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CURRENT DRIVE VIA LANDAU DAMPING OF KINETIC ALFVEN WAVE IN TOROIDAL GEOMETRY

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ABSTRACT

It is found that Landau-damping of low-phase-velocity ($v_p \ll v_{te}$, the electron thermal speed) waves is profoundly affected by toroidicity. The proportion α_u of the wave energy imparted to the untrapped particles significantly exceeds their numerical fraction. Computed values of α_u versus v_p/v_{te} are presented. The implications of these results on low-phase-velocity-wave current drive are assessed.

1. INTRODUCTION

Radio-frequency-induced non-inductive current drive may play an important role for attaining steady-state Tokamak operation [1]. *Superthermal* (parallel phase velocity v_p greater than the electron thermal speed v_{te}) operation is favored for obtaining high current-drive efficiencies because of low collisionality and absence of trapped-particle effects [2 − 6]. The high momentum-transfer efficiency of *subthermal* ($v_p/v_{te} < 1$) wave current drive [7], although able to surmount the increased collisional losses, could be severely curtailed by the trapped-particle effects [5, 6].

Trapped electrons constitute a fraction $\varphi_t = \exp\left[-(v_p/v_{te})^2/2\varepsilon\right]$ of the electrons resonant with the wave; the corresponding fraction of the untrapped particles is given by $\varphi_u = 1 - \varphi_t$, where $\varepsilon = r/R_0$, r is the radius, $R = R_0(1 + \varepsilon\cos\theta)$ and R_0 is the torus major radius. It is generally assumed that the energy imparted to the trapped particles is irrevocably lost. This assumption has been called to question [3] on the grounds that the trapped particles are able to store canonical angular momentum via Ware pinch [8]; the stored canonical angular momentum is later released via inverse Ware pinch and drives useful plasma current. In a recent study, based on the rigorous conservation of the canonical angular momentum, it is shown that the released canonical momentum is redistributed among the plasma electrons and ions in the ratio $\tilde{\nu}_{ee}/\tilde{\nu}_{ei}$, where $\tilde{\nu}_{ee}$ and $\tilde{\nu}_{ei}$ are the *extant* collision frequencies for the trapped electrons with parallel velocity v_p [9].

Also, it is customary to assume that the energy absorbed by the trapped and the passing particles via Landau damping is proportional to their numerical abundance φ_t/φ_u [2 − 6]. Landau damping, however, is sensitive to the particle's velocity $v_{\|}$ along the magnetic field. The extreme variations in $v_{\|}$ encountered in a toroidal geometry, most particularly by the trapped population, is likely to be accompanied by corresponding modifications in Landau damping. This paper examines the relative Landau damping contributed by the trapped and the untrapped particles using the theory developed by Grishanov and Nekrasov [10]. It is found that the wave energy is imparted preferentially to the untrapped particles so that higher current-drive efficiencies in comparison with the current projections would be possible. The fraction α_u of the wave energy absorbed by the untrapped particles is computed for a variety of parameters. The implications of these results for the subthermal current drive are discussed.

2. REVIEW OF THE THEORY

The problem of wave damping by trapped and passing particles, respectively, is analyzed in Ref.10 for arbitrary frequencies for large aspect ratio Tokamaks. A simplified outline valid for low frequency ($\omega \ll \omega_c$) waves (retaining Landau damping, but neglecting cyclotron damping) starts with the linearized drift-kinetic equation [11]

$$\frac{\partial f_0}{\partial t} + \frac{h_\theta v_\parallel}{r}\frac{\partial f_0}{\partial \theta} + \frac{inh_\phi v_\parallel}{R} f_0 - \frac{h_\theta v_\perp \sin\theta}{2R}\left(v_\perp \frac{\partial f_0}{\partial v_\parallel} - v_\parallel \frac{\partial f_0}{\partial v_\perp}\right) = -\nu f_0 + Q_0 \ , \qquad (1)$$

where

$$Q_0 = -\frac{e}{m} E_\parallel \frac{\partial F}{\partial v_\parallel} \ , \qquad (2)$$

$$f = F + \sum_l f_l \exp\left[i(n\phi - \omega t + l\sigma)\right] \ , \qquad (3)$$

ϕ and θ are the toroidal and poloidal angles, f is the particle distribution function, F is the steady-state distribution assumed to be Maxwellian, σ is the space-velocity angle ($v_1 = v_\perp \cos\sigma$, $v_2 = v_\perp \sin\sigma$, $v_3 = v_\parallel$), l is the space-velocity harmonic number, $h_\phi = |B_\phi/B|$, $h_\theta = |B_\theta/B|$, n is the toroidal wave number, ν is the collision frequency, $E_\parallel = \sum_m E_m \exp(im\theta)$ is the electric field along the magnetic field direction and $v_\parallel$, $v_\perp$ are the velocity components along and perpendicular to the magnetic field direction. Since no net current is contributed by the $l \neq 0$ space-velocity components, only the f_0 term is retained in Eq.(1). The plasma current is given by

$$j_\parallel = 2\pi e \int_{-\infty}^{\infty} v_\parallel \, dv_\parallel \int_0^{\infty} f_0 v_\perp \, dv_\perp \ . \qquad (4)$$

The substitutions

$$v_\perp = u \sin\gamma \ , \qquad (5)$$

and

$$v_{\parallel} = u \cos\gamma \ , \tag{6}$$

transform Eqs.(1) and (4) to

$$(\nu - i\omega) f_0 + \frac{h_\theta u \cos\gamma}{r} \frac{\partial f_0}{\partial \theta} + \frac{i n h_\phi u \cos\gamma}{R_0(1+\varepsilon\cos\theta)} f_0 + \frac{h_\theta \sin\theta u \sin\gamma}{2R_0(1+\varepsilon\cos\theta)} \frac{\partial f_0}{\partial \gamma} = Q_0 \ , \tag{7}$$

and

$$j_{\parallel} = 2\pi e \int_0^\infty u^3 \, du \int_0^\pi f_0 \sin\gamma \cos\gamma \, d\gamma \ . \tag{8}$$

A second set of transformations

$$\theta' = \theta \ , \tag{9}$$

and

$$\Lambda = \sin^2\gamma \, (1+\varepsilon\cos\theta) = \frac{2\mu B_0}{u^2} \ , \tag{10}$$

would provide the convenient form for distinguishing between passing and trapped particles. In Eq.(10) μ is the magnetic moment and B_0 is the magnetic field at the axis. Equations (5)-(8) become

$$v_{\perp} = \sqrt{\frac{\Lambda}{1+\varepsilon\cos\theta}} \ , \tag{11}$$

$$v_{\parallel} = s\sqrt{1 - \frac{\Lambda}{1+\varepsilon\cos\theta}} \ , \tag{12}$$

$$\frac{\partial f_0^{(s)}}{\partial\theta} + i\left[\frac{nq}{1+\varepsilon\cos\theta} - \frac{s(\omega+i\nu)r}{h_\theta u\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\theta}}}\right] f_0^{(s)} = \frac{\partial f_0^{(s)}}{\partial\theta} + i\chi^{(s)} f_0^{(s)} = G_0 \ , \tag{13}$$

and

$$j_{\parallel} = \frac{\pi e}{1+\varepsilon\cos\theta} \int_0^\infty u^3 \, du \int_0^{1+\varepsilon\cos\theta} d\Lambda \sum_s s f_0^{(s)} \ , \tag{14}$$

where

$$G_0 = -\frac{r}{h_\theta}\frac{e}{m}\frac{E_{\parallel}}{v_{\parallel}}\frac{\partial F}{\partial v_{\parallel}} = \frac{e r E_{\parallel}}{m h_\theta v_{te}^2} F \ , \tag{15}$$

$$\chi^{(s)} = \frac{nq}{1+\varepsilon\cos\theta} - \frac{s(\omega+i\nu)r}{h_\theta u\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\theta}}} \ , \tag{16}$$

$q = r h_\phi / R_0 h_\theta$ is the safety factor, $s = \pm 1$ for $v_{\parallel} \gtrless 0$ and the s summation extends over $s = \pm 1$. Equation (13) possesses the solution

$$f_0^{(s)} = \exp\left(-i\int_{-\theta_m}^{\theta} \chi^{(s)}\, d\eta\right)\left[\int_{-\theta_m}^{\theta} G_0 \exp\left(i\int_{-\theta_m}^{y} \chi^{(s)}\, d\eta\right) dy + C^{(s)}\right] , \qquad (17)$$

where θ_m defines the maximum azimuthal extent of the particle's excursion. For the passing particles $\theta_m = \pi$ while for the trapped particles $\theta_m = \cos^{-1}[(\Lambda - 1)/\varepsilon]$. The integration constants $C^{(s)}$ are determined by the boundary conditions

$$f_0^{(1)}(\theta_m) = f_0^{(-1)}(\theta_m) , \qquad (18a)$$

$$f_0^{(1)}(-\theta_m) = f_0^{(-1)}(-\theta_m) , \qquad (18b)$$

for the trapped particles and

$$f_0^{(s)}(\theta_m) = f_0^{(s)}(-\theta_m) , \qquad (19)$$

for the untrapped particles. The boundary conditions together with Eqs. (14) and (17) give the current $j_{\|}$ for an electric field excitation $E_{\|} = E_m \exp(im\theta)$ as

$$j_{\|} = \frac{\omega_{pe}^2 \epsilon_0}{i\omega} \sum_s E_m \left[\Psi_{u,m}^{(s)} + \Psi_{t,m}^{(s)}\right] , \qquad (20)$$

where

$$\Psi_{u,m}^{(s)} = \sqrt{\frac{2}{\pi}} \frac{isr\omega}{h_\theta v_{te}(1 + \varepsilon\cos\theta)} \int_0^{\infty} U^3 \exp(-U^2)\, dU \int_0^{1-\varepsilon} d\Lambda$$
$$\times \left[\int_{-\pi}^{\theta} \frac{\exp\left(imy - i\int_y^{\theta} \chi^{(s)}\, d\eta\right)}{1 - \exp\left(-i\int_{-\pi}^{\pi} \chi^{(s)}\, d\eta\right)} dy\right.$$
$$\left. + \int_{\theta}^{\pi} \frac{\exp\left(imy - i\int_{-\pi}^{\theta} \chi^{(s)}\, d\eta - i\int_y^{\pi} \chi^{(s)}\, d\eta\right)}{1 - \exp\left(-i\int_{-\pi}^{\pi} \chi^{(s)}\, d\eta\right)} dy\right] , \qquad (21)$$

$$\Psi_{t,m}^{(s)} = \sqrt{\frac{2}{\pi}} \frac{isr\omega}{h_\theta v_{te}(1+\varepsilon\cos\theta)} \int_0^\infty U^3 \exp(-U^2)\, dU \int_{1-\varepsilon}^{1+\varepsilon\cos\theta} d\Lambda$$

$$\times \left[\int_{-\theta_m}^{\theta_m} \frac{\exp\left(imy - i\int_{-\theta_m}^{\theta} \chi^{(s)}\, d\eta - i\int_{y}^{\theta_m} \chi^{(-s)}\, d\eta\right)}{\exp\left(-i\int_{-\theta_m}^{\theta_m} \chi^{(s)}\, d\eta - i\int_{-\theta_m}^{\theta_m} \chi^{(-s)}\, d\eta\right)} dy \right.$$

$$- \int_{-\theta_m}^{\theta} \frac{\exp\left(imy - i\int_{y}^{\theta} \chi^{(s)}\, d\eta - i\int_{-\theta_m}^{\theta_m} \chi^{(-s)}\, d\eta\right)}{\exp\left(-i\int_{-\theta_m}^{\theta_m} \chi^{(s)}\, d\eta - i\int_{-\theta_m}^{\theta_m} \chi^{(-s)}\, d\eta\right)} dy$$

$$\left. - \int_{\theta}^{\theta_m} \frac{\exp\left(imy - i\int_{-\theta_m}^{\theta} \chi^{(s)}\, d\eta - i\int_{y}^{\theta_m} \chi^{(s)}\, d\eta\right)}{\exp\left(-i\int_{-\theta_m}^{\theta_m} \chi^{(s)}\, d\eta - i\int_{-\theta_m}^{\theta_m} \chi^{(-s)}\, d\eta\right)} dy \right], \tag{22}$$

ϵ_0 is the permittivity of free space and $U^2 = u^2/2v_{te}^2$. The power absorbed per unit volume is given by $P = \Re\left[j_\| E_\|^*\right]$. Integrating over ϕ and θ, gives the fraction of the power imparted by the wave to the untrapped particles at radius r as

$$\alpha_u(r) = \frac{\Re \int_{-\pi}^{\pi} \sum_s \Psi_{u,m}^{(s)} (1+\varepsilon\cos\theta)\exp(-im\theta)\, d\theta}{\Re \int_{-\pi}^{\pi} \sum_s \left[\Psi_{u,m}^{(s)} + \Psi_{t,m}^{(s)}\right] (1+\varepsilon\cos\theta)\exp(-im\theta)\, d\theta}. \tag{23}$$

Integrals of the type $\int \chi^{(s)}\, d\eta$ in Eqs.(21) and (22) can be handled analytically [10]. The first term in Eq.(16) has a standard integral. The second term is reducible to an elliptic integral of the third kind via the substitution $\tau^2 = (1-\cos\eta)/(1+\varepsilon\cos\eta)$ giving

$$\int_0^\theta \frac{d\eta}{\sqrt{1-\frac{\Lambda}{1+\varepsilon\cos\eta}}} = \frac{\sqrt{2}(1+\varepsilon)}{\sqrt{1+\varepsilon-\Lambda}} \int_0^{\tau_\theta} \frac{d\tau}{(1+\varepsilon\tau^2)\sqrt{\left(1-\frac{1-\varepsilon}{2}\tau^2\right)\left(1-\frac{\varepsilon\Lambda}{1+\varepsilon-\Lambda}\tau^2\right)}}, \tag{24}$$

where $\tau_\theta = \sqrt{(1-\cos\theta)/(1+\varepsilon\cos\theta)}$. The remaining integrals are computed using Gauss integration techniques.

3. COMPUTATIONAL RESULTS

The parameters used in the computations are $R_0 = 5\,m$, plasma radius $a = 1.25\,m$, $\rho = r/a$, aspect ratio $A = R_0/a = 4$, $q = (1 - \rho^2 + \rho^4/3)^{-1}$, $n_e = 2 \times 10^{20}\,m^{-3}$, $T_e = 25\,keV$, $m = 0$ and $n = 8$. Subsequent to the choice of v_p, the frequency of operation is given by $\omega = nv_p/R_0$. The collision parameter ν is assumed to be given by the Spitzer electron-ion momentum transfer collision frequency. Finite ν, although needed to facilitate the convergence of the integrals in Eqs.(21) and (22), does not play a critical role in the overall results.

Figure 1 is a plot of α_u, the fractional energy absorbed by the untrapped electrons and $\beta_u = \varphi_u/(\varphi_u + \varphi_t)$, their fractional abundance versus v_p/v_{te} for $\rho = 0.2$. As expected, $\kappa_u = \alpha_u/\beta_u \approx 1$ for larger phase-velocity waves ($v_p/v_{te} \gtrsim 0.4$). For the low-phase-velocity-wave ($v_p/v_{te} \lesssim 0.4$) current drive, κ_u exceeds unity because of the dominant presence of trapped electrons with an inherently diminished capacity for Landau damping. These results would lead to an upward revision of the current-drive efficiency by the *subthermal* schemes. For typical kinetic-Alfven-wave current drive [12, 13, 9] parameters with $v_p/v_{te} \approx 0.1$, $\alpha_u \approx 0.25$ and $\kappa_u \approx 2.8$, i.e., the power coupled into the untrapped electrons exceeds their relative abundance almost by a factor of three.

Figure 2 shows α_u, β_u and κ_u as a function of ρ assuming $v_p/v_{te} = 0.1$. A broad maxima with $\kappa_u \approx 2.9$ occurs at $\rho \approx 0.25$. The initial increase in κ_u is caused by the increase in the trapped particle population at larger ρ. The subsequent flattening is presumably due to the enlargement of the trapped particle excursions between the bounce points for fixed v_p/v_{te} and increasing ρ; the larger trajectories being less prone to reduction in Landau damping. The changes in n_e and T_e with ρ were deliberately ignored in order to study the effect of ρ on κ_u.

The foregoing computations were repeated for the azimuthal wave numbers $m = \pm 1$ without any notable new findings. Also computations performed by varying T_e, A and n contribute no additional insights.

4. DISCUSSION AND CONCLUSIONS

Using the theory developed by Grishanov and Nekrasov [10], we find significant enhancement in the energy absorbed by the passing particles in comparison with their numerical abundance. For the case of kinetic-Alfven-wave current drive this enhancement is of the order of three, so that almost a quarter of the wave energy is deposited in the passing particles constituting less than one-tenth of the population resonant with the wave at $v_p/v_{te} \approx 0.1$.

Furthermore, in Ref.(9) it is pointed out that the strict requirements of the conservation of the canonical angular momentum would lead to substantial recovery of the wave momentum, initially imparted to the trapped particles, for the purpose of current drive. The trapped electrons suffer inward Ware pinch [8] as they gain momentum from the wave. In *steady state*, an equal number of trapped electrons undergo inverse Ware pinch via collisions with the *bulk-plasma* population. The fraction of the momentum initially imparted to the trapped particle population is given by $\alpha_t = 1 - \alpha_u$. The inverse Ware pinch transfers the fraction

$$\alpha_{t\to u} = \frac{\tilde{\nu}_{ee}}{\tilde{\nu}_{ee} + \tilde{\nu}_{ei}}(1-\sqrt{\varepsilon})\alpha_t \approx \frac{K(\xi)\nu_{ee}}{\nu_{ee}+\nu_{ei}}(1-\sqrt{\varepsilon})\alpha_t \approx \frac{K(\xi)}{1+.5Z}(1-\sqrt{\varepsilon})(1-\alpha_u) \quad (25)$$

back to the circulating *bulk-plasma* electrons, where $(1-\sqrt{\varepsilon})$ is the fraction of the circulating electrons in the plasma bulk, ν_{ee} and ν_{ei} are Spitzer collision frequencies and $K(\xi)$ is a correction factor due to the magnetic field effects on ν_{ei}. Anomalous magnetic field effects lead to an enhancement of ν_{ei} for the trapped electrons with $r_{ce} \lesssim \lambda_D$, where r_{ce} and λ_D are the electron gyroradius and plasma Debye length, respectively $[14-16]$. If one assumes that the wave momentum given to the trapped electrons with $r_{ce}/\lambda_D \leq \xi$ is irretrievably lost while the remainder is collisionally redistributed between the bulk-plasma electrons and ions in the ratio ν_{ee}/ν_{ei}, one obtains

$$K(\xi) = exp\left(-\frac{\omega_{ce}^2}{\omega_{pe}^2}\frac{\xi^2}{4}\right) . \quad (26)$$

The currently available information $[14-16]$ is insufficient to ascribe a precise value to ξ; we assume a conservative figure of $\xi = \sqrt{2}$. Further assuming $\omega_{ce}^2/\omega_{pe}^2 = 1/2$

and $Z = 1.5$ gives $K(\xi) \approx 0.78$ and $\alpha_{t\to u} \approx 0.30$, so that the net fraction of the wave momentum contributing to the kinetic-Alfven-wave current drive becomes

$$\alpha_u^T = \alpha_u + \alpha_{t\to u} \approx 0.55 \ . \tag{27}$$

This value of α_u^T is substantially the same as was found in Ref.9 which did not include the mutually cancelling effects arising from (i) increase in the fraction of energy absorbed by the untrapped electrons and (ii) anomalous magnetic-field effects on ν_{ei}. One concludes that a current-drive efficiency of $R_0 n_{20} I/P \approx 2$ (similar to that found in Ref.9) would be feasible using the subthermal kinetic-Alfven-wave current drive. This efficiency figure is at least a factor of five higher than the alternative approaches such as lower-hybrid and fast-wave current drives.

A more precise analysis would self-consistently balance the effects of momentum transfer by the wave, Ware pinch, the inverse Ware pinch, collisional dissipation, anomalous collisionality in a magnetic field, together with the intricacies of toroidal transport. The objective of this Research Note is to point out the enhancement in the energy absorbed by the untrapped particles from the subthermal phase-velocity waves resulting in heightened prospects for the subthermal wave current-drive schemes.

Note that the form of the drift-kinetic equation used in this paper assumes the familiar form

$$\frac{\partial f_0}{\partial t} + v_\parallel \frac{\partial f_0}{\partial x_\parallel} + \left[\frac{e}{m}E_\parallel - \mu \nabla B\right] \frac{\partial f_0}{\partial v_\parallel} = -\nu f_0 \tag{28}$$

using the transformations $\eta = \theta$, $v'_\parallel = v_\parallel$ and $\mu = v_\perp^2/2B$ in Eq.(1). In Eq.(28)

$$v_\parallel \frac{\partial f_0}{\partial x_\parallel} = \frac{h_\theta v_\parallel}{r} \frac{\partial f_0}{\partial \theta} + \frac{h_\phi v_\parallel}{R} \frac{\partial f_0}{\partial \phi} \ , \tag{29}$$

and

$$\nabla B = \frac{h_\theta}{r} \frac{\partial B}{\partial \theta} + \frac{h_\phi}{R} \frac{\partial B}{\partial \phi} = \frac{h_\theta}{r} \frac{B_0 R_0 \epsilon \sin\theta}{(1 + \epsilon\cos\theta)^2} \ . \tag{30}$$

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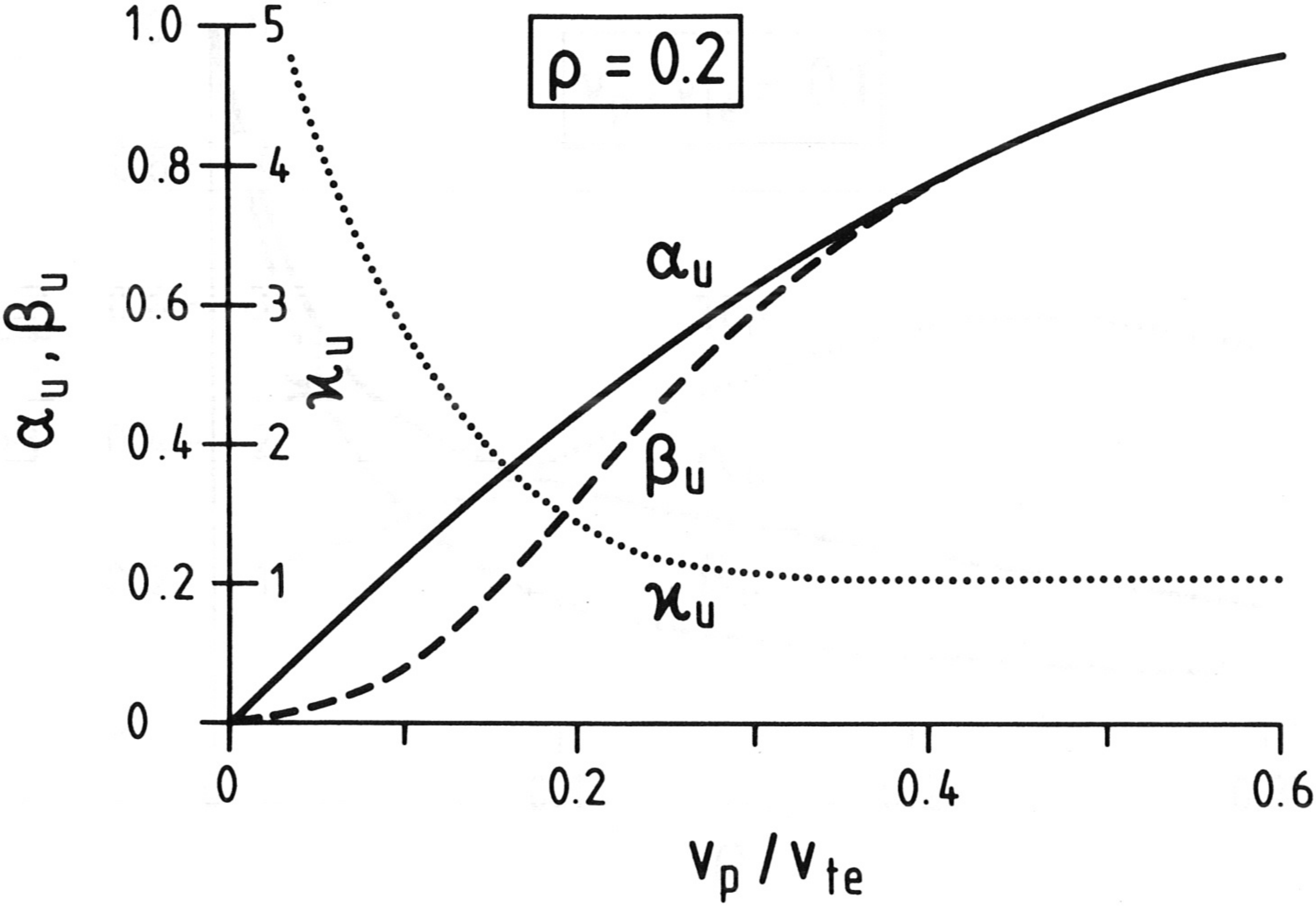

Fig. 1 α_u, β_u and κ_u as a function of v_p/v_{te}.

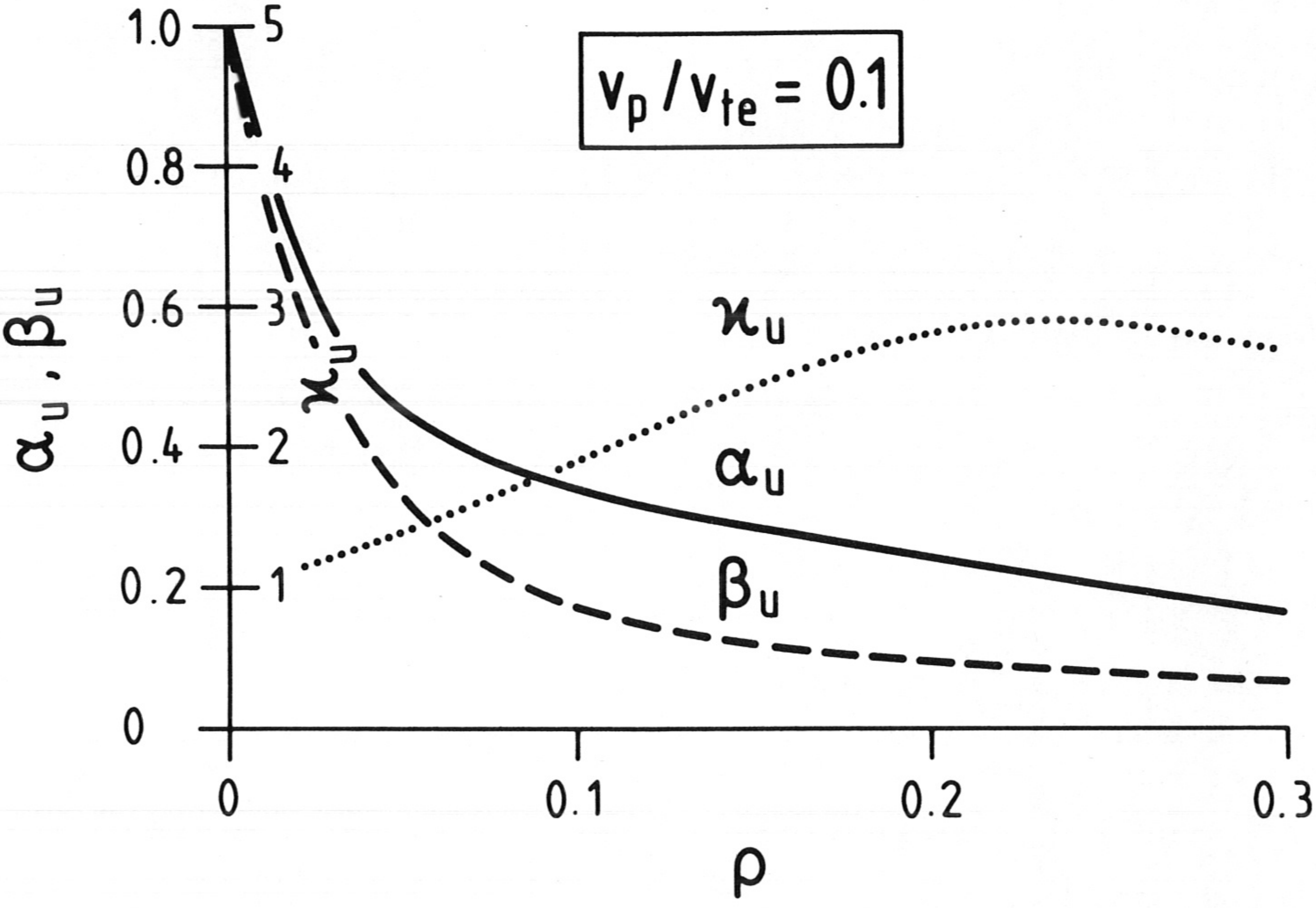

Fig. 2 α_u, β_u and κ_u versus ρ.