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Dimensional analysis of empirical electron heat diffusivities and comparison with microturbulence induced transport

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ABSTRACT. Empirical scaling laws for the electron heat diffusivity χ_e in the OH, L and H regimes are expressed in dimensionless form. It is found that particle orbit, collisionality and finite beta effects enter in all regimes. The different scalings are shown to result from changes in the finite β contribution, i.e. in magnetic turbulence induced transport. A unified scaling law $\chi_e \propto a^{-\frac{1}{2}} A_i^{-\frac{1}{2}} n_e^{\alpha-\frac{1}{2}} T_e^{\alpha-\frac{1}{2}} (q/B_t)^{2\alpha}$ with $\alpha = -\frac{1}{2}$ (OH) and $\alpha = \frac{1}{2}$ (L and H) is presented which also holds for the intermediate region between OH and L confinement. Purely electrostatic drift wave models including gyrokinetic effects and collisions yield B_t, A_i and T_e scalings incompatible with the empirical results. Inclusion of electromagnetic effects and pressure-driven terms, e.g. due to resistive ballooning modes, is shown to be necessary.

At present, there is still a gap between the theoretical scaling relations of transport coefficients based on drift instabilities /1/ and the empirical scaling laws determined from local transport analysis of tokamak plasmas in various confinement regimes /2-4/. The origin of these discrepancies is investigated by dimensional analysis of the empirical diffusivities and comparison with coefficients predicted by microturbulence models.

The following empirical scaling relations for the electron heat diffusivity χ_e and the diffusion coefficient in Ohmically heated (OH) and auxiliary-heated (L and H) plasmas were determined from ASDEX divertor discharges /4/:

$$\chi_e^{OH}(r) \propto A_i^{-\frac{1}{2}} B_t n_e(r)^{-1} T_e(r)^{-1} q(r)^{-1} \quad (1)$$

$$\chi_e^{L,H}(r) \propto A_i^{-\frac{1}{2}} B_t^{-1} q(r) \quad (2)$$

where A_i is the ion mass number and B_t is the toroidal magnetic field taken at the major radius R. The L and H scalings exhibit different numerical factors. Relation (2) holds for heating powers much higher than the Ohmic input prior to the auxiliary heating ($P_{AUX}/P_{OH} >> 1$). Outstanding features of the L and H scalings in the asymptotic limit are the lack of density and temperature dependences and the inverse B_p scaling ($qB_t^{-1} = r(B_pR)^{-1}$, $\tau_E^{L,H} \propto I_p$).

The thermal diffusivities are expressed in dimensionless form by the ansatz

$$\frac{\chi_e}{D_B} \propto \left(\frac{\rho_e}{a}\right)^{\alpha_1} \left(\frac{a}{\lambda_e}\right)^{\alpha_2} \beta_e^{\alpha_3} q^{\alpha_4} \left(\frac{m_e}{m_i}\right)^{\alpha_5} \quad (3)$$

with

$$D_B = \frac{c\, T_e}{16\, e\, B_t}\,, \quad \frac{\rho_e}{a} = \frac{c\, m_e^{\frac{1}{2}}\, T_e^{\frac{1}{2}}}{e\, B_t\, a}\,,$$

$$\frac{a}{\lambda_e} = \frac{a}{v_{Te}\tau_{ee}}\,, \quad \beta_e = \frac{8\pi n_e T_e}{B_t^2}\,, \tag{4}$$

where a is the plasma radius, D_B is the Bohm diffusion coefficient, ρ_e is the electron gyroradius, λ_e is the mean free path for electron-electron collisions, and β_e is the beta value due to the electron pressure. The empirical scalings can be represented by

$$\frac{\chi_e^{OH}}{D_B} \propto \frac{\rho_e}{a}\left(\frac{a}{\lambda_e}\right)^{\frac{1}{2}} \beta_e^{-\frac{3}{2}}\; q^{-1} \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \tag{5}$$

$$\frac{\chi_e^{L,H}}{D_B} \propto \frac{\rho_e}{a}\left(\frac{a}{\lambda_e}\right)^{\frac{1}{2}} \beta_e^{-\frac{1}{2}}\; q \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \tag{6}$$

They differ only in their dependence on β_e and q. Obviously, particle orbit, collisional and finite β contributions are important in all regimes. Equations (5) and (6) can be combined by writing

$$\frac{\chi_e}{D_B} \propto \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{\rho_e}{a} \left(\frac{a}{\lambda_e}\right)^{\frac{1}{2}} \beta_e^{-1} \left(\beta_e q^2\right)^{\alpha} \tag{7}$$

with $\alpha = -\frac{1}{2}$ in the Ohmic and $\alpha = \frac{1}{2}$ in the L, H case. The change in scaling is merely due to the factor $(\beta_e q^2)^{\alpha}$. The exponent rises with the auxiliary heating power and beta (starting from $\alpha = -\frac{1}{2}$ for $P_{AUX}/P_{OH} = 0$) and saturates at $\alpha = \frac{1}{2}$ for $P_{AUX}/P_{OH} \approx 4$. The different scalings in the OH and L, H regimes result from changes in the finite pressure contribution and thus have to be attributed to magnetic turbulence induced transport.

Including only gyrokinetic effects yields $\chi_e \propto D_B \rho_e \propto T_e^{\frac{3}{2}} B_t^{-2}$, which completely disagrees with the empirical scalings. Incorporation of collision terms cannot cure the wrong B_t scaling ($\lambda_e \propto T_e^2 n_e^{-1}$), whereas finite-beta effects do. Both OH and L, H diffusivities become B_t independent owing to the factor β_e^{-1}.

From Eq. (7) the following unified scaling law is obtained:

$$\chi_e(r) \propto a^{-\frac{1}{2}} A_i^{-\frac{1}{2}} n_e(r)^{\alpha-\frac{1}{2}} T_e(r)^{\alpha-\frac{1}{2}} \left(\frac{q(r)}{B_t}\right)^{2\alpha} \qquad (8)$$

with $-\frac{1}{2} \leqq \alpha \leqq \frac{1}{2}$. It holds for the OH ($\alpha = -\frac{1}{2}$) and L, H regimes ($\alpha = \frac{1}{2}$) and for the intermediate region between OH and L confinement ($-\frac{1}{2} < \alpha < \frac{1}{2}$). With increasing α the density and temperature dependences become weaker and finally disappear in the L and H regimes. Simultaneously, the inverse B_p scaling is approached. The smooth transition from OH to L confinement agrees with experimental findings /5/ when α is approximated by the expression $\alpha = 0.25\ P_{AUX}/P_{OH} - 0.5$ for $P_{AUX}/P_{OH} \leqq 4$. The intermediate χ_e does not simply result from the superposition of Ohmic and pure L scalings.

The global scaling law proposed in Ref. /6/

$$\frac{1}{\tau_E} = \left[\left(\frac{1}{\tau_E^{OH}}\right)^2 + \left(\frac{1}{\tau_E^{AUX}}\right)^2\right]^{\frac{1}{2}}$$

corresponds to

$$\chi_e = \left[(\chi_e^{OH})^2 + (\chi_e^{AUX})^2\right]^{\frac{1}{2}}$$

with $\chi_e^{AUX} = C^{AUX} f(r)\ \beta_e\ q^2$. It is interesting that this term appears also in Eq. (7), but it enters in a different way

$$\chi_e \propto \chi_e^{OH} \left(\frac{\chi_e^{AUX}}{C^{AUX} f}\right)^{\alpha+\frac{1}{2}} \qquad (9)$$

The global energy confinement time in the intermediate region thus scales as

$$\tau_E \propto \tau_E^{OH} \left(\frac{\tau_E^{AUX}}{\tau_n} \right)^{\alpha+\frac{1}{2}} \qquad (10)$$

with $\tau_n \propto a^2/C^{AUX}$.

Turning now to microinstability-based transport models, we focus on the question whether drift wave turbulence can be responsible for the empirical diffusivities in Eqs. (1) and (2) and the particle orbit, collisional and finite beta contributions (see Eq. (7)). The usual way of estimating the anomalous transport coefficients is to start with the strong turbulence expression

$$\chi_e \simeq \frac{\gamma_{lin}}{k_\perp^2} \qquad (11)$$

The linear growth rate holds under the assumption of linear electron dynamics. This can be justified by the fact that the important non-linear effects are due to the ion equations. With $\gamma_{lin} = \delta(k_\perp)\, \omega_{*e}$ valid for drift instabilities one obtains

$$\chi_e \simeq \frac{\delta(k_\perp)\, \omega_{*e}}{k_\perp^2} \qquad (12)$$

Here, δ is the non-adiabatic electron response, $k_\perp$ is the wave number $\perp \vec{B}$, and $\omega_{*e} = k_\theta c T_e/(e B_t r_n)$ is the electron diamagnetic drift frequency with the poloidal wave number $k_\theta = m/r$ and the density gradient scale length $r_n = |d \ln n/dr|^{-1}$. From gyrokinetic models which neglect collisions and finite pressure effects it follows that

$$\chi_e \propto \frac{T_e}{B_t r_n} \frac{1}{k_\perp} \qquad (13)$$

Replacing $k_\perp$ by the fastest growing wave number of most drift instabilities given by $k_\perp \rho_s \simeq 0.3$, where $\rho_s = m_i^{\frac{1}{2}} c T_e^{\frac{1}{2}}/(eB_t)$ is the ion gyroradius evaluated at the electron temperature, then yields

$$\chi_e \propto A_i^{\frac{1}{2}} \frac{T_e^{\frac{3}{2}}}{B_t^2 r_n} \qquad (14)$$

Compared with the empirical scaling laws, this formula has the wrong dependence on B_t, T_e and A_i. Taking into account only particle orbit effects is thus totally insufficient. If collisions are included, a ν_{ei}-dependent factor appears, which, however, does not remove the B_t^{-2} scaling. Note that the different B_t laws cannot be reconciled either by a factor q^2 since this would introduce a wrong B_p variation. It is thus concluded that purely electrostatic drift wave turbulence is not consistent with the empirical scalings in the OH, L and H regimes. Electromagnetic effects have to be included. Taking into account the fluctuations of the parallel vector potential $\tilde{A}_\parallel$ can introduce a β dependence.

The OH regime and intermediate region between OH and L confinement of ASDEX are characterized by $P_{AUX}/P_{OH} = 0$ to 4.0, $\beta = 0.3$ to 0.6 % and $\beta_p = 0.3$ to 0.6 (for $I_p = 380$ kA). In this beta range the growth rate and k spectrum of drift instabilities are just slightly modified by the electromagnetic terms /7/. The anomalous transport due to electrostatic fluctuations ($\tilde{\Phi}$) does not differ much from that in the $\beta = 0$ limit. A substantial change can occur, however, in the electron heat diffusivity owing to the magnetic fluctuations ($\tilde{A}_\parallel$). Incorporating both $\tilde{\Phi}$ and $\tilde{A}_\parallel$ induced by drift instabilities and applying $\rho_e^2 \omega_{pe}^2 \propto \beta_e$ yields a scaling as $\chi_e/D_B \propto \beta_e^{-1}$ /8,9/. Consequently, the inverse β_e dependence in Eq. (7) can be explained by drift wave turbulence if electromagnetic effects are included. It should be mentioned that the β_e^{-1} scaling is also obtained from microtearing /10,11/ and drift-tearing modes /3/.

It seems to be impossible to reconcile the factor $(\beta_e q^2)^\alpha$ in the empirical scaling law with the theoretical diffusivities inferred from drift wave turbulence alone. The dependence on $\beta_e q^2$ with positive exponent α is indicative of pressure-driven modes, since a similar factor $\left[\beta q^2 R/(L_p s)\right]^\mu$ appears in the diffusivities derived from resistive ballooning /12/ or interchange instabilities /7/. Here, $L_p = |d \ln p/dr|^{-1}$ is the pressure gradient scale length and $s = (r/q)\partial q/\partial r$ is the dimensionless shear.

For the Ohmic case, a factor close to $(\beta_e q^2)^{-\frac{1}{2}}$ was deduced from a model based on drift-tearing instabilities and magnetic reconnection /3/. These modes are driven by the current density gradient. It is likely that the α increase with heating power and beta in the intermediate region reflects the growing contribution of pressure-driven modes, e.g. resistive ballooning modes, to magnetic turbulence induced transport.

The correlation with beta is not only visible in power scans but also in the course of high power L discharges. The L-mode transport is delayed by about 10 ms against the onset of neutral injection, and the Ohmic confinement is only recovered at about 80 ms after the end of the beam heating /13, 14/. These time delays are correlated with the evolution of the measured β_p which rises on a 10 ms time scale due to the large heating power and which slowly declines to the Ohmic level in about 70 ms after the end of the injection.

Note that ideal ballooning modes were shown to be stable in Ohmic and L plasmas /15, 16/. According to ideal MHD and kinetic models these modes only grow unstable above a beta threshold of typically 1.3 % /17/ which clearly exceeds the beta values in the OH and intermediate regimes. Consequently, ideal ballooning instabilities cannot cause the observed anomalous fluxes in these plasmas.

It is concluded that the empirical χ_e scalings in the OH, L and H regimes are incompatible with purely electrostatic drift wave turbulence. Inclusion of electromagnetic effects yields the β_e^{-1} scaling but fails to explain the factor $(\beta_e q^2)^\alpha$ which is indicative of resistive ballooning or interchange instabilities. The different scalings in the OH and L, H regimes result from changes in this finite pressure contribution and thus in magnetic turbulence induced transport.

It seems that drift instabilities and resistive ballooning instabilities or a combination of both are responsible for the anomalous transport. These modes induce electrostatic and magnetic fluctuations (corresponding to $\tilde{n}_e$ and $\tilde{B}_r$) which are simultaneously present in the hot plasma. The anomalous particle flux is attributed to electrostatic turbulence whereas the anomalous electron heat conduction can be explained by magnetic turbulence and ergodic magnetic fields. This is consistent with the correlation between the electron heat diffusivity and the diffusion coefficient observed in all confinement regimes if the electrostatic and magnetic fluctuation levels are coupled ($\tilde{\Phi} \propto \tilde{A}_{\parallel}$).

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