^2} ,$$

$\hat{n}_i$ is the number of ions within the sphere in velocity space with radius $|v_{\|}|$, $< v_i^2 >$ is the average v^2 for these particles, v_{te} and v_{ti} are the electron and ion thermal speeds, $u_{/e,/i} = v_e/(\sqrt{2}v_{te,i})$ is the normalized electron velocity, q is the safety factor, Z is the ionic charge, $\rho = r/a$, a is the plasma radius and R_0 is the torus major radius. Combining the capture probabilities of the electron during its traverse to and from the trapping position θ_t, P_{cap} per unit radian at (r, θ) becomes

$$P_{cap}(u_{\|/e}, \theta, u_{\perp/e}) = \Theta(\theta) \left[\exp\left(-\int_0^{\theta} \Theta(\theta')\, d\theta' \right) + \kappa(\theta_t) \exp\left(-\int_{\theta}^{\theta_t} \Theta(\theta')\, d\theta' \right) \right] ,$$

where

$$\kappa(\theta_t) = 1 - \int_0^{\theta_t} \Theta(\theta') \exp\left(-\int_0^{\theta'} \Theta(\theta'')\, d\theta'' \right) d\theta' ,$$

$$\Theta(\theta) = \frac{qR_0\nu_a(\theta)}{\sqrt{2}v_{ti}u_{\|/i}(\theta)} , \tag{2}$$

$$u_{\|e,i}(\theta) = \left[u_{\|e,i}^2(0) - \frac{\varepsilon(1 - \cos\theta)}{1 + \varepsilon\cos\theta} u_{\perp e,i}^2(0) \right]^{1/2} ,$$

and θ_t is the trapping angle for the velocity $(u_{\|/e}, u_{\perp/e})$. The captured fraction becomes

$$\alpha_{cap}(u_{\|/e}, \theta, u_{\perp/e}) = \frac{1}{\pi^{3/2}} \exp\left[-u_{/e}^2(0) \right] P_{cap}(u_{\|/e}, \theta, u_{\perp/e}) .$$

Integrating over the parallel velocity gives

$$\alpha_{cap}(\theta, u_{\perp/e}) = \int_{u_{\|a}}^{u_{\|b}} \alpha_{cap}(u_{\|/e}, \theta, u_{\perp/e})\, du_{\|/e} \ ,$$

where $\varepsilon = r/R_0$, the integration limits $u_{\|a} = \sqrt{\varepsilon(1-\cos\theta)/(1+\varepsilon\cos\theta)}\,u_{\perp/e}$ and $u_{\|b} = \sqrt{2\varepsilon/(1-\varepsilon)}\,u_{\perp/e}$ being set by the trapping condition.

The captured electrons join the stagnant population and acquire the parallel velocity distribution of ions through collisions. The parallel escape probability and escape rate out of the stagnant pool are given by

$$P_{esc}(u_{\|/i}, \theta, u_{\perp/e}) = \exp\left[-\int_0^{\theta} \Theta(\theta')\, d\theta'\right] , \tag{3}$$

and

$$\dot{P}_{esc}(u_{\|/i}, \theta, u_{\perp/e}) = c_i P_{esc}(u_{\|/i}, \theta, u_{\perp/e}) \ ,$$

respectively, where

$$c_i = \frac{(v_{\|i}/r)\nu_a}{(v_{\|i}/r) + \nu_a} \ ,$$

is the escape frequency; the particle flux determined by $v_{\|i}/r$ being mitigated by the Maxwellianization rate ν_a. Integrating over the parallel velocities gives

$$\dot{P}_{esc}(\theta, u_{\perp/e}) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} c_i P_{esc}(u_{\|/i}, \theta, u_{\perp/e}) \exp(-u_{\|/i}^2)\, du_{\|/i} \ .$$

A stagnant electron can also escape from the stagnant pool by an increase in its gyroradius through energy exchange collisions with ions at frequency ν_{ie}. The corresponding escape rate equals $\nu_{ie} < P_{esc}(\theta, \sqrt{T_i/T_e}u_{\perp/e}) >$, where the average is performed over the perpendicular electron velocities corresponding to the ion temperature; the perpendicular escape channel might assume significance in high collisionality plasmas where the parallel escape route becomes inefficient. The resultant net escape time becomes

$$\tau_{esc}(\theta, u_{\perp/e}) = \left[\dot{P}_{esc}(\theta, u_{\perp/e}) + \nu_{ie} < P_{esc}(\theta, \sqrt{T_i/T_e}u_{\perp/e}) >\right]^{-1} \ .$$

The number of electrons escaping the stagnant pool is balanced by the induction of an equal number of non-stagnant electrons into the stagnant pool. The captured electrons spend a time τ_{esc} in the stagnant phase and $\tau_b/2$ in transit between the stagnation periods, where τ_b, the bounce time for trapping at θ is given by

$$\tau_b(\theta, u_{\perp/e}) = \frac{4R_0\sqrt{\varepsilon(1+\varepsilon\cos\theta)}}{v_{te}u_{\perp/e}(0)\sin(\theta/2)}\, F\left(\frac{\theta}{2}\,\middle|\,\frac{(1+\varepsilon\cos\theta)}{(1+\varepsilon)\sin^2(\theta/2)}\right) ,$$

F being the elliptic integral of the first kind. It has been implicitly assumed that both capture and escape occur substantially at the trapping angle θ_t. The stagnant fraction α_s becomes

$$\alpha_s(\theta, u_{\perp/e}) = \frac{\tau_{esc}(\theta, u_{\perp/e})}{\tau_{esc}(\theta, u_{\perp/e}) + .5\tau_b(\theta, u_{\perp/e})}\,\alpha_{cap}(\theta, u_{\perp/e}) .$$

The contribution to the particle diffusivity is

$$D_e(\theta, u_{\perp/e}) = v_d^2(\theta, u_{\perp/e})\tau_{esc}(\theta, u_{\perp/e})\,\alpha_s(\theta, u_{\perp/e}) , \tag{4}$$

where the radial component of the drift velocity is given by

$$v_d(\theta, u_{\perp/e}) = \frac{1}{2}\frac{(1+\varepsilon)\sin\theta}{(1+\varepsilon\cos\theta)}\frac{v_{te}^2 u_{\perp/e}^2}{\omega_{ce}R_0} . \tag{5}$$

Integrating over θ and $u_{\perp/e}$ gives the total diffusivity at r as

$$\hat{D}_e(r) = 4\pi\int_0^\pi d\theta \int_0^\eta D_e(\theta, u_{\perp/e})u_{\perp/e}\,du_{\perp/e} . \tag{6}$$

Similarly, the contribution of radial electron drift to the electron thermal diffusivity may be expressed in the form

$$\chi_e(\theta, u_{\perp/e}) = \frac{(1+\varepsilon)u_{\perp/e}^2(0)}{(1+\varepsilon\cos\theta)}\,D_e(\theta, u_{\perp/e}) , \tag{7}$$

where the entire kinetic energy of the electron is assumed to be contained in $u_{\perp/e}$ at the trapping location θ.

Further, the radial displacement of the trapped electron entails a loss

$$\Delta p_\varphi = \frac{e}{2\pi}\frac{\partial\Phi}{\partial\rho}\,\Delta\rho = eRB_\theta\,\Delta\rho = \frac{mr\omega_{ce}}{q}\,v_d\tau_{esc} ,$$

in the canonical angular momentum

$$p_\varphi(\theta, u_{\perp/e}) = \frac{ea^2 B_{\varphi 0}}{2(q_a - q_0)} \ln \left[\frac{q_0 + (q_a - q_0)A^2}{q_0 + (q_a - q_0)\rho^2} \right] \pm mR \left[\frac{\varepsilon(1-\cos\theta)}{1+\varepsilon\cos\theta} \right]^{1/2} v_\perp(0) ,$$

where $A = R_0/a$. The net fractional loss is $\Delta p_\varphi \alpha_s / \hat{p}_\varphi(r)$ where

$$\hat{p}_\varphi(r) = 4\pi \int_0^\pi d\theta \int_0^\infty \alpha_t(\theta, u_{\perp/e}) p_\varphi(\theta, u_{\perp/e}) u_{\perp/e}\, du_{\perp/e}$$

$$= \sqrt{\frac{2\varepsilon}{1+\varepsilon}} \frac{ma^2\omega_{ce}}{2(q_a - q_0)} \ln \left[\frac{q_0 + (q_a - q_0)A^2}{q_0 + (q_a - q_0)\rho^2} \right] ,$$

α_t being the trapped-particle fraction. The canonical-angular-momentum diffusivity may be expressed as

$$\Psi_e(\theta, u_{\perp/e}) = \frac{2\Delta p_\varphi(\theta, u_{\perp/e})\alpha_s(\theta, u_{\perp/e})}{\hat{p}_\varphi(r)} v_d^2 \tau_{esc} , \tag{8}$$

where the multiplication by two accounts for the averaging over τ_{esc}^2 in a Poisson distributed collision process. The diffusivities $\hat{\chi}_e(r)$ and $\hat{\Psi}_e(r)$ are obtained from Eqs.(7) and (8), respectively, upon integrating over θ and $u_{\perp/e}$ as in Eq.(6).

Fig.2 shows the diffusivities as a function of plasma radius for several T_0 in a hydrogen plasma with $n_e = n_{e0}(1-\rho^2)$, $T_e = T_{e0}(1-\rho^2)$, $q = q_0 + (q_a - q_0)\rho^2$, $Z = \exp(\rho \ln Z_a)$, $B_{\varphi 0} = 5\,T$, $n_{e0} = 10^{20}\,m^{-3}$, $q_0 = 1$, $q_a = 3.5$, $Z_a = 3$, $R_0 = 2\,m$, $A = 3$ and $T_{i0} = T_{e0} = T_0$. For normal operating conditions, the diffusivity contributed by the Matsuda anomaly is inconsequential in the plasma interior. However, the relatively large collisionality near the plasma edge triggers an explosive buildup; the diffusivity values grow by several orders of magnitude within a span of a few centimeters.

Such violent episodes might have been anticipated from the $\exp(-\int \Theta\, d\theta)$ dependence of capture and escape probabilities. In Eq.(3), $P_{esc} \to 0$ as $\int \Theta\, d\theta \approx \Theta\Delta\theta \gg 1$. The Matsuda anomaly extends to parallel electron velocities of the order of ξv_{ti} or within an angle

$$\Delta\theta \sim \frac{m}{M} \frac{\xi^2}{u_{\perp/e}^2(0)} \frac{(1+\varepsilon\cos\theta_t)^2}{\varepsilon(1+\varepsilon)\sin\theta_t} , \tag{9}$$

from the banana-orbit turning point θ_t, where $\xi \sim \mathcal{O}(1)$. For a given radius, Eqs.(1), (2) and (9) yield

$$\Theta\Delta\theta \sim q R_0 Z T_i^{-2} n_e \ln B_0 . \tag{10}$$

The T_i^{-2} dependence of $\Theta\Delta\theta$ predisposes the particle and thermal fluxes to the sudden onset/extinction of the stagnation catastrophe with decreasing/increasing ion temperature at the plasma edge. The phenomenon, though instigated by purely collisional processes, has the appearance of a bifurcation and might be responsible for the L-H mode transitions[6-8] in Tokamaks. The poloidal variation of diffusivity (Fig.3), in agreement with the observed asymmetry of the heat flux to the divertor[8], gives further support to this possibility. The large contribution for $\theta \lesssim 1$ arises from the $\Delta\theta \sim (\sin\theta_t)^{-1}$ dependence in Eq.(9) resulting in low escape probabilities for small θ. The reduction in transport near $\theta = 0$ is due to the vanishing of v_d in Eq.(5).

The diffusivities increase with q, R_0 and B_0 as expected from Eq.(10). More complex effects accompany variations in n_e and Z_a because of the simultaneous changes in ν_a, ν_{ie} and η. The T_e effect on diffusivity is similarly complicated by the opposing contributions from the changes in the collisionality, gradient drift and the Debye length. Details of these results will be presented in a more comprehensive communication.

The foregoing analysis employs the approximate form of ν_a given by Ware[2], the precise determination using Rostoker's equation[9] in the manner of Matsuda[1] being prohibitively difficult. Possibly significant features like drift, rotation, recycling and radial E field have been ignored in this analysis. Also, the approximate forms of the particle distribution functions as well as the neglect of the stagnation effects on particle distributions puts limitations on the accuracy of the present analysis. The assumption $\theta_{cap} \approx \theta_t \approx \theta_{esc}$, used in this analysis, implies banana-regime transport so that $N_c = q\pi R_0 \nu_{ei}/v_{te} \ll 1$. For $N_c \gtrsim 1$, an escaping electron would suffer repeated collisions and must seek a fresh escape after each stagnation encounter. The effective escape time would increase by a factor of $\sim O(N_c^2)$ with a concomitant enhancement of diffusivity.

The experimental measurements are subject to uncertainties as well, principally due to the lack of accurate information regarding the crucial parameter T_i near the separatrix, where T_i is known to have steep gradients[7] with an e-folding length of $1-2\,cm$. Thus a precise comparison of theory and experiment is not possible. However, the values of $T_i \sim 100\,eV$ for the onset of L-H transitions are within the range of experimental results. For example, Fig.7 of Ref.7 shows that $T_i \approx 400\,eV$ five centimeters *inside* the separatrix.

The principal conclusions of this Letter may be summarized as:

(1) The Matsuda anomaly has inconsequential bearing on the bulk-plasma diffusivity including any deleterious effects on the recovery of canonical-angular momentum proposed as an efficient mechanism for low-phase-velocity current-drive schemes[4,5]. Thus the exclusion of momentum recovery from the trapped electrons with $u_{\perp/e} < \eta$ in Ref.5 is overly pessimistic. The current-drive-efficiency figure in Ref.5 is to be upgraded by about 20%, making it by far the most promising current-drive scheme to date.

(2) The stagnation of the trapped particles for low T_i can lead to considerable diffusion fluxes at the plasma edge. The stagnation effect is readily circumvented by an ion-temperature increase of the order of $100\,eV$.

(3) The stagnation catastrophe bears a close resemblence to the L-H mode transition in Tokamaks, satisfying the transition criteria given in Ref.8, namely, (i) the theory is capable of bifurcation, (ii) the transition has a threshold involving the edge temperature, (iii) the edge gradients would become steep at the transport barrier, (iv) the asymmetry of the heat flux to the divertor is reduced, and (v) the transition is not critically dependent upon the heating method.

(4) The plasma convection associated with stagnation might be linked to edge turbulence. The large electron diffusivity would also influence the radial-electric-field distribution at the edge. The precise delineation of these effects, however, is outside the scope of the present study.

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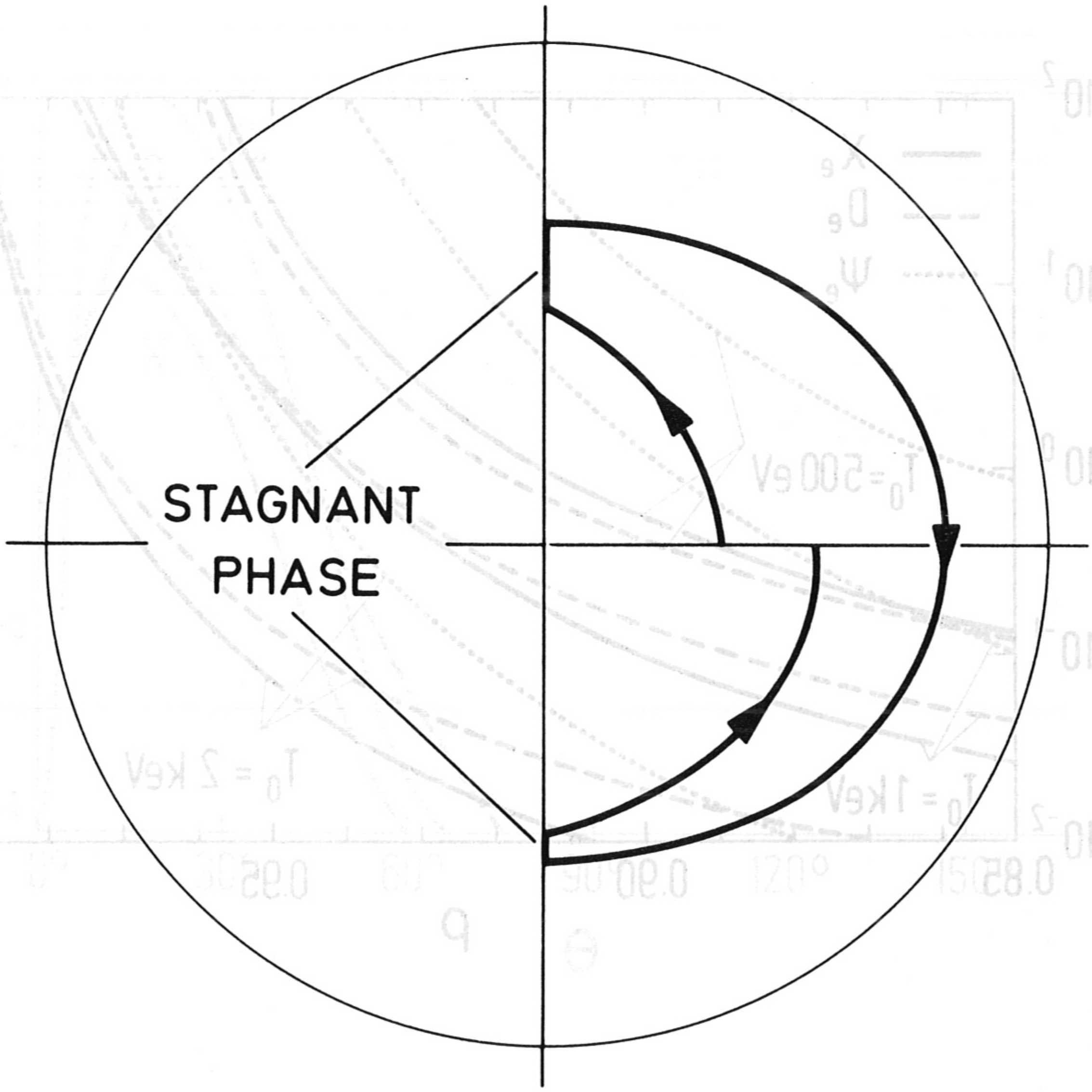

Fig.1 Banana orbit modified by the Matsuda anomaly.

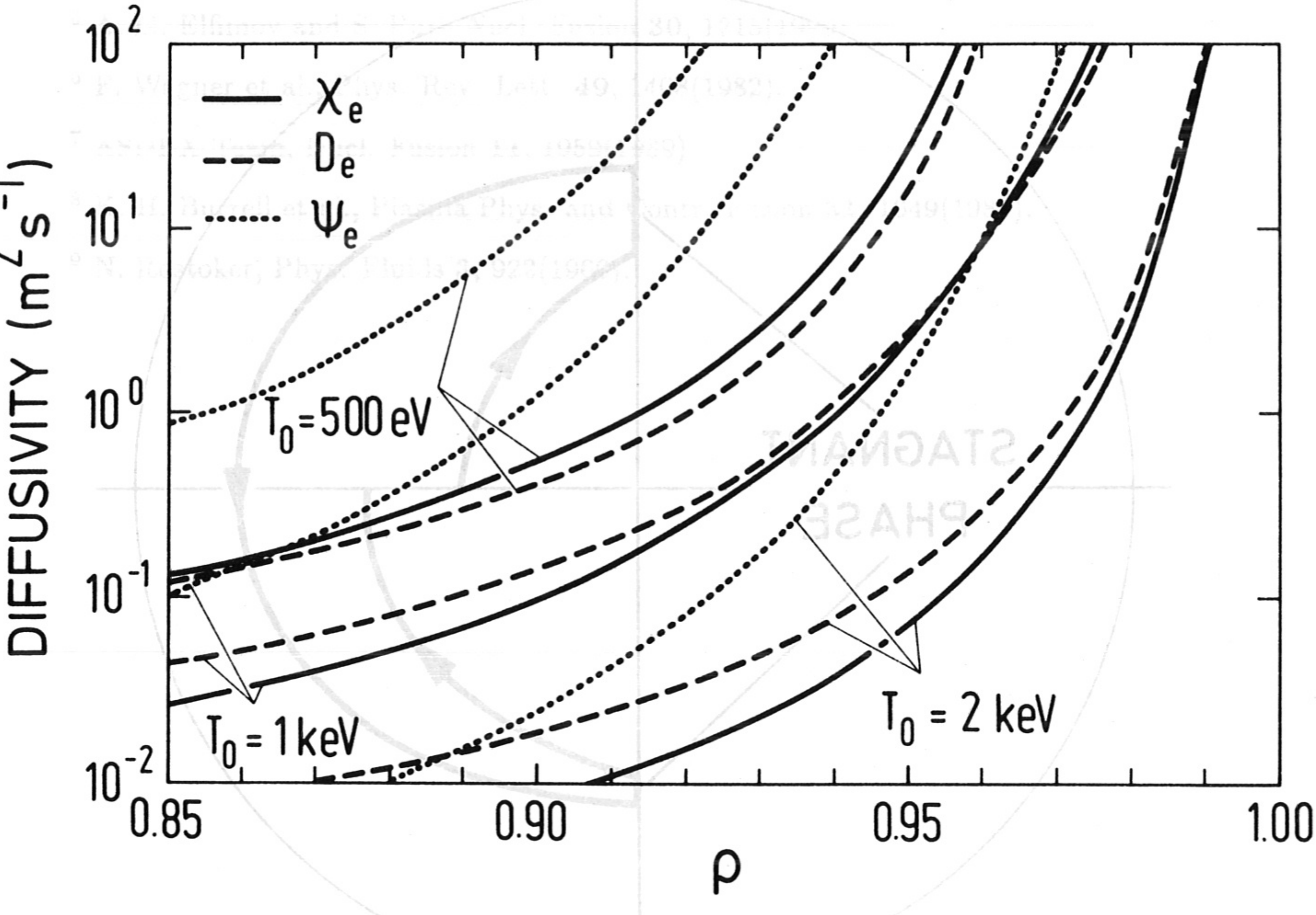

Fig.2 Diffusivity versus ρ as a function of T_0.

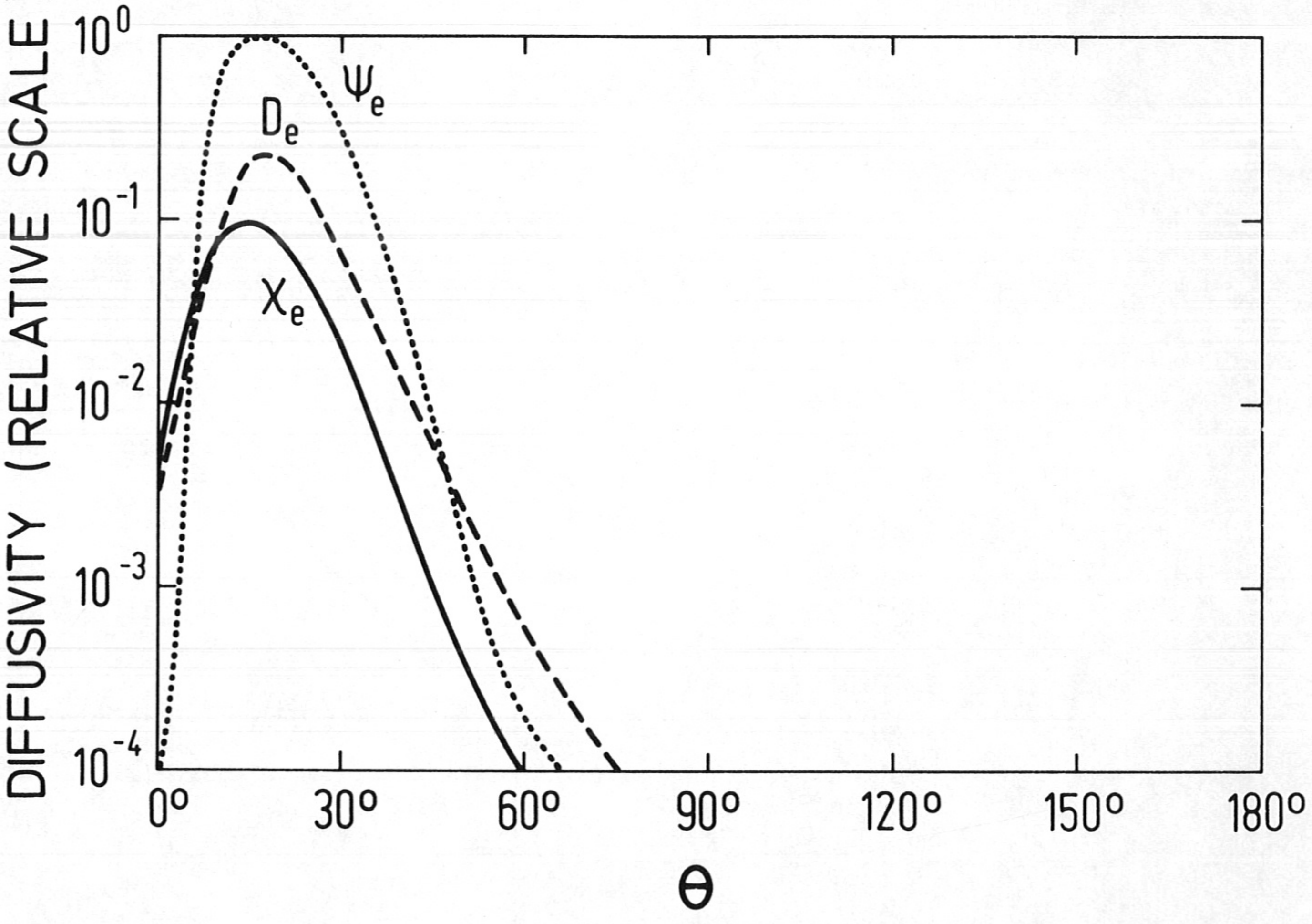

Fig.3 Diffusivity variation with the poloidal angle.